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OPTIMIZATION OF VIBRATORY CONVEYING UPWARD BY INCLINED TRACK WITH POLYHARMONIC NORMAL VIBRATIONS

Abstract. The paper is devoted to the research of vibratory conveying of piece goods along an inclined track, performing harmonic longitudinal and polyharmonic normal vibrations. It is considered the conditions of reaching maximum conveying velocity at specified values of frequency and amplitude of longitudinal vibrations – the conditions of maximum dimensionless conveying velocity, depending on several dimensionless parameters in the moving modes without hopping. These dimensionless parameters are the inclination angle parameter – a ratio of an inclination angle tangent to a frictional coefficient, the intensive vibration coefficient – a ratio of the longitudinal amplitude of vibration to the amplitude of the first harmonic of normal vibration and frictional coefficient. Maximal conveying velocity is achieved at the certain values of normal vibration amplitudes and values of phase difference angles between longitudinal and normal vibrations, which are called optimal, and their values are dependent on these two dimensionless parameters, while maximum normal vibration acceleration should be equal to the gravitational acceleration. The research was made by approximate harmonic balance method and by numerical step-by-step integration method, which allows performing calculations with any given accuracy. The results obtained by the two methods are compared.

To determine the maximal and optimal values of elevation angles, there are calculated the maximal value of the inclination angle parameter at which the value of dimensionless velocity is equal to zero, and the value of the inclination angle parameter at which a particle moves to a specified height in the minimum time. The optimal values of amplitudes of harmonics of polyharmonic normal vibration are determined in dependence on the inclination angle parameter with the number of harmonics from four to seven. The graphs of these dependencies are presented and the most important values of dimensionless parameters are presented in the table.

Keywords: vibratory conveying, polyharmonic normal vibration, harmonic longitudinal vibration, electromagnetic drive.

Introduction

Vibratory conveying devices are widely used in different industrial enterprises, especially for parts feeding, orienting, or internal transportation in automated assembly systems, robotized industrial production lines [1], [2]. For these purposes, the most popular devices are vibratory conveyors, bowl feeders, vibratory manipulators with electromagnetic drive due to their high reliability and productivity, design's simplicity. They must transport parts not only horizontally, but also along an inclined plane or a spiral trough upwards. The most common and simple vibratory conveying devices provide the linear vibration of conveying track, that is suitable for conveying of relatively small and not very massive piece goods. For conveying the massive heavy goods or fragile and explosive products, it is necessary to ensure the moving of products without hopping. However, the vibratory conveying devices with linear vibration do not allow achieving high conveying velocity and large elevation angles of conveying track in the

moving modes without hopping. The increasing of conveying velocity and elevation angles in non-hopping modes, and as a result the increase of productivity, can be reached in vibratory devices with asymmetric longitudinal vibration or with independent longitudinal and normal vibrations of conveying track.

Problem statement

For reaching the high conveying velocity, the conveying devices with electromagnetic drives should provide large amplitude of longitudinal vibrations. That is why the electromagnetic drive of longitudinal vibration should operate in near-resonant mode, in which vibration is sinusoidal. At the same time, the amplitude of normal vibration should be stable to ensure the moving without hopping, and the electromagnetic drive of normal vibration should be operated far from resonance, where vibration can be non-sinusoidal, in particular polyharmonic. In author's papers [3], [4] there was considered vibratory conveying by harmonic longitudinal and polyharmonic normal vibrations, represented by equations:

$$x = A\sin(\omega t + \varepsilon),$$

$$y = \sum_{i=1}^{n} A_i \sin[(i\omega t + \frac{\pi}{2}(i-1)], \tag{1}$$

where A – amplitude of longitudinal vibration, A_i – amplitude of i-th harmonic of normal vibration, ω – frequency of vibration, n – number of harmonics, ε – phase difference angle, t – time.

These studies were carried out by approximate harmonic balance method, and their results are reliable when longitudinal vibration amplitude is essentially larger than normal vibration amplitudes. The numerical methods of investigation, for example step-by-step integration method, using the methods solving of nonlinear equations, provide the possibility to determine the optimal parameters of vibrations to reach the maximal velocity for any cases. It was found that the maximal conveying velocity is achieved at a certain ratio of amplitudes of normal vibration's harmonics. Moreover, this ratio, which was called optimal, depends on the inclination angle of conveying track. However, the dependence of the optimal ratio of normal vibration's harmonics on inclination angles was researched for vibration with two and three harmonics only.

Review of Modern Information Sources on the Subject of the Paper

The investigation of vibratory conveying of a material point by two-component independent longitudinal and normal vibrations has been made in many papers. In particular, the simplest case –elliptical vibration – harmonic longitudinal and normal vibrations with a phase difference between them, when conveying track vibrates by elliptical trajectory, is considered in [1], [5]–[7]. It was proved that elliptical vibration can achieve higher conveying velocity and elevation angles comparing to linear vibration, especially in non-jumping modes of moving. An even higher value of velocity and elevation angles can be reached by using polyharmonic vibration, in particular biharmonic vibration. Vibratory conveying by biharmonic longitudinal vibration is considered in [2], [8], but conveying velocity, in this case, is much less than in the case of two-component independent vibrations. Vibratory conveying by two-component biharmonic vibrations was considered in [9], conveying by biharmonic normal vibration with three harmonics was considered in [10], but optimal parameters of vibration were not determined here. The optimal relative to velocity law of two-component vibrations with a limited value of acceleration has obtained by Lavendel [11]. According to this law, longitudinal and normal vibrations should be functions with piecewise constant acceleration. Nevertheless, this law is difficult to implement by an electromagnetic drive.

The author's research [3, 4] shows that using harmonic longitudinal and polyharmonic normal vibrations can sufficiently increase conveying velocity and elevation angles at certain ratios of amplitudes and phase differences angles.

Objectives and Problems of Research

The mean conveying velocity v can be calculated by formula [2]:

$$v = A\omega V, \tag{1}$$

where V – dimensionless conveying velocity – parameter, depending on several dimensionless parameters. The effectiveness of the process of vibratory conveying can be estimated by the value of V, which determines a velocity per unit frequency and amplitude of longitudinal vibration.

In this paper, the dependence of dimensionless conveying velocity on dimensionless vibration parameters with harmonic longitudinal and polyharmonic normal vibrations with the number of harmonics from 2 to 7 is investigated. It is shown that maximal conveying velocity can be reached with the definite values of normal harmonics ratio, which depend on the inclination angle of conveying track. The dependencies of dimensionless conveying velocity on dimensionless vibration parameters are investigated by numerical methods that ensure the specified accuracy. The optimal values of the ratio of harmonic amplitudes of normal vibrations and the values of maximal and optimal elevation angles are researched.

Optimization of conveying velocity by harmonic balance method

According to the investigations of two-component vibratory conveying [2, 3, 5], the value of dimensionless V velocity depends on the next dimensionless parameters: K_α – the inclination angle parameter, K_β – the intensive vibration parameter, G – the gravitational overload parameter, which are calculated as follows:

$$K_\alpha = \frac{\tan\alpha}{\mu}, \qquad\qquad K_\beta = \frac{A}{A_1\mu}, \qquad\qquad G = \frac{A_m\omega^2}{g\cos\alpha},$$

where g – the gravitational acceleration, A_m – amplitude of normal vibration in a moment when its acceleration is maximum.

The harmonic balance method allows determining the optimal values of amplitudes of harmonics acceleration w_i with sufficient accuracy only with a fairly sufficiently intense longitudinal vibration, when $A_x \gg A_m$ or $K_\beta \to \infty$. In this case, according to the investigations [2], the particles move in the mode with one sliding motion upward and one sliding motion downward during the vibration period without sticks relative to the track. The optimal phase difference angle $\varepsilon = 90°$ and amplitudes of harmonic acceleration w_i should satisfy the equation [3]:

$$1 + \Theta - \frac{2}{\pi}(\psi + \sum_{i=1}^{n}\frac{w_i}{i}\sin i\psi) = 0, \tag{3}$$

where $\psi = (2\pi+\tau_+-\tau_-)/2$; $w_i = \dfrac{i^2 A_i\omega^2}{g\cos\alpha}$ – dimensionless normal vibration acceleration amplitude of i-th harmonic; $\tau = \tau_+$ and $\tau = \tau_-$ – the initial moments of the sliding motion upward and sliding motion downward respectively, $\tau = \omega t$.

The conditions of a particles moving without hopping with maximum velocity are described by the system of equations:

$$\sum_{i=1}^{n} w_i \sin(i\tau_m + \frac{\pi(i-1)}{2}) = 1; \tag{4}$$

$$\sum_{i=1}^{n} i w_i \cos(i\tau_m + \frac{\pi(i-1)}{2}) = 0, \tag{5}$$

where τ_m – the moment of maximum normal acceleration.

The dimensionless velocity in this case is calculated by formula [3]:

$$V = -\cos(\psi). \tag{6}$$

The optimal values of amplitudes w_i can be determined by the method of Lagrange multipliers with the additional n equations:

$$\frac{1}{i}\sin i\psi + \lambda[i\cos(i\tau_m + \frac{\pi(i-1)}{2}) - i^2\sin(i\tau_m + \frac{\pi(i-1)}{2})] = 0, \tag{7}$$

where λ – unknown Lagrange multiplier.

The Eqs. (3), (4), (6), (7) form the system of $n+3$ equations with $n+3$ unknown variables $\sum_{i=1}^{n} w_i$, ψ, τ_m, λ. However, this system of equations can have many solutions, to choose a desirable solution, it is necessary to introduce the additional inequalities, for example, $w_1<2$; $w_2>w_3$; $0<\tau_m<\pi/2$, etc.

**Investigation of dimensionless velocity *V* dependence on dimensionless parameters
by the step-by-step integration method**

The equation of a particle's motion without hopping on a track, vibrating according to Eqs. (1), using the values dimensionless distance $\chi = x\,/A$, can be represented by analogy with the equation describing the non-hopping motion with elliptical vibration [12], as follows:

$$\chi'' = l_\pm \sin(\tau - \kappa_\pm) - \delta_\pm + \sum_{i=2}^{n} \frac{w_i}{w_1 q_\pm} \sin\{i\tau - [i\varepsilon - \frac{\pi}{2}(i-1)]\}\,, \tag{8}$$

where $l_\pm = \dfrac{\sqrt{1 + K_\beta^2 \pm 2K_\beta \cos\varepsilon}}{K_\beta}$; $\kappa_\pm = \arctan\dfrac{\sin\varepsilon}{\cos\varepsilon \pm K_\beta}$; $\delta_\pm = \dfrac{\Theta \pm 1}{K_\beta w_1 l_\pm}$; $q_\pm = K_\beta l_\pm$. The signs "+" and

"–" correspond to the particle sliding upward and downward respectively.

The initial moments of the sliding motion upward $\tau = \tau_+$ and sliding motion downward $\tau = \tau_-$ are determined as the roots of the system of two nonlinear equations:

$$\cos(\tau_+ - \kappa_+) - \cos(\tau_- - \kappa_+) + \delta_+(\tau_+ - \tau_-) +$$
$$+\sum_{i=2}^{n} \frac{w_i}{iw_1 q_+}\{\cos[i\tau_+ - (i\varepsilon - \frac{\pi(i-1)}{2})] - \cos[i\tau_- - (i\varepsilon - \frac{\pi(i-1)}{2})]\} = 0\,; \tag{9}$$

$$\cos(\tau_- - \kappa_-) - \cos(\tau_+ - \kappa_-) + \delta_-(2\pi + \tau_- - \tau_+) +$$
$$+\sum_{i=2}^{n} \frac{w_i}{iw_1 q_-}\{\cos[i\tau_- + (i\varepsilon - \frac{\pi(i-1)}{2})] - \cos[i\tau_+ + (i\varepsilon - \frac{\pi(i-1)}{2})]\} = 0\,. \tag{10}$$

Dimensionless velocity can be found by formula:

$$V = \frac{S_+ + S_-}{2\pi}\,, \tag{11}$$

where S_+ and S_- – dimensionless distances of a particle moving upward and backward respectively during the vibration period, that are calculated by equations:

$$S_+ = l_+\{\cos(\tau_+ - \kappa_+)(\tau_- - \tau_+) + \sin(\tau_+ - \kappa_+) - \sin(\tau_- - \kappa_+) - \delta_+(\tau_- - \tau_+)^2/2 +$$
$$+\sum_{i=2}^{n} \frac{w_i}{i^2 w_1 q_+}\{i\cos[i\tau_+ - (i\varepsilon - \frac{\pi(i-1)}{2})](\tau_- - \tau_+) -$$
$$-\sin[i\tau_- - (i\varepsilon - \frac{\pi(i-1)}{2}) - \sin[i\tau_+ - (i\varepsilon - \frac{\pi(i-1)}{2})]\}\}\,; \tag{12}$$

$$S_- = l_-\{\cos(\tau_- - \kappa_-)(\tau_+ - \tau_-) + \sin(\tau_- - \kappa_-) - \sin(\tau_+ - \kappa_-) - \delta_-(2\pi + \tau_+ - \tau_-)^2/2 +$$
$$+\sum_{i=2}^{n} \frac{w_i}{i^2 w_1 q_-}\{i\cos[i\tau_- + (i\varepsilon - \frac{\pi(i-1)}{2})](2\pi + \tau_+ - \tau_-) -$$
$$-\sin[i\tau_+ + (i\varepsilon - \frac{\pi(i-1)}{2}) - \sin[i\tau_- + (i\varepsilon - \frac{\pi(i-1)}{2})]\}\}\,. \tag{13}$$

Having given the values parameters K_α and K_β and a set of dimensionless amplitudes of harmonics acceleration w_i, satisfying the condition $G = 1$, substituting them into Eq. (8), we calculate the values of $l_\pm$, $\kappa_\pm$, $\delta_\pm$, $q_\pm$. Then we determine the roots τ_+ and τ_- of system Eqs. (9) and (10) in the range from 0 to 2π, changing the value of the angle ε from 60° to 90°. Then we substitute the values of τ_+ and τ_- into Eqs. (12) and (13), and then substitute S_+ and S_- into Eq. (11), thus determining the value of velocity V, depending on parameters K_α and K_β and the optimal values of w_i and ε, providing a maximum of V.

**Dependence of dimensionless velocity *V* and optimal dimensionless amplitudes of harmonics *w_i*
on inclination angle parameter *K_α***

Dependence of dimensionless velocity V on the inclination angle parameter K_α was studied by harmonic balance method in [3], the graphs of dependence $V(K_\alpha)$ were presented for the number of

harmonics $n = 1$–7. However, the dependence of w_i on parameter K_α has been not studied for $n > 3$. That is why the values of V at $K_\alpha > 0.7$ and $n \geq 4$ are incorrect and somewhat underestimated.

The dependence of velocity V on parameters K_α and K_β was studied by the harmonic balance method and the step-by-step integration method. The approximated harmonic balance method gives accepted results, comparable to the results of step-by-step integration method, for the values $K_\beta > 40...50$.

Calculations were performed using MathCAD. The dependencies of dimensionless velocity V on inclination angle parameter K_α for number of harmonics $n = 1 - 7$ are shown in Fig. 1. Dimensionless velocity V increases with the increasing number of harmonics n and decreases with increasing of inclination angle parameter.
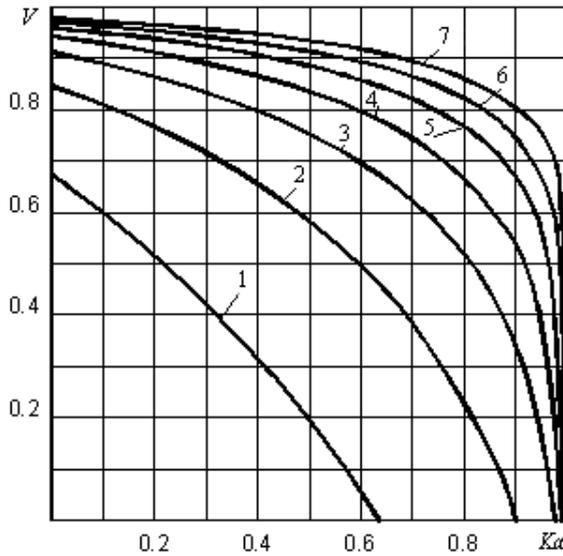

Fig. 1. Dependence of dimensionless velocity V on inclination angle parameter K_α with values of parameters $K_\beta \to \infty$, $G = 1$ and different number of harmonics n from 1 to 7

Graphs of optimal amplitudes w_i dependencies on parameters K_α for numbers of harmonics $n = 2$ and 3 were determined in the author's works [13]. Graphs of such dependencies for $n > 3$ are shown in Fig. 2. Continuous lines in Fig. 2 *a* show graphs for $n = 4$ and dashed lines show graphs for $n = 5$; continuous lines on Fig. 2 *b* show graphs for $n = 6$ and dashed lines show graphs for $n = 7$. These results were obtained by an approximate harmonic balance method, they are valid for $G = 1$; $K_\beta \to \infty$. As seen in Fig. 2, with increasing of K_α the optimal values of the lower harmonics ($i \leq n / 2$) amplitudes increase but the values of higher harmonics ($i > n / 2$) amplitudes decrease. However, at the range $K_\alpha > 0.9$ optimal amplitudes of all harmonics decrease sharply with increasing of K_α.

The values of dimensionless velocity V for horizontal conveying ($K_\alpha = 0$) with intensive vibration ($G = 1$, $K_\beta > 40...50$) with optimal ratio of the amplitudes of harmonics with their number from 1 to 7 and with optimal difference phase angles are shown in Table 1.

To determine the value of dimensionless velocity V at various values of K_β, the step-by-step integration method should be applied, which allows obtaining the result with a given accuracy.

Table 1

Maximal dimensionless velocity and optimal normal acceleration amplitudes

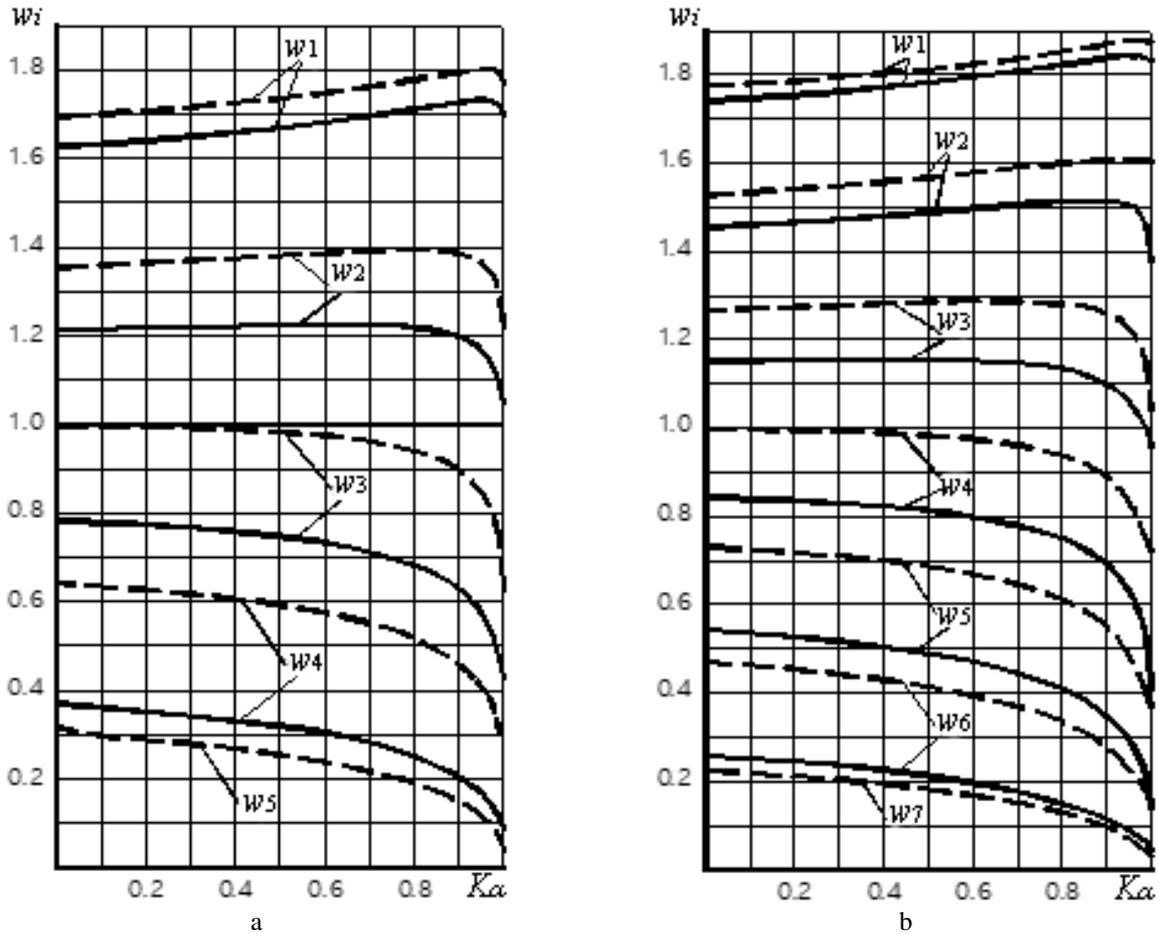| Number of harmonics n	V (at $K_\alpha = 0$)	w_1	w_2	w_3	w_4	w_5	w_6	w_7
1	0.6736	1	–	–	–	–	–	–
2	0.8469	1.355	0.643	–	–	–	–	–
3	0.9122	1.527	0.998	0.471	–	–	–	–
4	0.9433	1.628	1.213	0.783	0.370	–	–	–
5	0.9604	1.694	1.354	0.998	0.643	0.329	–	–
6	0.9709	1.740	1.454	1.152	0.844	0.543	0.258	–
7	0.9776	1.774	1.527	1.266	0.998	0.730	0.470	0.225

Fig. 2. Dependence of optimal amplitudes of dimensionless normal acceleration w_i on inclination angle parameter K_α (when parameters $K_\beta \to \infty$, $G = 1$) with different number of harmonics: a – $n = 4$ – continuous lines, $n = 5$ – dashed lines; b – $n = 6$ – continuous lines, $n = 7$ – dashed lines

Dependence of dimensionless velocity V on parameter K_β

The step-by-step integration method allows calculating the dependence of dimensionless velocity V on parameters K_α and K_β with different values of amplitudes w_i and angle ε. The initial values of w_i were taken from approximate results by harmonic balance method (Fig. 2) for specific values of K_α and n, and varied in small intervals, observing the condition $G = 1$. The value of angle ε varied from 60° to 90°. For the calculations, a MathCAD-program was developed, at the input of which the values of parameters K_β, K_α, n, initial values of w_i and ε were set, and at the output, the value of V have obtained, as well as the optimal values of w_i and ε.

The dependence of velocity V on parameter K_β at $K_\alpha = 0$ (the optimal horizontal conveying without hopping) with different numbers of harmonics of normal vibrations n from 2 to 7 and optimal values of amplitudes w_i and angle ε, is shown in Fig. 3 by continuous lines. Similar dependence at $K_\alpha = 0.9$ (with inclinations angles close to the limiting values) is shown by dashed lines.

An important property of vibratory conveying of a material point with two-component vibration of conveying track is that the dimensionless velocity V is practically independent on K_β with an increase of K_β, starting from $K_\beta > 40...50$. The optimal value of angle ε depends on the values of parameters K_α and K_β.

Determining of limiting and optimal values of inclination angle parameter K_α

The limiting value of inclination angle parameter $K_\alpha = K_{\alpha\ lim}$ is the maximal value of K_α at which dimensionless velocity V equals zero. The value of $K_{\alpha\ lim}$ at the number of harmonics $n = 2$ and $n = 3$ are determined in [4]. When $n > 4$ the value of $K_{\alpha\ lim} \to 1$, i.e. the inclination angle becomes equal to the angle

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of friction. However, the use of angles corresponding to $K_\alpha > 0.9$ is undesirable, since the value of the friction coefficient is not always accurately known, besides, it may change during conveying and particles cannot slide upward. The values of parameter $K_{\alpha\,lim}$ for different numbers of harmonics n are given in Table 2.

Table 2

The limited and optimal values of the inclination angle parameter K_α

Number of harmonics n	$K_{\alpha\,lim}$	$K_{\alpha\,opt}$	V (at $K_\alpha = K_{\alpha\,opt}$)	$V\,K_{\alpha\,opt}$
1	0.636	0.35	0.37	0.129
2	0.900	0.57	0.524	0.299
3	0.977	0.71	0.614	0.436
4	0.996	0.79	0.634	0.540
5	0.999	0.84	0.736	0.618
6	0.999	0.87	0.779	0.678
7	0.999	0.90	0.807	0.726

The optimal value of inclination angle parameter $K_\alpha = K_{\alpha\,opt}$ is the value of K_α at which a particle moves to a specified height in the minimum time. It is determined under the condition of the maximum value of the product $V \cdot K_\alpha$. The values of optimal inclination angle parameter $K_\alpha = K_{\alpha\,opt}$ with different n were determined in [4], but as the dependence of w_i on parameter K_α has been not taken into account here, the results for $n \geq 4$ are incorrect and underestimated. Dependencies of the product $V \cdot K_\alpha$ on the inclination angle parameter K_α for the different number of normal harmonic amplitudes n are shown in Fig. 4. The values of optimal inclination angle parameter $K_{\alpha\,opt}$, values of the product $V \cdot K_\alpha$ and velocity V at $K_\alpha = K_{\alpha\,opt}$ are given in Table 2.
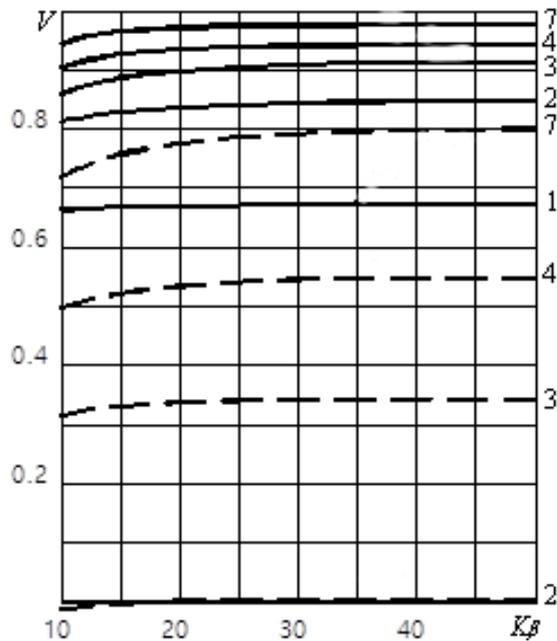

Fig. 3. Dependence of dimensionless velocity V on intensive vibration parameter K_β with values of parameters $G = 1$ at $K_\alpha = 0$ (continuous lines) and at $K_\alpha = 0.9$ (dashed lines)
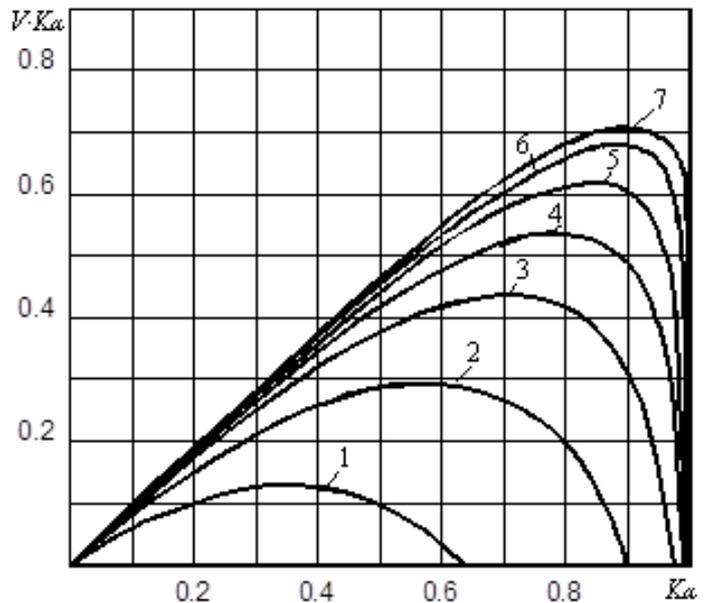
Fig. 4. Dependence of the product $V \cdot K_\alpha$ on inclination angle parameter K_α with different number of harmonics n

Conclusions

The dimensionless velocity V dependence on dimensionless parameters (the inclination angle parameter and the intensive vibration parameter) for vibratory conveying with harmonic longitudinal and normal polyharmonic vibrations of track for non-hopping modes of parts moving (when parts sliding

without detachment from the conveying track) is investigated. The limiting and optimal values of the inclination angle parameter have been determined, which allows, knowing the frictional coefficient of moving particles, to determine the maximal and optimal values of the inclination angle of conveying track. The latter is the angle, at which a particle moves to a specified height in the minimum time. The limiting and optimal values of the inclination angle increase with an increase of the number of harmonics.

Velocity V increases with an increase of the number of harmonics n, but with each subsequent harmonic this increase decreases. A significant increase of V with an increase of number of higher harmonics is observed only at inclination angles close to the limiting ones. Velocity decreases with the increase of the inclination angle parameter K_α (increasing of inclination angle when moving upward). Velocity increases with the increase of intensive vibration parameter K_β (increasing of longitudinal vibration amplitude), but practically does not change, starting at $K_\beta > 50\ldots80$. The maximum value of velocity is achieved at certain, optimal values of amplitudes of normal vibration harmonics. These values depend on the inclination angle parameter K_α, with the increasing of parameter K_α the optimal values of the lower harmonics ($i \leq n/2$) amplitudes increase but values of the higher harmonics ($i > n/2$) amplitudes decrease.

For further research, it seems important to create a method for calculating the parameters of polyharmonic normal vibration for practical use in industrial conveyor devices with an electromagnetic drive.

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