

Thermomechanical behavior of an electrically conductive cylindrical implant under the action of external unstable electromagnetic fields

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A physical and mathematical model for determining the thermostressed state of an electrically conductive cylindrical implant under the action of external unstable electromagnetic fields, in particular the impulse with amplitude modulation, is proposed. This model allows predicting the maximum allowable values (according to physiological norms) of temperature and stress intensities in the considered implant depending on the parameters of external unstable electromagnetic fields and the time of their action.

Keywords: *electrically conductive implant, cylinder, unstable electromagnetic field, temperature, stress intensities, physiological criteria.*

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1. Introduction

In the practice of surgical dentistry and orthopedics of the upper and lower extremities, implants of different configurations are often used to connect fractures and to create a rod-base to which is attached either a single tooth or a whole denture-bridge for two implants, as well as to connect fractures of the extremities [1]. The geometric shape of such implants can be modeled with a solid cylinder. Implants made of electrically conductive non-ferromagnetic materials or their alloys, which are stainless steels with appropriate alloying impurities of rare earth elements: chromium, titanium, nickel, vanadium, tungsten [2–5], are mostly used. In the course of life, a patient with an existing implant (in the jaw or extremities) may be exposed to external unstable electromagnetic fields (EMF). Such EMFs are created by appropriate medical devices (UWF devices, computer tomographs, etc.) or household or industrial devices [6]. Their effect on the cylindrical implant in question is manifested by two physical factors – Joule heat and volumetric ponderomotive forces [7]. These physical factors create in the implant a non-stationary temperature field and mechanical stresses and stress intensities. In order to ensure comfortable physiological conditions of the patient's life, it is necessary to be able to predict the thermomechanical behavior of the implant available to him.

In this paper, a physical and mathematical model for determining the thermally stressed state of an electrically conductive cylindrical implant under the action of external unstable EMFs of characteristic types is proposed [8]. This model allows to predict the determination of the maximum allowable values of temperature and stress intensity according to physiological norms depending on the parameters of external unstable EMF [9] and the time of their action.

2. Basic relations of the physical and mathematical models of the implant

To determine the thermally stressed state of the cylindrical implant in question, we will proceed from the consistently related Maxwell relations and the dynamic thermoelasticity problem. The rod implant is modeled with a continuous electrically conductive cylinder of radius R and length L . Taking into account that $R/L \ll 1$, we will consider the implant in the approach of a long cylinder. The side surface of the cylindrical implant, which is in direct contact with the bone, is considered insulated and free from external surface load.

A cylindrical implant is considered in a cylindrical coordinate system (r, φ, z) , the axis Oz of which coincides with the axis of the cylinder. The cylinder material is homogeneous, isotropic and non-ferromagnetic. Physical and mechanical characteristics of the implant material are constant in the considered temperature change range.

The influence of external unstable EMF on the implant is manifested by the appearance of Joule heat sources Q and volume-distributed non-stationary ponderomotive forces \mathbf{F} . These two physical factors create additional non-stationary temperature T and mechanical stresses described by the components σ_{rr} , $\sigma_{\varphi\varphi}$ and σ_{zz} of the dynamic stress tensor $\hat{\sigma}$ in the implant. The components σ_{jj} ($j = r, \varphi, z$) of the tensor $\hat{\sigma}$ are considered as the sum of two components $\sigma_{jj} = \sigma_{jj}^Q + \sigma_{jj}^F$. The components σ_{jj}^Q are due to the Joule heat Q , and the components σ_{jj}^F are due to the ponderomotive forces \mathbf{F} . According to the obtained total stresses σ_{jj} , we determine the stress intensities σ_i according to the formula [10]

$$\sigma_i = \sqrt{(3I_2 - I_1^2)/2}. \quad (1)$$

Here ($j = 1, 2$) is the j -th invariant of the stress tensor $\hat{\sigma}$.

The physiologically comfortable condition of the patient with the implant under the action of external unstable EMF will be maintained under certain conditions. Such conditions correspond to the fact that the maximum values of temperature T and intensities of mechanical stresses σ_i on the surfaces of the implant which are in contact with the bone will not exceed the limit physiologically acceptable values T_* and σ_{i*} . These values correspond to the experimentally established values of the limits of temperature and mechanical pain thresholds. The formulated criteria of physiologically admissible values of temperature and intensity of stresses are mathematically written down by expressions

$$T_{\max} \leq T_*; \quad \sigma_{i\max} \leq \sigma_{i*}. \quad (2)$$

Values T_{\max} and $\sigma_{i\max}$ are calculated on the side surface $r = R$ of the cylinder.

Based on the formulated physical and mathematical model, the calculation model for determining the temperature T , stresses σ_{jj} and stress intensities σ_i consists of three stages. In the first stage, based on Maxwell's relations from the equation

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} - \sigma \mu \frac{\partial H_z}{\partial t} = 0 \quad (3)$$

under the boundary condition on the side surface of the cylinder

$$H_z(R, t) = H_{z0}(t), \quad (4)$$

conditions of axisymmetry on the axis of the cylinder $r = 0$

$$\frac{\partial H_z(0, t)}{\partial r} = 0 \quad (5)$$

and zero initial condition

$$H_z(r, 0) = 0 \quad (6)$$

we determine the axial component $H_z(r, t)$ of the magnetic field vector $\mathbf{H} = \{0; 0; H_z(r, t)\}$. Here $H_{z0}(t)$ is experimentally established law of change in time t of an axial component H_z on a surface $r = R$ of the cylinder; σ is the electrical conductivity coefficient, μ is the magnetic permeability of the cylinder material.

In the second stage, taking into account the known Joule heat $Q(r, t)$ from the heat conduction equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{1}{\kappa} \frac{\partial T}{\partial t} = -\frac{1}{\lambda} Q \quad (7)$$

subject to heat insulation of the surface $r = R$

$$\frac{\partial T(R, t)}{\partial r} = 0, \quad (8)$$

conditions of axisymmetry of temperature on the axis of the cylinder $r = 0$

$$\frac{\partial T(0, t)}{\partial r} = 0 \quad (9)$$

and zero initial condition

$$T(r, 0) = 0 \quad (10)$$

we write the temperature expression $T(r, t)$. Here, κ , λ are the coefficients of thermal diffusivity and heat conductivity of the implant material.

In the third stage, according to the obtained temperature $T(r, t)$ and the component $F_r(r, t)$ of the ponderomotive force, we determine the radial component $u_r(r, t)$ of the displacement vector $\mathbf{u} = \{u_r; 0; 0\}$ from the equation

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} - \frac{1}{c_1^2} \frac{\partial^2 u_r}{\partial t^2} = \alpha \frac{1 + \nu}{1 - \nu} \frac{\partial T}{\partial r} - \frac{(1 + \nu)(1 - 2\nu)}{E(1 - \nu)} F_r \quad (11)$$

under boundary conditions

$$(1 - \nu) \frac{\partial u_r}{\partial r} + \nu \frac{u_r}{r} - \alpha(1 + \nu)T = 0 \quad (12)$$

on the side surface $r = R$ of the implant, the axisymmetry conditions

$$\frac{\partial u_r(0, t)}{\partial r} = 0 \quad (13)$$

on the axis of the cylinder and the zero initial condition

$$u_r(r, 0) = 0. \quad (14)$$

Here $c_1 = (E(1 - \nu)) / (\rho(1 + \nu)(1 - 2\nu))^{-1/2}$ is the speed of the elastic wave of expansion of the cylindrical implant during its isothermal deformation; α , ν coefficients of linear heat expansion and Poisson, E is the Young's modulus, ρ is the density of implant material.

According to the found total radial displacements $u_r(r, t) = u_r^Q(r, t) + u_r^F(r, t)$ from the relations

$$\sigma_{rr} = \frac{E}{(1 + \nu)(1 - 2\nu)} \left[(1 - \nu) \frac{\partial u_r}{\partial r} + \nu \frac{u_r}{r} - \alpha(1 + \nu)T \right], \quad (15)$$

$$\sigma_{\varphi\varphi} = \frac{E}{(1 + \nu)(1 - 2\nu)} \left[(1 - \nu) \frac{u_r}{r} + \nu \frac{\partial u_r}{\partial r} - \alpha(1 + \nu)T \right], \quad (16)$$

$$\sigma_{zz} = \nu_n (\sigma_{rr} + \sigma_{\varphi\varphi}) - \alpha ET \quad (17)$$

we determine the total stresses σ_{jj} ($j = r, \varphi, z$) and the corresponding stress intensities σ_i by formula (1).

Based on the obtained expressions of temperature T and stress intensity σ_i and their values on the cylinder surface $r = R$, we analyze according to the accepted criteria (2) the maximum allowable values of the parameters of external unstable EMF, which provide physiologically comfortable conditions for the patient.

3. Constructions of solutions of initial-boundary value problems under the action of homogeneous nonstationary EMF

To construct the solutions of the initial-boundary value problems of electrodynamics (3)–(6), heat conductivity (7)–(10) and thermoelasticity (11)–(14) formulated above, we apply the approximation of all key functions $\Phi(r, t) = \{H_z(r, t), T(r, t), u_r(r, t)\}$ in radial variable r by cubic polynomials [11]:

$$\Phi(r, t) = \sum_{i=0}^3 a_i^\Phi(t) r^i. \quad (18)$$

The coefficients a_i^Φ of the approximation polynomials (18) are given as a linear combination

$$a_i^\Phi(t) = a_{i1}^\Phi \Phi_1(t) + a_{i2}^\Phi \Phi_2(t) + a_{i3}^\Phi \Phi^+(t) \quad (19)$$

of the limit value $\Phi^+(t)$ of the desired functions on the surface $r = R$ of the implant and the integral (with respect to the radial variable) characteristics

$$\Phi_s(t) = \frac{s+1}{R^{s+1}} \int_0^R \Phi(r, t) r^s dr, \quad s = 1, 2 \quad (20)$$

of the key functions $\Phi(r, t)$.

The equation for integral characteristics $\Phi_s(t)$ is obtained by integrating the original equations (3), (7), (11) according to formula (20) and using expressions (18), (19) in analytical transformations.

As a result, we obtain the following representations of the key functions in a continuous cylinder in terms of their integral characteristics and surface values (as they differ from zero):

- for the axial component $H_z(r, t)$ of the vector \mathbf{H}

$$H_z(r, t) = H_{z1}(t) (20 - 100r_*^2 + 80r_*^3) + H_{z2}(t) (-30 + 180r_*^2 - 150r_*^3) + H_z^+(t) (1 - 10r_*^2 + 10r_*^3); \quad (21)$$

- for temperature $T(r, t)$ under the condition of heat insulation of the cylinder surface $r = R$

$$T(r, t) = T_1(t) (-19 + 90r_*^2 - 60r_*^3) + T_2(t) (14 - 60r_*^2 + 40r_*^3); \quad (22)$$

- for the radial component $u_r(r, t)$ of the displacement vector u

$$u_r(r, t) = u_{r1}(t) (20 - 100r_*^2 + 80r_*^3) + u_{r2}(t) (-30 + 180r_*^2 - 150r_*^3) + \alpha_1 r_* T(r, t) + \alpha_2 r_* F_r(r, t). \quad (23)$$

Here $r_* = r/R$ is the dimensionless radial coordinate, $\alpha_1 r_*$, $\alpha_2 r_*$ are the corresponding cubic polynomials with respect to the variable r_* . Based on the obtained expressions of the axial component (21)–(23), we write the solution of the considered problem under the action of a homogeneous nonstationary EMF.

4. Computer analysis of thermomechanical behavior of the implant under the action of unstable EMF

The action of an external unstable EMF is given by the value of the axial component H_z of the vector \mathbf{H} on the surface $r = R$ of the cylinder as a function [8]

$$H_z^+(t) = k H_0 (e^{-b_1 t} - e^{-\beta_2 t}) \cos \omega t. \quad (24)$$

Here k is the normalization factor; H_0 is the amplitude of electromagnetic oscillations with circular frequency ω ; β_1 and β_2 are parameters characterizing the times t_{incr} of increase of the amplitude of electromagnetic oscillations of the frequency ω from zero to its maximum value H_0 and t_{decr} is the decrease of this amplitude from the value H_0 to zero. Time of action of unstable EMF is $t_i = t_{incr} + t_{decr}$. Unsteady EMF, which is mathematically described by expression (24), has the character of impulsed EMF with amplitude modulation [8, 9, 12–17].

Substituting the expression (24) of the function $H_z^+(t)$ in the relation (3)–(23), we obtain the expressions of Joule heat Q , radial component F_r of the vector of ponderomotive force \mathbf{F} and temperature T , radial displacements u_r and corresponding stresses σ_{jj} ($j = r, \varphi, z$) under the action of unstable EMF.

For numerical analysis, stainless steel (with doped impurities of chromium, nickel and titanium) was chosen for the implant material. Electrophysical and thermophysical characteristics of steel were selected according to the data given in the literature [8, 11]. For calculations, it was assumed that the radius of the cylinder is $R = 1$ mm, and the duration of the electromagnetic action are $t_i = 10$ s, 100 s, 300 s, 600 s. Accordingly, the value H_0 is chosen equal to $H_0 = 1$ A/m, 10 A/m, 10^2 A/m, 10^3 A/m. The circular frequency is equal to the value $\omega = 6.3 \cdot 10^5$ rad/s belonging to the radio frequency range. This value is outside the resonant EMF frequencies of the cylindrical implant under consideration.

The results of calculations of changes in time t of Joule heat Q , radial component F_r of ponderomotive force, temperature T and stress intensities σ_i on surfaces $r = R$; $0.5R$; $0.25R$ (lines 1–3) for the duration $t_i = 10$ s of action of unstable EMF are shown in Figs. 1–4.

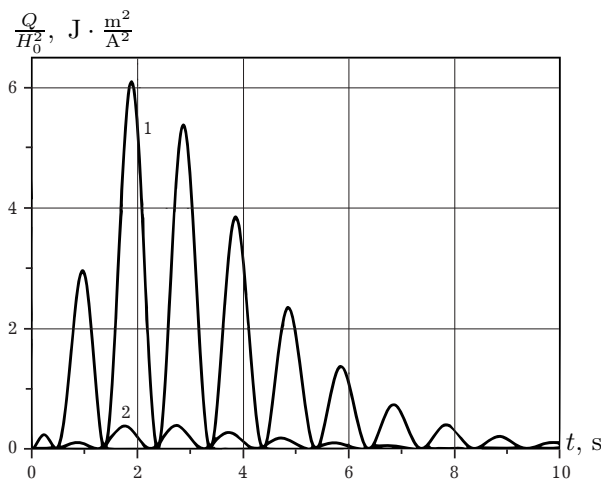


Fig. 1. Change in time of Joule heat generation in a solid cylinder.

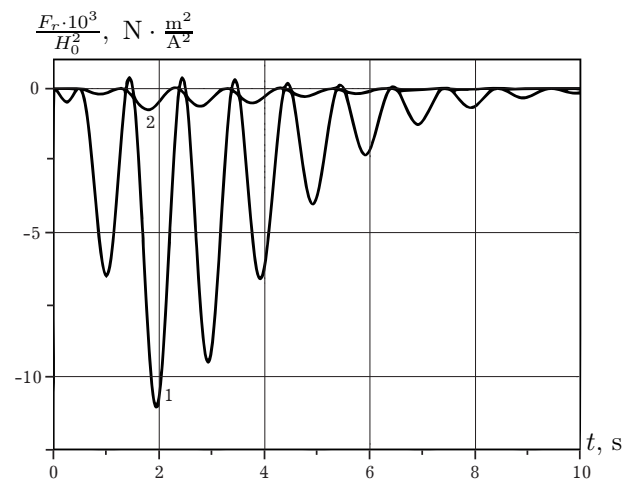


Fig. 2. Change in time of the ponderomotive force in a continuous cylinder.

The change in time of Joule heat Q for each specified value of the radial coordinate has the character of a sequence of heat impulses during the considered electromagnetic action. In each of these heat impulses, the maximum value of the amount of heat released in the cylinder corresponds to the nature of the change in time of function (24), which describes the unstable nature of the external EMF. There is a pronounced heating of the cylinder, which is close to the surface.

The change in time of the radial component of the ponderomotive force F_r during the considered electromagnetic action has the character of a sequence of force impulses. The maximum values of the ponderomotive force in the cylinder similarly correspond to the nature of the change in time of the function (24).

The nature of the change in time of the temperature corresponds to its increase in accordance with the nature of the change in time of the heat impulses caused by the Joule heat.

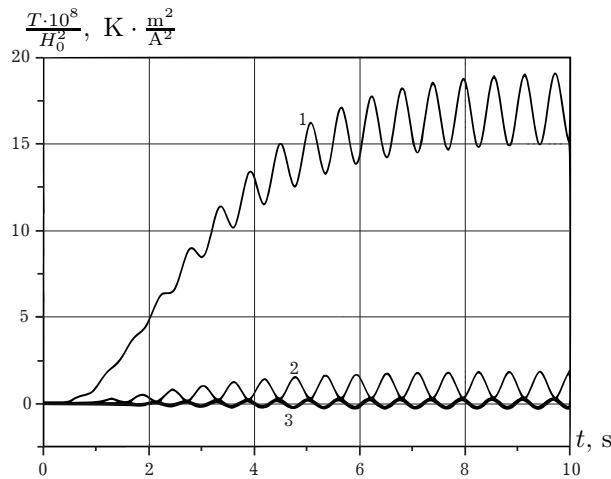


Fig. 3. Change in time of the total temperature in a solid cylinder.

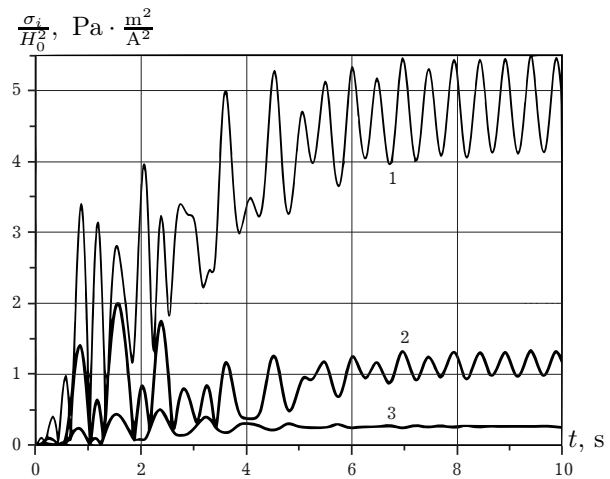


Fig. 4. Change in time of intensity of stresses σ_i in the continuous cylinder.

Note that the temperature T in Fig. 3 corresponds to the total temperature due to both the Joule heat Q and the process of energy dissipation caused by ponderomotive force F_r and taken into account according to the method described in [8, 11, 12].

Stress intensities on all considered cylindrical surfaces have an oscillating nature of change in time, which is due to the nature of change in time of the total temperature and ponderomotive force. Stress intensities reach maximum values on the implant surface and decrease sharply when approaching the axis $r = 0$. The maximum values of all considered physical quantities are reached on a surface $r = R$ of the cylinder. The maximum values of Joule heat Q and ponderomotive force F_r are reached in time $t \approx 0.1t_i$, and temperature T and stress intensity σ_i are reached in time $t \approx 0.5t_i$.

Based on the above given dependences of change in time t and in radial coordinate r for temperature T (Fig. 3) and stress intensity σ_i (Fig. 4) it is possible to conduct a qualitative and quantitative analysis of these physical quantities depending on the amplitude-frequency characteristics of the considered unstable EMF and the duration t_i of its action.

The results of the numerical analysis are presented in Table 1.

Based on the data in Table 1, it is possible to predict the thermomechanical behavior of a cylindrical electrically conductive implant under the conditions of the patient staying with such an implant in external unstable EMFs having the character of impuled EMFs with amplitude modulation.

5. Conclusion

A physico-mathematical model for determining the thermally stressed state of an electrically conductive cylindrical implant under the action of external unstable EMFs is proposed. Criteria for physiologically comfortable condition of a patient with an implant are formulated. These criteria take into account the known experimentally established limits of pain thermal and mechanical thresholds. On the basis of these criteria, it is possible to determine the maximum allowable values of amplitude-frequency

Table 1.

t_i, s	$H_0, A/m$	T, K	σ_i, Pa
10	1	$1.7 \cdot 10^{-7}$	5
	10	$1.7 \cdot 10^{-5}$	$5 \cdot 10^2$
	10^2	$1.7 \cdot 10^{-3}$	$5 \cdot 10^4$
	10^3	0.17	$5 \cdot 10^6$
100	1	$1.7 \cdot 10^{-6}$	50
	10	$1.7 \cdot 10^{-4}$	$5 \cdot 10^3$
	10^2	$1.7 \cdot 10^{-2}$	$5 \cdot 10^5$
	10^3	1.7	$5 \cdot 10^7$
300	1	$5.1 \cdot 10^{-6}$	150
	10	$5.1 \cdot 10^{-4}$	$1.5 \cdot 10^4$
	10^2	$5.1 \cdot 10^{-2}$	$1.5 \cdot 10^6$
	10^3	5.1	$1.5 \cdot 10^8$
600	1	$1.02 \cdot 10^{-5}$	300
	10	$1.02 \cdot 10^{-3}$	$3 \cdot 10^4$
	10^2	$1.02 \cdot 10^{-1}$	$3 \cdot 10^6$
	10^3	10.2	$3 \cdot 10^8$

characteristics and duration of action of external unstable EMFs for safe stay in them of the person with the considered implant.

A method for constructing solutions of initial-boundary value problems to find key functions — the axial component of the magnetic field vector, temperature, radial displacements and stress intensities in a cylindrical implant due to the action of external unstable EMFs — is proposed. This technique allows us to obtain expressions of key functions in the form convenient for computer analysis.

The maximum values of temperature, ponderomotive force and stress intensity in a cylindrical electrically conductive implant depending on the amplitude-frequency characteristics and duration of action of the unstable EMF are numerically analyzed. On the basis of the formulated criteria and the carried-out numerical researches, it is possible to analyze maximum admissible values of amplitude-frequency characteristics and duration of action of unstable EMF at observance of which the patient with the available implant will be provided with physiological comfort.

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Термомеханічна поведінка електропровідного циліндричного імпланта за дії зовнішніх неусталених електромагнітних полів

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Запропоновано фізико-математичну модель визначення термонапружного стану електропровідного циліндричного імпланта за дії зовнішніх неусталених електромагнітних полів, зокрема імпульсних із модуляцією амплітуди. Дана модель дозволяє прогнозувати гранично допустимі, згідно з фізіологічними нормами, значення температури та інтенсивності напружень у розглядуваному імпланті залежно від параметрів зовнішніх неусталених електромагнітних полів та часу їх дії.

Ключові слова: *електропровідний імплант, циліндр, неусталене електромагнітне поле, температура, інтенсивності напружень, фізіологічні критерії.*