Modeling a signal generated by microparticles moving in the aerodynamic flow

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The article presents a model of a signal generated by microparticles moving in an aerodynamic flow. This model is based on the Lorentz–Mie scattering theory. It is shown that the visibility and the signal/noise ratio of the Doppler signal are determined by the degree of amplitude and polarization matching of the scattered waves. These parameters also depend on the degree of phase matching of "elementary" Doppler signals. Using this signal model, it is possible to calculate the shape of the aperture of the receiving optics for a specific type of laser Doppler anemometer. The use of such an aperture will increase the visibility, the signal-to-noise ratio and the measurement accuracy of the aerodynamic flow velocity using a laser Doppler anemometer.

Keywords: *signal, laser, microparticle, model, scattered radiation, flow rate.*

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1. Introduction

Under experimental studies of the aerodynamic characteristics of aircraft models that flow around the gas flow, the methods of laser Doppler anemometry are used when studying of gas turbines, compressors, screws and other objects [1]. Using laser Doppler anemometers (LDA) you can get information about the velocity field around the object of study [2]. The reliability of this information depends on the accuracy of the measurements [3].

In turn, the accuracy of measurements is determined by matching the speed of particles and flow [4].

To reduce the error due to the lag of the microparticles from the flow, particles of a given size are introduced into it [5]. In addition, the difficulty of measuring high velocities of aerodynamic flows is that the energy of the pulsed Doppler signal is small. Manufacturers of such high-tech and expensive devices claim high accuracy (0.5%) of the measured speed [6]. But the accurate measurement of the LDA signal frequency is possible only at high signal-to-noise ratio and signal visibility [7].

To determine ways to increase the signal-to-noise ratio and signal visibility, you must have a reliable model of its generation.

2. Main results and discussions

In the LDA of a differential type the measuring volume is generated transmitting optics on the intersection of laser beams, Fig. 1, which has the shape of an ellipsoid of rotation.

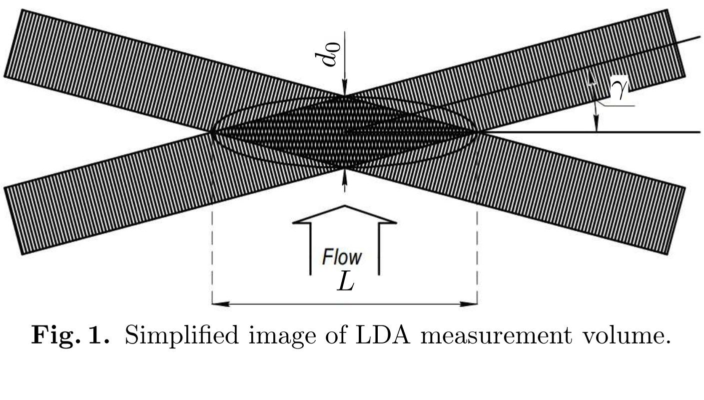

Fig. 1. Simplified image of LDA measurement volume.

Laser beams interfere in the local area of their intersection. The model of the intensity distribution of laser radiation in the measuring volume is written as [8]:

$$I = 2I_0 \exp\left\{-2\left[x^2 + y^2\cos^2(\gamma/2) + z^2\sin^2(\gamma/2)\right]/b_0^2\right\}\left\{\cosh\left[2yz\sin(\gamma)/b_0^2\right] + \cos\left[\frac{2\pi}{\lambda}\sin(\gamma/2)y\right]\right\},$$

where I_0 is laser intensity at the center of the measuring volume; d_0 is diameter of measuring volume; $b_0 = d_0/2$; x, y, z are Cartesian coordinates relative to the center of the measuring volume; γ is laser beam angle; λ is length of the wave of laser radiation.

Graphically, the intensity distribution in the measuring volume is presented in Fig. 2.

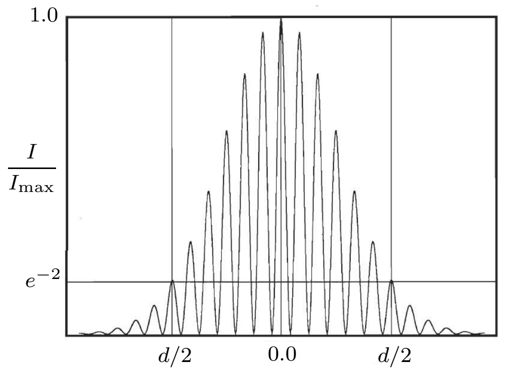

Fig. 2. The intensity distribution of laser radiation in the measuring volume.

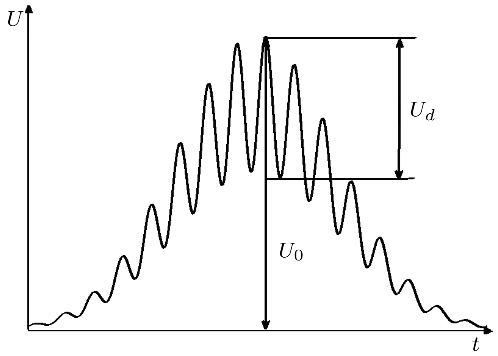

Fig. 3. The shape of the Doppler signal, when a microparticle passes the center of the measuring volume: U_d is high frequency component of a Doppler signal; U_0 is constant signal component.

When crossing the measuring volume with a microparticle, it scatters laser radiation from two beams. This radiation is received by the receiving optics on the photodetector. A running pattern is generated on its surface. As a result of optical heterodyning at the output of the photodetector, a signal is generated, the shape of which, without taking into account the influence of noise, is shown in Fig. 3.

The signal generation model can be constructed on the basis of the Rayleigh, Rayleigh–Gans, Lorentz–Mie scattering theories, or the Fraunhofer diffraction theory. But since the size of the microparticles that are introduced into the flow is in the range of $1-20\,\mu$m, the Lorentz–Mie theory or the diffraction theory can be used to construct the signal model.

In [8], using diffraction theory, it was shown that the signal visibility is determined by the ratio of the intensities of scattered waves from two laser beams. But this approach does not take into account the influence of polarization and the phase of the scattered waves on the LDA signal.

In [9] it was shown that a more reliable model of signal generation can be obtained on the basis of the Lorentz–Mie scattering theory, which allows taking into account the influence of polarization-phase effects on it.

When modeling the LDA signal based on the Lorentz–Mie theory, we will assume that monodisperse spherical particles are in the aerodynamic flow. At each moment of time in the center of the measuring volume is not more than one microparticle. Linearly polarized laser beams are equal in intensity.

As a result of scattering by a moving microparticle of laser radiation, two beams of radiation appear, which have different frequencies due to the Doppler effect. This radiation within the angular aperture is directed to the photodetector.

Let the scattered radiation be received within a small angular aperture. The angular dimensions of the aperture angle are such that the intensity and state of polarization of the two scattered waves are constant within it. The model of such an "elementary" Doppler signal can be represented as:

$$U_i = U_0 + U_{di} = \frac{e\eta k_1}{h\nu}RC\Delta S_i(E_{\varphi1}^2 + E_{\theta1}^2 + E_{\varphi2}^2 + E_{\theta2}^2) + \frac{e\eta k_1}{h\nu}RC\Delta S_i\sqrt{E_{\varphi1}^2E_{\varphi2}^2 + E_{\theta1}^2E_{\theta2}^2 + 2E_{\varphi1}^2E_{\varphi2}^2E_{\theta1}^2E_{\theta2}^2\cos(\Delta_{1i}-\Delta_{2i})}\cos(\omega_d + \varphi_{di}),$$

where $C = \sqrt{(\varepsilon\varepsilon_0)/(\mu\mu_0)}$; e, h, and ν are electron charge, Planck's constant and radiation frequency; ε, ε_0 and μ, μ_0 are relative and absolute dielectric and magnetic flux permeability; η, k_1, R are quantum efficiency, gain and load impedance of the photodetector; ΔS_i is area within which scattered radiation is collected; Δ_{1i}, Δ_{2i} are phase difference between E_φ and E_θ components of the scattered waves; ω_d is Doppler frequency; φ_{di} is the phase of the "elementary" Doppler signal, which is found from the expression:

$$\varphi_{di} = \operatorname{Arctan} \frac{E^2_{\varphi 1i} E^2_{\varphi 2i} \sin \Delta_{1i} + E^2_{\theta 1i} E^2_{\theta 2i} \sin \Delta_{2i}}{E^2_{\varphi 1i} E^2_{\varphi 2i} \cos \Delta_{1i} + E^2_{\theta 1i} E^2_{\theta 2i} \cos \Delta_{2i}}.$$

The components of the scattered waves are found in accordance with the scattering theory of Lorentz–Mie from the following expressions [10]:

$$\dot{E}_{\varphi_{ki}} = \frac{E_{k0} e^{-k_c r_i}}{k_c r_i} E_1(\theta_{ki}) \sin e^{-j\delta^k_{1i}} e^{-j\nu_k t}, \qquad \dot{E}_{\theta_{ki}} = \frac{E_{k0} e^{-k_c r_i}}{k_c r_i} E_2(\theta_{ki}) \cos e^{-j\delta^k_{2i}} e^{-j\nu_k t},$$

where $k = 1, 2$, E_{k0} is laser beams electric field modulus; $E_1(\theta_{ki})$, $E_2(\theta_{ki})$ are modules of scattered waves complex amplitudes; k_c is wave vector module; ν_k is scattered waves frequency; δ^k_{1i}, δ^k_{2i} are the initial phases of the scattered waves that are equal:

$$\delta^k_{1i} = \operatorname{Arctan} \frac{\operatorname{Im} E_1(\theta_{ki})}{\operatorname{Re} E_1(\theta_{ki})}, \qquad \delta^k_{2i} = \operatorname{Arctan} \frac{\operatorname{Im} E_2(\theta_{ki})}{\operatorname{Re} E_2(\theta_{ki})}.$$

The modules of the complex amplitudes of the scattered waves are calculated using recurrent formulas, which are represented as convergent series:

$$E_1 = \sum_{l=1}^{l} \big[c_l(\rho, m) Q_l(\cos\theta_i) + b_l(\rho, m) S_l(\cos\theta_i)\big], \quad E_2 = \sum_{l=1}^{l} \big[c_l(\rho, m) S_l(\cos\theta_i) + b_l(\rho, m) Q_l(\cos\theta_i)\big],$$

where $\rho = \pi d/\lambda$, m is wave parameter and complex refractive index of a microparticle; $c_l(\rho, m)$, $b_l(\rho, m)$ are amplitudes of partial waves:

$$c_l(\rho, m) = q_l C_l = c'_l + jc''_l; \qquad b_l(\rho, m) = q_l B_l = b'_l + jb''_l; \qquad q_l = j(-1)^l \frac{2l-1}{l(l+1)}.$$

where

$$C_l = \frac{\psi_l(\rho)\, \psi'_l(m\rho) - m\, \psi'_l(\rho)\, \psi_l(m\rho)}{\xi_l(\rho)\, \psi'_l(m\rho) - m\, \xi'_l(\rho)\, \psi_l(m\rho)}, \qquad B_l = -\frac{\psi'_l(\rho)\, \psi_l(m\rho) - m\, \psi_l(\rho)\, \psi'_l(m\rho)}{\xi'_l(\rho)\, \psi_l(m\rho) - m\, \xi_l(\rho)\, \psi'_l(m\rho)},$$

$$\xi_l(z) = \psi_l(z) + j\chi_l(z),$$

where: z takes the value of ρ or $m\rho$.

Functions $\psi_l(z)$ and $\chi_l(z)$ are found with the help of series that recurrent formulas satisfy:

$$\eta_l(z) = \frac{2l-1}{z} \eta_{l-1}(z) - \eta_{l-2}(z), \qquad \eta'_l(z) = \eta_{l-1}(z) - \frac{l}{z} \eta_l(z).$$

Functions $Q_l(\cos\theta_i)$ and $S_l(\cos\theta_i)$ are dependent on angle θ_i between the laser beam and the scattered wave and are calculated using Legendre polynomials using recurrent formulas:

$$Q_l(\cos\theta_i) = \frac{2l-1}{l-1} \cos\theta_i Q_{l-1}(\cos\theta_i) - \frac{l}{l-1} Q_{l-2}(\cos\theta_i),$$
$$S_l = \frac{l^2}{2l+1} Q_{l+1}(\cos\theta_i) - \frac{(l+1)^2}{2l+1} Q_{l-1}(\cos\theta_i).$$

Calculation the components of the scattered field can be performed only with the help of a computer due to an excessively large amount of calculations.

The visibility of the LDA signal is determined by the ratio $V = U_d/U_0$ and can vary from 0 to 1. It is determined by the degree of amplitude and polarization matching of the scattered waves, as well as the degree of phase matching of the "elementary" Doppler signals. Visibility and signal-to-noise ratio can be calculated as follows:

$$V = k_a k_p k_f, \qquad S/N = \frac{\eta}{h\nu\Delta f} k_a k_p^2 k_f^2 \int_{\Delta\Omega} \sqrt{(E_{\varphi 1} + E_{\theta 1})^2 + (E_{\varphi 2} + E_{\theta 2})^2}\, r_i^2\, d\Omega,$$

where k_a is amplitude matching coefficient; k_p is polarization matching coefficient; k_f is phase matching coefficient of "elementary" Doppler signals; r_i is distance traveled by the scattered waves.

The degree of amplitude matching when receiving scattered radiation in a small angular aperture k_{ai} and within the limits of the final angular aperture $\Delta\Omega$ according to this model we will define as:

$$k_{ai} = \frac{2\sqrt{I_{s1i} I_{s2i}}}{(I_{s1i} + I_{s2i})}, \qquad k_a = \frac{2\int_{\Delta\Omega} \sqrt{I_{s1i} I_{s2i}}\, d\Omega}{\int_{\Delta\Omega} I_{s1} d\Omega + \int_{\Delta\Omega} I_{s2i}\, d\Omega},$$

where $I_{s1i} = C\big(E_{\varphi 1i}^2 + E_{\theta 1i}^2\big)$ and $I_{s2i} = C\big(E_{\varphi 2i}^2 + E_{\theta 2i}^2\big)$ are scattered wave intensities.

The degree of polarization matching of the scattered radiation when receiving radiation in a small angular aperture k_{pi} and in the final aperture $\Delta\Omega$ we find as follows:

$$k_{pi} = \frac{E_{\varphi 1i}^2 E_{\varphi 2i}^2 + 2E_{\varphi 1i} E_{\varphi 2i} E_{\theta 1i} E_{\theta 2i} \cos(\Delta_{1i} - \Delta_{2i}) + E_{\theta 1i}^2 E_{\theta 2i}^2}{(E_{\varphi 1i}^2 + E_{\theta 1i}^2)(E_{\varphi 2i}^2 + E_{\theta 2i}^2)},$$

$$k_p = \frac{\int_{\Delta\Omega} \sqrt{(E_{\varphi 1i}^2 + E_{\theta 1i}^2)(E_{\varphi 2i}^2 + E_{\theta 2i}^2) k_{\mathrm{pi}}}\, d\Omega}{\int_{\Delta\Omega} \sqrt{(E_{\varphi 1i}^2 + E_{\theta 1i}^2)(E_{\varphi 2i}^2 + E_{\theta 2i}^2)}\, d\Omega}.$$

Phase matching "elementary" Doppler signals when receiving scattered radiation in the angular aperture $\Delta\Omega$ is represented by the following expression:

$$k_f = \frac{\left| \int_{\Delta\Omega} \sqrt{k_{pi} I_{s1i} I_{s2i}} e^{-j\varphi_{di}}\, d\Omega \right|}{\int_{\Delta\Omega} \sqrt{k_{pi} I_{s1i} I_{s2i}}\, d\Omega}.$$

Based on the LDA Doppler signal generation model, obtained on the basis of the Lorentz–Mie scattering theory, the apertures shape calculations of the receiving optics were performed. The use of such apertures allows one to increase the degree of amplitude, polarization, phase matching and, accordingly, the visibility of the Doppler signal, the signal-to-noise ratio and measurement accuracy.

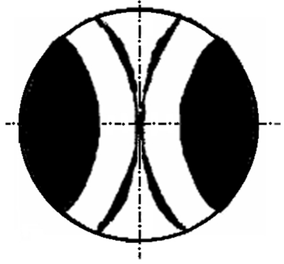

Fig. 4. The shape of the receiving aperture under condition of $k_{ai} > 0.3$.

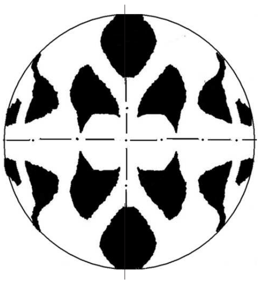

Fig. 5. The shape of the receiving under condition of $k_{pi} > 0.5$.

The increase in the modulation depth of the Doppler signal due to the matching of the scattered waves in intensity is considered in [11].

Thus, Fig. 4 shows the shape of the aperture, in which scattered radiation is received with a degree of amplitude matching not worse than $k_{ai} > 0.3$. Areas of aperture in which $k_{ai} < 0.3$ they are diaphragmized. The use of such a diaphragm allows one to increase the visibility of the signal from 0.04 to 0.45 for the LDA type Power Sight TR-SS-1D-561.

Fig. 5 shows the shape of the aperture of the receiving optics, when receiving radiation within which, the degree of polarization matching of the scattered waves will be no worse, than $k_{pi} > 0.5$.

Based on the calculations of the LDA signal phase, the following peculiarity of its change was established. Thus, the phase of the Doppler signal when receiving radiation in a narrow aperture angle can take only two values of 0 or 180°.

This leads to the fact that signals received from different areas of the aperture are in antiphase. They compensate each other and the visibility of the total signal decreases. For some sizes of microparticles or for some angles between probe beams, the signal visibility approaches zero. For example, Fig. 6*a* shows the dependence of the change in the LDA signal phase, with the angle between the beams of 14° on the aperture angle α. Scattered radiation is received in a narrow ring $\Delta\alpha = 1'30''$. For α from 0 to 2° $\varphi_d = 0°$, and when $2° < \alpha < 5.7°$ $\varphi_d = 180°$, further when $\alpha > 5.7°$, $\varphi_d = 0°$. Visibility when receiving radiation in the full aperture is $V = 0.01$, which is explained by the mutual compensation of signals. To increase the visibility to a value of 0.59, it is necessary to use a receiving aperture, the shape of which is shown in Fig. 6*b*. Signals when receiving radiation in this aperture will be phased.

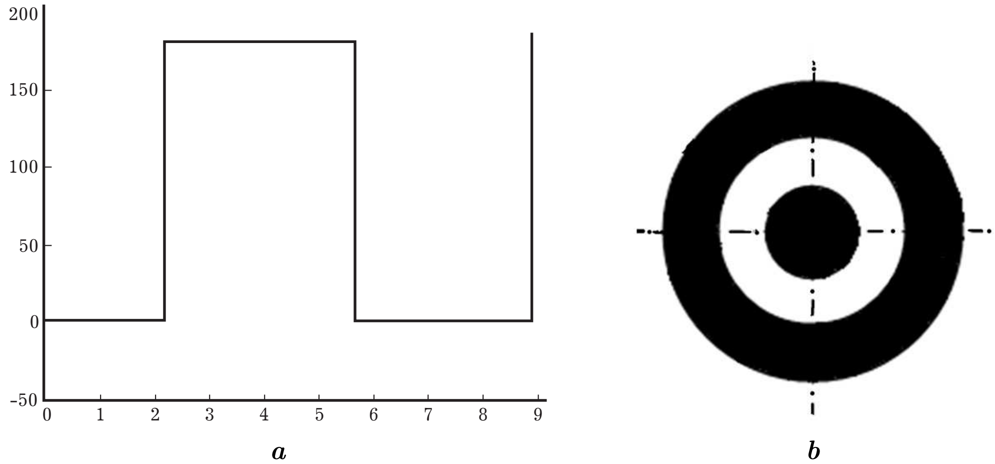

Fig. 6. The dependence of the phase signal $\varphi_d = f(\alpha)$ (***a***); the shape of the aperture, which provides phase matching "elementary" Doppler signals (***b***).

3. Conclusions

Thus, depending on the parameters of the optical LDA scheme, the size and the refractive index of the microparticles, it is possible to calculate the shape of the receiving aperture. The use of such an aperture will allow the reception of scattered radiation with a high degree of amplitude, polarization matching of scattered waves, as well as with a high degree of phase matching of "elementary" Doppler signals. A high value of the signal visibility and its signal-to-noise ratio will provide the required accuracy of measuring the velocity of the aerodynamic flow.

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Моделювання сигналу, який утворюється мікрочастинками, що рухаються в аеродинамічному потоці

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У статті подано модель сигналу, який утворюється мікрочастинками, що рухаються у аеродинамічному потоці. Ця модель побудована на основі теорії розсіяння Лоренца–Мі. Показано, що коефіцієнт глибини модуляції та відношення сигнал/завада допплерівського сигналу залежать від амплітудного та поляризаційного узгодження розсіяного випромінювання. Ці параметри залежать також від узгодження "елементарних" допплерівських сигналів за фазою. За допомогою поданої моделі сигналу можна розрахувати форму апертури приймальної оптики для конкретного типу лазерного допплерівського анемометра. Застосування такої апертури дасть змогу підвищити коефіцієнт глибини модуляції, відношення сигнал/завада сигналу та точність вимірювання швидкості лазерним допплерівським анемометром.

Ключові слова: *сигнал, лазер, мікрочастинка, модель, розсіяне випромінювання, швидкість потоку.*

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