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PARAMETERS FOR CALCULATION OF THREE-DIMENSIONAL ELECTROMAGNETIC FIELD BY ASYMPTOTIC EXPANSION METHOD

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Abstract: The paper deals with an approximate analytical solution of a three-dimensional problem of the theory of electromagnetic field, which is based on the use of asymptotic expansion under the condition of a strong skin-effect for a field produced by a closed current-carrying loop located near a conducting half-space. It is noted that each member of an asymptotic series is determined with an error, the value of which depends on the value of a small parameter and increases with increasing the index of series member resulting in limited number of its members. It is identified that when using the method of asymptotic expansion, the number of members of a series can be limited by the relatively small number, which is determined by the specified limits of the allowable accuracy of calculation (relative error). The authors determine the optimal number of asymptotic series members, and indicate that calculation accuracy depends on the value of a small parameter, and for a specific conducting material it depends on the field frequency and the minimum distance from external field sources to a conducting body.

Key words: analytical and asymptotic calculation methods, 3D electromagnetic fields, strong skin-effect

1. Introduction

Despite the widespread use of numerical methods for calculating three-dimensional electromagnetic fields, analytical or numerical-analytical approaches, which allow the most significant characteristics of electromagnetic systems to be taken into account, remain effective. Analytical, accurate and approximate methods of calculating an alternating electromagnetic field are used in solving inverse problems of the theory of electromagnetic field and optimization of electromagnetic systems. For such applications, the difficulties associated with a significant increase in the volume of calculations and the simultaneous need to ensure high accuracy of field calculation, for example, in the problems of extending the field from the surface [1–3].

The need to solve the inverse problems of field theory and optimize the geometry of electromagnetic systems arises in the development of devices for electrical engineering equipment. They occur in the creation of technological devices for heat treatment of metals [4], magnetic pulse treatment of metal products [5], treatment of metallic materials with electric current and high intensity electromagnetic field [6].

In the above examples, in in electroconductive media of the elements of devices that are affected by the electromagnetic field, there is a strong skin-effect when the field and the induced current exist in a thin surface layer of the conducting body [7]. In this case, one usually uses a mathematical model, in which the body of real shape is locally replaced by an electroconducting half-space, and the external field is produced by sources located outside the body in the dielectric region. The application of the analytical solution of such a problem, obtained in [8, 9], in the general formulation is also associated with rather cumbersome calculations. Therefore, even in this case, the development and implementation of approximate methods for calculating fields in specific applications are relevant.

An effective approach for determining three-dimensional electromagnetic fields using the approximate asymptotic method in [10–12] is used to solve inverse problems to find the spatial geometry of field inductors in the field of heat treatment using induction heating of moving metal strips The electromagnetic field is considered in the case of a strong skin-effect in the extended interpretation, when a strong skin-effect means not only a small value of the depth of penetration of the field into the conductive medium compared to body size, but also the fact that the depth of field penetration is a small value compared to the characteristic dimensions in the entire electromagnetic system, including the dimensions of the current-carrying loop and the distance from the loop to the media dividing boundary. The ratio of penetration depth to the characteristic dimensions of electromagnetic system is believed to be a small parameter, but not necessarily to tend to zero.

When obtaining the geometry of the inductors for induction heating of moving strips, calculations were performed for a specific number of members in the asymptotic series, and the accuracy of the results was checked separately. However, in the method of asymptotic expansion, the number of members of the series is limited not only by the required accuracy of the calculation. The error has a lower limit, which is determined by the value of a small parameter. Thus excess of the certain number of series members can lead to growth of the general error of calculation.

The aim of this work is to analyze the error estimation using the asymptotic expansion method and to choose the optimal number of members of the asymptotic series depending on the value of a small parameter under a strong skin-effect in the system "AC loop - conductive half-space".

2. Mathematical model

2.1. Analytical solution of a three-dimensional problem

The study is based on the exact analytical solution of the linear problem presented in [8, 9] for a three-dimensional electromagnetic field produced in the system "arbitrary current-carrying loop - electroconducting half-space". An analytical solution is found in both areas: dielectric, where a closed loop l of the sinusoidal current $\dot{I}_0$ is located, and electro conductive with specific electrical conductivity γ and relative magnetic permeability μ, where eddy currents flow. The problem does not impose restrictions on the geometry of the loop and its orientation relative to the interface, the electrophysical properties of the medium and the field frequency ω. In this regard, the task is general.

Fig. 1 shows a current-carrying loop element and the orientation relative to the flat surface of the media division of single tangent vectors to the output loop t and to the loop t_1 mirrored from the surface.

In the dielectric region, the vector $\dot{\mathbf{A}}_e = \dot{\mathbf{A}}_0 + \dot{\mathbf{A}}_1 + \dot{\mathbf{A}}_2$ and scalar $\dot{\phi}_e$ potentials, as well as the induction of the magnetic field $\dot{\mathbf{B}}_e = \dot{\mathbf{B}}_0 + \dot{\mathbf{B}}_1 + \dot{\mathbf{B}}_2$ and the electric field strength $\dot{\mathbf{E}}_e = \dot{\mathbf{E}}_0 + \dot{\mathbf{E}}_1 + \dot{\mathbf{E}}_2$ are presented in the form of loop integrals :

$$\dot{\mathbf{A}}_e = \dot{\mathbf{A}}\frac{\mu_0 \dot{I}_0}{4\pi}\oint_l \left(\frac{\mathbf{t}}{r} - \frac{\mathbf{t}_1}{r_1} - \mathbf{t}_1 \frac{\partial \dot{G}_e}{\partial z}\right) dl\,, \qquad (1)$$

$$\dot{\phi}_e = i\omega \frac{\mu_0 \dot{I}_0}{4\pi}\oint_l (\mathbf{t}_1 \cdot \mathbf{e}_z)\dot{G}_e dl\,, \qquad (2)$$

$$\dot{\mathbf{E}}_e = -\frac{\mu_0 i\omega \dot{I}_0}{4\pi}\oint_l \left(\frac{\mathbf{t}}{r} - \frac{\mathbf{t}_1}{r_1} - \mathbf{e}_z \times \left[\mathbf{t}_1 \times \nabla \dot{G}_e\right]\right) dl\,, \qquad (3)$$

$$\dot{\mathbf{H}}_e = -\frac{\dot{I}_0}{4\pi}\oint_l \left[\frac{\mathbf{t}\times\mathbf{r}}{r^3} - \frac{\mathbf{t}_1\times\mathbf{r}_1}{r_1^3} - \mathbf{t}_1 \times \nabla\left(\frac{\partial \dot{G}_e}{\partial z}\right)\right] dl\,, \qquad (4)$$

where $\mathbf{e}_z$ is the unit vector in the direction of the axis z. Potentials are written under Lorenz gauge condition.

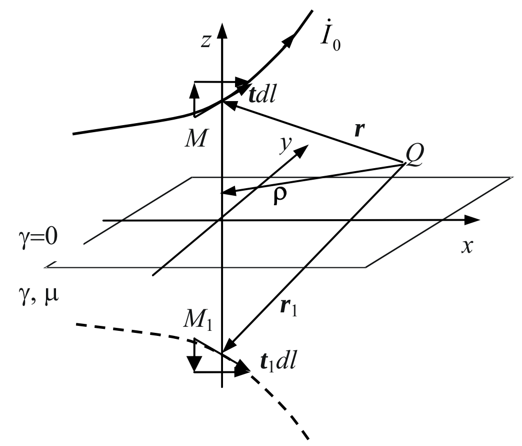

Fig. 1. Element of arbitrary current-carrying loop

The first two summands in expressions (1) - (4) for the vector potential, magnetic induction and electric field strength do not depend on the properties of the conductive medium and are determined by the alternating current of the output loop and the current of the loop mirrored from the interface. The third summands in all vector fields, as well as the scalar potential $\dot{\varphi}_e$, are completely determined by a single function $\dot{G}_e$ that depends on the depth of field penetration $\delta = \sqrt{2/(\omega\mu_0\mu\gamma)}$

$$\dot{G}_e = 2\int_0^\infty \frac{e^{-\vartheta(z - z_{M1})} J_0(\vartheta\rho_{QM})}{\vartheta + \frac{1}{\mu}\sqrt{\vartheta^2 + i\omega\mu\mu_0\gamma}} d\vartheta = = 2\int_0^\infty \frac{e^{-\vartheta(z - z_{M1})} J_0(\vartheta\rho_{QM})}{\vartheta + \sqrt{\left(\frac{\vartheta}{\mu}\right)^2 + i\left(\frac{\sqrt{2}}{\mu\delta}\right)^2}} d\vartheta, \qquad (5)$$

From (5) it is seen that the numerator of the subintegral function $\dot{G}_e$ depends on the components of the vector $\mathbf{r}_1 = \mathbf{r}_{M1} - \mathbf{r}_Q$ connecting the observation point Q with the point M_1 on the mirrored loop. The denominator of the function $\dot{G}_e$ includes separately the product of the depth of field penetration by the relative magnetic permeability $\delta\mu$, and the relative magnetic permeability μ.

Note that the presented expressions written for the case when an external electromagnetic field is created by a single current loop, using the principle of superposition, are easily extended to the case of an arbitrary system of loops, i.e an arbitrary external field. The result is also generalized for the case of an arbitrary time dependence of current using the Fourier transform.

The presented expressions allow us to determine a three-dimensional electromagnetic field at any point in the region where the external field sources are located.

2.2. Expanding the function $\dot{G}_e$ as an asymptotic series

Since a strong skin-effect is being considered, a small parameter that follows directly from (5) is a value that is defined as the ratio of two dimensional values: $\delta\mu$ and distance r_1:

$$\varepsilon_1 = \frac{\mu\delta}{\sqrt{2}r_1}. \quad (6)$$

For a nonmagnetic medium ($\mu = 1$), this parameter coincides with the ratio of the characteristic value of the depth of field penetration to the distance r_1. For ferromagnetic materials, the value of the parameter ε_1 may be much larger. But in this case we will assume that the entered parameter remains small, for example, for higher frequencies than for nonmagnetic media.

To use an approximate method for calculating an electromagnetic field in a dielectric half-space, it suffices to find an approximate representation of the function $\dot{G}_e$ that determines the potentials and vectors of the field. For the analysis to be performed, we introduce a dimensionless variable

$$\chi = \frac{\mu}{\sqrt{\omega\mu\mu_0\gamma}}\vartheta = \frac{\mu\delta}{\sqrt{2}}\vartheta \quad (7)$$

and represent an expression for $\dot{G}_e$ as follows:

$$\dot{G}_e = \frac{2}{\sqrt{i}}\int_0^\infty \frac{\exp\left(-\frac{\cos\beta_1}{\varepsilon_1}\chi\right)J_0\left(\frac{\sin\beta_1}{\varepsilon_1}\chi\right)}{w_1(\chi)}d\chi. \quad (8)$$

Here, the dimensionless function $w_1(\chi)$ in the denominator of the subintegral is

$$w_1(\chi) = \frac{\chi}{\sqrt{i}} + \sqrt{1 + \left(\frac{\chi}{\mu\sqrt{i}}\right)^2}. \quad (9)$$

The geometric representation of the quantity β_1 included in the factors of the numerator in (8) is explained in Fig. 2. This is an angle between the vertical axis and the direction of the vector from the source point M_1 to the observation point Q.

Peculiarities of improper integral (8) and its subintegral function, which, based on Laplace's approach to estimation of functions of this kind, allow us to substantiate the use of asymptotic expansion for the function $\dot{G}_e$ [13].

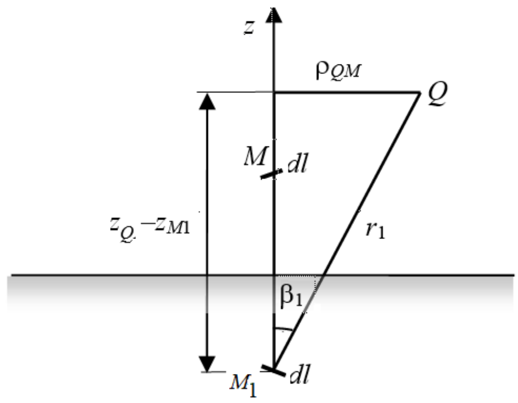

Fig. 2. Mutual location of source and observation points

The variable χ with respect to which the integration is performed varies in the range from 0 to ∞, and the multiplier $1/w_1(\chi)$ can be expanded into a power series with respect to χ within the convergence radius $\chi \le \chi_c$, which, depending on the value μ, is within $1 \le \chi_c \le \sqrt{2}$.

$$\frac{1}{w_1} = \sum_{n=0}^{\infty} a_n(\mu)\left(\frac{\chi}{\sqrt{i}}\right)^n. \quad (10)$$

At $\chi/\varepsilon_1 \ge 1$, the numerator of the subintegral function in (8) decreases rapidly, and at large values of χ, it changes faster than any power function. The magnitude of the improper integral (8) for small ε_1 is determined mainly by the behavior of the subintegral function near the origin of coordinates. The integral of the product of power and exponential functions exists even outside the convergence domain

$$\dot{G}_e^{(n)} = = \frac{2}{\sqrt{i}}\int_0^\infty a_n(\mu)\left(\frac{\chi}{\sqrt{i}}\right)^n \exp\left(-\frac{\cos\beta_1}{\varepsilon_1}\chi\right)J_0\left(\frac{\sin\beta_1}{\varepsilon_1}\chi\right)d\chi = \quad (11)$$
$$= 2(-1)^n a_n(\mu)\left(\frac{\varepsilon_1}{\sqrt{i}}\right)^{n+1} r_1^{n+1}\frac{\partial^{(n)}}{\partial z^n}\left(\frac{1}{r_1}\right).$$

However, a series composed of integrals from the members of subintegral function expansion is divergent for any ε_1 [13]. To use the series, it is necessary to restrict it to a fixed number of members N. In this case, $\dot{G}_e$ is replaced by the function $\dot{G}_{eN}$ with an error that decreases with decreasing ε_1 [14].

$$\dot{G}_e \approx \dot{G}_{eN} = \sum_{n=0}^{N}\dot{G}_e^{(n)} = = \sum_{n=0}^{N} 2(-1)^n a_n(\mu)\left(\frac{\varepsilon_1}{\sqrt{i}}\right)^{n+1} r_1^{n+1}\frac{\partial^{(n)}}{\partial z^n}\left(\frac{1}{r_1}\right), \quad (12)$$

where the multiplier, which contains a small parameter is associated with a constant propagation $p=\sqrt{i\omega\mu\mu_0\gamma}$ by the ratio $\frac{\varepsilon_1 r_1}{\sqrt{i}}=\frac{\mu}{p}$

3. Characteristics of asymptotic approximation

3.1 Estimation of an asymptotic series expansion error

Let us represent the function $\dot{G}_e$ by the sum of the finite number of its first N terms $\dot{G}_e^{(n)}$ and the remainder R_N

$$\dot{G}_e=\sum_{n=0}^{N}\dot{G}_e^{(n)}+R_N(\mu,\varepsilon_1)=\dot{G}_{eN}+R_N(\mu,\varepsilon_1). \quad (13)$$

The remainder R_N depends on both the number N and the quantities μ,ε_1. Therefore, when studying the approximate method of calculation, it is necessary to determine not only the influence of the value of a small parameter, but also the number of members of the asymptotic series.

Asymptotic series (12) is divergent with the feature generally characteristic of asymptotic series, that when the number of series members increases, the error first decreases, reaching a minimum, and then the addition of new members only increases it. This feature is clearly illustrated in [15] and shown in Fig. 3.

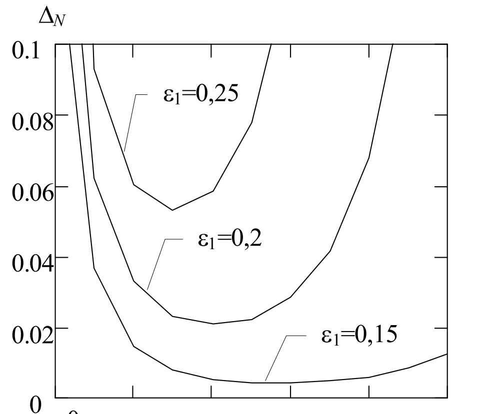

Fig. 3. Relative error $\Delta_N=|R_N|/|\dot{G}_e|$ depending on the number of members in the series N ($\mu=5;\ \rho=0$).

With ε_1 decreasing (for example, with increasing a field frequency or for materials with higher electrical conductivity, or at points most distanced from the interface), the minimum error decreases and is achieved when the number of the asymptotic series members increases. This feature determines the usefulness of asymptotic series.

Bounded power series, to which series (12) belongs, are called asymptotic series of the Poincare type [14]. For them, the error does not exceed the first rejected member and, therefore, quickly approaches zero at $\varepsilon\to 0$. However, each member of an asymptotic series is determined with an error, whose value depends on the value of the small parameter and the number of the series member. Thus, the estimate of the expansion accuracy in the magnitude of the first rejected member is approximate and this also requires an estimate of the error of each member of the series.

In [15], it is proposed to estimate the error of each member of the series, taking into account the distance between the source point on the mirrored loop and the observation point at $\rho=0$. In this case, there is only the maximum value of the error and at the same time greatly simplifies the expressions for its estimation. The relative value of the error Δ_n for a member of a series with a number n is determined by the expression

$$\Delta_n=\frac{\left|\dot{G}_e^{(n)}-\dot{G}_e^{(n)}\right|}{\left|\dot{G}_e^{(n)}\right|}=\frac{e^{-1/\varepsilon_1}\sum\limits_{k=0}^{n}\frac{1}{k!\varepsilon_1^k}}{1-e^{-1/\varepsilon_1}\sum\limits_{k=0}^{n}\frac{1}{k!\varepsilon_1^k}}. \quad (13)$$

The relative error of calculation of each expansion member at $n\to\infty$ tends to infinity. As the number n of series members increases, so does the error of its determination, although its relative contribution to the total sum decreases. The value of the relative error for the first ten members of the series at different values of the parameter ε_1 is shown in Fig. 4.

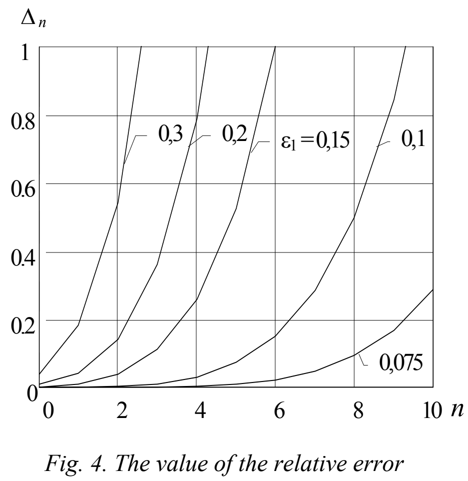

Fig. 4. The value of the relative error of asymptotic series members.

It is seen that with the number n of the series members increasing, there is also an increase in the error of its determination, although at the same time there is a decrease in its relative contribution to the total sum.

3.2. Choosing the number of members of an asymptotic series

When using the asymptotic method, an important practical task is the optimal choice of the number of members of an asymptotic series depending on the value of the small parameter ε_1. The choice can be based on the analysis of the value of the remainder $R_N(\mu, \varepsilon_1)$. Two approaches can be used here: the first is based on the estimation of the last considered member of the series, the second - directly on the estimation of the remainder R_N at a given allowable error.

Choosing the number of members to estimate the error of the last member of a series. The possibility of applying the first method is due to the fact that $R_N(\mu, \varepsilon_1)$ in (12) does not exceed the last rejected member of the series, which, however, itself is determined with a certain error [9]. In this case, if a certain series member is calculated with an error exceeding a certain allowable value, for example, if the error becomes comparable to the value of the expansion member, then taking into account such a member of the series does not increase the accuracy of calculation. Moreover, if the relative error in determining the values of series members increases with increasing their number, then the assessment of the achievement, by any member, of the relative error limit will determine the limit value of the number of series members, beyond which increasing their number will only increase the total error.

The number of members of a series should be limited to the value of N at which their further increase leads only to an increase in the total error. It follows that the number of members of a series should not exceed the value at which the relative error of the last member of the series does not exceed a given value C_N, for example, equal to one. In this case, the condition for determining the number of series members $n = N$ can be written in the form

$$\Delta_n(\varepsilon_1) \le C_N . \tag{14}$$

Having compared the data presented in Fig. 4, with the dependences in Fig. 3, we can see that the minimum error of the approximate calculation is achieved when the number of members of the series meets the condition $C_N \approx 1$. This confirms the assumption that the estimate of the optimal number of members of the series can be performed on the basis of the estimate of the error of the last considered member.

The limit number of members of the series found from (13) for different allowable values of the relative error Δ_n for the last considered member of the series at $n = N$ is shown in Fig. 5.

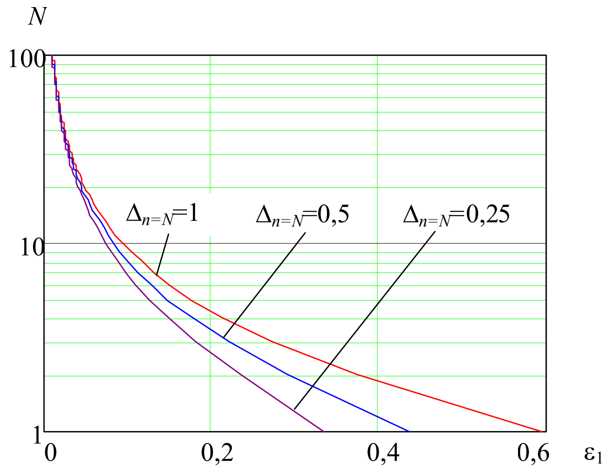

Fig. 5. The limit number of members of an asymptotic series.

As we can see from Fig. 5 when the value of the small parameter ε_1 decreases (at higher frequencies or for materials with higher electrical conductivity), there is a rapid increase in the possible number of members of the asymptotic series. However, the value of the considered members of the series at large values n is very small. This fact is illustrated by the data in Fig.6, which shows the value of the limit members of the series, which are obtained by fulfilling condition (14).

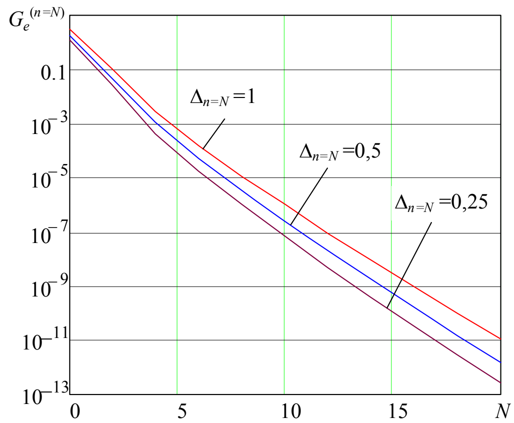

Fig. 6. The limit of the boundary members of the asymptotic series selected under condition (14).

As Fig. 6 shows, the calculation accuracy can significantly exceed the required or even reasonable level. Calculations without further limitation of the number of series members are unnecessarily complicated.

Choosing the number of members for a given error of an asymptotic series. The approach is to estimate the error of the whole asymptotic series $\Delta_N(N, \varepsilon_1)$.

Assume that it is sufficient that the calculation be performed with an accuracy at which the error does not necessarily have to be less than the specified $\Delta_{\min}$. Furthermore, the approximate method of calculation, whose accuracy depends on the value of the small parameter ε_1 being used, we then define the error limit $\Delta_{\max} > \Delta_{\min}$, the excess of which indicates the inadmissibility of using this method.

Consider, as an example, a nonmagnetic medium $\mu = 1$ for which we develop a series of dependences $\Delta_N(N, \varepsilon_1) = |R_N| / |\dot{G}_e|$ as functions of the number of considered series members for different values of a small parameter.. At the same time we choose specific error limits, for example, $\Delta_{\min} = 10^{-3}$ i $\Delta_{\max} = 10^{-1}$. The dependencies $\Delta_N(N, \varepsilon_1)$ are shown in Fig. 7.

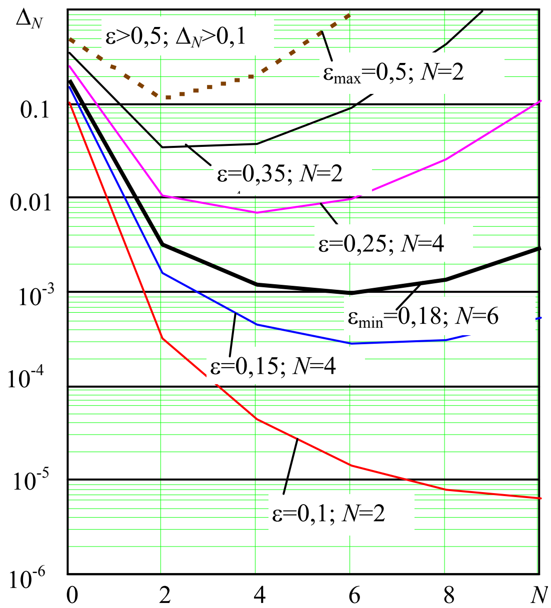

Fig. 7. Choosing the number of series members for $\Delta_{\min} = 10^{-3}$, $\Delta_{\max} = 10^{-1}$

The Figure highlights two curves, among others. One curve, which is shown by a solid bold line at $\varepsilon_{\min} = 0{,}18$, corresponds to the dependence for which the minimum value of the error is equal to the limit value $\Delta_N(N, \varepsilon_{\min}) = \Delta_{\min}$ and which is realized for the number of series members $N = 6$. At lower values of the small parameter $\varepsilon_1 < \varepsilon_{\min}$, the error limit value $\Delta_{\min}$ will be implemented when the number of members of the series does not exceed the set value, i.e. $N \le 6$. The Figure shows the dependencies when we can be limited to a smaller number of series members: $\varepsilon_1 = 0{,}15 \;\; N = 4; \;\; \varepsilon_1 = 0{,}1 \;\; N = 2$.

The other curve in Fig. 7, which is shown by a bold dotted line at $\varepsilon_{\max} = 0{,}5$, corresponds to the dependence for which the minimum value of the error is equal to the limit value $\Delta_N(N, \varepsilon_{\max}) = \Delta_{\max}$ and which is realized for the number of series members $N = 2$. With the values of a small parameter $\varepsilon_1 > \varepsilon_{\max}$, for any number of series members, the calculation error exceeds $\Delta_{\max}$. This proves that it must not be to use the approximate method for such values of a small parameter at the chosen limit value of an error $\Delta_{\max}$.

In the intermediate range of values of the small parameter $\varepsilon_{\min} < \varepsilon_1 < \varepsilon_{\max}$, the minimum achievable calculation errors are already within $\Delta_{\min} < \Delta_N < \Delta_{\max}$ In this case, these minimum errors are realized for the number of members of the series, which also do not exceed the maximum number $N = 6$ corresponding to the value of the number of members of the series at $\varepsilon_1 = \varepsilon_{\min}$.

The main result of this analysis is that when using the method of asymptotic expansion, the number of members of a series can be limited to a relatively small number, which is determined by the allowable accuracy of the calculation (relative error).

The number of expansion members is determined by the value of a small parameter $\varepsilon_1 = \dfrac{\mu}{h\sqrt{2\pi f \mu\mu_0\gamma}}$, which depends on the electrophysical parameters of the conductive medium μ, γ, the field frequency f and the minimum distance of the current-carrying loop to the interface *h*. For a specific electrically conductive material, the obtained results allow us to indicate the required number of expansion members depending on the field frequency and the minimum distance of the current-carrying conductor from the body surface.

Consider an example where the conductive medium is aluminum with electrophysical parameters $\mu = 1, \gamma = 3{,}7 \cdot 10^7 \,\text{Ом}^{-1} \cdot \text{м}^{-1}$. Setting specific values of the small parameter ε_1, including the set $\varepsilon_{\min} = 0{,}18$ and $\varepsilon_{\max} = 0{,}5$, we find a dependence of the field frequency on the minimum height of the current-carrying conductor at $\varepsilon_1 = \text{const}$ and respectively for a specific number of series members, at which the minimum error for the given ε_1 is realized. Such dependences are shown in Fig. 8.

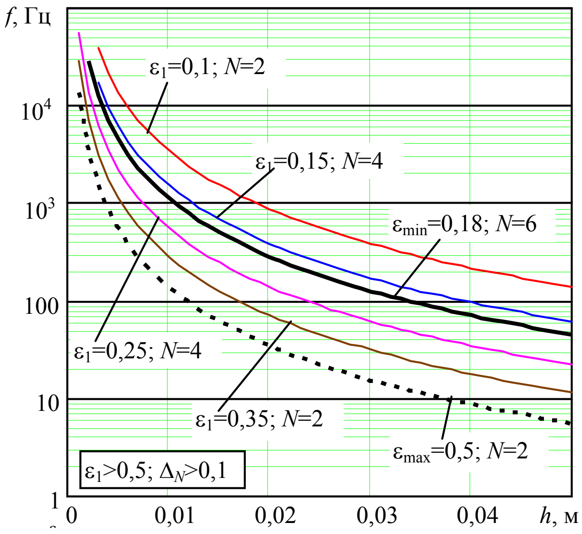

Fig. 8. The dependence of the field frequency on the minimum height of the current loop at the given number of series members for aluminum ($\Delta_{min} = 10^{-3}$, $\Delta_{max} = 10^{-1}$)

When performing practical calculations, these dependences allow us to choose the required number of members of an asymptotic series and specify an estimate of the calculation accuracy. Usually the material and field frequency are known. Further, taking account a geometrical configuration, it is necessary to define only the minimum distance from a loop to a conducting surface and given the presented dependences to find calculation parameters. In that case, if a current dependence on time differs from sinusoidal, it is possible to use a frequency spectrum of the current and for each frequency (or for characteristic frequencies) to apply the described approach.

4. Conclusion

1. With a strong skin-effect, provided that the depth of the skin layer is small compared not only to the characteristic size of the conductive body, but also the distance from the body to external field sources, an effective method of calculating three-dimensional problems of the electromagnetic field theory is the method of asymptotic expansion.

2. The error of the calculation method, which is determined by a small parameter proportional to the ratio of the depth of field penetration to the distance between the conductive body and the external field sources, also depends on the number of considered members of a series, the number of which must be limited.

3. It is established that the number of members of a series can be limited to a relatively small number, which is determined by the specified limits of the allowable accuracy of the calculation. This allows, depending on the value of a small parameter, and for a particular conductive material depending on the field frequency and the minimum distance from the circuit to the conductive body, the determination of the optimal number of members of an asymptotic series and specification of an estimate of calculation accuracy.

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ПАРАМЕТРИ ДЛЯ РОЗРАХУНКУ ТРИВИМІРНОГО ЕЛЕКТРОМАГНІТНОГО ПОЛЯ МЕТОДОМ АСИМПТОТИЧНОГО РОЗКЛАДАННЯ

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Представлено наближений аналітичний розв'язок тривимірної задачі теорії електромагнітного поля, який оснований на використанні асимптотичного розкладання за умови сильного скін-ефекту для поля, створеного замкнутим струмовим контуром, розташованим поблизу електропровідного півпростору. Зазначено, що кожен член асимптотичного ряду визначається з похибкою, величина якої залежить від значення малого параметра і зростає зі збільшенням номера члена ряду, що обумовлює обмеженість кількості його членів. Встановлено, що при використанні методу асимптотичного розкладання число членів ряду може бути обмежено відносно невеликою кількістю, яка визначається заданими межами припустимої точності розрахунку (відносною похибкою). Визначено оптимальне число членів асимптотичного ряду та вказано оцінку точності розрахунку в залежності від величини малого параметру, а для конкретного електропровідного матеріала в залежності від частоти поля і мінімальної відстані від джерел зовнішнього поля до електропровідного тіла

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