

## RESEARCH OF DYNAMICS OF THE VIBRATING MACHINE'S DOWNLOADING WITH THE VERTICAL MOVEMENT

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**The purpose of research.** The main purpose of the work is to develop mathematical software for studying the dynamics of vibration machines of volumetric processing, and calculate the influence of various factors on the efficiency of the process based on applied systems of automated mathematical calculations, including MathCad and MatLab. **Method.** The research was carried out on the basis of a class of vibrating machines with unbalanced drive type and spring suspension. A mathematical model of loading vibrating processing machines with vertical perturbation is constructed, which is represented by a layering of flat beams that perform vertical oscillations. Methods of nonlinear mechanics were used to build a mathematical model. **Results.** The dynamic processes at vibration compaction of loading and separation of its fractions for the purpose of increase of intensity of the given technological processes are investigated. The dependences for determining the influence of physical and mechanical properties of loading components on the process dynamics are obtained. **Scientific novelty.** New approaches for construction of mathematical models of research of processes in machines of vibration processing, in particular vibration consolidation and separation are considered. In the course of research, differential equations were used to determine the change in the amplitude and frequency of loading from the influence of external and internal parameters of the vibration processing process. **Practical significance.** The obtained mathematical model makes it possible to investigate the dynamics of loading of the vibrating machine, to determine the amplitude and frequency of oscillations of loading of the vibrating machine depending on its properties and the properties of the vibrating machine. The obtained solutions of nonlinear differential equations to describe the motion of loading in vibrating machines allow to automatically determine the amplitudes, frequencies and trajectories of different components of loading depending on the parameters of the system "vibrating machine - loading", by algorithmizing them in applied systems of automated mathematical calculations.

**Key words:** unbound machining tool, vibrating machine, surface treatment, asymptotic methods, unbalanced drive, nonlinear mechanics.

### *Introduction.*

Among high-efficiency technologies with rather small energy and material consumption the significant part is made by those which use various vibration processes assisting the increase of processing quality, level of mechanization and automation of many labour-consuming operations, economic efficiency and productivity of work etc.

The area of use of vibration technologies is wide enough and also tends to the further growth in machine-building, mining, building other areas of industry. In mechanical engineering, vibration is used for various types of intermediate and finishing, grinding, hardening, sorting, transportation, etc. In the processing and food industries, vibration technologies are used for the processes of loosening, sifting, grinding of raw materials, creation of dispersion mixtures, cleaning of root crops.

Of course, the increase of efficiency, safety of use and optimization of various modes of operation of the equipment for realization of vibration technologies assumes first of all improvement of its design parameters and operational characteristics. As a result, the change of these indicators requires research conducted in computer-aided design systems (to optimize design parameters) and mathematical calculation systems (to optimize production indicators).

### *Analysis of modern information sources on the subject of research.*

The development of the computer technology and computer-aided design systems necessitates the creation of new methods of design, construction and manufacture of vibrating machines in combination with traditional methods based on experimental research, improvement and modernization of existing equipment [1]. The study of the motion of individual components of the vibrating machine, in particular,

its working bodies, is also quite common thing [2], [3], but does not give an exhaustive picture of the dynamics of the vibrating machine during its operation.

The use of mathematical modeling in the research of dynamic processes in vibrating machines is a common method [4], [5], [6], which precedes their design. However, the linearization of these models, to simplify the course of research, often narrows the results in terms of universality of their application for a wide range of vibrating machines for similar purposes and principles of operation, and leads to an incomplete theoretical picture of reproduction of real physical processes. Along with the methods listed above, methods of studying dynamic processes in vibrating machines for various purposes, based on the use of mathematical numerical methods and computer technology, are also used in practice [7]. At authors presented work there are developed the a number of nonlinear mathematical models of vibrating machines for various purposes [8] - surface hardening and cleaning, separators, crushers and mills. Based on these models, the research of the dynamics of these machines was carried out to determine the optimal parameters of their designs and the choice of modes of their operation.

### ***The purpose of research.***

The purpose of this paper consists in creating the mathematical software for the automated research of vibration machines dynamics, loading for vibrating seal, loading or separation of its compound for the intensity increase of these technological processes.

### ***The main material.***

Vibration-type machines are distinguished by their diversity in structural use and the ability to use equipment of the same type for various technological operations. Therefore, the developed analytical solutions for the study and determination of performance should take into account these nuances. It has been developed for this purpose a software of dynamics research of volumetric processing vibration machines (nonlinear mathematical models of the vibration machine working body movement and its loading), which have made possible the programs creation of the process efficiency factors research on the basis of CAM systems, in particular MathCad and MatLab. The task of creation of mathematical maintenance for the automated research of the vibrating machines loading dynamics is considered below. It is designed to study the vibration compaction of the load or the separation of its components in order to increase the intensity of these production processes.

The object of study is a vibrating machine with a spring suspension and unbalanced drive. This installation has advantages over other types of machines, which consists in high reliability of component designs, simplicity of service, compact overall dimensions. Unlike machines of pneumatic or hydraulic type, which are sensitive to overloads, the vibrating exciter of the vibrating machine can work in a fairly wide range of changes in operating parameters. Vibration-type machines work as follows: the vibrator, which acts as an imbalance, drives the working container, which is mounted on an elastic spring suspension. The perturbing force generated at start-up is necessary to dampen the resistance forces acting on the loading layer. As a result, an intensive process of interaction of the loaded fractions takes place in the working container. The intensity of the processing is determined by the parameters of the system, in particular the perturbation forces, physical and mechanical characteristics of the processing material and the amplitude-frequency characteristics of the machine. During vibration volumetric processing, the perturbation forces change in magnitude and direction, as a result of which the loaded particles are evenly mixed and transported in the desired direction. The ability to move the container in a horizontal or vertical plane allows controlling this process.

In case of vibration condensation of the container loading or separation of its fractions for density and size it is enough to ensure only vertical direction of the container fluctuation of the vibration machine - that is to give to loading only vertical indignation. The result and intensity of such vibration processing of loading will be determined by amplitude and frequency of loading fluctuations which by the turn will depend on a type of indignation and parameters of system (vibration machine and loading).

In the fig. 1 are submitted some basic circuits of vibration machines which provide vertical (or dominant vertical) fluctuation of the vibration machine's container - vertical force of loading indignation. The circuits in fig. 1a, 1b, 1c represent vibration machines, which have exciter drive (shaft with unbalanced weight fastens to the working container and its rotation predetermines varying movement of the container in a vertical plane). In fig. 1d the circuit of vibration machine is submitted which container changes vertically for the account of crank mechanism. In vibration machine represented on fig. 1a fluctuation only along a vertical axis of the container is achieved by the terminator of movement 5 and it's directing 4 (different constructive performance). Such design switches off completely fluctuation of the container along its horizontal axis. In vibration machine represented on fig. 1b the suspension bracket consists of vertical and horizontal springs, and rigidity of horizontal springs is much more then rigidity of vertical one that predetermines dominant vertical fluctuations compared with insignificant horizontal.

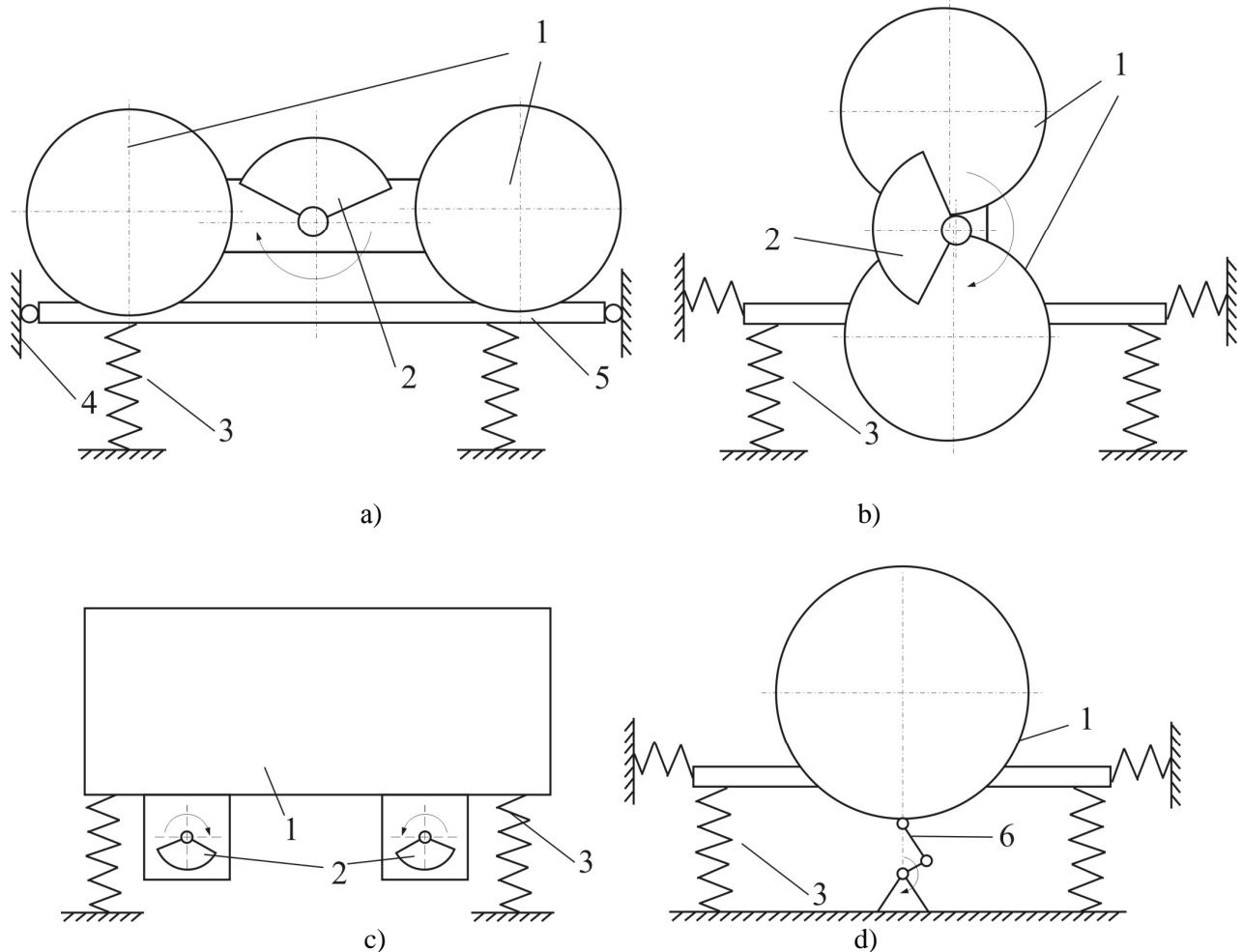


Fig. 1. Vibration machine for realization of vibration condensation of loading or separation of its compound (1 - working container, 2 - driving exciters, 3 - spring suspension bracket, 4 - directings of horizontal movement terminator, 5 - horizontal terminator of movement with rollers, 6 - crank mechanism)

In vibration machine represented on fig. 1c presence of two exciter drives independent between itself and attached not to the bottom of the container (the fig. 1a - b), but to its lateral walls predetermines universality of application of the machine: at the rotation of exciters in one direction - the container carries out flat movement in vertical plane and the machine can be used for vibration volumetric processing of products; at the rotation of exciters of identical weight with identical eccentricity and angular speed in opposite directions horizontal compound indignation from left and right exciters are mutually compensated - container moves only vertically. Besides, the presence permits two independent exciter at

the expense of redistribution of amplitude of fluctuations of the container to carry out separation of loading compound of vibration machines not only along a vertical axis of symmetry of the container, but also along horizontal. Rather vibration machine which circuit represented on a fig. 1d, it can have the limiter of horizontal fluctuations as well as in vibration machine represented on the circuit 1a. Besides instead of a spring suspension bracket in vibration machine which circuit represented on a fig. 1, there can be other type of a suspension bracket, in particular pneumatic.

Thus, in vibration machines with vertical movement of the container to loading the container enclosed only vertical indignation. The purpose of this work is to study the dynamic processes of vibration machines of the vibration type. It has been previously investigated that the mathematical model of bulk homogeneous load can be a layering of flat elastic beams, which in some way interact with each other and with the walls of the container. However, the proposed type of kinematic perturbation of the system leads to another motion of the medium - these are transverse, vertical oscillations. The nature of this movement has not been previously investigated. As in the mentioned works, the environment will be modeled also as a layering of flat beams, for them we will consider cross oscillations. This movement of the load provides this type of kinematic perturbation and restrictions on the movement of the container. The differential equation of motion of the conditionally selected layer of the loaded medium takes the form:

$$u_{tt} + a^2 u_{zzzz} = \Theta f(u, u_t, \dots, u_{zzz}, m), \quad (1)$$

where  $a^2 = \frac{EI}{rF}$  ( $E$ ,  $I$ ,  $F$ ,  $r$  - parameters which characterize physically-mechanical properties

of environment),  $u(t, z)$  - vertical moving of a layer loading with coordinate  $z$  at the any moment of time  $t$ ;  $\Theta f(\dots)$  - analytical  $2\pi$  periodic till  $m$  function which takes into account: a) a deviation of the elastic characteristics of loose loading from the linear law of elasticity; b) influence dissipative and other nature of viscous-elastic forces on dynamics of process; c) influence of kinematic indignation on vertical fluctuations of environment ( $m$ - frequency of kinematic indignation,  $\Theta$ - small parameter which specifies the maximum order of smallness of the mentioned above nonlinear and external forces in comparison with linearly elastic forces of environment).

Accepting to attention a kind of contact of loading and walls of the container we shall consider that for the differential equation (1) the following regional conditions are carried out:

$$u(z, t)|_{z=0} = u(z, t)|_{z=l} = 0, \quad (2)$$

where  $l$  - geometrical parameter of the container.

In particular: a) if to take into account only that fact that the material of loading satisfies close to the technical law of elasticity, the function  $f(\dots)$ - accepts a look:

$$f(\dots) = -a_1^2 \frac{\Pi^2}{\Pi z^2} (u_{zz})^3; \quad (3)$$

b) If the forces of resistance and dissipative forces satisfy the law of Bolotin,  $f(\dots)$ - accepts a look:

$$f = u_t (B + B_0 u^2); \quad (4)$$

c) If to take into account only kinematic indignations, the specified function with sufficient accuracy is represented by the sum Furie by harmonics  $k \gg m \gg t$  ( $k = 1, 2, \dots$ ).

The physical contents of regional conditions (4) answers contact of stratifications of beams to walls of the container, which approximately can be simulated immovable articulate fastening.

We shall proceed to construction solution of the equation (1). With this purpose we shall consider the not indignant equation, which answers (1), that is equation

$$u_{tt} + a^2 u_{zzzz} = 0. \quad (5)$$

Separating in (5) replaceable according to the formula  $u(z, t) = Z(z) \times \cos(\omega t)$  for a finding of unknown function  $Z(z)$  is received the differential equation:

$$Z_{zzzz} - u^4 Z = 0, \quad (6)$$

$$\text{where } u^4 = \frac{w^2}{a^2}.$$

The function  $Z(z)$ , proceeding from (3), satisfies regional conditions  $z(0) = z(l) = 0$ ,  $z_{zz}(0) = z_{zz}(l) = 0$ .

The generalized solution of equation (6) is expressed through function of Bogoliybov-Krylov:

$$K_1(uz) = \frac{1}{2}(chu_z + \cos uz), \quad K_2(uz) = \frac{1}{2}(shu_z + \sin uz), \quad K_3(uz) = \frac{1}{2}(chu_z - \cos uz),$$

$$K_4(uz) = \frac{1}{2}(shu_z - \sin uz) \text{ as follows: } Z(z) = \sum_{i=1}^4 S_i K_i(uz). \quad (7)$$

Satisfying the condition (7), we shall receive:

$$Z(z) = a_n \sin \frac{n \pi z}{l}, \quad (8)$$

where  $a_n$  - any constant

Given the last equation, the single-frequency and frequency solutions of the undisturbed equation (6) will look like:

$$w_n = a \frac{n^2 \pi^2}{l^2}, \quad (9)$$

$$u_n(t, z) = a_n \cos(w_n t + a_n) \sin \frac{n \pi z}{l}. \quad (10)$$

The presence of different dissipative forces and resistance forces leads to accelerated damping of high-frequency oscillations and the formation of a dynamic process with one (in most cases the first) frequency. Therefore, in this system we shall consider so-called one-frequency modes of fluctuations with frequency equal by first -  $n = 1$  (main frequency). Last appreciably facilitates a technique of research of the indignant equation (1), and at finding of its solution an index  $k$  we shall omit. We shall accept the solution of regional task (1-2) as

$$u(t, z) = a \cos(y) \sin \frac{\pi z}{l}, \quad y = \omega t + g, \quad (11)$$

where  $a$  - amplitude,  $g$  - initial phase one-frequency of dynamic process (replaceable in time).

In view of properties of completeness of system functions

$$\{Z_n(z)\} = \left\{ \sin \frac{n \pi z}{l} \right\}, \text{ for the finding of the laws of change of amplitude and frequency of}$$

fluctuations of a loading layer in a not resonant case ( $\omega \neq m$ ) the differential equation are received:

$$\Delta^2 \Delta^2 f_1(z, a, q, j) \sin \frac{\pi z}{l} \cos j \, dj \, dq \, dz, \\ j \Delta^2 \Delta^2 f_1(z, a, q, j) \sin \frac{\pi z}{l} \sin j \, dj \, dq \, dz, \quad q = m, \quad (12)$$

where

$$f_1(z, a, q, j) = f_1^{\frac{a}{\omega}} \cos j \sin \frac{\pi z}{l}, -a \omega \sin j \sin \frac{\pi z}{l}, \frac{p}{l} a \cos j \cos \frac{\pi z}{l}, \dots, -\frac{a \omega^3}{\epsilon l} \cos j \sin \frac{\pi z}{l}, q \frac{\ddot{}}{\omega}.$$

For a case of the main resonance  $\omega \gg m$  the laws of change of amplitude both the differences of phases of the own and compelled fluctuations are determined from system of the differential equations:

$$\begin{aligned} \ddot{a} &= \frac{1}{2lwp} \int_0^{2p/l} \ddot{f}_1(z, a, q, j+q) \sin \frac{pz}{l} \cos(j+q) dq dz, \\ j &= w - m + \frac{1}{2alwp} \int_0^{2p/l} \ddot{f}_1(z, a, q, j+q) \sin \frac{pz}{l} \sin(j+q) dq dz, \end{aligned} \quad (13)$$

where  $j = y - q$ .

As an example we shall consider cross fluctuations of loading only in view of its nonlinear-elastic properties, that is for a case, if the function  $f(\dots)$  looks like (3). Amplitude-frequency characteristic of cross fluctuations of loading changes according to parities:

$$\begin{aligned} \ddot{a} &= 0, \\ j &= a \frac{p}{c} \frac{\ddot{a}}{l} + \frac{9}{32} a_1 \alpha^2 \times \frac{p}{c} \frac{\ddot{a}^4}{l}, \end{aligned} \quad (14)$$

From the last formula it can be seen that the natural frequency of the loading layer depends not only on its physical and mechanical properties, but also on the amplitude (initial perturbation), given that for the accepted assumptions the system is conservative.

If in addition to take into account action of external periodic indignation, that is the right part of the equation (1) looks like

$$f(\dots) = -a_1^2 \frac{p^2}{l^2} (u_{zz})^3 + e \times b_0 \sin mx, \quad (15)$$

The equation which describe dynamic process in close to a resonant case, accept a look:

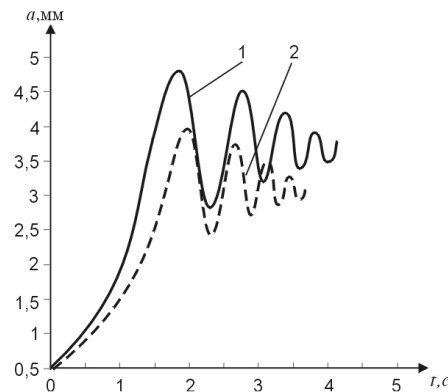
$$u(z, t) = a \times \sin \frac{pz}{l} \cos \frac{a}{c} \times \frac{p}{c} \frac{\ddot{a}}{l} + j \frac{\ddot{a}}{l}, \quad (16)$$

$$\begin{aligned} \ddot{a} &= -\frac{4l^2 \times e \times b_0}{p(a \times p^2 + m \alpha^2)} \cos j, \\ j &= a \frac{p^2}{l^2} - m + \frac{15}{28} \alpha_1 \alpha^2 \frac{p^4}{l^4} + \frac{4l^2 \times e \times b_0}{p \alpha (p^2 a + l^2 m)} \sin j. \end{aligned}$$

In the given dependences  $e b_0$  - is expressed through amplitude of external indignation  $b$ .

$$\frac{e}{c} b_0 = \frac{b}{r F} \frac{\ddot{a}}{l}.$$

In the fig. 2 is represented, as an example, resonant curve of development of amplitude of loading fluctuations at its following physically-mechanical parameters:  $E = 2 \times 10^7 H \times m^{-2}$ ,  $l = 1m$ ,  $r F = 7,2kg \times m^{-1}$ .



**Fig. 2.** Diagram of the dependence of the amplitude of vertical oscillations of the layer of the medium at the resonant state for  $b_0 = 0,0014 \text{ m}^2 \times \text{kg}^{-1}$  (curve 1),  $b_0 = 0,0009 \text{ m}^2 \times \text{kg}^{-1}$  (curve 2).

### Conclusions.

The use of different types of calculations and the obtained software allows to study the dynamics of the loading layer of the vibrating machine, to determine the frequency and amplitude of oscillations of the layer depending on its physical and mechanical properties and design characteristics of the vibrating machine. Solutions of the obtained nonlinear differential equations describing the movement of the loading layer in the middle of the container allow to determine automatically the trajectory, amplitude, frequency, load, optimize the vibration processing, correct calculations and use these calculations in further studies of technological vibrating machines.

### References

1. Dzhedzhula, O.M. (2019). Osoblivosti konstruyuvannya vibracijnych zmischuvachiv [Features of vibration mixers design]. *Vibratsiyi v tekhnitsi ta tekhnolohiyakh – Vibrations in engineering and technology*, 4 (95), 24–30. [in Ukrainian]. DOI: 10.37128/2306-8744-2019-4-3
2. Lawinska, K. and Modrzewski, R. (2017). Analysis of sieve holes blocking in a vibrating screen and a rotary and drum screen. *Journal «Fizykochemiczne Problemy Mineralurgii – Physicochemical Problems of Mineral Processing»*, 53, 812–828. DOI: <https://doi.org/10.5277/ppmp170212>
3. Zhou, N. (2015). Dynamic characteristics analysis and optimization for lateral plates of the vibration screen. *Journal of Vibroengineering (J. VIBROENG)*, 17(4), 1593–1604. <https://www.jvejournal.com/article/15826>
4. Subach A.P. (1991). *Dynamica processov i mashyn obyemnoy obrobotky [Dynamics of processes and machines of volumetric processing]*. Riga: Zinatne [in Russian].
5. Symonenko T. E. (2011). O razrabotke matematycheskoi modeli dlia obrabotky nezakreplennykh detalei [The development of a mathematical model for processing loose parts]. *Visnyk Donbaskoi derzhavnoi mashynobudivnoi akademii – Bulletin of Donbass State and Machine-Building Academy*, 2 (23), 201–205 [in Russian]. [http://www.dgma.donetsk.ua/science\\_public/ddma/2011-2-23/article/11STEUPD.pdf](http://www.dgma.donetsk.ua/science_public/ddma/2011-2-23/article/11STEUPD.pdf)
6. Oryshchenko S.V. and Matsiuk B.V. (2013). Doslidzhennia dynamiky vibratsiinoho hrokhota ta otsinka yoho efektyvnosti [Research of the dynamics of vibrating screen and evaluation of its effectiveness]. *Vibratsiyi v tekhnitsi ta tekhnolohiyakh – Vibrations in engineering and technology*, 3 (71), 120–125. [in Ukrainian]. <http://vibrojournal.vsau.edu.ua/files/pdfa/2000.pdf>
7. Ivanov, K. and Vaisberg, L. (2015). New Modelling and Calculation Methods for Vibrating Screens and Separators. *Journal «Lecture Notes in Control and Information Sciences»*, 22, 55–61. DOI: [10.1007/978-3-319-15684-2\\_8](https://doi.org/10.1007/978-3-319-15684-2_8)
8. Topilnytskyi V., Rebot D., Sokil M., Velyka O., Liaskovska S., Verkhola I., et al. (2017). Modeling the dynamics of vibratory separator of the drum type with concentric arrangement of sieves. *Eastern-European Journal of Enterprise Technologies. Applied Mechanics*, 2, 7 (86), 26–35. DOI: <https://doi.org/10.15587/1729-4061.2017.95615>
9. Mitropolskii Yu. A. (1997). *Nelineynaya mekhanika. Odnokhastotnye koljebaniya [Nonlinear mechanics. Single frequency oscillations]*. Kyiv: Ins.matematiki NAN Ukrainy [in Russian].

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## ДОСЛІДЖЕННЯ ДИНАМІКИ ЗАВАНТАЖЕННЯ ВІБРАЦІЙНИХ МАШИН З ВЕРТИКАЛЬНИМ ЗБУРЕННЯМ

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**Мета.** Розробити математичне забезпечення для дослідження динаміки вібраційних машин об’ємного оброблення, та розрахунку впливу різних чинників на ефективність процесу на базі прикладних систем автоматизованих математичних розрахунків, зокрема MathCad та MatLab. **Методика.** Дослідження проводились на основі класу вібромашини з дебалансним типом приводу і пружинною підвіскою.

Побудована математична модель завантаження вібраційних оброблювальних машин з вертикальним збуренням, яка представлена нашаруванням плоских балок, що здійснюють вертикальні коливання. Для побудови математичної моделі використано методи нелінійної механіки. **Результати.** Досліджено динамічні процеси при вібраційному ущільненні завантаження і сепарації його фракцій з метою підвищення інтенсивності даних технологічних процесів. Отримано залежності для визначення впливу фізико-механічних властивостей складових завантаження на динаміку процесу. **Наукова новизна.** Розглянуто нові підходи для побудови математичних моделей дослідження процесів в машинах вібраційної обробки, зокрема вібраційного ущільнення та сепарації. Отримані диференціальні рівняння для визначення зміни амплітуди та частоти завантаження від впливу зовнішніх та внутрішніх параметрів процесу віброобробки. **Практична значущість.** Побудована математична модель дає можливість дослідити динаміку завантаження вібромашини, визначити амплітуду та частоту коливань завантаження вібромашини в залежності від його властивостей та властивостей вібраційної машини. Отримані розв'язки нелінійних диференціальних рівнянь опису руху завантаження у вібраційних машинах дають змогу автоматизовано визначати амплітуди, частоти та траєкторії руху різних складових завантаження в залежності від параметрів системи „вібраційна машина - завантаження”, шляхом їх алгоритмізації у прикладних системах автоматизованих математичних розрахунків.

**Ключові слова:** не зв'язаний інструмент оброблення, вібраційна машина, поверхнева обробка, асимптотичні методи, дебалансний привід, нелінійна механіка.

### References

1. Джеджула О. М. Особливості конструювання вібраційних змішувачів. *Вібрації в техніці і технологіях*. 2019. № 4 (95). С. 24–30. DOI: 10.37128/2306-8744-2019-4-3
2. Lawinska K., Modrzewski R. Analysis of sieve holes blocking in a vibrating screen and a rotary and drum screen. *Journal «Fizykochemiczne Problemy Mineralurgii – Physicochemical Problems of Mineral Processing»*. 2017. Vol. 53. P. 812–828. DOI: <https://doi.org/10.5277/ppmp170212>
3. Zhou N. Dynamic characteristics analysis and optimization for lateral plates of the vibration screen. *Journal of Vibroengineering (J. VIBROENG)*. 2015. Vol. 17 (4). P. 1593–1604. <https://www.jvejournal.com/article/15826>
4. Субач А. П. Динамика процессов и машин объемной обработки. Рига : Зинатне, 1991. 240 с.
5. Симоненко Т. Е., Барсуков В. А. О разработке математической модели для обработки незакрепленных деталей. *Вісник Донбаської державної машинобудівної академії*. 2011. № 2 (23). С. 201–205. [http://www.dgma.donetsk.ua/science\\_public/ddma/2011-2-23/article/11STEUPD.pdf](http://www.dgma.donetsk.ua/science_public/ddma/2011-2-23/article/11STEUPD.pdf)
6. Орищенко С. В., Мацюк Б. В. Дослідження динаміки вібраційного грохота та оцінка його ефективності. *Вібрації в техніці і технологіях*. 2013. № 3 (71). С. 120–125. <http://vibrojournal.vsau.edu.ua/files/pdfa/2000.pdf>
7. Ivanov K., Vaisberg L. New Modelling and Calculation Methods for Vibrating Screens and Separators. *Journal «Lecture Notes in Control and Information Sciences»*. 2015. Vol. 22. P. 55–61. DOI: [10.1007/978-3-319-15684-2\\_8](https://doi.org/10.1007/978-3-319-15684-2_8)
8. Topilnyskyi V., Rebot D., Sokil M., Velyka O., Liaskovska S., Verkhola I., et al. (2017). Modeling the dynamics of vibratory separator of the drum type with concentric arrangement of sieves. *Eastern-European Journal of Enterprise Technologies. Applied Mechanics*, 2, 7 (86), 26–35. DOI: <https://doi.org/10.15587/1729-4061.2017.95615>
9. Митропольский Ю. А. Нелинейная механика. Одночастотные колебания. К. : Ин-т математики НАН Украины, 1997. 385 с.