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LORENTZ FORCE IN VORTEX ELECTRIC FIELD

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Abstract In our time, the functions of the law of interaction of moving charged bodies have been taken over entirely by the relativistic theory under the guise of a pseudo-slogan about the inability of Galileo's transformations. In contrast, the article adapts the Lorentz force to the case of moving charges in the all-possible range of velocities in the usual three-dimensional Euclidean space and time. This takes into account the finite rate of propagation of an electric field and the law of conservation of electric charge. On this basis, the trajectory of the free motion of electron in a non-uniform electric field generated by a positively charged spherical body is simulated.

Keywords: Lorentz force, moving charges, Eucli–dean space, finite rate of propagation of the electric field.

1. Introduction

The main use of the Lorentz force (its special case is the Ampere force) is electric machines. It is also widely used in electronic devices to influence charged particles (electrons, ions), such as in electron beam tubes, as well as in mass spectrometry, MHD generators. In charged particle accelerators, it sets the orbit in which these particles move. The Lorentz force is successfully used in non-contact measurement of the velocity of a conductive fluid. When molten metal or conductive fluid moves through a magnetic field, eddy currents appear inside the fluid, the interaction of which with the resultant field leads to the appearance of a Lorentz force, which in turn depends on the conductivity and velocity of the fluid. Recently, the Lorentz force has found an interesting application in electrodynamic mass accelerators as a promising weapon – electromagnetic guns (railguns). The use of gunpowder has reached its limit – the speed of the charge released with their help is limited to 2.5 km/s, while a modern electromagnetic gun accelerates a conductive projectile to 7.2 km/s. Defeat the target is equated to nuclear. But there is a bright side to this story as well – the possibility to use new weapons in the future to combat the asteroids threatening our planet. In most cases, the electromagnetic field is heterogeneous and very complex in its structure. Therefore, the dynamics of the motion of charged bodies in such a field is a complex problem of theoretical electricity.

Many scientific publications are devoted only to the dynamics of electron's motion in an electric field, but the vast majority of them cover the range of pre-relativistic velocities. Sufficient accuracy for moving charges in such a range is provided by Coulomb's law of electric interaction and magnetic interaction of the so-called Lorentz force.

At relativistic velocities one has to resort to complex equations of the theory of relativity, which are not always applicable and which in most cases cannot be used in practice. Therefore, there is a reasonable question, "Why don't we simplify the problem by adapting the Coulomb law to the case of moving charges in the usual three-dimensional space and physical time?".

It is clear that such a solution to the problem is not in favor of the relativists, and therefore they declared Galileo's transformation as unsounded, despite the reservations made over a hundred years ago by Henry Poincare on the Lorentz transformations [1]. In [1] we read: "Poincare's particular view of the new theory was not given serious importance. Much later, as far as in the second half of XX century it became apparent that Poincare was entirely correct in claiming that no physical experience could confirm the truth of some transformations and reject others as inadmissible.

The origins of the misunderstanding of Poincare's views lie in the disclosure of the conditional nature of simultaneity. This resulted in the possibility of misunderstanding this theory with the main focus on the "failure" of Galileo's transformations. This misunderstanding is reflected in the accepted logic of constructing the theory of relativity, when new properties of motion at high speeds are deduced from the relativistic properties of space and time".

One hundred years later, C. Karavashkin is much more radical [2]: "The statements of relativists about the inability of classical physics as a whole, as well as Newton's law of universal gravitation to describe dynamic processes, are erroneous. The only requiment to adapt Newton's law to the realm of dynamic fields is to correctly take into account the finite speed of the gravitational field propagation. Only a return to the original three-dimensional linear space plus the time of classical physics will return scientists to the path of studying precisely physical processes, and not juggling symbols and searching for nonexistent covariance.

2. The Lorentz force

The expression of Lorentz force in vector form is well known

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{1}$$

where **F** is the force vector; **E** is the vector of electric field intensity in instantaneous electrical interaction; B is the magnetic induction vector; v is the velocity vector; q is the charge of a moving body.

How the special theory of relativity (STR) disposes of the Lorentz force can be judged by [3]. They write an expression for the Coulomb force of a point charge Qrelative to a frame of reference K', moving in vacuum at a velocity $\mathbf{u}$ relative to a frame K in which the charge Q is at rest and the charge q moves at a velocity v relative to it (Q). If you enter a notation

$$\mathbf{E} = \frac{Q\gamma\mathbf{r}}{\left(r^2 + \gamma^2 \frac{(\mathbf{r}\cdot\mathbf{u})^2}{c^2}\right)^{\frac{3}{2}}}; \quad \mathbf{B} = \frac{1}{c}[\mathbf{u} \times \mathbf{E}], \tag{2}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}, \tag{3}$$

then formula (1) looks as follows

$$\mathbf{F} = q\left(E\mathbf{r} + \frac{1}{c^2}\left(\mathbf{v} \times E\gamma[\mathbf{u} \times \mathbf{r}]\right)\right) = \\ = q\mathbf{E} + \frac{q}{c}[\mathbf{v} \times \mathbf{B}]. \tag{4}$$

Hence, it is obvious that the magnetic field is a relativistic effect associated with the delay of the electric field displacement (due to the finite speed of interaction propagation) when its source moves with the speed $\mathbf{u}$, or, purely kinematically, due to the transformation of the expression for the interaction force when passing from one inertial frame of reference to the other. When the charge creating the field is at rest, the expression for the Lorentz force transforms into the Coulomb law. Similar results can be reached bypassing the STR in physical space and time [4] without assocoating oneself with an inertial or non-inertial frame of reference, what will be also shown below by us, but much simpler and more generalized.

3. Coulomb's law of moving charges

The law of electrical interaction of point-charged bodies was established experimentally in 1785 by Sh. Coulomb. Coulomb theory laid the foundations of electrostatics. The law describes the electrical force of interaction of fixed charged bodies

$$\mathbf{F} = k\frac{q_1 q_2}{r^2}\mathbf{r}_0, \tag{5}$$

where **F** is the vector of forces of the interacting charged bodies with the charges q_1 and q_2, r is the distance between the electrical centers of the bodies; $\mathbf{r}_0$ is the single vector; k is the electrical constant

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987742 \cdot 10^9 \text{ Nm}^2\text{C}.$$

The limitation by immobility of interacted charges can be interpreted as propagation of an electric field at unlimited speed, or the so-called instantaneous electrical interaction. In fact, according to modern ideas, both electric and gravitational fields propagate with the maximum possible physical velocity c = 3.108 m/s. Therefore, to adapt the law (5) to the actual conditions, it is sufficient to consider the fact of a temporary electrical delay! Otherwise, at a frozen moment of time t, we will take the distance not to the actual location of the interacting bodies, but to the point of the trajectory taking into account the time delay Δt.

The mathematical dependences of the time delay of the field wave are obtained from purely geometric representations. Without loss of generalization, one of the charged bodies, for example q_1, is considered stationary, the other q_{21} – moving along the direction of the velocity vector **v**.

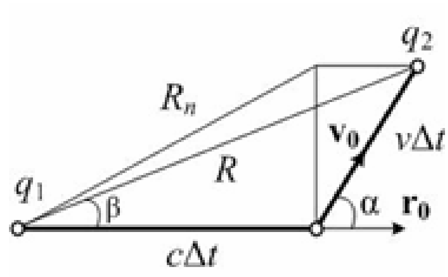

Fig. 1. Geometrization of a field interaction time delay.

Let the distance between the bodies at a frozen moment t be R. But their interaction will be determined by some angle delay β due to the time delay Δt of the passage of the field wave to the corresponding point of the trajectory at the speed of light c. The distance covered by the moving body during this time will be $v\Delta t$,where v is its instantaneous velocity. Time Δt can be easily found by the geometric constructions of Fig. 1 as

$$\Delta t = R / \sqrt{c^2 + v^2 + 2cv\mathbf{r}_0 \cdot \mathbf{v}_0}, \tag{6}$$

where $\mathbf{v}_0$ is a single velocity vector.

Then the real distance between the interacting bodies, given the time delay, will be

$$r = c\Delta t = R \,/\, \sqrt{1 + \frac{v^2}{c^2} + 2\frac{v}{c}\mathbf{r}_0 \cdot \mathbf{v}_0} \,. \qquad (7)$$

Substituting (7) into (5), we obtain an expression of the adapted to the Coulomb law in the case of motion

$$\mathbf{F} = k\frac{q_1 q_2}{R^2}\left(1 + \frac{v^2}{c^2} + 2\frac{v}{c}\mathbf{r}_0 \cdot \mathbf{v}_0\right)\mathbf{r}_0 \,. \qquad (8)$$

The desired orientation of the vectors can be found from the corresponding coordinate equations of mechanical motion.

Law (8) has been successfully tested in problems of the dynamics of electron motion in an electric nonuniform field.

Specific cases. 1. The direct trajectory of motion runs through interacting point bodies. Then $\alpha = 0$ when the bodies move away and $\alpha = \pi$ when they approach. Therefore, (8) is simplified

$$\mathbf{F} = k\frac{q_1 q_2}{R^2}\left(1 \pm \frac{v}{c}\right)^2 \mathbf{r}_0 \,, \qquad (9)$$

and the "+" sign indicates the distancing of bodies, and the sign "–", on the contrary, their approaching.

When approaching in the natural direction with a limit speed ($v = c$), the electrical interaction disappears by itself ($F \to 0$). On the other hand, when braking (against electrical action ($v = -v$) the relativistic factor increases fourfold ($F \to 4F$). This explains the effects such as electrical delay of signals.

2. A moving body travels across the force action. In that case $\alpha = \pi/2$. Then (8) takes a slightly different form

$$\mathbf{F} = k\frac{q_1 q_2}{R^2}\left(1 + \frac{v^2}{c^2}\right)\mathbf{r}_0 \,, \qquad (10)$$

3. *Stationary bodies*. Given $\mathbf{v} = 0$, expressions (8)–(10) degenerate to the original law (5). There is no such direct transition in the general theory of relativity (GTR). They have to go up into variational principles, change the scalar curvature of space to the usual Lagrange energy function, and pass with it all through the mathematical mazes inherent in them, thus making a clear substitution of concepts. But back to the Lorenz force. For this purpose, we also express the time interval Δt according to the same geometric constructions of Fig. 1, but somewhat differently

$$\Delta t = R_n \,/\, \sqrt{c^2 + v_n^2}\,; \quad v_n = v \sin\alpha \,, \qquad (11)$$

where v_n is the normal velocity component.

Then, the real distance (7) of the interaction will be

$$r = cdt = R_n \,/\, \sqrt{1 + \frac{v_n^2}{c^2}} \,. \qquad (12)$$

Now on the basis of (11), (12), formula (8) can be written in the form

$$\mathbf{F} = q_2\left(E + v_n \frac{k_q q_1 v_n}{R_n^2 c^2}\right)\mathbf{r}_0 \,; \qquad (13)$$

moreover, the module of the electric field intensity vector is written by definition

$$E = k\frac{q_1}{R_n^2} \,. \qquad (14)$$

Let us use the Biot-Savart law in the form [4]

$$\mathbf{B} = \frac{\varepsilon_0}{\upsilon_0}(\mathbf{v} \times \mathbf{E}) = \frac{\mathbf{v} \times \mathbf{E}}{c^2} \qquad (15)$$

and $c = \sqrt{\nu_0 / \varepsilon_0}$, where υ_0 is the magnetic reluctivity $\nu_0 = \varepsilon_0 c^2 = 7.957749 \cdot 10^5$ m/H.

If (12), (14), (15) are taken into consideration, then expression (13) coincides with Lorentz force expression (1)!

Formulas (5), (10), (13) comprehensively reveal the physical essence of the first two terms of the relativistic coefficient (8) out of three. It remains to be ascertain for the last of them. It is clear that the intensity of the electric field, the value of the motion velocity is not enought to determine the force as a vector, this requires a more spatial orientation of the values involved. It is this orientation the third term depending on the spatial angle α is responsible for.

If consider the increment of squares of distances of instantaneous charge interaction

$$(\Delta R)^2 = R^2 - \left(R\big|_{\alpha=\pi/2}\right)^2 , \qquad (16)$$

then on the basis of (6), (7) we can obtain

$$2\frac{v}{c}\mathbf{r}_0 \cdot \mathbf{v}_0 = \frac{(\Delta R)^2}{r^2} \,. \qquad (17)$$

Thus, the relativistic effect of motion in electricity is successfully encoded in the magnetic field as the relativistic effect of the electric field. The creators of magnetism knew this [5], but over time, this truth has been forgotten by electricians. However, relativists, judging by expression (4) and commentary to it, remember and creatively operate with this truth. This is very important to us because we have come to the same understanding of the physical process with an unconventional approach.

Example. Fig. 2 shows the results of simulating a trajectory of electron attraction by a charged sphere

($Q = +3.16 \cdot 10^{-4}\ C$). The differential equations written according to Newton's second law and (9) have been integrated

$$\frac{dv}{dt} = \frac{q_e}{m_e}\frac{kQ}{R^2}\left(1-\frac{v}{c}\right)^2; \quad \frac{dR}{dt} = -v, \qquad (18)$$

where q_e, m_e are the parameters of the electron $\left(q_e / m_e = 0{,}17588 \cdot 10^{12}\right)$, under initial conditions: $v = 0$; $R = 10$ m.

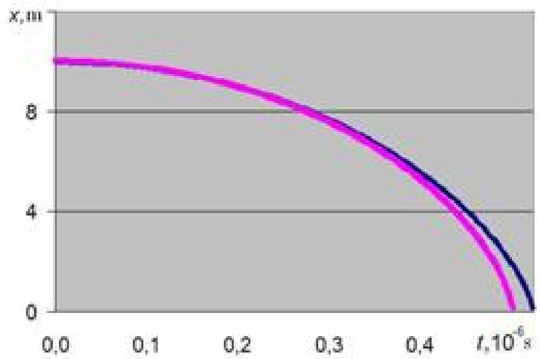

Fig. 2. Time dependence of the trajectory of motion x = R (t). Lower curve – the result obtained by the classical Coulomb law, higher curve – by the law of moving charges.

Based on the analysis of the results obtained, it is possible to quantify the conclusion of classical electronics that "in electrovacuum devices, the electron velocity does not exceed 0.1 s. The analysis of digital data (Fig. 2) showed that at a speed of 0.1 s in this problem we have a difference in speeds of 4600 km/s with a relative error of 16 %, which can be only attributed to the striking non-uniformity of the electric field.

4. Conclusion

1. The Lorentz force is represented as an adapted vector of Coulomb's law of electrical interaction in the case of moving masses. The adaptation is realized in real space and time bypassing the theory of relativity. This takes into account the finite rate of electric field propagation and the law of charge conservation.

2. On the basis of this adaptation, the trajectory of the electron motion in a non-uniform electric field has been simulated, bypassing the concept of a magnetic field.

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СИЛА ЛОРЕНЦА У ВИХРОВОМУ ЕЛЕКТРИЧНОМУ ПОЛІ

Василь Чабан

У наш час функції закону взаємодії рухомих наладованих тіл перебрала на себе цілковито теорія відносности, прикриваючись псевдогаслом про неспроможність перетворень Галілея. Усупереч цьому в статті адаптовано силу Лоренца на випадок рухомих ладунків у всеможливому діапазоні швидкостей у звиклих три вимірному Евклідовому просторі і часі. При цьому враховано скінченну швидкість поширення електричного поля і закон збереження електричного ладуну. На цій підставі просимульовано траєкторію вільного руху електрона в нерівномірному електричному полі, згенерованому позитивно наладованимним сферичним тілом.

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