COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING
Vol. 10, No. 1, 2020

APPLICATION OF FREQUENCY STABILITY CRITERION FOR ANALYSIS OF DYNAMIC SYSTEMS WITH CHARACTERISTIC POLYNOMIALS FORMED IN $j^{1/3}$ BASIS

Orest Lozynskyi[1], Yaroslav Marushchak[2], Andriy Lozynskyi[1], Bohdan Kopchak[1], Lidiya Kasha[1]
[1] Lviv Polytechnic National University, Lviv, Ukraine
[2] Rzeszow University of Technology, Rzeszow, Poland
orest.y.lozynskyi@lpnu.ua, jamaru@prz.edu.pl, andrii.o.lozynskyy@lpnu.ua, bohdan.l.kopchak@lpnu.ua, lidiia.v.kasha@lpnu.ua

Abstract: This paper considers the stability of dynamical systems described by differential equations with fractional derivatives. In contrast to a number of works, where the differential equation describing the system may have a set of different values of fractional derivatives, and the characteristic polynomial is formed on the basis of the least common multiple for the denominators of these indicators, this article proposes forming such a polynomial in a specific $j^{\frac{1}{3}}$ basis and studying the stability of systems with such fractional description based on the resulting rotation angles of $H_n(j^{\frac{l}{m}}\omega)$ vector at a frequency change from zero to infinity.

This technique is similar to the investigation of system stability by frequency criteria used for a similar problem in describing the system by differential equations in integer derivatives.

The application of characteristic polynomials formed in the $j^{\frac{1}{3}}$ basis for the description of the processes in dynamic systems and the analysis of the stability of such systems on the basis of the frequency criterion are the essence of the scientific novelty of this paper.

The article contains the following sections: problem statement, work purpose, presentation of the research material, conclusions, list of references.

Key words: dynamic system, fractional derivative, stability, characteristic polynomial, frequency stability criterion.

1. Problem statement.

The application of fractional order controllers in technical systems together with the creation of process models using fractional order derivatives have formed new classes of problems in such systems. These are the formation of the mathematical apparatus for the study of transition processes in systems with fractional order derivatives, on the one hand, and the analysis of the stability of such systems, on the other. Although the approximation of fractional order systems by systems with integer order derivatives is quite often used for the study of transition processes, the application of this approach for stability analysis requires additional research. According to Matignon's stability theorem [1], the location area of the roots of a characteristic polynomial ensuring stable operation of the system varies depending on the base index of the degree of the characteristic polynomial $j^{1/\alpha}$. The fractional order system can be stable even in the presence of a pair of complex conjugated roots in the right plane. At the same time, the presence of roots in the right half-plane for systems with derivatives of the integer order clearly indicates the instability of such a system. The transformed complex ω_R - Riemann plane [2–4] or the Lyapunov stability criterion [5–6] are most frequently used to study the stability of the fractional order system. It should be noted that the traditional transformation of the characteristic polynomial

$$Q(s) = a_1 s^{\frac{\alpha_1}{\beta_1}} + a_2 s^{\frac{\alpha_2}{\beta_2}} + \ldots + a_n s^{\frac{\alpha_n}{\beta_n}}$$

in the form

$$Q(s) = a_1 (s^{\frac{1}{m}})^n + a_2 (s^{\frac{1}{m}})^{n-1} + \ldots + a_n (s^{\frac{1}{m}})^1 + a_{n+1},$$

where m is calculated on the basis of the least common multiple of $\beta_1, \beta_2 \ldots \beta_n$, can lead to a high-order polynomial, and the analysis of the stability of the system will be significantly complicated. It is the high order of the obtained characteristic polynomial that creates significant difficulties in applying the frequency criteria for the analysis of the stability of systems with integer derivatives in fractional order systems. Thus, in [7], to eliminate the problem of high orders, it is proposed to analyze the argument of a modified characteristic polynomial of a system of the form $\sum_{k=0}^{m} a_{m-k} \cdot (s^{\frac{1}{\alpha}})^k \Big/ a_0 \cdot (s+\lambda)^{m/\alpha}$, where λ is a natural number. The system will be stable if the argument of the modified polynomial is equal to zero. In [8], the stability criterion for a system with fractional order derivatives is formulated as follows: «the stability of the system requires that the hodograph, starting at the positive part of the real axis $\omega = 0$ while ω growing to ∞ should pass sequentially through the $m \cdot \pi / (2 \cdot \alpha)$ sectors of the complex plane, where *m* is the order of the polynomial, α is determined from the characteristic polynomial basis $s^{1/\alpha}$.

It should be noted that it is possible to apply the fractional order characteristic polynomial to describing the processes in electromechanical systems [9, 10]. Moreover, the order of the polynomial itself is not high, and the basis of such a polynomial can be either $s^{1/2}$ or $s^{1/3}$, $s^{2/3}$.

The purpose of this work is the formation of characteristic polynomials of systems in the $j^{\frac{1}{3}}$ basis of fractional values and the analysis of the stability of such systems based on the frequency criterion.

2. Presentation of the research material.

Integer order characteristic polynomial of a closed dynamic system of the form:

$$H(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s + a_n$$

after the transition to the frequency domain by replacement $s \rightarrow j\omega$ can be written as:

$$H(j\omega) = a_0 (j\omega - p_1) \cdot (j\omega - p_2) \cdot \\ \cdot (j\omega - p_3)..(j\omega - p_n),$$

where $p_1, p_2, p_3 ... p_n$ are the roots of the characteristic equation $H(s{=}j\omega){=}0$.

Having moved from the Cartesian coordinate system to the polar one, we obtain

$$H(j\omega) = a_0 \prod_{i=1}^{n} (r_i e^{j\varphi_i}) = a_0 e^{j\sum_{i=1}^{n}\varphi_i} \cdot \prod_{i=1}^{n} r_i ,$$

where $\varphi(\omega) = \sum_{i=1}^{n} \varphi_i(\omega)$ is the argument (arg) of the complex function $H(j\omega)$.

In order to write similar expressions for polynomials of fractal systems, it is necessary to solve the issue of choosing the basis for a characteristic polynomial in its frequency representation.

Let us consider the case of $j^{1/3}$ value as the basis for a characteristic polynomial and investigate the relationship of the roots of a fractional characteristic polynomial in the frequency representation under the condition of the stability of fractional systems.

It is known that the trigonometric form of a complex number is generally represented as follows: $z = a + jb = r \cdot (\cos\varphi + j \sin\varphi)$, where $r = \sqrt{a^2 + b^2}$; $\varphi = arctg \frac{b}{a}$. Representation of a complex number in trigonometric form is required for the application of the well-known Moivre formula which enables calculation of any fractional degree *d* of a complex number, i.e. for $d = \frac{l}{m}$, where *m* is the degree of the root of a complex number and *l* is the degree of a complex number (*m* and *l* are integers) .

Then, taking into account the Moivre formula, we can write for the case when $l = 1$

$$\sqrt[m]{z} = \sqrt[m]{|z|} \cdot (\cos \frac{\arg z + 2k\pi}{m} + j \sin \frac{\arg z + 2k\pi}{m}),$$

where $k = 0,\ 1,\ 2, \ldots (m-1)$.

For *m=3* we obtain

$$\sqrt[3]{j} = \sqrt[3]{1} \cdot (\cos \frac{\frac{\pi}{2} + 2k\pi}{m} + j \sin \frac{\frac{\pi}{2} + 2k\pi}{m}),$$

and when $k = 0 \rightarrow \sqrt[3]{j} = \cos \frac{\pi}{2 \cdot 3} + j \sin \frac{\pi}{2 \cdot 3}$.

Now, a polynomial in the $j^{\frac{1}{3}}$ basis can be formed which looks as follows:

$$H_n(j^{\frac{1}{3}}\omega) = a_0 (j^{\frac{1}{3}}\omega)^n + a_1 (j^{\frac{1}{3}}\omega)^{n-1} + \\ + a_2 (j^{\frac{1}{3}}\omega)^{n-2} + \ldots + a_{n-1} (j^{\frac{1}{3}}\omega) + a_n$$

and, having written it as

$$H_n^*(j^{\frac{1}{3}}\omega) = a_0 (j^{\frac{1}{3}}\omega - p_1) \cdot (j^{\frac{1}{3}}\omega - p_2) \ldots \\ \ldots (j^{\frac{1}{3}}\omega - p_{n-1})(j^{\frac{1}{3}}\omega - p_n)$$

we can analyze the options for the placement of roots on the complex plane $jV(\omega)$, $U(\omega)$ and the resulting angles of vector rotation $(j^{\frac{1}{3}}\omega - p_i)$ to meet the condition of the frequency criterion of stability given in [8].

It is worth noting that according to this criterion, the total angle of the vector rotation $H_n^*(j^{\frac{l}{m}}\omega))$ at the frequency changing from 0 to infinity must be equal to

$$\varphi = n\frac{\pi \cdot l}{2 \cdot m},$$

where n is the exponent of the polynomial.

3. Analysis of the influence of the location of the roots of characteristic polynomial $H_n^*(j^{\frac{1}{3}}\omega)$ on the system stability.

3.1. All roots are real numbers and are located in the left half-plane.

The analysis of the displacement of the vector $j^{\frac{1}{3}}\omega - p_1$ for the first root under the condition ω=0÷∞ is shown in Fig.1

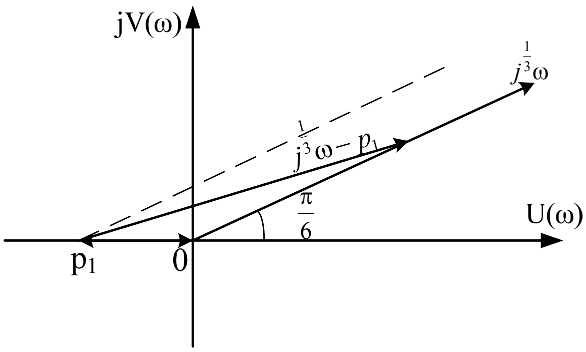

Fig. 1.

For frequency $\omega = 0$, the vector $(j^{\frac{1}{3}}\omega - p_1)$ occupies the position $+p_1$. As the frequency increases to $\omega = \infty$, the angle of rotation of the vector $(j^{\frac{1}{3}}\omega - p_1)$ will be equal to $+\frac{\pi}{6}$ (counterclockwise rotation). Therefore, the total angle of the vector $H_n^*(j^{\frac{1}{3}}\omega)$ rotation with taking into account all n roots will be:

$$\Delta\varphi = (n-1)\frac{\pi}{6} + 1\cdot\frac{\pi}{6} = n\cdot\frac{\pi}{6}.$$

Thus, by analogy with the Mikhailov criterion and in accordance with criterion [8] in this case for a stable system, the resulting angle of the vector $H_n^*(j^{\frac{1}{3}}\omega)$ rotation at the frequency changing from zero to infinity takes the value $n\cdot\frac{\pi}{6}$.

3.2. At least one real root is placed in the right half-plane.

For this case, the analysis of the displacement of the vector $j^{\frac{1}{3}}\omega - p_1$ for the first root under the condition ω = 0 ÷ ∞ is shown in Fig. 2.

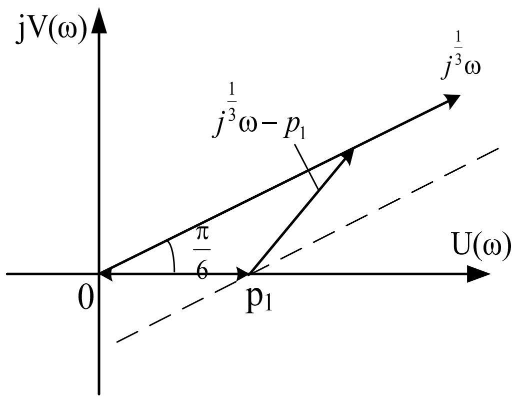

Fig. 2.

If the frequency $\omega = 0$, then the vector $(j^{\frac{1}{3}}\omega - p_1)$ takes the position "$-p_1$". If $\omega \to \infty$, the angle of the vector $(j^{\frac{1}{3}}\omega - p_1)$ rotation equals $5\cdot\frac{\pi}{6}$ clockwise, i.e. the angle is negative. Provided that the remaining roots are located in the left half-plane, the total angle of the vector rotation can be calculated as follows:

$$\Delta\varphi = (n-1)\frac{\pi}{6} - 5\cdot\frac{\pi}{6} = n\cdot\frac{\pi}{6} - \pi.$$

Here $\Delta\varphi \neq n\cdot\frac{\pi}{6}$, i.e. such a value of Δφ occurs due to the instability of the system.

3.3. All roots are complex-conjugated numbers with a negative real part.

For this case, the analysis of the vector displacement for the first two complex-conjugated roots under the condition ω = 0 ÷ ∞ is shown in Fig. 3.

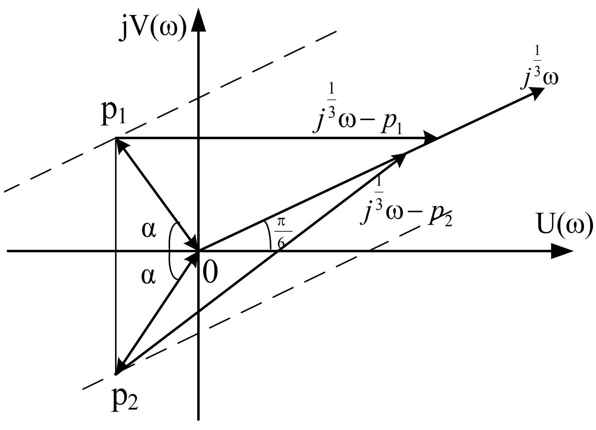

Fig. 3.

The resulting angle of the vector ($j^{\frac{1}{3}}\omega - p_1$) rotation, provided that the frequency changes from 0 to ∞, is equal to ($\frac{\pi}{6} + \alpha$) counterclockwise and the angle of the vector ($j^{\frac{1}{3}}\omega - p_2$) rotation is equal to ($\alpha - \frac{\pi}{6}$) clockwise. Thus, the resulting angle of the rotation of vectors of the pair of complex conjugated roots is equal to

$$\Delta\varphi_{pair} = \frac{\pi}{6} + \alpha - (\alpha - \frac{\pi}{6}) = 2 \cdot \frac{\pi}{6}.$$

With regard to the rest of similar roots, the angle of vector $H_n^*(j^{\frac{1}{3}}\omega)$ rotation will be written as follows:

$$\Delta\varphi = (n-2)\frac{\pi}{6} + \frac{2\pi}{6} = n \cdot \frac{\pi}{6}.$$

Therefore, we may conclude that such a value of $\Delta\varphi$ is associated with a stable system.

3.4. At least one pair of roots is complex-conjugated numbers with a positive real part (the roots are in the middle of AOB sector).

For this case, the analysis of the vector displacement for the first two complex-conjugated roots under the condition ω = 0 ÷ ∞ is shown in Fig. 4.

Here, the angle of the vector ($j^{\frac{1}{3}}\omega - p_1$) rotation, while ω is changing from zero to ∞, is equal to $\alpha + 5 \cdot \frac{\pi}{6}$ clockwise (negative angle). The angle of the vector ($j^{\frac{1}{3}}\omega - p_2$) rotation, if $0 \le \omega \le \infty$, is equal to $5 \cdot \frac{\pi}{6} - \alpha$, also clockwise (negative angle). Therefore, the total angle of the vectors rotation for such a pair of roots is as follows:

$$\Delta\varphi_{pair} = -\alpha - \frac{5\pi}{6} - \frac{5\pi}{6} + \alpha = -2 \cdot \frac{5\pi}{6}.$$

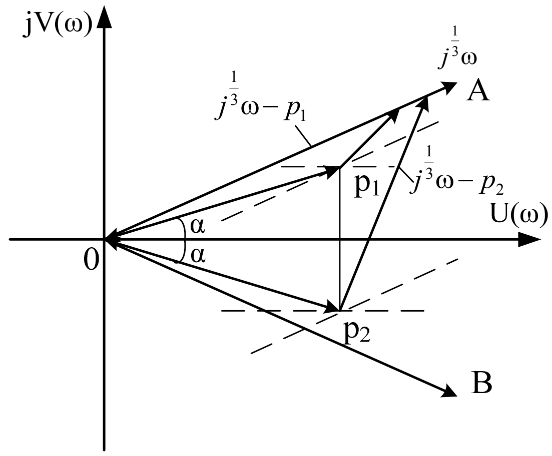

Fig. 4.

Taking into account that the remaining roots are stable, the resulting angle of the vector $H_n^*(j^{\frac{1}{3}}\omega)$ rotation is equal to

$$\Delta\varphi = (n-2)\frac{\pi}{6} - (\frac{5\pi}{6} + \alpha) - (\frac{5\pi}{6} - \alpha) = $$
$$= n \cdot \frac{\pi}{6} - 12 \cdot \frac{\pi}{6} < n\frac{\pi}{6}.$$

With regard to the obtained result, it can be concluded that such a system is not stable.

3.5. Complex-conjugated roots with a positive real part (the roots are outside the AOB sector).

For this case, the analysis of the vector displacement for the first two complex-conjugated roots under the condition ω = 0 ÷ ∞ is shown in Fig. 5.

It can be seen from this figure that the angle of rotation of the vector ($j^{\frac{1}{3}}\omega - p_1$) is equal to +($\frac{7\pi}{6} - \alpha$) (counterclockwise), and the angle of rotation of the vector ($j^{\frac{1}{3}}\omega - p_2$) is equal to -($\frac{5\pi}{6} - \alpha$) (clockwise).

Thus, in the case when the pair of complex-conjugated roots is outside the AOB sector, the resulting rotation angle of the pair of roots will be equal to

$$\Delta\varphi_{pair} = \frac{7\pi}{6} - \alpha - (\frac{5\pi}{6} - \alpha) = 2 \cdot \frac{\pi}{6}.$$

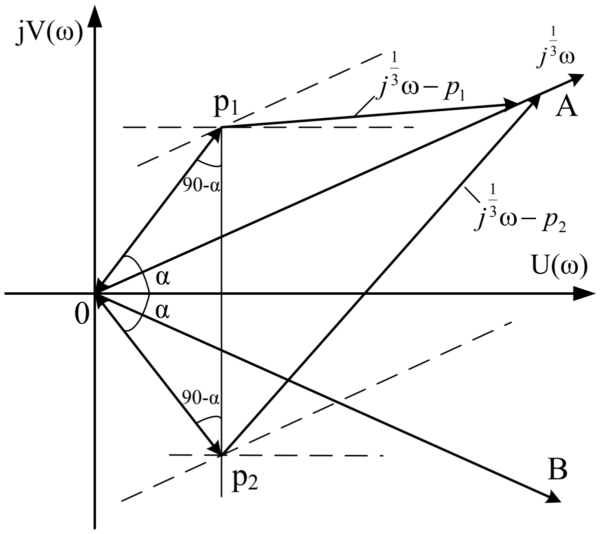

Fig. 5.

Taking into account the remaining roots, the total angle of rotation of the vector $H_n^*(j^{\frac{1}{3}}\omega)$ will be equal to

$$\Delta\varphi = (n-2)\frac{\pi}{6} + \frac{2\pi}{6} = n\frac{\pi}{6}.$$

By analogy with the value of Δφ for stable systems, we can conclude that such a fractional order system is stable even with a positive real part of complex conjugated roots.

The latter result does not fit into the classical understanding of the condition of root placement in terms of the system stability when describing by the transfer functions of the integer order. It is obvious that for describing the model of fractional systems the condition of a non-stable system is transformed into the condition of finding the root in the corresponding sector of the right plane, and not in the whole right half-plane. Therefore, it is probably necessary to check the angles of rotation of the hodograph $H_n(j^{\frac{l}{m}}\omega)$ for different orders of characteristic polynomials n under the condition of changing the frequency within $0 \le \omega \le \infty$. This will once again verify the correctness of the criterion in [8], if the total angle of vector $H_n(j^{\frac{l}{m}}\omega)$ hodograph rotation based on the formation of the $j^{\frac{l}{m}}$ basis will be equal to $\Delta\varphi = n \cdot \frac{\pi \cdot l}{2 \cdot m}$, where n is the order of the characteristic polynomial in the $j^{\frac{l}{m}}$ basis.

Let us investigate hodographs for the systems of different order, which are represented by a characteristic polynomial in the $j^{1/3}$ basis.

a. Second-order hodograph

The characteristic polynomial in the $j^{1/3}$ basis can be written as follows:

$$H^*{}_2(j^{\frac{1}{3}}\omega) = a_0(j^{\frac{1}{3}}\omega)^2 + a_1(j^{\frac{1}{3}}\omega) + a_2.$$

Obviously, here $n=2$.

Let us move on to representing this expression in the j basis, using the formula

$$j^{\frac{1}{3}} = \frac{1}{2}(\sqrt{3} + j).$$

Then $U(\omega) = \frac{1}{2}a_0\omega^2 + \frac{1}{2}a_1\omega\sqrt{3} + a_2$ is a real part,

$V(\omega) = \frac{1}{2}a_0\sqrt{3}\omega^2 + \frac{1}{2}a_1\omega$ is an imaginary part.

For $\omega = 0$ we have $U(\omega) = a_2$, $V(\omega) = 0$.

For $\omega = \infty$ we have $U(\omega) = \infty$ and $V(\omega) = \infty$.

So,

$$tg\varphi = \left. \frac{\omega^2(\frac{1}{2}a_0\sqrt{3} + \frac{1}{2}a_1\frac{1}{\omega})}{\omega^2(\frac{1}{2}a_0 + \frac{1}{2}a_1\sqrt{3}\frac{1}{\omega} + a_2\frac{1}{\omega^2})} \right|_{\omega\to\infty} = \sqrt{3}.$$

Then $\varphi = arctg\sqrt{3} = 60^o = 2\frac{\pi}{6}$. The magnitude of the rotation angle of the vector $H_2(j^{\frac{l}{3}}\omega)$ obtained earlier indicates that such a system is stable. Therefore, the conclusion is that the analysis of the stability of fractional systems can be performed on the hodograph of the characteristic polynomial in the j basis, which corresponds to the characteristic polynomial in the $j^{1/3}$ basis.

b. Third-order hodograph

Let us write the characteristic polynomial in the $j^{1/3}$ basis for the case when n=3:

$$H^*{}_3(j^{\frac{1}{3}}\omega) = a_0(j^{\frac{1}{3}}\omega)^3 + a_1(j^{\frac{1}{3}}\omega)^2 + a_2(j^{\frac{1}{3}}\omega)^1 + a_3).$$

Then we move on to the characteristic polynomial in the j basis. Using the formula $j^{1/3}$, we write:

$$H_3^*(j\omega) = a_0 j\omega^3 + a_1\frac{1}{2}(1 + j\sqrt{3})\omega^2 +$$
$$+ a_2\frac{1}{2}(\sqrt{3} + j)\omega + a_3.$$

Therefore, the expressions of the real and imaginary part of this expression are as follows:

$$U(\omega) = a_1 \frac{1}{2}\omega^2 + a_2 \frac{1}{2}\sqrt{3}\omega + a_3 ,$$

$$V(\omega) = a_0\omega^3 + a_1 \frac{1}{2}\sqrt{3}\omega^2 + a_2 \frac{1}{2}\omega .$$

For $\omega = 0$ we have $U(\omega) = a_3$, $V(\omega) = 0$.

If $\omega = \infty$, the resulting rotation angle of the vector $H_3^*(j\omega)$ can be determined as follows:

$$tg\phi = \left.\frac{\omega^3 (a_0 + a_1 \frac{1}{2}\sqrt{3}\frac{1}{\omega} + a_2 \frac{1}{2}\frac{1}{\omega^2})}{\omega^3 (a_1 \frac{1}{2}\frac{1}{\omega} + a_2 \frac{1}{2}\sqrt{3}\frac{1}{\omega^2} + a_3 \frac{1}{\omega^3})}\right|_{\omega\to\infty} = \infty .$$

Hence, $\Delta\varphi = arctg\infty = \frac{\pi}{2} = 3\frac{\pi}{6}$. Taking into account the signs of $V(\omega)$ i $U(\omega)$, the angle is in the first quadrant, i.e. $\Delta\varphi = \frac{3\pi}{6}$.

This value of the hodograph rotation angle corresponds to the condition of stability.

c. Fifth-order hodograph

Let us write the characteristic polynomial in the $j^{1/3}$ basis for the case when *n=5*:

$$H_5(j^{\frac{1}{3}}\omega) = a_0(j^{\frac{1}{3}}\omega)^5 + a_1(j^{\frac{1}{3}}\omega)^4 + a_2(j^{\frac{1}{3}}\omega)^3 + $$
$$+ a_3(j^{\frac{1}{3}}\omega)^2 + a_4(j^{\frac{1}{3}}\omega)^1 + a_5.$$

Let us move on to the characteristic polynomial in the j basis. Considering that

$$j^{\frac{1}{3}} = \frac{1}{2}(\sqrt{3} + j) ;$$

$$j^{\frac{2}{3}} = \frac{1}{2}(1 + j\sqrt{3}) ;$$

$$(j^{\frac{3}{3}})^3 = j ;$$

$$j^{\frac{4}{3}} = j\frac{1}{2}(\sqrt{3} + j) = \frac{1}{2}(j\sqrt{3} - j) ;$$

$$j^{\frac{5}{3}} = j \cdot j^{\frac{2}{3}} = j\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1}{2}(j - \sqrt{3}) ,$$

characteristic polynomial $H_5(j^{\frac{1}{3}}\omega)$ in j basis will have the form:

$$H_5^*(j\omega) = a_0 \frac{1}{2}(j - \sqrt{3})\omega^5 + a_1 \frac{1}{2}(j\sqrt{3} - 1)\omega^4 +$$
$$+ a_2 j\omega^3 + + a_3 \frac{1}{2}(1 + j\sqrt{3})\omega^2 + a_4 \frac{1}{2}(\sqrt{3} + j)\omega \ + a_5 .$$

Having grouped the real and imaginary part, we obtain

$$U(\omega) = -a_0 \frac{1}{2}\sqrt{3}\omega^5 - a_1 \frac{1}{2}\omega^4 + a_3 \frac{1}{2}\omega^2 +$$
$$+ a_4 \frac{1}{2}\sqrt{3}\omega \ + a_5,$$

$$V(\omega) = a_0 \frac{1}{2}\omega^5 + a_1 \frac{1}{2}\sqrt{3}\omega^4 + a_2\omega^3 +$$
$$+ a_3 \frac{1}{2}\sqrt{3}\omega^2 + a_4 \frac{1}{2}\omega.$$

According to these expressions, let us analyze the hodograph in the *j* basis.

If $\omega = 0$, $U(\omega) = a_5$, $V(\omega) = 0$.

If $\omega = \infty$, by writing the expression for $tg\varphi$ as

$$tg\varphi = \left.\frac{\omega^5 (a_0 \frac{1}{2} + a_1 \frac{1}{2}\sqrt{3}\frac{1}{\omega} + a_2 \frac{1}{\omega^2} + a_3 \frac{1}{2}\sqrt{3}\frac{1}{\omega^3} + a_4 \frac{1}{2}\frac{1}{\omega^4})}{-\omega^5 (a_0 \frac{1}{2}\sqrt{3} + a_1 \frac{1}{2}\frac{1}{\omega} - a_3 \frac{1}{2}\frac{1}{\omega^2} - a_4 \frac{1}{2}\sqrt{3}\frac{1}{\omega^4} - a_5 \frac{1}{\omega^5})}\right|_{\omega\to\infty} =$$
$$= \frac{+a_0 \frac{1}{2}}{-a_0 \frac{1}{2}\sqrt{3}} = -\frac{1}{\sqrt{3}},$$

we obtain $\Delta\varphi = arctg(-\frac{1}{\sqrt{3}}) = 5\frac{\pi}{6}$.

Therefore, the hodograph of the vector $H_5(j^{\frac{1}{3}}\omega)$ in the representation in the j basis when changing the frequency from 0 to ∞ will return to the angle $\frac{5\pi}{6}$, and this corresponds to the stability condition shown above.

Thus, the obtained values of hodographs of vectors $H_n(j^{\frac{1}{3}}\omega)$ at $\omega = 0$ and $\omega = \infty$ do not contradict the conditions of the above-mentioned criterion of stability of the system with fractional derivatives.

4. Conclusion

The analysis of the influence of root location in the complex plane $jV(\omega)$ and $U(\omega)$ on the stability of the system and the study of the resulting rotation angles of vectors $H_n^*(j^{\frac{1}{3}}\omega)$ for $n = 2$, 3, 5 when changing the frequency from zero to infinity has been carried out. It has confirmed the condition of the frequency criterion of system stability described by fractional order equations for the case of characteristic polynomials formed in the $j^{\frac{1}{3}}$ basis.

5. References

[1] D. Matignon, "Stability result on fractional differential equations with applications to control processing," in *Proc. International Meeting on Automated Compliance Systems and the International Conference on Systems, Man, and Cybernetics (IMACS-SMC '96)*, pp. 963–968, Lille, France, 1996.

[2] A. G. Radwan, A. M. Soliman, and A. S. Elwakil, "On the stability of linear system with fractional order elements," *Chaos Solutions & Fractals*, no. 40, pp. 2317–2328, 2009.

[3] M. S. Tavazoei, and M. Haeri, "A note on the stability of fractional order systems," *Mathematics and Computers in Simulation*, no. 79, pp. 1566–1576, 2009.

[4] M. Rivero, S. Rogosin, J. A. T. Machado, and J. Trujillo, "Stability of fractional order systems. *Mathematical problems in engineering. New Challenges in Fractional Systems,* vol. 2013, 2013.

[5] S. Liang, C. Peng, and Y. Wang, "Improved linear matrix inequalities stability criteria for fractional order systems and robust stabilization synthesis: the 0<α<1 case", *Control Application*, vol.30, no 4, 2013.

[6] A. Banzaonia, A. Hmamed, F. Mesquine, B. Enhayoun, and F. Tadeo, "Stabilization of continuous-time fractional positive systems by using Lyapunov function", *IEEE Trans. Aut. Control,* vol. 59, no. 8, pp. 2203–2208, 2014.

[7] M. Buslowicz, "Stability analysis of linear continuous-time fractional systems of commensurate order", *Journal Automation, Mobile Robotics and Intelligent Systems,* vol. 3, no. 1, pp. 12–17, 2009.

[8] O. Yu. Lozynskyy, P. I. Kalenyuk, and A. O. Lozynskyy, "Frequency criterion for stability analysis of the systems with derivatives of fractional order", *Mathematical Modelling and Computing*, vol. 7, no. 2, 2020. (Unpublished).

[9] Y. Marushchak, B. Kopchak, and L. Kasha, "Robust stability of fractional electromechanical systems", *Electrical Power and Electromechanical Systems*, Lviv, Ukraine: Publishing House of Lviv Polytechnic National University, no. 900, pp. 47–51, 2018. (Ukrainian)

[10] O. Lozynskyy, A. Lozynskyy, B. Kopchak, Y. Paranchuk, P. Kalenyuk, and Y. Marushchak, "Synthesis and research of electromechanical systems described by fractional order transfer functions," in *Proc. International Conference on Modern Electrical and Energy Systems (MEES-2017)*, pp. 16–19, Kremenchuk, Ukraine, 2017.

ЗАСТОСУВАННЯ ЧАСТОТНОГО КРИТЕРІЮ СТІЙКОСТІ ДЛЯ АНАЛІЗУ ДИНАМІЧНИХ СИСТЕМ З ХАРАКТЕРИСТИЧНИМИ ПОЛІНОМАМИ, СФОРМОВАНИМИ В БАЗИСІ $J^{1/3}$

Орест Лозинський, Ярослав Марущак,
Андрій Лозинський, Богдан Копчак, Лідія Каша

В даній статті розглянуто питання стійкості динамічних систем, які описуються диференціальними рівняннями з дробовими похідними. На відміну від ряду робіт, де диференціальне рівняння, яке описує систему, може мати набір різних значень показників дробових похідних, а характеристичний поліном формується на основі найменшого спільного кратного для знаменників цих показників, в даній статті пропонується сформувати такий поліном в конкретному базисі $j^{\frac{1}{3}}$ і далі проводити дослідження стійкості систем з таким дробовим описом на основі результуючих кутів повороту вектора $H_n(j^{\frac{l}{m}}\omega)$ при зміні частоти від нуля до нескінченності

Така методика є аналогічною до дослідження стійкості систем за частотними критеріями, які використовуються для подібної задачі при описі системи диференціальними рівняннями в цілочисельних похідних.

Саме застосування для опису процесів в динамічних системах характеристичних поліномів сформованих в базисі $j^{\frac{1}{3}}$ і аналіз стійкості таких систем на основі частотного критерію становлять суть наукової новизни даного матеріалу.

Стаття містить наступні розділи: постановка проблеми, мета роботи, виклад основного матеріалу, висновки, список літератури.

Orest Lozynskyi – Honoured Scientist of Ukraine, Professor, Dr. Sc. in Engineering, professor of the Department of Electromechatronics and Computerized Electromechanical Systems at Lviv Polytechnic National University. Graduated from Lviv Polytechnic National University, Department of Electric drive and Automation of Industrial Installations.

His research activities are directed towards solving scientific problems and applied tasks in the field of automation, synthesis, analysis and optimal control of the modes of dynamic systems.

Yaroslav Marushchak – Professor, Dr. Sc. in Engineering, professor of the Department of Electrical and Based Computer Engineering at Rzeszow University of Technology. Graduated from Lviv Polytechnic National University, Department of Electric Drive and Automation of Industrial Installations.

His research and teaching interests are in the areas of control theory of the electromechanical systems.

Andriy Lozynskyy – D.Sc. (2004), Professor (in 2006). Graduatied in 1993 from electromechanical faculty of Lviv Polytechnic Institute, specialty "Electric drive and automation of industrial enterprise and technological complexes". After completing of post-graduate studying he worked at the department of Electrical Machines and Apparatus at the Lviv Polytechnic National University. Since May 2004 he was a Deputy Pro-Rector for Research at Lviv Polytechnic National University. Since December 2014 – Director of Institute of Power Engineering and Control Systems. Fellowship of the Cabinet of Ministers of Ukraine (2001–2002). Team leader of the Grant of President of Ukraine for support of scientific researches of young scientists in 2006. Supervisor of 1 Dr.Sc. and 4 Ph.D. dissertations. Author and coauthor of 4 monographs, over 160 scientific articles.

Research interests: modern control systems; neuro- and fuzzy logic control and coordinate identification in vector controlleda.c. electric drive;renewable energy systems and e-mobility.

Bohdan Kopchak – Associate professor, Dr. Sc., Associate Professor of the Department of Electromechatronics and Computerized Electromechanical Systems at Lviv Polytechnic National University. Graduated from Lviv Polytechnic National University, Department of Electric Drive and Automation of Industrial Installations.

Research and teaching interests are in the areas of fractional order control systems, control of autonomous induction generator of wind power sets.

Lidiya Kasha – PhD, Associate Professor of the Department of Electromechatronics and Computerized Electromechanical Systems at Lviv Polytechnic National University, Ukraine. Research interests: mathematical modeling of processes in electromechanical energy converters, computer-aided design of electrical machines; research and design of switched reluctance motors.

Received: 18.03.2020. Accepted: 15.04.2020.