



ЕЛЕКТРОНІКА ТА ІНЖЕНЕРІЯ

**ANALYTICAL AND NUMERICAL METHODS
FOR CALCULATION THE DEEP OF PENETRATION
THE WELDING SEAM FORMED BY THE ELECTRON BEAM
GENERATED BY GLOW DISCHARGE ELECTRON GUNS**

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The article is devoted to the problem of defining the focal diameter of electron beam, formed by the glow discharge electron guns, as well as the necessary pressure in the gun chamber for realising the welding process. Taking into account, that glow discharge electron guns are widely used in industry for welding of different metals, and that for providing the high quality of welding joints estimation of energetic parameters in beam focus is very important, proposed methods are very important for effective elaboration and designing of the novel glow discharge electron guns constructions for specific technological operations. With known focal beam diameter and thermodynamic parameters of welding details material the deep of penetration of welding seam, as well as the necessary pressure in discharge chamber have been estimated. Two proposed methods are generally based on the analytical solving of explicit equation and on numerical solving of sophisticated non-linear equation. Obtained simulation results with and without taking into account the space charge of own beam electrons are also given.

Key words: *electron beam welding; deep of welding seam; glow discharge electron gun; beam focal diameter.*

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1. Introduction

High Voltage Glow Discharge Electron Guns (HVGDEG) are widely used in industry today for realizing different technological operation, including welding of thin-wall items in the soft vacuum [1, 2]. The main advantages of using the HVGDEG in welding technologies are follows [1, 2].

1. Relative simplicity of guns' construction and possibility of its disassembling for changing the spare details and repairing.

2. Relative simplicity and cheapness of technological evacuation equipment.

3. High productivity of welding technological process, especially in the case of forming the seams with the linear or ring geometry by using the profile electron beams, formed in the High Voltage Glow Discharge (HVGD) with the suitable spatial geometry of electrodes.

4. Simplicity of changing and control of electron beam current both aerodynamically by changing the pressure in discharge chamber and electrically by applying the potential on the additional electrode and, as a result, changing the concentration of ions and electrons in the discharge gap.

But it should be pointed out, that for effective designing of novel constructions of HVGDEG for advanced welding technologies and for its industrial application choosing and analyzing the dependences of welding seam deep on acceleration voltage, current of electron beam and its focal diameter are generally necessary. This technological problem is caused by the fact, that the quality of welding joints in electron-beam technologies is strongly depended on the full power of electron-beam and its power density [3].

Therefore, the aim of this article is finding and analyzing the analytical dependence for the length of welding seam deep h_s on acceleration voltage U_{ac} and beam current I_b . Since HVGD current is depended on acceleration voltage and pressure in discharge gap p_g , finally analytical dependence $h_s(U_{ac}, p_g)$ should be found and analyzed. Since the equation $h_s = h_s(U_{ac}, p_g)$ is non-linear and dependence, which can't be written in an explicit analytical form, obtained equation is solved numerically by using the Stephenson iteration method [4].

2. Analyze and statement of the problem

The main basic mathematical models for defining the deep of penetration of welding seam, as well as general classification of these models, are briefly described in the monography [3]. These models are generally based on solving the thermodynamic partial differential equation (PDE):

$$\frac{\partial T(x, y, z, t)}{\partial t} - a^2 \frac{\partial^2 T(x, y, z, t)}{\partial x^2} + \frac{\partial^2 T(x, y, z, t)}{\partial y^2} + \frac{\partial^2 T(x, y, z, t)}{\partial z^2} - \frac{\partial}{\partial t} = 0, \quad a = \frac{\lambda_m}{c\rho} \quad (1)$$

where t – time; x, y, z – space coordinate; a – coefficient of thermal diffusion; λ_m – thermal conductivity of welding metal; c – isobaric heat capacity of welding metal; and ρ – weight density of metal.

Generally, 4 methods of solving thermodynamic PDE equation (1) are considered in the literature [3–5].

1. Using of numerical methods for solving the equation (1) with defined boundary and initial conditions. For example, well-known numerical Crank – Nicolson method can be used [4]. This method usually given the high precision of solution, but it is generally very sophisticated. In additional, analyzing of analytical dependences $d_s(U_{ac}, I_b)$ is impossible. Furthermore, the accuracy of numerical solution is strongly depended on the discretization steps in the spatial coordinates [4].

2. Analytical solving of equation (1) with considering its nuclear, which basically is written as follows [3, 5]:

$$F(x, t) = \frac{1}{(2a\sqrt{\rho t})^n} \exp\left(-\frac{|x|}{4a^2 t}\right) \quad (2)$$

This method has some advantages relatively to the first one, because obtained analytical solution can be analyzed for finding the optimal energetic parameters of electron gun. But, in any case, forming of functional rows with using equation (2) is also sophisticated task [5]. In additional, convergence of such functional rows is usually the specific separate mathematical problem [5].

3. Excluding the time parameter from equation (1) and simplifying in to the Boltzmann Thermodynamic Equation (BTE) for stationary thermodynamic task. In the general form BTE is written as follows:

$$P_b = \frac{S_c (T_s - T_a) l_m(T)}{l_m(T)}, \quad P_b = U_{ac} I_b \quad (3)$$

where S_c – the square of welding contact; T_s – the temperature of welding seam; T_a – temperature of air in the welding chamber; l_m – the length of welded items. Equation (3) can be rewritten as explicit dependence $T_s(U_{ac}, I_b)$ as follows:

$$T_s(U_{ac}, I_b) = \frac{U_{ac} I_b l_m(T_s)}{l_m(T_s) S_c} + T_a. \quad (4)$$

Using of equation (4) is the simple and suitable mathematic model for defining the average temperature of seam in electron beam welding process [3], but necessary geometry parameter of welding seam, namely, the deep of penetration, is still unknown. For defining this parameter using of additional mathematical relations is needed. Also, the problem is, that the length of welded items l_m and the thermal conductivity of welding metal λ_m are the parameters, which are usually strongly depended on the temperature of welding place T_s . When dependences $l_m(T_s)$ and $\lambda_m(T_s)$ are strong non-linear, equation (4) can be solved only numerically relatively to T_s .

4. Finding, with taking into account considered boundary conditions, explicit analytical dependence $h_s(U_{ac}, I_b)$. Such relations were deeply considered and analyzed in the monography [3] and some of them will be presented at the next part of this article.

3. Analytical relations for defining the deep of welding seam with known energetic parameters of electron beam

For estimation the deep of seam penetration in the technology of electron-beam welding such approximation are generally used [3].

1. Generalized estimation, based on the presumptions, proposed by Hablanian [3, 6]:

$$h_s = 0.2 \sqrt{\frac{2a}{v_w r_b}} * \frac{U_{ac} I_b}{4 l_m T_m (1 + \frac{L_m}{c T_m})}, \quad (5)$$

where v_w – velocity of welding, r_b – focal radius of electron beam, T_m – the temperature of melting of welding metal, L_m – specific heat of metal fusion.

2. Improved estimation for the seams with high deep of penetration, proposed by Lopatko, Kartashov and Tkachov [3, 7]:

$$h_s = \frac{0.132 U_{ac} I_b}{l_m T_m \frac{v_w r_b}{a} + \frac{L_m}{c T_m}}^{0.815}. \quad (6)$$

Since in the HVGDEG the focal beam radius is usually in the range of few mm [1, 2], estimation with using the equation (5) in such physical conditions is usually corrected. For direct calculation the beam current with known required value of h_s , which is defined by the particularities of realized technological process, equation (5) can be rewritten as follows:

$$I_b(h_s) = \frac{20 h_s l_m T_m \frac{v_w r_b}{a} + \frac{L_m}{c T_m} \sqrt{\frac{v_w r_b}{2a}}}{U_{ac}}. \quad (7)$$

The disadvantage of equations (5)–(7) is that electron beam focal radius r_b is always strongly depended on the acceleration voltage U_{ac} and the beam current I_b [8, 9]. Corresponded analytical relations will be considered in the next sections of the article.

4. Simplified estimation of operation pressure in high voltage glow discharge electron gun without taking into account the space charge of beam electrons

In the monography [10] the following relation for calculation the focal radius r_b have been proposed:

$$r_b(U_{ac}) = \frac{r_c}{\alpha} \sqrt{\frac{\Delta U}{U_{ac}}}, \quad (8)$$

where r_c – the radius of cathode emission zone; α – the convergence angle of electron beam, defined by the cathode sphere radius and transverse radius of HVGD electrodes system; ΔU – dissipation of the beam electrons by the velocity, which for the standard regimes of HVGD lighting is usually in range of 30 V [10].

Dependence of beam current on the acceleration voltage and on pressure in the volume of discharge lighting also presented in papers [1, 2] and in monography [10]. It is the result of analytical solving the Poisson equation for specific physical conditions of HVGD lighting and written as follows [10]:

$$I_b = AU_{ac}^k p_g^m, \quad (9)$$

where A, k, m – semiempirical coefficients, defined by the cathode material and using operation gas [1, 2].

With taking into account (8), (9), the equation (7) is rewritten as follows:

$$p_g(h_s, U_{ac}) = \frac{20h_s | T_m \left(\frac{\Delta U}{U_{ac}} \right)^{0.25} \left[1 + \frac{L_m}{cT_m} \sqrt{\frac{v_w r_c}{2aa}} \right]^{\frac{1}{\alpha}}}{AU_{ac}^{1.25+k}}. \quad (10)$$

Obtained equation (10) is simple and generally can be used for the rough estimation of the pressure of operation gas with known value of deep of penetration of welding seam. But, by the physical point of view, such estimation is generally correct only for the small values of beam current, range of 10–50 mA. For larges values of beam current using of relation (8) is incorrect, since the focal beam diameter have the significantly larges value. The reason of enlarging the focal beam diameter for highest value of beam current is influence of intrinsic space charge of beam electrons [1, 2, 8, 9].

5. Estimation of operation pressure in high voltage glow discharge electron gun with taking into account the space charge of beam electrons

From the basic theory of the electron beam physic the following relation for defining the focal radius of electron beam is well-known [8, 9]:

$$\ln \frac{r_c}{r_b} = 1.04 * 10^{-3} \frac{U_{ac}^{1.5}}{l_b} \operatorname{tg}(a), \quad (11)$$

Writing explicitly the dependence $r_b(r_c, \alpha, U_{ac}, I_b)$ form equation (11) and substituting the obtained result into equation (7), the following relation for defining the pressure in the HVGD discharge gap p_g can be obtained:

$$f(p_g, h_s, U_{ac}) = \frac{20h_s | T_m \left[1 + \frac{L_m}{cT_m} \sqrt{\frac{v_w r_c}{2aa}} \right]^{\frac{1}{\alpha}}}{U_{ac} \sqrt{2a \exp \left[1.04 * 10^{-3} \operatorname{tg}(a) U_{ac}^{1.5} \right] AU_{ac}^k p_g^m}} - AU_{ac}^k p_g^m = 0, \quad (12)$$

Relation (12) is non-linear and can be solved numerically. For obtaining numerical solution of equation (12) the iteration Stephenson method was used. Corresponded iteration formula is written as follows:

$$p_{g_{n+1}} = p_{g_n} - \frac{f^2(p_{g_n})}{f(p_{g_n} + f(p_{g_n})) - f(p_{g_n})}. \quad (13)$$

During solving the equation (12) by using the iterative relation (13) it is assumed, that the values h_s and U_{ac} are constant. The precision of solution was chosen as $\Delta p_g = 0.001 \text{ Pa}$ and with obtaining such level of precision the iterative process has been interrupted.

The pervious investigations of relation (12) with analyzing function $f(p_g, h_s, U_{ac})$ and study its derivation are shown, that for range of values by acceleration voltages 10–30 kV and by the deep of welding seam 0.0002–0.01 m the equation (12) always have only one solution and the iterative process (13) converges steadily.

Obtained simulation results are presented and analyzed at the next part of the article.

6. Results of simulation and its discussion

Testing simulation results was obtaining for the welding of titanium plane items in the hydrogen medium. Corresponded thermodynamic and technological parameters of calculation model for this process, that have been used for providing the calculation tests, are presented at the table. All thermodynamic parameters for melting the titanium items in hydrogen medium were take correspondently to handbooks [11, 12].

**Corresponded parameters for mathematic model of technological process
of welding the titanium items in hydrogen**

Types of parameters	Parameter	Value
Thermodynamic	Coefficient of thermal diffusion, a	$25 \cdot 10^6 \frac{\text{m}^2}{\text{s}}$
	Thermal conductivity of titanium, λ_m	$15 \frac{\text{W}}{\text{m} \cdot \text{K}}$
	Temperature of melting of titanium, T_m	1941 K
	Specific heat of titanium fusion, L_m	$18,8 \cdot 10^3 \frac{\text{J}}{\text{mol}}$
	Isobaric heat capacity of titanium, c	$600 \frac{\text{J}}{\text{kg} \cdot \text{K}}$
Technological	Acceleration voltage U_{ac}	5 – 20 kV
	Welding speed, v_w	$2 \frac{\text{m}}{\text{min}}$
	Cathode radius, r_c	0.7 m
	Beam converge angle, α	0.2618 rad
Coefficients of HVGD current-voltage characteristic in equation (9)	Coefficient A	$97 \frac{\text{A}}{\text{V} \cdot \text{Pa}}$
	Coefficient k	$k = 1.4$
	Coefficient m	$m = 1.5$

Unknow semiempirical coefficients of HVGD current-voltage characteristic A , k and m for different pressure in the discharge gap was defined experimentally on the educational technological equipment of Electron Beam Technological Devices Laboratory, Electronic Devices and Systems Department, Faculty of Electronics, National Technical University of Ukraine “Igor Sikorsky Kiev Polytechnic Institute”, as well as on industrial electron-beam technological equipment at Private Actioner Society, Scientific and Industrial Association “Chervona Hvylya”.

The graphic dependences, obtained by direct solving of equation (10) with applying the simplified model without taking into account the influence of intrinsic space charge of beam electrons, are presented at Fig. 1, *a*. In the contrary, the graphic dependences, obtained by numerical solving of non-linear equation (12) with using iterative formula (13), are presented at Fig. 1, *b*.

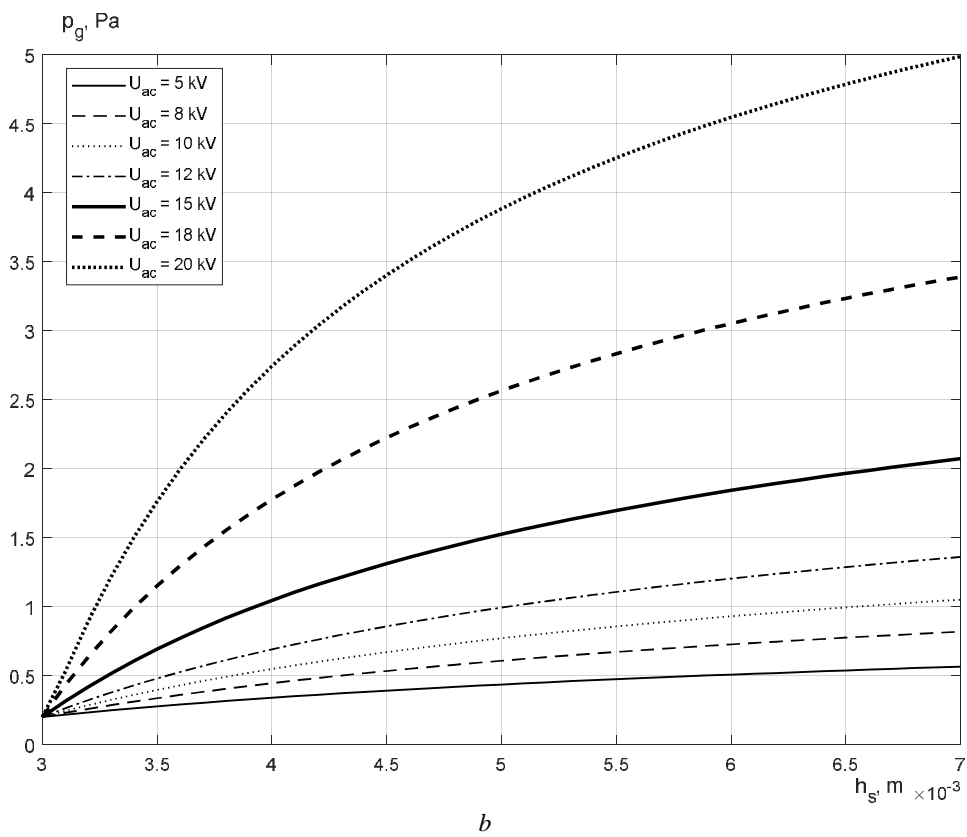
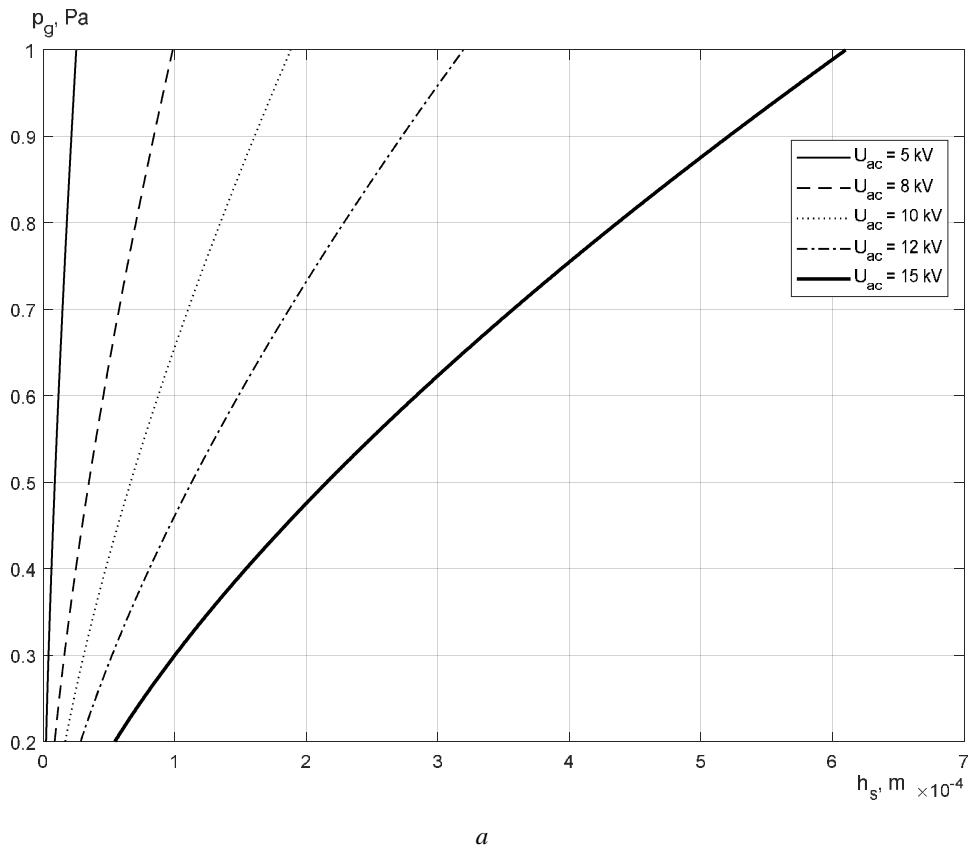


Fig. 1. Dependences of the deep of welding seam penetration on the acceleration voltage and pressure in discharge chamber. a – without taking into account the space charge of beam electrons; b – with taking into account the space charge of beam electrons

Generally clear, that with increasing the power of electron beam, namely, the acceleration voltage U_{ac} and beam current I_b , the deep of welding seam also became greatly. But the dependences, presented at Fig. 1, *a*, are basically steeper and have the largest derivation, therefore, corresponding to these dependences the value of h_s is generally higher. Furthermore, in dependences, presented at Fig. 1, *b*, reducing of deep of welding seam h_s with increasing the acceleration voltage U_{ac} is observed. It can be explained by the enlarging of beam focal radius.

In any case, using of simple formula (10) without providing long-time numerical calculations, allows obtain the corresponded results immediately. Necessity of using more cumbersome numerical calculations is defined by the specific engineering tasks for designing of welding HVGD electron gun and by the particularities of welding process [3].

Conclusion

Analytical dependences, which allows to estimate the preliminary value of the pressure in discharge chamber of HVGD electron sources, designed for electron-beam welding, are presented in the article. The basic parameters of proposed models are: acceleration voltage U_{ac} , electrical and geometrical parameters of considered HVGD electrodes system, thermodynamic properties of welding metals, as well as necessary deep of penetration of welding seam h_s . In the article two different models are proposed. Simple analytical equation (10) allows to provide the direct calculations, but influence of intrinsic space charge of beam electrons isn't taking into account in it. In contrary, numerical solving of sophisticated equation (12) with using iterative formula of Stephenson method (13) allow take into account the dependence of focal beam radius on acceleration voltage U_{ac} and beam current I_b . The testing calculations, provided for welding of titanium items in the hydrogen medium with using the relations (10) and (12), allows to estimate correctly the technological possibilities of HVGD electron guns, as well as find necessary value of pressure in the discharge chamber. The article may be interesting to the experts in the branch of technology of electron-beam welding, as well as in the branch of designing the industrial electron-beam welding equipment.

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АНАЛІТИЧНІ ТА ЧИСЕЛЬНІ МЕТОДИ ДЛЯ РОЗРАХУНКУ ГЛИБИНИ ПРОНИКАННЯ ЗВАРЮВАЛЬНОГО ШВА, ЯКИЙ ФОРМУЄТЬСЯ ЕЛЕКТРОННИМ ПУЧКОМ, ЩО ГЕНЕРУЄТЬСЯ ГАЗОРОЗРЯДНОЮ ЕЛЕКТРОННОЮ ГАРМАТОЮ

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Розглянуто проблему визначення фокального діаметра електронного пучка, який формується газорозрядною електронною гарматою, а також необхідного тиску в камері гармати для реалізації процесу зварювання. Зважаючи на те, що газорозрядні електронні гармати широко використовують у промисловості для зварювання різних металів, а також на те, що для забезпечення високої якості зварювальних з'єднань вкрай важлива оцінка енергетичних параметрів електронного пучка, запропоновані методи вельми важливі для подальшого ефективного розроблення нових газорозрядних електронних гармат для специфічних технологічних операцій. За умови відомого фокального діаметра електронного пучка та термодинамічних параметрів зварюваних деталей оцінені глибина проникнення зварювального шва та необхідний тиск у розрядній камері. Загалом два запропоновані методи ґрунтуються на аналітичному розв'язуванні простого рівняння та на чисельному розв'язуванні складного нелінійного рівняння. Також у статті наведено результати моделювання, отримані з урахуванням та без урахування просторового заряду електронного пучка.

Ключові слова: електронно-променеє зварювання; глибина зварювального шва; фокальний діаметр пучка.