MATHEMATICAL MODELING AND COMPUTING, Vol. 8, No. 3, pp. 515–525 (2021)

Modeling throttle bridge measuring transducers of physical-mechanical parameters of Newtonian fluids

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(Received 9 June 2021; Revised 28 August 2021; Accepted 30 August 2021)

The paper proposes a measuring transducer of the physical-mechanical parameters of a Newtonian fluid based on a throttle bridge measuring diagram with identical turbulent and laminar throttles in opposite arms. A mathematical model is built for the throttle bridge transducer of the combined parameter, which depends on the kinematic viscosity and density of the fluid. The problem of parametric optimization of the proposed measuring transducer is formulated and analytically solved in the paper. The authors calculated the transform function of the measuring transducer of the combined parameter of jet fuel.

Keywords: *kinematic viscosity, density, Newtonian fluid, model, throttle, measuring transducer.*

2010 MSC: 65C20, 76A02, 94A05 **DOI:** 10.23939/mmc2021.03.515

1. Introduction

Newtonian fluids' physical-mechanical parameters often characterize their fractional and chemical composition and quality indicators, both at the production and finished products. The quality of many petroleum products, such as reactive, motor, diesel fuels, oil fuel, is normalized by the values of kinematic viscosity and density. Currently, the quality of liquid fuels is usually analyzed by using laboratory methods. However, continuous determining of viscosity and density of liquids in their manufacturing and operation conditions is a complex task, which still requires the development of highly sensitive and accurate measuring tools [1].

Promising for solving the problems of measuring the properties of fluids is the hydro-gas-dynamic throttle method, based on which many measuring transducers are built [2–11]. The main components of hydro-gas-dynamic measuring transducers are throttle elements. Depending on the fluids' behavior and design characteristics, they can be laminar or turbulent. The transform functions and characteristics of hydro-gas-dynamic measuring transducers depend on the type of diagram, type of throttle elements and their connection order in the diagram, and the operation mode parameters of the gauge [2]. Thus, transducers built on the laminar throttle make it possible to measure the dynamic viscosity of Newtonian fluids [3], the characteristic viscosity of polymer solutions [4,5], small and micro-flowrates of gases and liquids [6,7], plastic viscosity and yield stress of viscous-plastic fluids, consistency coefficient, and flow behavior index of pseudo-plastic fluids, etc. [8]. Turbulent throttle elements (orifice plates, nozzles) are widely used in flowmeters of medium and large flowrates of liquids and gases [9]. Combining laminar and turbulent throttles in measuring transducers makes it possible to measure fluids' kinematic viscosity, density, etc. [10, 11].

While synthesizing the hydrodynamic measuring transducers, the use of the structural-parametric method is effective. According to this method, for the specified measuring purposes, first of all, one chooses the optimal measurement diagram and then selects and calculates throttle elements with such design parameters, at which the specified characteristic of the transducer reaches an extremum [12].

This study aims to build a mathematical model of the hydrodynamic transducer of the combined parameter, which depends on the kinematic viscosity and density of the Newtonian fluid; solve analytically the problem of parametric optimization of the bridge measuring transducer; calculate the throttles' design characteristics of the measuring transducer, and estimate the uncertainty of the measuring result of the output of the measuring transducer of liquid fuel's properties.

2. Mathematical model of throttle bridge measuring transducer of the combined parameter of a Newtonian fluid

Mathematical models of throttle transducers of physical-mechanical parameters of liquids are built on mathematical models of throttle elements used in the measuring diagram. The mathematical model of each throttle is determined by its type, design, and working conditions in the diagram. Such a model is a flowrate characteristic of the throttle: the dependence between flowrate and pressure drop on the throttle element while the fluid is flowing [13]. Usually, laminar and turbulent throttle elements are used in hydrodynamic transducers.

The flowrate characteristic of a sufficiently long laminar throttle is described by Poiseuille's law [14]

$$Q = \frac{\pi R_L^4 \Delta P}{8\nu L} = \frac{\pi R_L^4 \rho \Delta P}{8\mu L}, \tag{1}$$

the flowrate characteristic of a short laminar throttle is determined as [13]

$$Q = \frac{4\pi L\mu\rho}{m}\left[\left(1 + \frac{m R_L^4 \Delta P}{16 L^2 \nu^2 \rho}\right)^{0.5} - 1\right]^{0.5}. \tag{2}$$

The flowrate characteristic of a turbulent throttle is described by the formula [13]

$$Q = \pi \alpha R_T^2 \left(2\Delta P \rho\right)^{0.5}, \tag{3}$$

where Q is mass flowrate of a liquid; ΔP is the pressure drop on a throttle; μ, ν are dynamic and kinematic viscosity of a liquid; ρ is liquid density; R_L and L are radius and length of the capillary tube; $m = (\alpha_L + \xi_L)/2$ is empirical dimensionless coefficient; α_L is Coriolis's coefficient; ξ_L is local resistance factor, which depends on the shape of the inlet, the way of fixing the capillary in the inlet chamber and the sharpness of the capillary inlet edge; R_T is the radius of the passing hole of the turbulent throttle; α the discharge coefficient, which depends on the design of the turbulent throttle and Reynolds number.

Bridge diagram of elements connection is widespread in measuring technology. Hydrodynamic transducers based on the bridge diagram allow one to create devices invariant to specific parameters of the fluid, or vice versa — with high sensitivity to other parameters of the fluid [4,5,12].

Considering the advantages of bridge measuring diagrams, we will use it to synthesize the transducer of the combined physical-mechanical parameter of Newton liquids in the form of a product of a square of kinematic viscosity by density $B_P = \nu^2 \rho$.

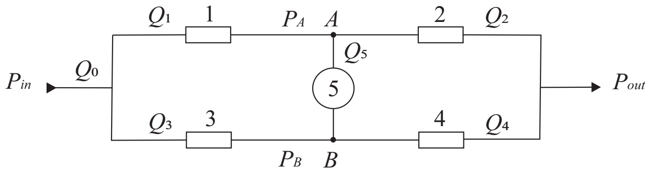

Fig. 1. Bridge throttle measuring diagram: throttles (sensitive elements) 1, 2, 3, 4; secondary transducer (measuring device) 5.

Hydrodynamic transducers, built on the bridge diagram, contain throttles that operate on the same fluid, have a supply diagonal, and the output diagonal $A - B$. We can see from Fig. 1, the bridge throttle measuring diagrams of the hydrodynamic measuring transducers contain four throttle elements 1, 2, 3, and 4 in the basic version.

There is a secondary transducer 5 in the output diagonal $A - B$, the hydraulic resistance of which is determined by the formula [15]

$$r = \frac{(P_A - P_B)}{Q_5 \rho}, \tag{4}$$

where Q_5 is mass flowrate of liquid in the diagonal $A{-}B$ of the bridge diagram; P_A is the liquid pressure between the throttles 1 and 2; P_B is the liquid pressure between the throttles 3 and 4. Depending on the type of secondary transducer, hydraulic resistance may vary within $[-\infty, +\infty]$. Bridge diagrams may be balanced or unbalanced depending on the presence or absence of pressure difference between points A and B during measuring for each measured value. Unbalanced bridge measuring circuits can operate in a constant pressure (pressure difference) mode or a constant flow mode. Unbalanced hydrodynamic measuring bridges do not require equalizing, are reliable, fast-response, and have wide functionality for measuring physical-mechanical parameters and characteristics of liquids [2].

Let us build a model of an unbalanced bridge measuring diagram based on oppositely placed laminar and turbulent throttle elements, in the output diagonal of which a pressure difference transducer is installed with hydraulic resistance $\pm\infty$ and flowrate $Q_5 = 0$ (see Fig. 1):

$$\begin{cases} Q_1 = Q_2; \\ Q_3 = Q_4; \\ Q_0 = Q_1 + Q_3; \\ Q_1 = \sqrt{2}\pi\alpha_1 R_1^2 \left[(P_{in} - P_A)\,\rho\right]^{0.5}; \\ Q_2 = \dfrac{\pi R_2^4}{8L_2\nu}\left(P_A - P_{out}\right); \\ Q_3 = \dfrac{\pi R_3^4}{8L_3\nu}\left(P_{in} - P_B\right); \\ Q_4 = \sqrt{2}\pi\alpha_4 R_4^2 \left[(P_B - P_{out})\,\rho\right]^{0.5}; \\ \Delta P = P_A - P_B, \end{cases} \tag{5}$$

where Q_1, Q_2, Q_3, Q_4 are mass flowrates of liquid through the throttles $1-4$, respectively; Q_0 is mass flowrate of liquid through the bridge diagram; R_1, R_4 are the radiuses of the holes of the turbulent throttles 1 and 4; α_1, α_4 are the discharge coefficients of turbulent throttles 1 and 4, respectively; R_2, R_3 are the radiuses of the cylindric channels of the laminar throttles 2 and 3; L_2, L_3 are the length of the laminar throttles 2 and 3, respectively; P_{in}, P_{out} are the pressures at the inlet and the outlet of bridge diagram; ΔP is the pressure difference in opposite vertices A and B of the bridge diagram.

The investigations in [11] prove that using the same turbulent and laminar throttles in opposite arms of the bridge diagram provides maximum transducer sensitivity. Thus, the mathematical model (5) of the bridge diagram is simplified to the form

$$\begin{cases} Q = Q_1 = Q_2 = Q_3 = Q_4; \\ Q_0 = 2Q; \\ L = L_2 = L_3; \\ R_T = R_1 = R_4; \\ R_L = R_2 = R_3; \\ Q_1 = \sqrt{2}\pi\alpha R_T^2 \left[(P_{in} - P_A)\,\rho\right]^{0.5}; \\ Q_2 = \dfrac{\pi R_L^4}{8L\nu}\left(P_A - P_{out}\right); \\ Q_3 = \dfrac{\pi R_L^4}{8L\nu}\left(P_{in} - P_B\right); \\ Q_4 = \sqrt{2}\pi\alpha R_T^2 \left[(P_B - P_{out})\,\rho\right]^{0.5}; \\ \Delta P = P_A - P_B. \end{cases} \tag{6}$$

In addition to the above, the following symbols are adopted in the model (6): R_L, L are the radius and the length of capillary tubes; R_T is the radius of the passing hole of turbulent throttles.

Let us determine the output quantity — pressure drop in the output diagonal $A-B$ of the hydraulic bridge diagram by solving the system (6). Fluid flowrate in serially connected laminar and turbulent (turbulent and laminar) throttles is the same for the chosen bridge diagram with the same laminar and turbulent throttles in opposite arms:

$$\sqrt{2}\pi\alpha R_T^2\left[\left(P_{in}-P_A\right)\rho\right]^{0.5}=\frac{\pi R_L^4}{8L}\frac{1}{\nu}\left(P_A-P_{out}\right), \tag{7}$$

$$\frac{\pi R_L^4}{8L}\frac{1}{\nu}\left(P_{in}-P_B\right)=\sqrt{2}\pi\alpha R_T^2\left[\left(P_B-P_{out}\right)\rho\right]^{0.5}. \tag{8}$$

Let us use symbols B_L and B_T for design complex parameters of laminar and turbulent throttles, respectively:

$$B_T=\sqrt{2}\pi\alpha R_T^2, \tag{9}$$

$$B_L=\pi R_L^4/8L, \tag{10}$$

then equations (7) and (8) will take the form

$$B_T\left[\left(P_{in}-P_A\right)\rho\right]^{0.5}=B_L\frac{1}{\nu}\left(P_A-P_{out}\right), \tag{11}$$

$$B_L\frac{1}{\nu}\left(P_{in}-P_B\right)=B_T\left[\left(P_B-P_{out}\right)\rho\right]^{0.5}. \tag{12}$$

Let us determine the pressures P_A and P_B between throttles by solving the equations (11) and (12):

$$P_A=P_{out}-\frac{1}{2}B_CB_P+\frac{1}{2}\sqrt{4B_CB_P\left(P_{in}-P_{out}\right)+B_C^2B_P^2}, \tag{13}$$

$$P_B=P_{in}+\frac{1}{2}B_CB_P-\frac{1}{2}\sqrt{4B_CB_P\left(P_{in}-P_{out}\right)+B_C^2B_P^2}. \tag{14}$$

Here $B_C=B_T^2/B_L^2$ is the design complex parameter of the bridge diagram; $B_P=\nu^2\rho$ is a combined physical-mechanical parameter.

Taking into account the formulas (9) and (10), let us determine the design complex parameter B_C of the bridge diagram with the pairwise identical laminar and turbulent throttles in opposite arms:

$$B_C=128\alpha^2R_T^4L^2/R_L^8. \tag{15}$$

Finally, from equations (13) and (14), we determine the output quantity of the transducer:

$$\Delta P=P_A-P_B=\sqrt{4B_CB_P\Delta P_s+B_C^2B_P^2}-\Delta P_s-B_CB_P, \tag{16}$$

where $\Delta P_s=P_{in}-P_{out}$ is the pressure difference in the supply diagonal of the bridge.

Note that the solution (16) of the system (6) was found for the condition of stabilization of pressure difference ΔP_s and infinite hydraulic resistance in the output diagonal $A-B$.

From equation (16), we can see that the pressure difference in the output diagonal of the bridge

$$\Delta P=f\left(B_P,\Delta P_s,B_C\right) \tag{17}$$

depends on the values of the parameter B_P, the pressure difference ΔP_s, and the design complex parameter B_C. The measured quantity — the combined parameter B_P change in the interval determined by kinematic viscosity and density of the Newtonian liquid in the specified ranges.

Thus, the authors built a mathematical model (6) of throttle bridge transducer to measure the combined parameter of liquids, the properties of which are regulated by kinematic viscosity and density.

3. Parametric optimization of the throttle bridge transducer

The synthesis of any measuring transducers is carried out taking into account the following parameters: average sensitivity; the uncertainty of the measurement result; performance; nonlinearity; sensitivity to uninformative parameters, etc. Solving the parametric optimization problem of the measuring transducer, we choose an optimization criterion — one of the above parameters or combining these parameters with weight coefficients [12].

The mentioned parameters for the chosen transducer diagram show that they significantly depend on the throttle characteristics and fluid parameters. Considering that the designed transducer should ensure high measuring sensitivity of each value B_P in a particular range, we recommended an average sensitivity S_d as the criterion for parametric optimization of the transducer.

Average sensitivity S_d, i.e., sensitivity within a specific measurement range, is one of the important metrological characteristics of any gauge. It is the ratio of the change of the output quantity x_{out} of transducer to the change of the input quantity x_{in} [12]

$$S_d = \frac{x_{out2} - x_{out1}}{x_{in2} - x_{in1}}, \tag{18}$$

where x_{out1}, x_{out2} are the output transducer values at the beginning and the end of the measurement range, respectively; x_{in1}, x_{in2} are the input transducer values at the beginning and the end of the measurement range, respectively. The value S_d characterizes the averaged change of the transducer output quantity when the measured quantity changes per unit in a given range.

Parametric optimization of the transducer will consist in choosing such design characteristics of turbulent and laminar throttles, for which the optimum criterion will acquire maximum value. In addition to that, one should consider the specified limits on the transducer supply parameters, the geometric dimensions of throttle elements, and the fluid flow behavior in throttles.

Above we noted that the parameter B_P is the input quantity for the designed transducer. The pressure drop ΔP in the output diagonal $A - B$ of the bridge is the output quantity. So, let us determine the average sensitivity by the formula

$$S_d = \frac{\Delta P_2 - \Delta P_1}{B_{P2} - B_{P1}}, \tag{19}$$

where ΔP_2, ΔP_1 are the pressure difference between the bridge vertices $A - B$ for the upper B_{P2} and the lower B_{P1} measuring limits B_P, respectively.

Let us determine the average sensitivity of the transducer, based on formulas (19) and (16):

$$S_d = \frac{B_C}{(B_{P2} - B_{P1})} \left(B_{P2}\sqrt{\frac{4\Delta P_s}{B_C B_{P2}} + 1} - B_{P1}\sqrt{\frac{4\Delta P_s}{B_C B_{P1}} + 1} \right) - B_C. \tag{20}$$

We can see that the sensitivity S_d depends on the power supply ΔP_s of the bridge, the limits B_{P1} and B_{P2} of the measured parameter, and the design complex parameter B_C of the hydrodynamic bridge. Fig. 2 shows the dependence of sensitivity S_d on the design complex parameter B_C for different measurement ranges B_P calculated by the formula (20) at $\Delta P_s = 50$ kPa. We can see from Fig. 2 that the average sensitivity S_d in each range has an extremum, and as the measurement range B_P increases, the optimal value S_d decreases.

Since the design complex parameter B_C is variable, we will find a derivative for investigating the sensitivity extremum

$$\frac{dS_d}{dB_C} = \frac{1}{B_C\,(B_{P2} - B_{P1})} \left(\frac{2\Delta P_s + B_C B_{P2}}{\sqrt{4\Delta P_s/(B_C B_{P2}) + 1}} - \frac{2\Delta P_s + B_C B_{P1}}{\sqrt{4\Delta P_s/(B_C B_{P1}) + 1}} \right) - 1. \tag{21}$$

Equating the derivative (21) to zero, we found the design complex parameter B_C by solving the obtained equation. Analysis of the second-order derivative d^2S_d/dB_C^2 shows that found value B_C is optimal, and the optimization criterion, transducer's sensitivity S_d, in the measurement range $B_{P1} \leqslant B_P \leqslant B_{P2}$ reaches the maximum value.

Fig. 2 shows the optimal design complex parameters B_C^{opt} of the bridge transducer calculated for the different specified values B_{P1}, B_{P2}. We can see from Fig. 2 that values B_C^{opt} decrease as the measurement range increases. Fig. 3 also confirms the decreasing dependence of optimal design complex parameter B_C^{opt} on the ratio B_{P2}/B_{P1}.

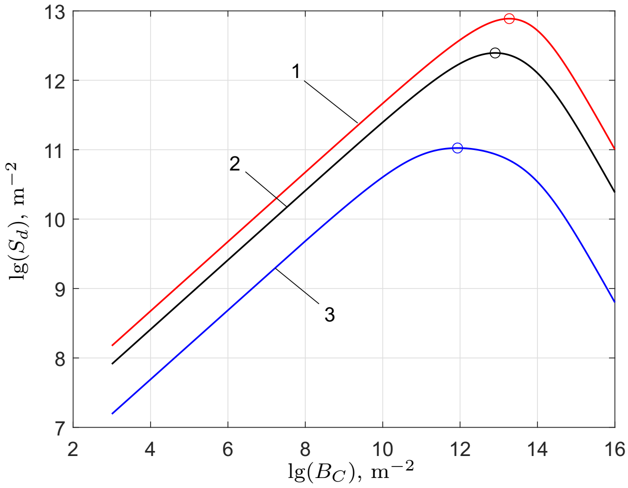

Fig. 2. Dependence of the sensitivity S_d of hydrodynamic bridge transducer on the design complex parameter B_C:
1, $B_{P1} = 1.897082 \cdot 10^{-9}$ N; $B_{P2} = 2.580313 \cdot 10^{-9}$ N;
2, $B_{P1} = 1.021878 \cdot 10^{-9}$ N; $B_{P2} = 1.997774 \cdot 10^{-8}$ N;
3, $B_{P1} = 1.021878 \cdot 10^{-9}$ N; $B_{P2} = 7.671857 \cdot 10^{-7}$ N.

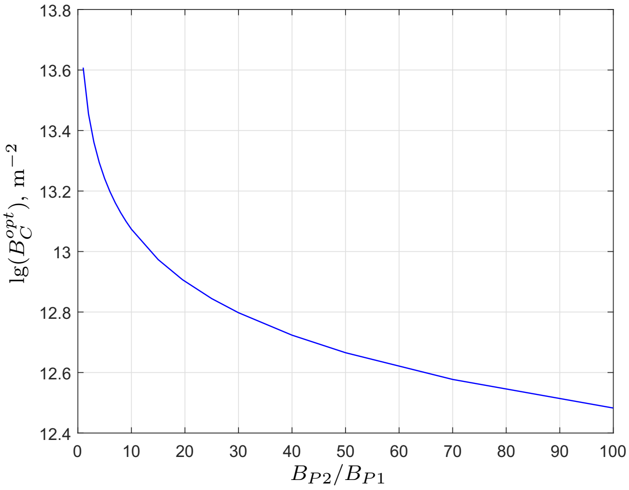

Fig. 3. Dependence B_C^{opt} on the ratio B_{P2}/B_{P1}, obtained for $B_{P1} = 1.021878 \cdot 10^{-9}$ N.

4. Calculation of design and mode characteristics of the transducer of liquid fuel properties

Let us use the built models to design a hydrodynamic bridge measuring transducer of the combined parameter $B_P = \nu^2\rho$ of jet fuel TS-1 [16]. The lower limit of measurement $B_{P1} = 0.948541 \cdot 10^{-9}$ N and the upper limit of measurement $B_{P2} = 5.1606264 \cdot 10^{-9}$ N for jet fuel at a temperature of 20°C is chosen according to the changes of kinematic viscosity and density of fuel during its production.

For a given measurement range and pressure difference in the supply diagonal ($\Delta P_s = 50$ kPa) according to the formula (16), the transform functions of the bridge transducer are calculated for different values of the design complex parameter B_C (see Fig. 4). A feature of these functions is that the pressure difference ΔP in chambers A and B between throttles in a given measurement range can be positive (curves 4 and 5) or negative (curves 1 and 2), depending on the design complex parameter B_C. According to curve 3 in Fig. 4 for the lower values B_P in a given measurement range, the pressure P_A in chamber A is less than the pressure P_B in chamber B; at a particular value of the parameter B_P: $P_A = P_B$ and $\Delta P = 0$; with a further increase B_P to the upper limit of measurement, the pressure in chamber A will be greater than the pressure P_B, that is $P_A > P_B$.

We calculated the transducer sensitivity for different values of the design complex parameter B_C for a given measurement range, as well as its optimal value $B_C^{opt} = 1.7938 \cdot 10^{13}$ (see Fig. 5). At this optimal value B_C^{opt}, the transducer's sensitivity reaches the maximum value $S_d = 6.70666 \cdot 10^{12}\ \frac{\text{Pa}}{\text{N}}$, which corresponds to the maximum difference of the output signal $\Delta P_{\max} = 28.249$ kPa for the upper ΔP_2 and lower ΔP_1 measuring limits of the parameter $\nu^2\rho$.

The problem of calculating the design characteristics of turbulent and laminar throttles of the bridge hydrodynamic transducer is to choose the diameter $2R_L$ and the length L of the capillary tube and orifice plate diameter, for which the design complex parameter B_C will be equal to the optimal value $B_C^{opt} = 1.7938 \cdot 10^{13}$. For correct using the mathematical models of throttles, their geometric dimensions must satisfy the following conditions: Reynolds number must be less than the upper limit number Re_1 for the laminar throttle, and more than some lower limit number Re_2 — for the turbulent one. The length of the capillary tubes should be sufficiently long for the laminar flow to be fully developed, and the pressure loss for the end effects can be neglected. Approximately sufficient length L_E for the laminar flow of Newtonian liquid can be calculated by the formula [17]

$$L_E = (1.1 + 0.11Re)\,R_L.$$

Therefore, the length L of the capillary tube should at least be greater than L_E.

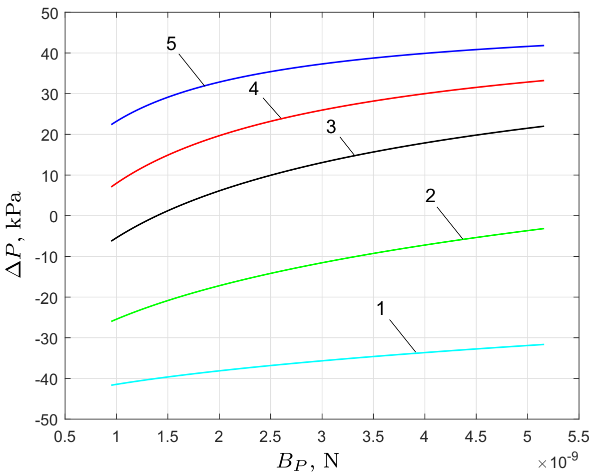

Fig. 4. Transform functions for different values of design complex parameter B_C:
1: $4 \cdot 10^{11}$, m^{-2}; 2: $4 \cdot 10^{12}$, m^{-2}; 3: $1.7938 \cdot 10^{13}$, m^{-2};
4: $4 \cdot 10^{13}$, m^{-2}; 5: $1 \cdot 10^{14}$, m^{-2}.

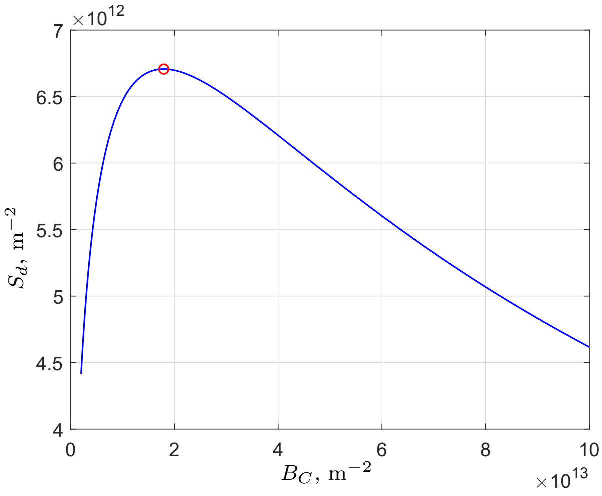

Fig. 5. Dependence of average sensitivity S_d on the design complex parameter B_C.

For a constant pressure drop ΔP_s on the bridge diagram, the mass flowrate of liquid in throttles is determined by the formula

$$Q = \frac{8\pi\alpha^2 R_T^4 L\nu\rho}{R_L^4}\left(\sqrt{1 + \frac{R_L^8 \Delta P_s}{32\alpha^2 R_T^4 L^2 \nu^2 \rho}} - 1\right). \tag{22}$$

Then Reynolds numbers Re^L and Re^T for flow in tubes and orifice plates must meet the requirements [18]

$$Re^L = \frac{2Q}{\pi R_L \mu_1} < Re_1, \quad Re^T = \frac{2Q}{\pi R_T \mu_2} > Re_2, \tag{23}$$

where μ_1, μ_2 are the lower and the upper limits of the dynamic viscosity of a liquid. The condition for the laminar flow of Newtonian liquid in the pipes is $Re_1 < 2000$. A transition away to turbulent flow can occur at $500 < Re_2 < 3000$ for the orifice plates.

Considering the formulated requirements for designing the measuring bridge transducer with optimal sensitivity, we calculated the main design characteristics of capillary tubes and orifice plates and the mode characteristics of the transducer, which are presented in Table 1.

The measuring transducer operates at a given temperature of 20°C. When the density ρ and kinematic viscosity ν change, the hydrodynamic resistances of the throttles also change. As a result, the output signal ΔP of the bridge diagram changes. It is registered by the differential pressure transducer 5 (see Fig. 2). The pressure difference transducer characteristics allow one to adjust the output signal for a given measurement range of B_P between 0 and $\Delta P_{\max}$. Thus, each particular value of the parameter B_P of the controlled liquid corresponds to a particular value of the output signal ΔP of the throttle bridge diagram.

Table 1. Characteristics of the measuring transducer.

Transducer characteristic	Symbol	Units	Value
Measurement range	B_P	N	$(0.948541 \div 5.160626) \cdot 10^{-9}$
Nominal density at temperature 20°C	ρ_{nom}	kg/m^3	810
Nominal kinematic viscosity at temperature 20°C	ν_{nom}	m^2/s	$1.66 \cdot 10^{-6}$
Nominal combined parameter at temperature 20°C	$B_{P_{nom}}$	N	$2.232036 \cdot 10^{-9}$
Diameter of capillary tubes	$2R_L$	m	$1.1 \cdot 10^{-3}$
Length of capillary tubes	L	m	0.30513
Diameter of the orifice plate	$2R_T$	m	$0.858 \cdot 10^{-3}$
Optimal design complex parameter	B_C^{opt}	m^{-2}	$1.7938 \cdot 10^{13}$
Averaged measuring sensitivity per range	S_d	Pa/N	$6.70666 \cdot 10^{12}$
Pressure difference in the supply diagonal of the bridge diagram	ΔP_s	kPa	50
Change of pressure difference in the output diagonal of the bridge within the measurement range	$\Delta P_{\max}$	kPa	28.249
Fluid flowrate through the transducer at B_{P1}	$Q_{\max}$	kg/s	$4.27741 \cdot 10^{-3}$
Fluid flowrate through the transducer at B_{P2}	$Q_{\min}$	kg/s	$3.96610 \cdot 10^{-3}$
Laminar throttle Reynolds number within the measurement range	from $Re^L_{\max}$ to $Re^L_{\min}$	—	from 1983.6 to 1597.6
Turbulent throttle Reynolds number within the measurement range	from $Re^T_{\max}$ to $Re^T_{\min}$	—	from 2543.1 to 2048.2
The discharge coefficient of turbulent throttle	α	—	0.61

5. Estimation of the uncertainty of the measurement result

To estimate the accuracy of determining the pressure drop, let us use the technique for estimating the uncertainty of measurement results [19].

Each value of pressure drop ΔP in a given measurement range $[B_{P1}, B_{P2}]$ depends on the input quantities according to the formulas (15) and (16):

$$\Delta P = f\left(\alpha, R_T, R_L, L, \Delta P_s\right). \tag{24}$$

Provided the input quantities α, R_T, R_L, L, ΔP_s are uncorrelated, the total uncertainty can be found by calculating and combining each of the contributing factors [19]:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[c_i u\left(x_i\right)\right]^2}, \tag{25}$$

where $u_c(y)$ is combined standard uncertainty associated with the output quantity estimate y; $u(x_i)$ are the components of standard uncertainty associated with the input quantities estimates x_i, which form a set $x_i = [\alpha, R_T, R_L, L, \Delta P_s]$; c_i is the contributing factor of the uncertainty of parameter x_i on the uncertainty of pressure drop, which is calculated by the formula [19]

$$c_i = \frac{\partial y}{\partial x_i}. \tag{26}$$

Taking into account the specified input quantities, let us write the equation of relative standard uncertainty of pressure drop ΔP [20]:

$$u_c = \left[(c_\alpha u_\alpha)^2 + (c_{R_T} u_{R_T})^2 + (c_{R_L} u_{R_L})^2 + (c_L u_L)^2 + (c_{\Delta P_s} u_{\Delta P_s})^2 \right]^{0.5}. \tag{27}$$

The values of the uncertainty of the parameters of equation (24) are calculated considering the characteristics of the measuring tools and presented in Table 2. Here are also the contributing factors of the components of the uncertainty of parameters that are part of the equation for determining the pressure drop ΔP. These factors are obtained by the formula (26) for the nominal value of the parameter $B_{P_{nom}}$ and the optimal design complex parameter B_C^{opt}.

Table 2. The components of the uncertainty of the parameters of the equation (24) and their contributing factors.

No.	Parameter	Components of standard uncertainty		Contributing factors of components of uncertainty	
		Symbol	Value	Formula	Value
1	α	u_α	0.005	$c_\alpha = \dfrac{\partial \Delta P}{\partial \alpha}$	$5.62450 \cdot 10^4$
2	R_T	u_{R_T}	$5 \cdot 10^{-6}$m	$c_{R_T} = \dfrac{\partial \Delta P}{\partial R_T}$	$1.59951 \cdot 10^8$
3	R_L	u_{R_L}	$5 \cdot 10^{-6}$m	$c_{R_L} = \dfrac{\partial \Delta P}{\partial R_L}$	$-2.49523 \cdot 10^8$
4	L	u_L	$1 \cdot 10^{-6}$m	$c_L = \dfrac{\partial \Delta P}{\partial L}$	$1.12442 \cdot 10^5$
5	ΔP_s	$u_{\Delta P_s}$	500 Pa	$c_{\Delta P_s} = \dfrac{\partial \Delta P}{\partial \Delta P_s}$	-0.18319

The overall result is obtained by summation of the contributions of the standard uncertainty of each input source from Table 2 to the combined uncertainty of result $u_c = 1511.17$ Pa. In the practice of technological measurements, expanded standard uncertainty $U_{\Delta P}$ is found by the formula [21]

$$U_{\Delta P} = 2u_c. \tag{28}$$

Consequently, the expanded standard uncertainty is $U_{\Delta P} = 3022$ Pa, which in relative units corresponds to 10.7% of the maximum pressure drop in the output diagonal of the bridge transducer. Calculations have shown that the uncertainty $U_{\Delta P}$ obtained for the nominal value $B_{P\,nom}$ is approximately maximum. Accordingly, the uncertainties for the other pressure drop values within the measurement range of the combined parameter $[B_{P1}, B_{P2}]$ are lower.

The obtained estimate of the uncertainty of the developed transducer confirms that it is possible to provide higher accuracy characteristics compared to known capillary devices of continuous measurement of liquids' viscosity and density [3–5, 10]. The authors' principles of designing transducers with maximum measuring sensitivity provide the least value of uncertainty in a given measurement range and reduce the nonlinearity of the transform function.

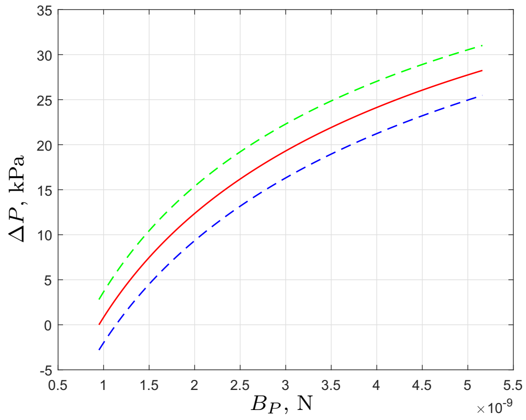

Fig. 6. Transform function of measuring transducer of combined parameter B_P.

Fig. 6 shows the transform function of the designed transducer with calculated uncertainty values.

We should also note that the specified uncertainty value is obtained without calibrating the throttle transducer. If one calibrates the developed transducer using reference liquids as it is usually done for measuring transducer of physical-mechanical parameters, it is possible to significantly reduce its measurement error and, accordingly, the uncertainty of the measurement result of the combined parameter.

6. Conclusions

To solve the problems of measuring the Newtonian liquids' properties, the authors proposed to use the hydrodynamic throttle method. A throttle bridge measuring diagram was used with identical turbulent and laminar throttles in opposite arms to synthesize the measuring transducer of the combined physical-mechanical parameter of Newtonian liquid. The mathematical model of throttle bridge measuring transducer of combined parameter, which depends on kinematic viscosity and fluid density, was build and investigated.

The problem of parametric optimization of measuring transducer is formulated and analytically solved in the paper. The authors determined such design characteristics of turbulent and laminar throttles of transducer that provide the maximum average sensitivity with the specified restrictions on the design characteristics of throttles and the fluids' behavior in the throttle. The transform function of the developed measuring transducer of the combined parameter of jet fuel was calculated. The expanded standard uncertainty of the output quantity of the developed measuring transducer of the combined physical-mechanical parameter of jet fuel was also estimated.

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Моделювання дросельних мостових вимірювальних перетворювачів фізико-механічних параметрів ньютонівських рідин

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У статті запропоновано вимірювальний перетворювач фізико-механічного параметра ньютонівської рідини на основі дросельної мостової вимірювальної схеми із однаковими турбулентними і ламінарними дроселями в протилежних плечах. Побудована математична модель дросельного мостового вимірювального перетворювача комбінованого параметра, який залежить від кінематичної в'язкості і густини рідини. У роботі сформульована та аналітично вирішена задача параметричної оптимізації запропонованого вимірювального перетворювача. Отримано розрахункову функцію перетворення вимірювального перетворювача комбінованого параметра реактивного палива.

Ключові слова: *кінематична в'язкість, густина, ньютонівська рідина, модель, дросель, вимірювальний перетворювач.*