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Passivity analysis of multiple time-varying time delayed complex variable neural networks in finite-time

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In this article, we investigate the problem of finite-time passivity for the complex-valued neural networks (CVNNs) with multiple time-varying delays. To begin, many definitions relevant to the finite-time passivity of CVNNs are provided; then the suitable control inputs are designed to guarantee the class of CVNNs are finite-time passive. In the meantime, some sufficient conditions of linear matrix inequalities (LMIs) are derived by using inequalities techniques and Lyapunov stability theory. Finally, a numerical example is presented to illustrate the usefulness of the theoretical results.

Keywords: *passivity, finite-time passivity, complex-valued neural networks, time-varying delays, Lyapunov functional, linear matrix inequality.*

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1. Introduction

Many researchers have been interested in studying the dynamic behavior of nonlinear systems in recent decades. The study of neural networks (NNs) has gotten a lot of attention recently because of their wide range of possible applications in optimization [1], pattern recognition [2], image processing [3], and other fields. Many publications, such as dissipativity [4], stability [5–7], multi stability [8], state estimation [9], and so on, provide the study results. Complex valued neural networks (CVNNs) are an extension of RVNNs that can be used for solving such problems. CVNNs are more challenging because they have complex-valued states, complex-valued connection weights, and complex-valued activation functions. CVNNs can be used to solve a wide range of physical problems, including electromagnetic waves, ultrasonic waves, quantum waves, light, and so on, according to their complex number property. Furthermore, CVNNs provide for the solution of problems that simple RVNNs cannot. Thus the stability analysis of CVNNs has become an energetic field of research. In this field, some exciting results have recently been proposed [10–13]. The activation function is very important in the complex domain. In real-valued neural networks, the activation functions are usually smooth and bounded. According to Liouville's theorem [14], every bounded entire function in the complex plane must be a constant function. This is clearly inappropriate. In other words, CVNN activation functions cannot be both bounded and analytic. As a result, activation functions are the main challenge for complex-valued neural networks. Due to the finite switching speed of amplifiers, time-delay plays an important role in the electronic circuit implementation of neural networks. Time-delay is very often a main source of oscillatory, resulting in system instability and lack of performance. As a result, research into systems with time-varying delays has gotten a lot of attention. Many neural network applications rely on the network's stability properties [15, 16]. Thus, many academics have developed an interest in studying the dynamical characteristics of time-delayed CVNNs [17–20]. The passivity theory, which is derived from circuit theory, is useful in analysing the stability of dynamical systems [21–23]. The main idea of passivity theory is that the passive properties of the system can keep the system internally stable. The passive control problem is another name for the passification problem. The goal of the passive control problem is to create a controller that results in a passive closed-loop system. Passivity and passification problems have been a hot topic of research in recent decades due to this fact. Many researches focused on the passivity problem of delayed neural networks due to the existence of time-delay.

Based on the functionality of Lyapunov–Krasovskii there were a number of results for passivity analysis of neural networks in the integer order (see the [24–28] and their references). Based on the master-slave idea, differential inclusions theory, and Lyapunov–Krasovskii stability theory, the topic of mixed H and passivity based synchronization requirements for memristor-based recurrent neural networks with time-varying delays was recently studied in [29]. It is worth noting that all of the above results were developed in the context of Lyapunov stability. In some practical processes, however, the major focus may be on the behavior of dynamical systems over a finite time interval. The passivity and passification of stochastic Takagi–Sugeno fuzzy systems with heterogeneous time-varying delays were investigated in [30]. In [31], the authors investigate the passivity and passification of time-delay systems. The authors of [32] used the Lyapunov–Krasovskii functional approach and weighting matrices to study passivity analysis of stochastic time-delay neural networks. In addition, the subject of passivity analysis for various neural networks has received a lot of attention [33–42]. To the best of authors knowledge, so far, no result on the finite-time passivity for complex valued neural network systems with time varying delay has been reported. This is the motivating factor behind our current investigation. Motivated by the earlier discussions, the contribution of this study is to investigate the passivity of complex-valued neural networks with time-varying delays. Using Schur complement lemma, some new passivity conditions are derived in the form LMIs by employing Lyapunov–Krasovkii functional method. The suitable controller has been designed, which ensures the given system to be passive. Finally numerical examples are given to illustrate the effectiveness of our proposed results. The rest of this paper is organized as follows: Problem description and preliminaries are given in section 2. The definition can be regarded as an extension of definition for integer order neural network system to complex system. In section 3, some new passivity conditions are derived. Numerical examples is given in section 4. Finally conclusion is presented in section 5.

2. Problem description and preliminaries

In this paper, we consider the n-dimensional CVNNs with multiple time-varying delays which can be described by

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-d(t))) + Eg(z(t-\tau(t))) + w(t) + u(t), \tag{1}$$

where $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$ is the state vector; $C = \operatorname{diag}\{c_1, c_2, \ldots, c_n\}$ is the positive diagonal matrix; $A = [a_{pq}]_{n\times n}$, $B = [b_{pq}]_{n\times n}$ and $E = [e_{pq}]_{n\times n}$ are respectively the feedback connection weight matrix and the delayed feedback connection weight matrices; $f(z(t)) = (f_1(z_1(t)), f_2(z_2(t)), \ldots, f_n(z_n(t)))^T$ is the activation function without delay, $f(z(t-d(t))) = (f_1(z_1(t-d(t))), f_2(z_2(t-d(t))), \ldots, f_n(z_n(t-d(t))))^T$ and $g(z(t-\tau(t))) = (g_1(z_1(t-\tau(t))), g_2(z_2(t-\tau(t))), \ldots, g_n(z_n(t-\tau(t))))^T$ are the activation functions with delays; the rest of the paper $f(z(t-d(t)))$ and $g(z(t-\tau(t)))$ can be represented by $f(z_d)$ and $g(z_\tau)$; $w(t) = [w_1(t), w_2(t), \ldots, w_n(t)]^T$ is the external input vector; $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in \mathbb{C}^n$ is represents the control input to be defined later; $z(\phi) = (\psi_1(\phi), \psi_2(\phi), \ldots, \psi_n(\phi))^T, \phi \in [-\tau, 0], \psi \in \mathcal{C}([-\tau, 0], \mathbb{C})$ is the initial state vector of (1).

Throughout this paper, the output vector $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T \in \mathbb{C}^n$ of the network (1) is given as follows:

$$s(t) = Lz(t) + Hw(t), \tag{2}$$

where $L, H \in \mathbb{C}^{n\times n}$ are complex-valued constant matrices.

Assumption 1. For any two constants τ and d, then the continuous time-varying delays $\tau(t)$ and $d(t)$ are satisfies

$$0 \leqslant \tau(t) \leqslant \tau, \quad 0 \leqslant d(t) \leqslant d.$$

Assumption 2. For $p = 1, 2, \dots, n$ and $z(t) = x(t) + iy(t)$, the activation functions $f_p(z_p(t))$, $f_p(z_d)$ and $g_p(z_\tau)$ can be separated into its real and imaginary parts as

$$f_p(z_p(t)) = f_p^R(x,y) + i f_p^I(x,y), \quad f_p(z_d) = f_p^R(x_d,y_d) + i f_p^I(x_d,y_d), \quad g_p(z_\tau) = g_p^R(x_\tau,y_\tau) + i g_p^I(u_\tau,v_\tau),$$

where $f_p^R(x,y)$, $f_p^I(x,y)$, $f_p^R(x_d,y_d)$, $f_p^I(x_d,y_d)$, $g_p^R(x_\tau,y_\tau)$, $g_p^I(x_\tau,y_\tau)$: $\mathbb{R}^2 \to \mathbb{R}$.

By using Assumptions 2, the CVNNs (1) can be separated into its real and imaginary parts as follows:

$$\begin{cases} \dot{x}(t) = -Cx(t) + A^R f^R(x,y) - A^I f^I(x,y) + B^R f^R(x_d,y_d) - B^I f^I(x_d,y_d) + E^R g^R(x_\tau,y_\tau) \\ \qquad -E^I g^I(x_\tau,y_\tau) + w^R(t) + u^R(t), \\ \dot{y}(t) = -Cy(t) + A^R f^I(x,y) + A^I f^R(x,y) + B^R f^I(x_d,y_d) + B^I f^R(x_d,y_d) + E^R g^I(x_\tau,y_\tau) \\ \qquad +E^I g^R(x_\tau,y_\tau) + w^I(t) + u^I(t), \end{cases} \tag{3}$$

and the output vector (2) can be written in the following manner:

$$\begin{cases} s^R(t) = L^R x(t) + H^R w^R(t) - L^I y(t) - H^I w^I(t), \\ s^I(t) = L^R y(t) + H^R w^I(t) + L^I x(t) + H^I w^R(t), \end{cases} \tag{4}$$

where A^R, A^I, B^R, B^I, E^R, E^I, L^R, L^I, H^R, H^I are the real and imaginary parts of the constant coefficient matrices A, B, E, L, H respectively; $w^R(t)$, $w^I(t)$, $u^R(t)$, $u^I(t)$ are real and imaginary parts of $w(t)$ and $u(t)$.

Assumption 3. The activation function $f_q(z(t))$ can be represented as follows by dividing it into its real and imaginary parts: $f_q(z(t)) = f_q^R(x,y) + i f_q^I(x,y)$, where $f_q^R(x,y)$, $f_q^I(x,y)$: $\mathbb{R}^2 \to \mathbb{R}$.

1. The partial derivatives of activation function $f_q(\cdot,\cdot)$ with respect to x, y: $\partial f_q^R/\partial x$, $\partial f_q^R/\partial y$, $\partial f_q^I/\partial x$, $\partial f_q^I/\partial y$ exist and are continuous.
2. The partial derivatives $\partial f_q^R/\partial x$, $\partial f_q^R/\partial y$, $\partial f_q^I/\partial x$, $\partial_q^I/\partial y$ are bounded, i.e, there exist positive constants λ_q^{RR}, λ_q^{RI}, λ_q^{IR}, λ_q^{II} such that

$$|\partial f_q^R/\partial x| \leqslant \lambda_q^{RR}, \quad |\partial f_q^R/\partial y| \leqslant \lambda_q^{RI}, \quad |\partial f_q^I/\partial x| \leqslant \lambda_q^{IR}, \quad |\partial f_q^I/\partial y| \leqslant \lambda_q^{II}.$$

Then, we can write

$$|f_q^R(x,y)| \leqslant \lambda_q^{RR}|x| + \lambda_q^{RI}|y|, \quad |f_q^I(x,y)| \leqslant \lambda_q^{IR}|x| + \lambda_q^{II}|y|.$$

Similarly, one can obtain

$$|f_q^R(x_d,y_d)| \leqslant \mu_q^{RR}|x_d| + \mu_q^{RI}|y_d|, \quad |f_q^I(x_d,y_d)| \leqslant \mu_q^{IR}|x_d| + \mu_q^{II}|y_d|,$$

where μ_q^{RR}, μ_q^{RI}, μ_q^{IR}, μ_q^{II} are positive constants.

Lemma 1 (Ref. [43] (Schur Complement)). *For any given matrix,*

$$\hat{\Theta} = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix} > 0,$$

where $\Theta_{11} = \Theta_{11}^T$, $\Theta_{22} = \Theta_{22}^T$ *is equivalent to any of the conditions listed bellow:*

$$\begin{aligned} &(i) \quad \Theta_{22} > 0, \quad \Theta_{11} - \Theta_{12}^T \Theta_{22}^{-1} \Theta_{12} > 0, \\ &(ii) \quad \Theta_{11} > 0, \quad \Theta_{22} - \Theta_{12} \Theta_{11}^{-1} \Theta_{12}^T > 0. \end{aligned}$$

Lemma 2 (Ref. [44]). *If* $\varrho_1, \varrho_2, \ldots, \varrho_n \geqslant 0$, $0 < \alpha_2 < \alpha_1$ *are real numbers, then*

$$\left(\sum_{p=1}^{n} \varrho_p^{\alpha_1} \right)^{\frac{1}{\alpha_1}} \leqslant \left(\sum_{p=1}^{n} \varrho_1^{\alpha_2} \right)^{\frac{1}{\alpha_2}}.$$

Lemma 3 (Ref. [45]). *For all* $\chi_1, \chi_2 \in \mathbb{R}^n$, *there exist a real number* $\delta > 0$ *and a symmetric matrix* $H \in \mathbb{R}^{n \times n}$ *such that* $2\chi_1^T \chi_2 \leqslant \delta \chi_1^T H \chi_1 + \delta^{-1} \chi_2^T H \chi_2$.

Definition 1 (Ref. [46]). *With respect to CVNNs* (1), *it is said that CVNNs* (1) *is finite-time passivity if there exists a positive definite function* $V(t)$ *such that*

$$w^T(t) s(t) \geqslant \dot{V}(t) + \xi (V(t))^{\rho}$$

for the external input vector $w(t)$ *and output vector* $s(t)$, *where* $\rho \in (0,1)$ *and* $\xi > 0$.

Definition 2 (Ref. [46]). *With respect to CVNNs* (1), *it is said that CVNNs* (1) *is finite-time output strict passivity if there exists a positive definite function* $V(t)$ *such that*

$$w^T(t) s(t) - \eta_1 s^T(t) s(t) \geqslant \dot{V}(t) + \xi (V(t))^{\rho}$$

for external input vector $w(t)$ *and output vector* $s(t)$, *where* $\rho \in (0,1)$, $\eta_1 > 0$ *and* $\xi > 0$.

Definition 3 (Ref. [46]). *With respect to CVNNs* (1), *it is said that CVNNs* (1) *is finite-time input strict passivity if there exists a positive definite function* $V(t)$ *such that*

$$w^T(t) s(t) - \zeta w^T(t) w(t) \geqslant \dot{V}(t) + \xi (V(t))^{\rho}$$

for the external input vector $w(t)$ *and output vector* $s(t)$, *where* $\rho \in (0,1)$, $\zeta > 0$ *and* $\xi > 0$.

3. Main results

In this section, we are going to design the necessary control mechanism that will ensure the CVNN's passive condition in finite-time. The designed controller is written as follows:

$$\begin{aligned} u^R(t) = &- K^R x(t) - B^R \mu^{RR} \mathrm{sgn}(x(t)) |x_d| - B^R \mu^{RI} \mathrm{sgn}(x(t)) |y_d| + B^I \mu^{IR} \mathrm{sgn}(x(t)) |x_d| \\ &+ B^I \mu^{II} \mathrm{sgn}(x(t)) |y_d| - E^R \rho^{RR} \mathrm{sgn}(x(t)) |x_\tau| - E^R \rho^{RI} \mathrm{sgn}(x(t)) |y_\tau| + E^I \rho^{IR} \mathrm{sgn}(x(t)) |x_\tau| \\ &+ E^I \rho^{II} \mathrm{sgn}(x(t)) |y_\tau| - k_1 |x(t)|^{\alpha} \mathrm{sgn}(x(t)), \end{aligned} \tag{5}$$

$$\begin{aligned} u^I(t) = &- K^I y(t) - B^R \mu^{IR} \mathrm{sgn}(y(t)) |x_d| - B^R \mu^{II} \mathrm{sgn}(y(t)) |y_d| - B^I \mu^{RR} \mathrm{sgn}(y(t)) |x_d| \\ &- B^I \mu^{RI} \mathrm{sgn}(y(t)) |y_d| - E^R \rho^{IR} \mathrm{sgn}(y(t)) |x_\tau| - E^R \rho^{II} \mathrm{sgn}(y(t)) |y_\tau| - E^I \rho^{RR} \mathrm{sgn}(y(t)) |x_\tau| \\ &- E^I \rho^{RI} \mathrm{sgn}(y(t)) |y_\tau| - k_2 |y(t)|^{\alpha} \mathrm{sgn}(y(t)), \end{aligned} \tag{6}$$

where K^R, K^I are control gain matrices, $k_1, k_2 > 0$ are positive constants and $0 < \alpha < 1$.

Theorem 1. *Suppose that the Assumptions 2–3 hold. CVNNs* (1) *is finite-time passive under the designed controller* (5) *and* (6), *if there exist positive symmetry matrices* P, P_1, Q_1, Q_2, Q_3, Q_4 *and nonzero constants* k_1, k_2, λ_1, λ_2, λ_3, λ_4 *such that the following LMI holds:*

$$\Lambda = \begin{bmatrix} \Xi_1 & \Xi_3 & 2P & 0 \\ \star & \Xi_2 & 0 & 2P_1 \\ \star & \star & -H^R & H^I \\ \star & \star & \star & -H^R \end{bmatrix} < 0, \tag{7}$$

where

$$\begin{aligned}
\Xi_1 &= (-PC - C^TP - 2K^RP) + \lambda_1 PA^RQ_1A^{R^T}P + \lambda_1^{-1}\mu^{RR^T}Q_1^{-1}\mu^{RR} - \lambda_2 PA^IQ_2A^{I^T}P \\
&\quad + \lambda_2^{-1}\mu^{IR^T}Q_2^{-1}\mu^{IR} + \lambda_4^{-1}\mu^{RR^T}Q_4^{-1}\mu^{RR}, \\
\Xi_2 &= (-P_1C - C^TP_1 - 2K^IP_1) + \lambda_3 P_1A^RQ_3A^{R^T}P_1 + \lambda_1^{-1}\mu^{RI^T}Q_1^{-1}\mu^{RI} + \lambda_2^{-1}\mu^{II^T}Q_2^{-1}\mu^{II} \\
&\quad + \lambda_3^{-1}\mu^{II^T}Q_3^{-1}\mu^{II} + \lambda_4 P_1A^IQ_4A^{I^T}P_1 + \lambda_4^{-1}\mu^{RI^T}Q_4^{-1}\mu^{RI}, \\
\Xi_3 &= \lambda_1^{-1}\mu^{RR^T}Q_1^{-1}\mu^{RI} + \lambda_2^{-1}\mu^{IR^T}Q_2^{-1}\mu^{II} + \lambda_3^{-1}\mu^{IR^T}Q_3^{-1}\mu^{II} + \lambda_4^{-1}\mu^{RR^T}Q_4^{-1}\mu^{RI}.
\end{aligned}$$

Proof. Let us consider the following Lyapunov functional

$$V(t) = x^T(t)Px(t) + y^T(t)P_1y(t). \tag{8}$$

Calculating the time derivative of (8) about the solutions of (3), we get

$$\begin{aligned}
\dot V(t) &= 2x^T(t)P\dot x(t) + 2y^T(t)P_1\dot y(t) \\
&= 2x^T(t)P\left[-Cx(t) + A^Rf^R(x,y) - A^If^I(x,y) + B^Rf^R(x_d,y_d) - B^If^I(x_d,y_d) + E^Rg^R(x_\tau,y_\tau)\right. \\
&\quad \left.-E^Ig^I(x_\tau,y_\tau) + w^R(t) + u^R(t)\right] + 2y^T(t)P_1\left[-Cy(t) + A^Rf^I(x,y) + A^If^R(x,y)\right. \\
&\quad \left.+B^Rf^I(x_d,y_d) + B^If^R(x_d,y_d) + E^Rg^I(x_\tau,y_\tau) + E^Ig^R(x_\tau,y_\tau) + w^I(t) + u^I(t)\right], \\
&= x^T(t)\left(-PC - C^TP\right)x(t) + 2x^T(t)PA^Rf^R(x,y) - 2x^T(t)PA^If^I(x,y) + 2x^T(t)\big(PB^Rf^R(x_d,y_d) \\
&\quad - B^If^I(x_d,y_d) + E^Rg^R(x_\tau,y_\tau) - E^Ig^I(x_\tau,y_\tau)\big) + 2x^T(t)Pw^R(t) + 2x^T(t)Pu^R(t) \\
&\quad + y^T(t)\left(-P_1C - C^TP_1\right)y(t) + 2y^T(t)P_1A^Rf^I(x,y) + 2y^T(t)P_1A^If^R(x,y) \\
&\quad + 2y^T(t)P_1\big(B^Rf^I(x_d,y_d) + B^If^R(x_d,y_d) + E^Rg^I(x_\tau,y_\tau) + E^Ig^R(x_\tau,y_\tau)\big) \\
&\quad + 2y^T(t)P_1w^I(t) + 2y^T(t)P_1u^I(t).
\end{aligned} \tag{9}$$

According to Lemma 1–3 and Assumption 3, it can be deduced that

$$\begin{aligned}
\dot V(t) &\leqslant x^T(t)\left(-PC - C^TP\right)x(t) + \lambda_1 x^T(t)PA^RQ_1A^{R^T}Px(t) + \lambda_1^{-1}f^{R^T}(x,y)Q_1^{-1}f^R(x,y) \\
&\quad - \lambda_2 x^T(t)PA^IQ_2A^{I^T}Px(t) + \lambda_2^{-1}f^{I^T}(x,y)Q_2^{-1}f^I(x,y) + 2PB^R|x^T(t)|[\mu^{RR}|x_d| + \mu^{RI}|y_d|] \\
&\quad - 2PB^I|x^T(t)|\left[\mu^{IR}|x_d| + \mu^{II}|y_d|\right] + 2PE^R|x^T(t)|\left[\rho^{RR}|x_\tau| + \mu^{RI}|y_\tau|\right] \\
&\quad - 2PE^I|x^T(t)|\left[\rho^{IR}|x_\tau| + \mu^{II}|y_\tau|\right] + 2x^T(t)Pw^R(t) + 2x^T(t)Pu^R(t) \\
&\quad + y^T(t)\left(-P_1C - C^TP_1\right)y(t) + \lambda_3 y^T(t)P_1A^RQ_3A^{R^T}P_1y(t) + \lambda_3^{-1}f^{I^T}(x,y)Q_3^{-1}f^I(x,y) \\
&\quad - \lambda_4 y^T(t)P_1A^IQ_4A^{I^T}P_1y(t) + \lambda_4^{-1}f^{R^T}(x,y)Q_4^{-1}f^R(x,y) + 2P_1B^R|y^T(t)|\left[\mu^{IR}|x_d| + \mu^{II}|y_d|\right] \\
&\quad + 2P_1B^I|y^T(t)|\left[\mu^{RR}|x_d| + \mu^{RI}|y_d|\right] + 2P_1E^R|y^T(t)|\left[\rho^{IR}|x_\tau| + \mu^{II}|y_\tau|\right] \\
&\quad + 2P_1E^I|y^T(t)|\left[\rho^{RR}|x_\tau| + \rho^{RI}|y_\tau|\right] + 2y^T(t)P_1w^I(t) + 2y^T(t)P_1u^I(t).
\end{aligned} \tag{10}$$

Then according to (5) and (6), it is obvious that

$$\begin{aligned}
\dot V(t) &\leqslant x^T(t)\Big[\left(-PC - C^TP - 2K^RP\right) + \lambda_1 PA^RQ_1A^{R^T}P + \lambda_1^{-1}\mu^{RR^T}Q_1^{-1}\mu^{RR} - \lambda_2 PA^IQ_2A^{I^T}P \\
&\quad +\lambda_2^{-1}\mu^{IR^T}Q_2^{-1}\mu^{IR} + \lambda_3^{-1}\mu^{IR^T}Q_3^{-1}\mu^{IR} + \lambda_4^{-1}\mu^{RR^T}Q_4^{-1}\mu^{RR}\Big]x(t) \\
&\quad + y^T(t)\Big[\left(-P_1C - C^TP_1 - 2K^IP_1\right) + \lambda_3 P_1A^RQ_3A^{R^T}P_1 + \lambda_1^{-1}\mu^{RI^T}Q_1^{-1}\mu^{RI}
\end{aligned}$$

$$
\begin{aligned}
&+\lambda_2^{-1}\mu^{II^T}Q_2^{-1}\mu^{II}+\lambda_3^{-1}\mu^{II^T}Q_3^{-1}\mu^{II}+\lambda_4 P_1 A^I Q_4 A^{I^T} P_1+\lambda_4^{-1}\mu^{RI^T}Q_4^{-1}\mu^{RI}\Big] y(t)\\
&+x^T(t)\left[\lambda_1^{-1}\mu^{RR^T}Q_1^{-1}\mu^{RI}+\lambda_2^{-1}\mu^{IR^T}Q_2^{-1}\mu^{II}+\lambda_3^{-1}\mu^{IR^T}Q_3^{-1}\mu^{II}+\lambda_4^{-1}\mu^{RR^T}Q_4^{-1}\mu^{RI}\right] y(t)\\
&+y^T(t)\left[\lambda_1^{-1}\mu^{RI^T}Q_1^{-1}\mu^{RR}+\lambda_2^{-1}\mu^{II^T}Q_2^{-1}\mu^{IR}+\lambda_3^{-1}\mu^{II^T}Q_3^{-1}\mu^{IR}+\lambda_4^{-1}\mu^{RI^T}Q_4^{-1}\mu^{RR}\right] x(t)\\
&+x^T(t)[2P]w^R(t)+y^T(t)[2P_1]w^I(t)-2k_1x^T(t)P\mathrm{sgn}(x(t))|x(t)|^{\alpha}\\
&-2k_2y^T(t)P_1\mathrm{sgn}(y(t))|y(t)|^{\alpha}. \qquad (11)
\end{aligned}
$$

Moreover, it has

$$
\begin{aligned}
-x^T(t)P\mathrm{sgn}(x(t))|x(t)|^{\alpha} &\leqslant -\lambda_{\min}(P)\left[\left(\sum_{p=1}^{n}|x_p(t)|\right)^{\alpha+1}\right],\\
-y^T(t)P_1\mathrm{sgn}(y(t))|y(t)|^{\alpha} &\leqslant -\lambda_{\min}(P_1)\left[\left(\sum_{p=1}^{n}|y_p(t)|\right)^{\alpha+1}\right].
\end{aligned}
$$

From Lemma 2, we can get:

$$
-\left(\sum_{p=1}^{n}|x_p(t)|\right)^{\alpha+1} \leqslant -\left[\left(\sum_{p=1}^{n}|x_p(t)|^2\right)\right]^{\frac{\alpha+1}{2}},\quad -\left(\sum_{p=1}^{n}|y_p(t)|\right)^{\alpha+1} \leqslant -\left[\left(\sum_{p=1}^{n}|y_p(t)|^2\right)\right]^{\frac{\alpha+1}{2}}. \qquad (12)
$$

Therefore,

$$
\dot{V}(t)-\bar{w}^T(t)\bar{S}(t) \leqslant \dot{V}(t)-\bar{w}^T(t)\bar{L}\bar{Z}(t)-\bar{w}^T(t)\bar{H}\bar{w}(t), \qquad (13)
$$

where

$$
\bar{w}(t)=\begin{bmatrix} w^R(t)\\ w^I(t)\end{bmatrix},\ \bar{S}(t)=\begin{bmatrix} s^R(t)\\ s^I(t)\end{bmatrix},\ \bar{Z}(t)=\begin{bmatrix} x(t)\\ y(t)\end{bmatrix},\ \bar{L}=\begin{bmatrix} L^R & -L^I\\ L^I & L^R\end{bmatrix},\ \bar{H}=\begin{bmatrix} H^R & -H^I\\ H^I & H^R\end{bmatrix}.
$$

From (13), which implies that

$$
\begin{aligned}
\dot{V}(t)-\bar{w}^T(t)\bar{S}(t) &\leqslant x^T(t)\Xi_1x(t)+y^T(t)\Xi_2y(t)+x^T(t)\Xi_3y(t)+y^T(t)\Xi_3^Tx(t)+x^T(t)[2P]w^R(t)\\
&\quad+y^T(t)[2P_1]w^I(t)+w^{R^T}(t)L^Rx(t)-w^{R^T}(t)L^Iy(t)+w^{I^T}(t)L^Ix(t)\\
&\quad+w^{I^T}(t)L^Ry(t)+w^{R^T}(t)H^Rw^R(t)+w^{I^T}(t)H^Iw^R(t)-w^{R^T}(t)H^Iw^I\\
&\quad+w^{I^T}(t)H^Rw^I-2k_1\lambda_{\min}(P)\left[\left(\sum_{i=1}^{n}|x_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}}-2k_2\lambda_{\min}(P_1)\left[\left(\sum_{i=1}^{n}|y_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}}\\
&\leqslant \eta^T\Lambda\eta-2k_1\lambda_{\min}(P)\left[\left(\sum_{i=1}^{n}|x_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}}-2k_2\lambda_{\min}(P_1)\left[\left(\sum_{i=1}^{n}|y_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}}\\
&\leqslant -2k_1\lambda_{\min}(P)\left[\lambda_{\max}^{-1}(P)V^R(t)\right]^{\frac{\alpha+1}{2}}-2k_2\lambda_{\min}(P_1)\left[\lambda_{\max}^{-1}(P_1)V^I(t)\right]^{\frac{\alpha+1}{2}}, \qquad (14)
\end{aligned}
$$

then it yields

$$
\begin{aligned}
\bar{w}^T(t)\bar{S}(t) &\geqslant \dot{V}(t)+\max\left[2k_1\lambda_{\min}(P)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P),2k_2\lambda_{\min}(P_1)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P_1)\right]\left(V^R(t)\right)^{\frac{-\alpha+1}{2}}+\left(V^I(t)\right)^{\frac{-\alpha+1}{2}}\\
&\geqslant \dot{V}(t)+\max\left[2k_1\lambda_{\min}(P)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P),2k_2\lambda_{\min}(P_1)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P_1)\right](V(t))^{\frac{-\alpha+1}{2}}, \qquad (15)
\end{aligned}
$$

where $\eta=\big(x(t)\ y(t)\ w^R(t)\ w^I(t)\big)^T$.

Therefore, from Definition 1, the CVNNs (1) is finite-time passive under the control inputs (5) and (6). The proof is completed here. ∎

Theorem 2. *Suppose that the Assumptions 1–3 hold. CVNNs* (1) *is finite-time output strictly passive under the designed controller* (5)–(6)*, if there exist positive symmetry matrices* P*,* P_1*,* Q_1*,* Q_2*,* Q_3*,* Q_4 *and nonzero constants* k_1*,* k_2*,* λ_1*,* λ_2*,* λ_3*,* λ_4*,* ζ_1 *such that the following LMI holds:*

$$\Lambda = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ \Xi_{21} & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & \Xi_{34} \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned}
\Xi_{11} &= \left(-PC - C^T P\right) + \lambda_1 P A^R Q_1 A^{R^T} P + \lambda_1^{-1} \mu^{RR^T} Q_1^{-1} \mu^{RR} - \lambda_2 P A^I Q_2 A^{I^T} P + \lambda_2^{-1} \mu^{IR^T} Q_2^{-1} \mu^{IR} \\
&\quad + \lambda_4^{-1} \mu^{RR^T} Q_4^{-1} \mu^{RR} + \zeta_1 (L^{R^T} L^R + L^{I^T} L^I), \\
\Xi_{22} &= \left(-P_1 C - C^T P_1\right) + \lambda_3 P_1 A^R Q_3 A^{R^T} P_1 + \lambda_1^{-1} \mu^{RI^T} Q_1^{-1} \mu^{RI} + \lambda_2^{-1} \mu^{II^T} Q_2^{-1} \mu^{II} \\
&\quad + \lambda_3^{-1} \mu^{II^T} Q_3^{-1} \mu^{II} + \lambda_4 P_1 A^I Q_4 A^{I^T} P_1 + \lambda_4^{-1} \mu^{RI^T} Q_4^{-1} \mu^{RI} + \zeta_1 (L^{R^T} L^R + L^{I^T} L^I), \\
\Xi_{12} &= \lambda_1^{-1} \mu^{RR^T} Q_1^{-1} \mu^{RI} + \lambda_2^{-1} \mu^{IR^T} Q_2^{-1} \mu^{II} + \lambda_3^{-1} \mu^{IR^T} Q_3^{-1} \mu^{II} + \lambda_4^{-1} \mu^{RR^T} Q_4^{-1} \mu^{RI} \\
&\quad - \zeta_1 \left(L^{R^T} L^I + L^{I^T} L^R\right), \\
\Xi_{21} &= \lambda_1^{-1} \mu^{RI^T} Q_1^{-1} \mu^{RR} + \lambda_2^{-1} \mu^{II^T} Q_2^{-1} \mu^{IR} + \lambda_3^{-1} \mu^{II^T} Q_3^{-1} \mu^{IR} + \lambda_4^{-1} \mu^{RI^T} Q_4^{-1} \mu^{RR} \\
&\quad - \zeta_1 \left(L^{I^T} L^R + L^{R^T} L^I\right), \\
\Xi_{13} &= 2P + \zeta_1 (L^{R^T} H^R + L^{I^T} H^I), \quad \Xi_{14} = \zeta_1 (-L^{R^T} w^I + L^{I^T} w^I), \quad \Xi_{23} = \zeta_1 (-L^{I^T} H^R + L^{R^T} H^I), \\
\Xi_{24} &= 2P_1 + \zeta_1 (L^{I^T} H^I + L^{R^T} H^R), \quad \Xi_{31} = \zeta_1 (-L^R + H^{R^T} L^R + H^{I^T} L^I), \\
\Xi_{32} &= \zeta_1 (L^I - H^{R^T} L^I + H^{I^T} L^R), \quad \Xi_{33} = \zeta_1 (H^R + H^{R^T} H^R + H^{I^T} H^I), \\
\Xi_{34} &= \zeta_1 (H^I - H^{R^T} H^I + H^{I^T} H^R), \quad \Xi_{41} = \zeta_1 (-L^I - H^{I^T} L^R + H^{R^T} L^I), \\
\Xi_{42} &= \zeta_1 (-L^R + H^{I^T} L^I + H^{R^T} L^R), \quad \Xi_{43} = \zeta_1 (-H^I - H^{I^T} H^R + H^{R^T} H^I), \\
\Xi_{44} &= \zeta_1 (-H^R + H^{I^T} H^I + H^{R^T} H^R).
\end{aligned}$$

Proof. In this proof, we can consider the same Lyapunov function as in Theorem 4. Then, one has

$$\dot{V}(t) - \bar{w}^T(t)\bar{S}(t) + \zeta_1 \bar{S}^T(t)\bar{S}(t) \leqslant \dot{V}(t) - \bar{w}^T(t)\bar{L}\bar{Z}(t) - \bar{w}^T(t)\bar{H}\bar{w}(t) + +\zeta_1 \bar{S}^T(t)\bar{S}(t).$$

That is to say,

$$\begin{aligned}
\dot{V}(t) - \bar{w}^T(t)\bar{S}(t) + \zeta_1 \bar{S}^T(t)\bar{S}(t) &\leqslant \dot{V}(t) - \bar{w}^T(t)\bar{L}\bar{Z}(t) - \bar{w}^T(t)\bar{H}\bar{w}(t) + \eta_1 \bar{S}^T(t)\bar{S}(t), \\
&\leqslant x^T(t)\Xi_{11}x(t) + y^T(t)\Xi_{22}2y(t) + x^T(t)\Xi_{12}y(t) + y^T(t)\Xi_{21}x(t) + x^T(t)\Xi_{13}w^R(t) \\
&\quad + y^T(t)\Xi_{24}w^I(t) + x^T(t)\Xi_{14}w^I(t) + w^{R^T}(t)\Xi_{31}x(t) + w^{R^T}(t)\Xi_{33}w^R(t) \\
&\quad + w^{R^T}(t)\Xi_{32}y(t) + w^{R^T}(t)\Xi_{34}w^I(t) + y^T(t)\Xi_{23}w^R(t) + w^{I^T}(t)\Xi_{41}x(t) \\
&\quad + w^{I^T}(t)\Xi_{43}w^R(t) + w^{I^T}(t)\Xi_{42}y(t) + w^{I^T}(t)\Xi_{44}w^I(t) \\
&\quad - 2k_1\lambda_{\min}(P)\left[\left(\sum_{i=1}^n |x_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\left(\sum_{i=1}^n |y_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}} \\
&\leqslant \eta^T\Lambda\eta - 2k_1\lambda_{\min}(P)\left[\left(\sum_{i=1}^n |x_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\left(\sum_{i=1}^n |y_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}} \\
&\leqslant -2k_1\lambda_{\min}(P)\left[\lambda_{\max}^{-1}(P)V^R(t)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\lambda_{\max}^{-1}(P_1)V^I(t)\right]^{\frac{\alpha+1}{2}},
\end{aligned} \tag{17}$$

then it yields

$$\begin{aligned}
\bar{w}^T(t)\bar{S}(t) - \zeta_1\bar{S}^T(t)\bar{S}(t) &\geqslant \dot{V}(t) + \max\left[2k_1\lambda_{\min}(P)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P), 2k_2\lambda_{\min}(P_1)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P_1)\right]\left(V^R(t)\right)^{\frac{-\alpha+1}{2}} \\
&\quad + \left(V^I(t)\right)^{\frac{\alpha+1}{2}} \geqslant \dot{V}(t) + \max\left[2k_1\lambda_{\min}(P)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P), 2k_2\lambda_{\min}(P_1)\lambda_{\max}^{\frac{-\alpha+1}{2}}(P_1)\right](V(t))^{\frac{\alpha+1}{2}}.
\end{aligned} \tag{18}$$

Therefore, from Definition 2 the CVNNs (1) is finite-time output strictly passive under the control inputs (5)) and (6). The proof is completed here. ■

Corollary 1. *Suppose that the Assumptions 1–3 hold. CVNNs* (1) *is finite-time input strictly passive under the designed controller* (5))–(6)*, if there exist positive symmetry matrices* P*,* P_1*,* Q_1*,* Q_2*,* Q_3*,* Q_4 *and nonzero constants* k_1*,* k_2*,* λ_1*,* λ_2*,* λ_3*,* λ_4*,* ζ *such that the following LMI holds:*

$$\Lambda_1 = \begin{bmatrix} \Xi_1 & \Xi_2 & 2P & 0 \\ \Xi_4 & \Xi_2 & 0 & 2P_1 \\ -L^R & L^I & -H^R+\zeta & H^I \\ -L^I & -L^R & -H^I & -H^R+\zeta \end{bmatrix} < 0. \tag{19}$$

Proof. Let us construct the same Lyapunov function as Theorem 4 and 5. Then,

$$\begin{aligned} &\dot{V}(t) - \bar{w}^T(t)\bar{S}(t) + \zeta\bar{w}(t)w(t) \\ &\quad\leqslant x^T(t)\Xi_1 x(t) + y^T(t)\Xi_2 2y(t) + x^T(t)\Xi_3 y(t) + y^T(t)\Xi_4 x(t) + x^T(t)[2P]w^R(t) \\ &\qquad + y^T(t)[2P_1]w^I(t) - w^{R^T}(t)L^R x(t) - w^{R^T}(t)L^I y(t) + w^{I^T}(t)L^I x(t) + w^{I^T}(t)L^R y(t) \\ &\qquad + w^{R^T}(t)(-H^R+\zeta)w^R(t) + w^{I^T}(t)(-H^I)w^R(t) + w^{R^T}(t)H^I w^I + w^{I^T}(t)(-H^R+\zeta)w^I \\ &\qquad - 2k_1\lambda_{\min}(P)\left[\left(\sum_{i=1}^n |x_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\left(\sum_{i=1}^n |y_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}}, \\ &\quad\leqslant \eta^T\Lambda_1\eta - 2k_1\lambda_{\min}(P)\left[\left(\sum_{i=1}^n |x_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\left(\sum_{i=1}^n |y_i(t)|^2\right)\right]^{\frac{\alpha+1}{2}}, \\ &\quad\leqslant -2k_1\lambda_{\min}(P)\left[\lambda_{\max}^{-1}(P)V^R(t)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\lambda_{\max}^{-1}(P_1)V^I(t)\right]^{\frac{\alpha+1}{2}}. \end{aligned} \tag{20}$$

Then it can be conclude that

$$\bar{w}^T(t)\bar{S}(t) - \zeta\bar{w}(t)w(t) \geqslant \dot{V}(t) - 2k_1\lambda_{\min}(P)\left[\lambda_{\max}^{-1}(P)V^R(t)\right]^{\frac{\alpha+1}{2}} - 2k_2\lambda_{\min}(P_1)\left[\lambda_{\max}^{-1}(P_1)V^I(t)\right]^{\frac{\alpha+1}{2}}.$$

Therefore, from Definition 3 the CVNNs (1) is finite-time input strictly passive under the control inputs (5) and (6). The proof is completed here. ■

Remark 1. In [47], Zeng and Xiao have investigated the finite-time passivity of neural networks with multiple time-varying delays. By employing inequalities technique and Lyapunov stability theory to guarantee the finite-time passivity under the delayed control scheme. In [48], have considered the passivity and stability analysis of complex valued neural networks with time varying delays, in which the uncertainty of norm bounded parameters to achieve more realistic system behaviors. In this paper, the delayed control scheme have been established, this can be used to ensure finite time passivity of complex valued neural networks with multiple time varying delays, in this case, the activation functions are not necessary to be restricted in order to be bounded.

Remark 2. It is worth noting that, if the CVNN (1) is finite-time passivity (strictly finite-time input and output passive) with the control inputs (5) and (6), then according to [47] the CVNN (1) can be realized stabilization in finite-time while the relevant requirements of Theorems 4, 5 or Corollary 1 are satisfied.

Remark 3. If we choose $w(t) = 0$ the CVNN (1) can be written as

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-d(t))) + Eg(z(t-\tau(t))) + u(t), \tag{21}$$

then according to Remark 3.5 and controllers (5) and (6), we can easily obtain that CVNN (21) is finite-time stable.

4. Numerical example

In this section, we present two simulation examples to demonstrate the utility of the theoretical conclusions.

Example 1. Consider the CVNN with multiple time-varying delays as follows:

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-d(t))) + Eg(z(t-\tau(t))) + w(t) + u(t), \tag{22}$$

where

$$z(t) = (z_1(t), z_2(t))^T, \qquad C = \operatorname{diag}\{1,1\},$$
$$A = \begin{pmatrix} 1.2 + i\,0.7 & 2.6 + i\,1.5 \\ 6.2 - i\,1.3 & -1.3 + i\,0.5 \end{pmatrix}, \qquad B = \begin{pmatrix} 1.3 + i\,0.4 & -1.5 + i\,0.8 \\ 0.3 + i\,2.5 & 3.2 + i\,1.7 \end{pmatrix},$$
$$E = \begin{pmatrix} 0.3 + i\,0.4 & 0.5 + i\,0.8 \\ 0.3 + i\,0.5 & 0.2 + i\,0.7 \end{pmatrix}, \qquad w(t) = \begin{pmatrix} 5\sin(t) + i\,2\sin(t) \\ 2\cos(t) + i\,3\cos(t) \end{pmatrix}.$$

The activation function is chosen as $f(z(t)) = \tanh(z(t))$, $f(z_d) = \tanh(z(t-d(t)))$, $g(z_\tau) = 0.5(|z(t-\tau(t))+1| - |z(t-\tau(t))-1|)$. Moreover, the time-varying delays are taken as $d(t) = 1-\sin(t)$, $\tau(t) = \frac{e^t}{e^t+1}$, such that the Lipchitz conditions in Assumption 3 are satisfied with $\lambda^{RR} = \lambda^{RI} = \lambda^{IR} = \lambda^{II} = 1$, $\mu^{RR} = \mu^{RI} = \mu^{IR} = \mu^{II} = 1$. In addition, choose the complex-valued output as

$$s(t) = Lx(t) + Hw(t), \tag{23}$$

where

$$s(t) = (s_1(t), s_2(t))^T, \quad L = \begin{pmatrix} 0.4 + 0.9i & 0 \\ 0 & 0.3 - 0.3i \end{pmatrix}, \quad H = \begin{pmatrix} 3.4 + 1.6i & 0 \\ 0 & -0.5 - 3.8i \end{pmatrix}.$$

Then the real and imaginary parts of the control protocol is chosen as in (5) and (6). By using schur complement Lemma 1 and Matlab LMI control toolbox, it can be found that the LMI condition (7) is satisfied with the following feasible solutions:

$$Q_1 = 10^3\begin{pmatrix} 1.016 & 0 \\ 0 & 1.025 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 507.46 & 0 \\ 0 & 508.46 \end{pmatrix}, \quad Q_3 = 10^3\begin{pmatrix} 0.0169 & 0 \\ 0 & 1.0169 \end{pmatrix},$$
$$Q_4 = 10^3\begin{pmatrix} 1.0269 & 0 \\ 0 & 1.169 \end{pmatrix}, \quad R_1 = \begin{pmatrix} -0.001 & 0 \\ 0 & -0.201 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 0.002 & 0 \\ 0 & 0.03 \end{pmatrix},$$
$$R_3 = \begin{pmatrix} 0.007 & 0 \\ 0 & 0.05 \end{pmatrix}, \quad R_4 = \begin{pmatrix} 0.03 & 0 \\ 0 & 0.121 \end{pmatrix}, \quad P = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.7102 \end{pmatrix},$$
$$P_1 = \begin{pmatrix} 0.0644 & -0.0058 \\ -0.0032 & 0.0181 \end{pmatrix}.$$

Therefore, the condition (7) is satisfied with the control inputs (5) and (6) which is shown in Fig. 8, such that the CVNNs (1) is finite-time passive under the control gain matrices

$$K^R = \begin{pmatrix} 2.91 & 0 \\ 0 & 23.78 \end{pmatrix}, \quad K^I = \begin{pmatrix} 2.71 & 0 \\ 0 & 22.78 \end{pmatrix}.$$

Meanwhile, it can be observe that the state trajectories of the CVNNs (1) and their outputs (2) are depicted in Figs. 1–5, which shows that CVNNs (1) may be finite-time passive. Suppose that choose $W^R(t) = W^I(t) = 0$, then it can be observe that the state trajectories of CVNNs (1) and its output (2) is shown in Figs. 6–7, it can be conclude that the CVNNs (1) is finite-time stable under the control inputs (5) and (6) which is shown in Figs. 8–9.

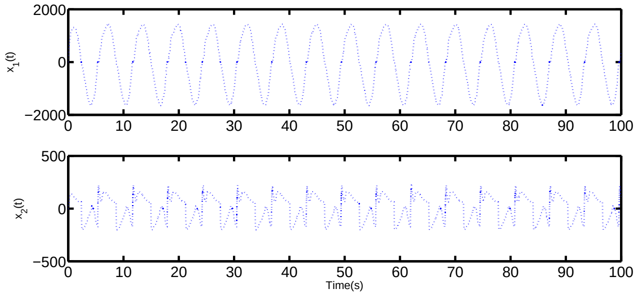

Fig. 1. The state trajectories of the real part of $z(t)$ in (22).

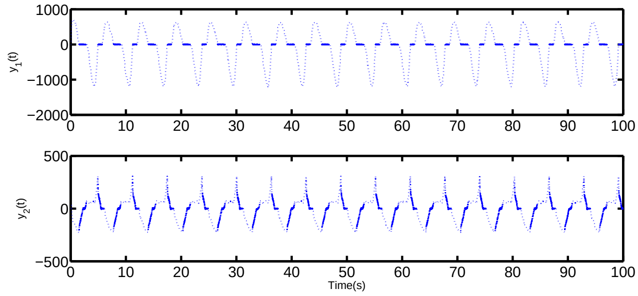

Fig. 2. The state trajectories of the imaginary part of $z(t)$ in (22).

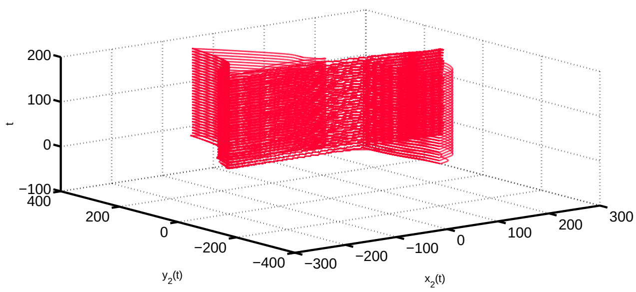

Fig. 3. Phase plot of the real and imaginary part of $z_2(t)$ in (22).

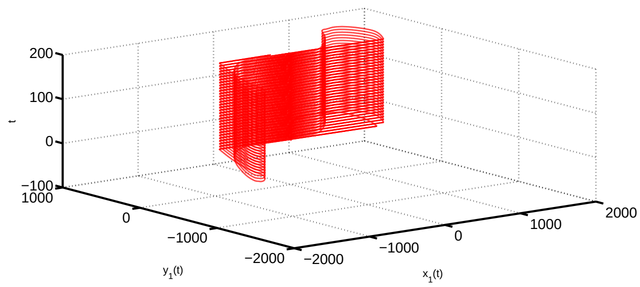

Fig. 4. Phase plot of the real and imaginary part of $z_1(t)$ in (22).

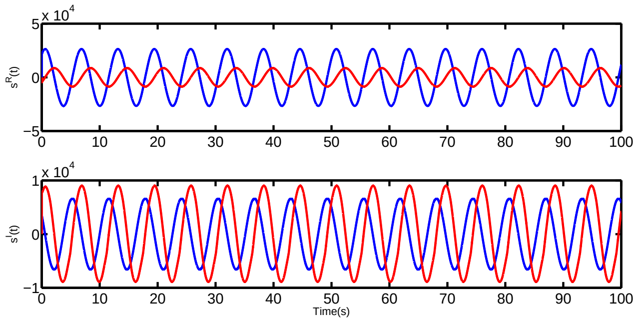

Fig. 5. Dynamics of the output vectors $s^R(t)$ and $s^I(t)$ in (23).

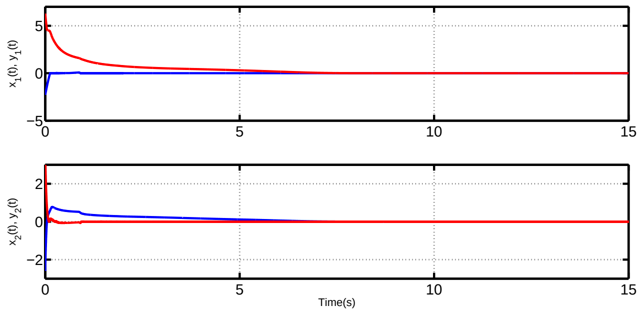

Fig. 6. The state trajectories of the real and imaginary parts of $z(t)$ in (22) when $w(t) = 0$.

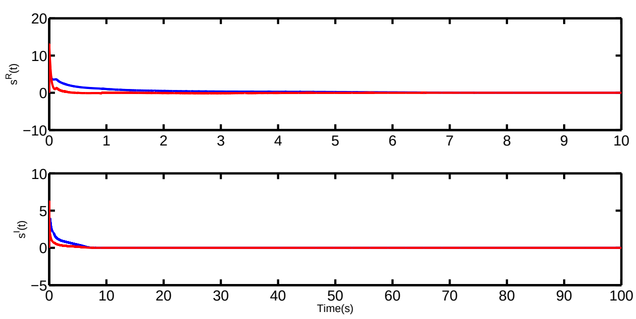

Fig. 7. Dynamics of the output vectors $s^R(t)$ and $s^I(t)$ of (23) when $w(t) = 0$.

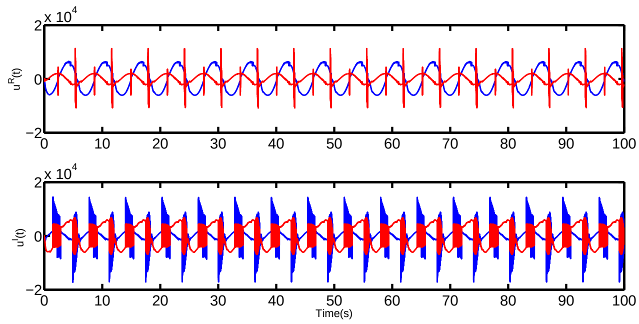

Fig. 8. Dynamics of the control inpts $u^R(t)$ and $u^I(t)$ in (5) and (6).

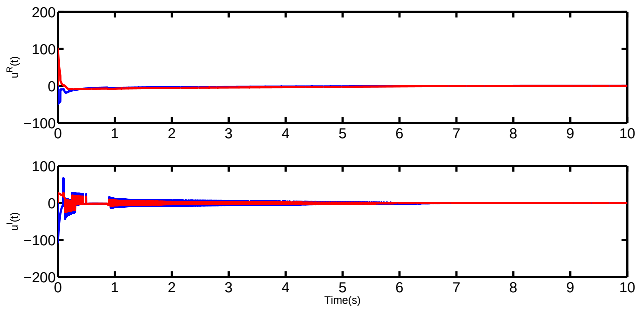

Fig. 9. Dynamics of the control inputs $u^R(t)$ and $u^I(t)$ in (5) and (6) when $w(t) = 0$.

5. Conclusion

This research paper is focused on the finite-time passivity of CVNNs with multiple time-varying delays. Moreover, different kinds of passivity concepts are proposed. In addition, we have designed the delayed feedback control law to guarantee the finite-time passivity and also we can achieve the finite-time stability for CVNNs without external disturbance. Meanwhile, the required conditions are provided using the Lyapunov stability theory and the inequality approaches, which may be solved by using Matlab Yalmip LMI toolbox. As a result, it is easy to select appropriate values for the established control laws. Finally, a simulation example is provided to show the effectiveness and feasibility of the derived results.

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Аналіз пасивності нестаціонарних комплексно-змінних нейронних мереж з обмеженою часовою затримкою

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У статті досліджується проблема скінченно-часової пасивності для комплексно-змінних нейронних мереж (КЗНМ) з декількома нестаціонарними затримками. Спочатку подано визначення, які стосуються скінченно-часових пасивностей КЗНМ; тоді відповідні керуючі входи реалізовані з метою гарантії скінченно-часової пасивності класу КНЗМ. Одночасно деякі достатні умови лінійних матричних нерівностей виведені з використанням теорії нерівностей та теорії стійкості Ляпунова. Нарешті, подано чисельний приклад для ілюстрації корисності отриманих теоретичних результатів.

Ключові слова: *пасивність, скінченно-часова пасивність, комплексно-змінні нейронні мережі, нестаціонарні затримки, функціонал Ляпунова, лінійна матрична нерівність.*