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RBF collocation path-following approach: optimal choice for shape parameter based on genetic algorithm

Saffah Z.[1], Hassouna S.[2], Timesli A.[2], Azouani A.[3,4], Lahmam H.[1]

[1] *Hassan II University of Casablanca, LIMAT Laboratory, Faculty of Sciences of Ben M'Sik, LIMAT Laboratory, Morocco*
[2] *Hassan II University of Casablanca, National Higher School of Arts and Crafts (ENSAM Casablanca), 20670 Casablanca, Morocco*
[3] *Sultan Moulay Slimane University, National School of Applied Sciences of Khouribga, LIPIM Laboratory, Morocco*
[4] *Freie Universität Berlin, Institut für Mathematik I, Arnimallee 7, 14195, Berlin, Germany*

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This paper presents a new method to solve a challenging problem and a topic of current research namely the selection of optimal shape parameters for the Radial Basis Function (RBF) collocation methods in both interpolation and nonlinear Partial Differential Equations (PDEs) problems. To this intent, a compromise must be made to achieve the conflict between accuracy and stability referred to as the trade-off or uncertainty principle. The use of genetic algorithm and path-following continuation allows us on the one hand to avoid the local optimum issue associated with RBF interpolation matrices, which are inherently ill-conditioned and on the other side, to map the original optimization problem of defining a shape parameter into a root-finding problem. Our computational experiments applied on nonlinear problems in structural calculations using our proposed adaptive algorithm based on genetic optimization with automatic selection of the shape parameter can yield more accuracy and a good precision compared to the same state of the art algorithm from literature with a fixed and given shape parameter and Finite Element Method (FEM).

Keywords: *large deformations, strong form, RBF collocation method, genetic algorithm, automatic choice of shape parameter.*

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1. Introduction

In recent years, the use of meshless methods has become important in many fields of engineering due to their simplicity in multivariate scattered data approximation [1] and their usefulness in solving many real-world engineering problems, the meshless methods are precisely discovered to attempt avoiding the lack of accuracy coming from the distortion of mesh as in FEM, where this distortion leads to the important lack of accuracy in the simulations, and also it attempt for minimizing the difficult step which is the mesh construction. These methods are a numerical method that require no mesh connections within the computational domain, they constitute a real revolution since the publication of Smooth Particle Hydrodynamics (SPH) method by the Astrophysicist Lucie [2] and the mathematicians Gingold and Monaghan [3]. The many interesting advantages of these methods are the absence of nodal connectivity, the mathematical simplicity and ease of implementation. So there is no need for numerical integrations, which reduces the computational cost compared to FEM. The meshless methods can be classified in two classes: (i) the meshless methods of Galerkin type [4, 5] and of collocation type [6–8]. The first class combines the meshless methods with the weak formulation of PDEs and often these methods require a domain partition. The Diffuse Element Method (DEM) introduced by Nayroles et al. in 1992 [9] is the first meshless method using the formulation of Galerkin type, in 1994 Belyrsho et al. [5] proposed the Element Free Galerkin Method using the Moving Least Squares method (MLS).

The collocation type methods, unlike the Galerkin type methods, use the strong form of PDEs. In 1990 a robust method has been proposed by Kansa [6, 10] which developed the Radial Basis Function (RBF) method for solving PDE of elliptic, parabolic, and hyperbolic types.

The history of RBFs goes back to 1971, where Hardy [11] innovated probably the most famous RBF, Multi-Quadratics (MQ) function, to deal with surface fitting on topography and irregular surfaces, Yoon [12] showed that the MQ RBF can also converges exponentially in a Sobolev space for PDEs and Madych [13] showed that the rate of kernels MQ, Inverse Multiquadric (IMQ), and Gaussian (GA) are exponential converge on reproducing kernel Hilbert space. Moreover, depending on how the RBFs are chosen, high-order or spectral convergence can be achieved [14, 15]. This method based on the interpolation theory using RBFs has been used successfully in the following works [16, 17], and it is becoming a best choice as a method for the numerical solution of PDEs [18,19]. The MQ RBF was extended by Ferreira et al. [20, 21] for the analysis of composite beams, plates and shells. Chen and his coworkers [22, 23] developed a new meshless method called the Method of Approximate Particular Solution (MAPS) which utilized RBF integrations to approximate the particular solution for PDEs while simultaneously satisfying the boundary conditions. More recent research conducted mostly by Fornberg and his coworkers [24–26], they showed that one may be able to overcome the conditioning problems of the traditional RBF approach by using other techniques such as the Contour–Pade algorithm. A review of the theory of RBF approximation is given by Powell [16].

RBF method is based on a shape parameter, the choice of this one has a significant impact on the accuracy of the RBF method, Carlson and Foley [27] showed that the influence of location of collocation data points is less than shape parameter on the accuracy of the optimal solution, the infinitely smooth RBFs typically leads to exponential convergence when the node density increases and the shape parameter decreases. These results have been observed numerically for the commonly used RBFs and proved theoretically for MQ and IMQ RBFs [28–31]. Kansa and Carlson [32] showed that variable shape parameters are also useful, they distribute some values in an interval to use them in variable shape parameter. Several strategies have been proposed for selecting the shape parameter. Hardy [11] demonstrated that an optimal shape parameter could be selected in $R2$ using $c = 0.815 \times d$, where $d = (1/N)\sum d_j$, and d_j is the distance from a given data point $x(j)$ and its neighbor close.Later, Franke [33] compared about 30 interpolation schemes in two dimensions, he found that two of the most accurate schemes were methods based on RBF interpolation so he proposed another form using $c = D/(0.8\sqrt{N})$, where D is the diameter of the smallest circle that includes all of the interpolation points. Both of these provided relatively good shape parameters, especially when the calculations are performed on single precision arithmetic. Fasshauer and McCourt [34] also proposed a new approach which can stably evaluate the RBF interpolants. Foley [35] proposed another scheme for the computation of a better value for the shape parameter c using another observation from [27], he showed that the optimal value for c is about the same for the MQ and the inverse MQ interpolants, and he proposed computing the RBF shape parameters by minimizing the Root–Mean–Square Error (RMSE) evaluated at a set of test points [35]. For different values of the shape parameters using a common set of training points, the shape parameter corresponding to the minimum RMSE were selected. Other methods, which tend to utilize a trail-and-error approach, yield results which are not optimally accurate or which ignore the effect of the precision of the solver.

In 1999 Rippa [36] presented an algorithm for selecting a good value for c by minimizing a cost function from which the data vector was sampled. This procedure is more advantageous than the procedure of Foley, as it does not require a set of test points. Recently in 2018 Chen et al. [37] present a novel sample solution approach for achieving a reasonably good shape parameter of the MQ-RBF in the Kansa method to resolve of problems with unknown analytical solution, the optimal shape parameter of the considered problem is chosen by considering the same problems using a guessing analytical solution. The obtained optimal shape parameter for the considered problem with sample solution is obtained numerically and is considered as an alternative optimal shape parameter for the original problem. At the level of the Moving Least Square Method (MLS), research on the optimal shape parameter of RBF is rare. Zheng et al. [38] presented an idea for measurement of the optimal shape parameter of Moving Least Squares approximation based on RBF, Hence the results obtained show that MLS using RBF with the optimal shape parameter are much better than the polynomial Least Squares.

Further efforts have utilized global optimization methods such as Genetic Algorithms (GAs) [39,40]. This latter is a search algorithm suitable for optimization problems due to its processing approach, its structure, and it is able to return a set of optimal solutions because of their simplicity and easy operations with minimum requirements. GAs have been used successfully in a wide variety of problems. Recently in 2016 Biazar and Hosami [41] present an algorithm that suggested to determine a valid interval in variable shape parameter. Esmaeilbeigi and Hosseini [40] presented a new approach based on the GA to find a good shape parameter in the resolution of partial differential equations by the Kansa method from where the results obtained show that the proposed algorithm based on the Genetic optimization is efficient and provides a reasonable shape parameter with acceptable solution precision. Afiatdoust and Esmaeilbeigi [39] propose to apply the genetic algorithm to determine good shape parameters of RBF for solving ordinary differential equations from where the results show that the algorithm provides reasonable shape parameters as well as precision acceptable in linear and nonlinear cases compared to other methods. Weikuan et al. [42] suggested a new RBF neural network method based on the Genetic Algorithms for weight optimization and the number of hidden neurons, then connected concurrently the weight using the least mean square method, the results showed that the GA is reliable for changing the neural network structure.

In our previous work [43], we developed a novel approach based on the RBF collocation method and a shape parameter search algorithm that determines the optimal shape parameter with a good precision of the results whatever the points distribution, in this work an optimization algorithm [44,45] is used to search the shape parameter and coupled with a high order algorithm to solve equilibrium equations in large deformations for a nonlinear elastic structure (geometrical nonlinearity). In this paper we propose a new method based on genetic algorithm, to find good variable shape parameters of kernel methods based on RBF, for the resolution of nonlinear problems. For the numerical analysis, we are interested by nonlinear problems in the structural calculation. In this context, we consider two bi-dimensional examples, one on a structure in tension (see section 6.1) and the other in bending (see section 6.2).

2. RBF collocation approximation

Consider the Euclidean norm $\|\cdot\|_2$ on $\mathbb{R}^d$ and a vector x in $\mathbb{R}^d$, the RBF have the form $\phi(\|x-x_j\|_2)$ which is defined strictly positive. The RBF collocation approximation can be written as:

$$s(x)=\sum_{j=1}^{N}\alpha_j\,\phi(\|x-x_j\|_2). \tag{1}$$

The nearest neighbors points x_j is a set of points to the point x, which are included in a local support domain $\Omega\subset\mathbb{R}^d$. The functions $\phi(\|x-x_j\|_2)$ contain the shape parameters c_i such as the multi-quadrics functions:

$$\phi_i(\|x-x_j\|_2)=\left(c_1^2+\|x-x_j\|_2^2\right)^q \tag{2}$$

and the exponential functions:

$$\phi_i(\|x-x_j\|_2)=e^{(c_2^2\,\|x-x_j\|_2^2)}. \tag{3}$$

By enforcing the interpolation constraints as follows:

$$s(x_i)=f(x_i), \tag{4}$$

which leads to determine the unknowns coefficients $\alpha=[\alpha_1,\dots,\alpha_N]^T$ using the interpolation matrix $R_{ij}=\phi(\|x_i-x_j\|)$, $i,j=1,\dots,N$ as follows:

$$\alpha = R^{-1} f, \tag{5}$$

where $f = [f(x_1), \ldots, f(x_N)]^T$.

The approximation of the function $s(x)$ is given:

$$s(x) = \langle \phi(x) \rangle R^{-1} f. \tag{6}$$

The choice of a good shape parameter c makes it possible to ensure the existence of R^{-1} and a well-conditioned R. As shown in the section 1, there are several research works which are interested in finding the best optimal value of c. Note that we cannot determine the theoretical value of c. Our contribution to this work is to develop a novel approach based on GA that determines the optimal parameter c with a good precision for nonlinear elastic problem.

3. Nonlinear elastic problem statement in strong form

The equations of equilibrium in strong formulation for a nonlinear elastic structure in large deformations [8, 43] assume that the displacements and forces imposed are proportional to a single scalar parameter λ called the load parameter as follows:

$$\begin{cases} (\operatorname{div} T)_i = \partial_j T_{ij} = 0 & \forall\, x \in \Omega \\ S_{ij} = D_{ijkl} \otimes \gamma_{kl}(U) & \forall\, x \in \Omega, \\ \gamma(U)_{ij} = \dfrac{1}{2}\left(\partial_j U_i + \partial_i U_j\right) + \dfrac{1}{2} \partial_i U_k \partial_j U_k & \forall\, x \in \Omega, \\ U_i = \lambda\, U_i^d(x) & \forall\, x \in \Gamma_1 = \partial\Omega_D, \\ T_{ij} \cdot N_j = \lambda\, F_i & \forall\, x \in \Gamma_2 = \partial\Omega_F, \end{cases} \tag{7}$$

where T and S represent the first and second tensor of Piola–Kirchhoff respectively, D is the elastic behavior tensor, Ω is the domain occupied by the structure, $\partial\Omega_U$ and $\partial\Omega_F$ are the boundary of an imposed displacement U^d and applied loading F respectively. N is the normal applied towards the outside of the boundary $\partial\Omega_F$.

We can rewrite the problem (7) in matrix form:

$$\begin{cases} [L]\,\{T\} = \{0\} & \text{in } \Omega, \\ \{T\} = [III] + [B(g(U))]\,\{S\}, \\ \{S\} = [D]\,\{\gamma\}, \\ \{\gamma\} = \left([II] + \dfrac{1}{2}\,[A(g(U))]\right)\{g(U)\}, \\ \{U\} = \lambda\,\{U_d\} & \text{on } \partial\Omega_U, \\ [N] \cdot \{T\} = \lambda\,\{F\} & \text{on } \partial\Omega_F, \end{cases} \tag{8}$$

where

$$[L] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} \end{bmatrix}; \quad [III] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}; \quad [B(g(U))] = \begin{bmatrix} U_{x,x} & 0 & U_{x,y} \\ 0 & U_{y,y} & U_{y,x} \\ 0 & U_{x,y} & U_{x,x} \\ U_{y,x} & 0 & U_{y,y} \end{bmatrix};$$

$$[II] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \quad [A(g(U))] = \begin{bmatrix} U_{x,x} & 0 & U_{y,x} & 0 \\ 0 & U_{x,y} & 0 & U_{y,y} \\ U_{x,y} & U_{x,x} & U_{y,y} & U_{y,x} \end{bmatrix};$$

$$[N] = \begin{bmatrix} N_x & 0 & N_y & 0 \\ 0 & N_y & 0 & N_x \end{bmatrix}; \quad \{g(U)\} = \begin{Bmatrix} U_{x,x} \\ U_{x,y} \\ U_{y,x} \\ U_{y,y} \end{Bmatrix}.$$

4. Numerical Path-following continuation and the local RBF approximation

Taylor series development is used to search the unknowns variables of problem (8):

$$\begin{cases} \{T(a)\} = \{T_0\} + \sum\limits_{p=1}^{N_0} a^p \{T_p\}, \\ \{S(a)\} = \{S_0\} + \sum\limits_{p=1}^{N_0} a^p \{S_p\}, \\ \{\gamma(a)\} = \{\gamma_0\} + \sum\limits_{p=1}^{N_0} a^p \{\gamma_p\}, \\ \{U(a)\} = \{U_0\} + \sum\limits_{p=1}^{N_0} a^p \{U_p\}, \\ \lambda(a) = \lambda_0 + \sum\limits_{p=1}^{N_0} a^p \lambda_p. \end{cases} \tag{9}$$

Where ($\{T_0\}$, $\{S_0\}$, $\{\gamma_0\}$, $\{U_0\}$ and λ_0) is a known solution point and "a" is the arclength along a solution arc $(u(a), \lambda(a))$. Note that $(u(a), \lambda(a))$ depends smoothly on the parameter "a" and $x = (u^T, \lambda)^T$. Using pseudoarc-length parametrization [8, 46] as shown in the previous paper [43], we add an equation to close our system and to obtain a nonsingular Jacobian for the our problem statement. In this context, we get an additional condition by projecting the pair $(U - U_0, \lambda - \lambda_0)$ on the tangent direction U_1, λ_1 as follows:

$$a = \langle U - U_0 \rangle \cdot U_1 + (\lambda - \lambda_0) \cdot \lambda_1. \tag{10}$$

Therefore, the following equations depending of the order p are obtained:

$$\begin{cases} \|U_1\|^2 + \lambda_1^2 = 1 & \text{for} \quad p = 1, \\ \langle U_p \rangle \cdot \{U_1\} + \lambda_p.\lambda_1 = 0 & \text{for} \quad p \geqslant 2. \end{cases} \tag{11}$$

We inject the developments in Eq. (9) into the non-linear problem (8) and identify the terms having the same powers of the parameter "a" to obtain a sequence of linear problems:
Problem at order 1:

$$\begin{cases} [L]\{T_1\} = \{0\}, \\ \{T_1\} = [G(g(U_0))]\{S_1\} + [\hat{S_0}]\{g(U_1)\}, \\ \{S_1\} = [D]\{\gamma_1\}, \\ \{\gamma_1\} = [H(g((U_0))] \{g(U_1)\}, \\ \{U_1\} = \lambda_1 \{U^d\}, \\ [N] \cdot \big\{[H(g(U_0))][D][G(g(U_0))] + [\hat{S_0}]\{g(U_1)\}\big\} = \lambda_1 \{F\}. \end{cases} \tag{12}$$

Problem at order p such that $2 \leqslant p \leqslant N_{\text{order}}$:

$$\begin{cases} [L]\{T_p\} = \{0\}, \\ \{T_p\} = [G(g(U_0))]\{S_p\} + [\hat{S_0}]\{g(U_k)\} + \{T_p^*\}, \\ \{S_p\} = [D]\{\gamma_p\}, \\ \{\gamma_p\} = [H(g(U_0))]\{g(U_p)\} + \{\gamma_p^*\}, \\ \{U_p\} = \lambda_k \{U^d\}, \\ [N] \cdot \big\{\{[H][D][G] + [\hat{S_0}]\} \{g(U_p)\}\big\} = \lambda_p \{F\} - [N] \big\{[H][D]\{\gamma_p^*\} + \{T_p^*\}\big\}, \end{cases} \tag{13}$$

where

$$[H(g(U_0))] = [II] + [A(g(U_0))], \quad [G(g(U_0))] = [III] + [B(g(U_0))],$$

$$\{\gamma_p^*\} = \frac{1}{2} \sum_{r=1}^{p-1} [A(g(U_r))] \{g(U_{p-r})\}, \quad \{T_p^*\} = \sum_{r=1}^{p-1} [B(g(U_r))] \{S_{p-r}\},$$

and $[\hat{S_0}]$ is a matrix which contains the stress components of the starting solution of each order:

$$\left[\hat{S_0}\right]=\begin{bmatrix} S_{11}^0 & S_{12}^0 & 0 & 0\\ 0 & 0 & S_{12}^0 & S_{22}^0\\ S_{12}^0 & S_{22}^0 & 0 & 0\\ 0 & 0 & S_{11}^0 & S_{12}^0 \end{bmatrix}. \tag{14}$$

We approximate the principle unknown $\{U_p\}$ by the local RBF method and we inject this approximation in the problems (11)–(13) to obtain after the assembly technique a compact problems as follows:
Order 1:

$$\begin{cases} \{U_1^*\}=[K_T]^{-1}\{F\},\\ \lambda_1=\frac{1}{\sqrt{\langle U_1^*\rangle\{U_1^*\}+1}},\\ \{U_1\}=\lambda_1\{U_1^*\}. \end{cases} \tag{15}$$

Order p for $2\leqslant p\leqslant N_{\text{order}}$:

$$\begin{cases} \{U_{\text{n-lin}}\}=[K_T]^{-1}\{F_p\},\\ \lambda_p=\frac{-\langle U_p^*\rangle\{U_1\}}{\langle U_p^*\rangle\{U_1\}+\lambda_1},\\ \{U_p\}=\lambda_p\{U_1^*\}+\{U_p^*\}, \end{cases} \tag{16}$$

where $[K_T]$ is the stiffness tangent matrix at the starting point ($\{T_0\}$, $\{S_0\}$, $\{\gamma_0\}$, $\{U_0\}$). The new starting point of the new solution branch is obtained by continuation procedure $\{T(a_{\max})\}$, $\{S(a_{\max})\}$, $\{\gamma(a_{\max})\}$, $\{U(a_{\max})\}$, where $a_{\max}$ is the convergence radius [8, 46]:

$$a_{\max}=\left(\varepsilon\frac{\|\{U_1\}\|}{\|\{U_{N_0}\}\|}\right)^{\frac{1}{N_0-1}}. \tag{17}$$

5. Estimation of optimum variable shape parameter in high-order RBF collocation approach

Genetic algorithms are part of evolutionary algorithms which are based on genetics and natural selection [47]. Their operation is extremely simple. We start from an initial population of arbitrarily chosen potential solutions (population). We assess their relative performance (fitness). These performances allow us to create another population of potential solutions by crossover, mutations, and selection which are simple evolutionary operators. This cycle must be repeated to find a satisfactory solution.

The purpose is to use a search algorithm using genetic algorithm to determine good shape parameter c in the RBF collocation method. In this study, the genetic algorithm is tested to determine good c for the simulation of large deformation problems. For a given distribution of points, this proposed strategy allows to determinate automatically and quickly the best value α, to build the shapes functions, with a good precision. We consider that the shape parameter $c=\alpha\times ds$ where α is a coefficient to determinate and ds is the average distance between points where $ds=\frac{1}{N}\sum_{j=1}^{N}d_j$ and d_j is the distance between the jth point and its nearest natural neighbor. The proposed genetic algorithm for the optimal value search of the coefficient α is presented in the Fig. 1. The idea is to minimize the relative error of the displacement at order 1 of the high order RBF collocation algorithm as shown this figure. This first order error estimator allows us to ensure a well-conditioned tangent matrix $[K_T]$ of the high order algorithm which is the same used in the other orders $k\geqslant 2$. For this reason, in the minimization of the relative error, we are limited to the displacement at order 1 to determine $\beta_{optimal}$.

In general, we can obtain the generation of the "Population" as follows:

$$\begin{aligned} \text{Population}&=\text{zeros}(N_{\text{population}},\text{digits}+1),\\ \text{Population}&=[\text{randi}([0\quad 9],[N_{\text{population}}\quad 1]),\dots,\\ &\qquad\text{randi}([0\quad 9],[N_{\text{population}}\quad \text{digits}])], \end{aligned} \tag{18}$$

where "$N_{\text{population}}$" represents the number of values α to be tested for each iteration, "digits" is the number of digits after the decimal point and "randi" is a random function between 1 and 40 where the shape parameter α varies in the interval $[1, 40]$. We also assume a single number after the decimal point which limited in the interval $[0, 9]$. The satisfactory solution is controlled by the displacement relative error which must be lower of ε. If the relative errors of the tested values are greater of the tolerance parameter ε, we change the values to be tested. We test the values of the new "Population" in the second iteration. The same procedure is repeated until the relative error is less than ε. This genetic algorithm strategy is similar to that used recently by Hassouna and Timesli (2021) [48].

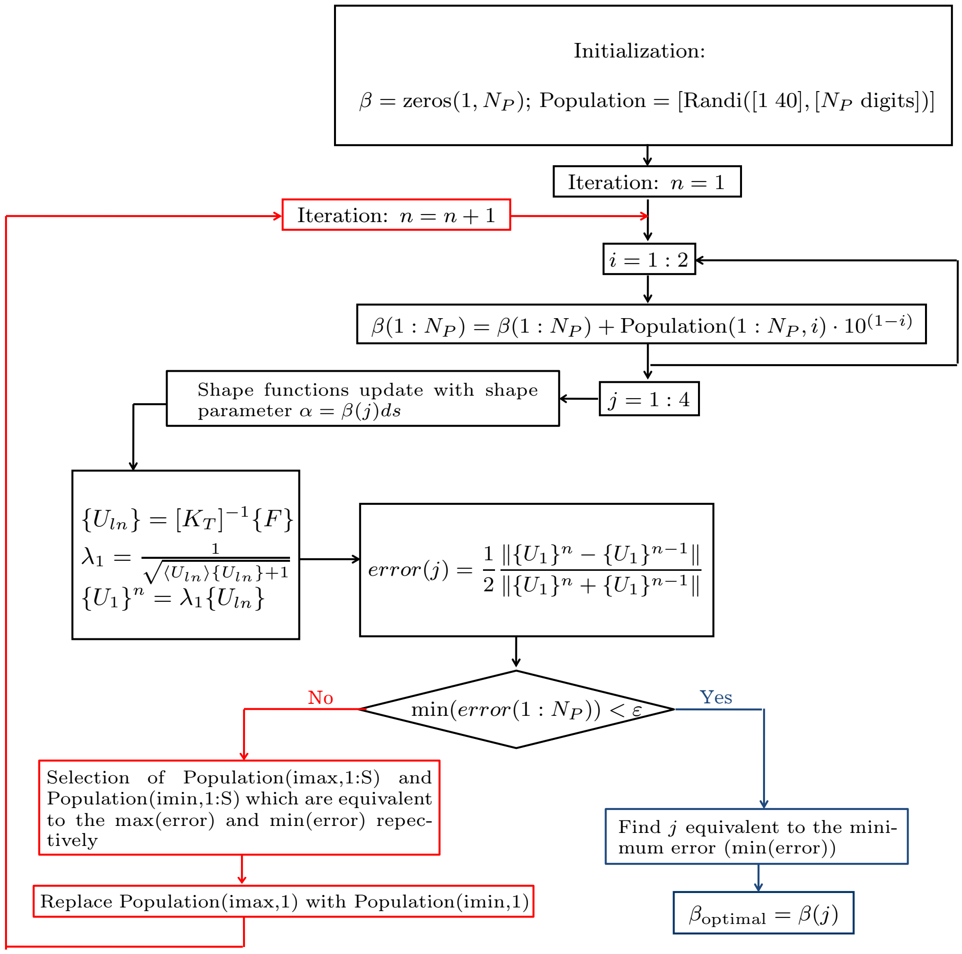

Fig. 1. Genetic algorithm strategy for finding the optimal value of β based on the first order error estimator.

6. Numerical results

In this application, using the proposed high order algorithm, we study the effect of the shape parameter on the solution of the nonlinear elastic problem. The accuracy of this numerical solution is controlled by the FEM method as a reference solution.

6.1. Bi-dimensional structure in tension

We propose to study a problem of elastic rectangular plate of length 100 mm and width 50 mm, this plate is fixed at $x = 0$ and subjected to a tensile force λF, with $F = 1$ MPa. The physical characteristics of the material are: Young's modulus $E = 200$ GPa and Poisson's ratio $\nu = 0.34$. The parameters of the high order algorithm are the truncation order $p = 15$ and the tolerance parameter $\eta = 10^{-8}$.

We performed a simulation using the proposed strong RBF approximation with the distributed points number 441 and the MQ function. Figure 3 represents the structure before and after deformation, where the representation of the initial state in black color and the deformed configuration in red color for $\lambda = 309870$. In the RBF method, the value of the shape parameter affects the accuracy and the

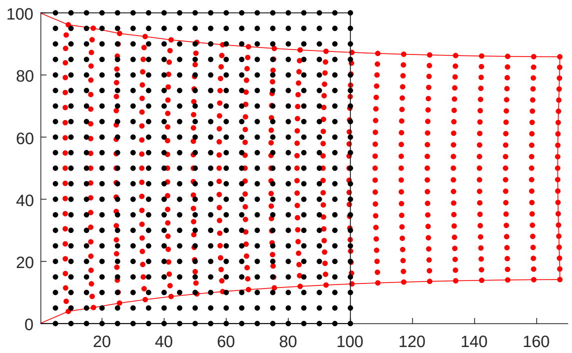

Fig. 2. Condition number for the MQ RBF interpolation.

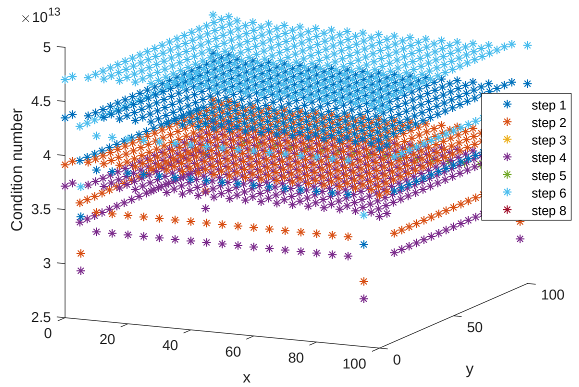

Fig. 3. Initial and deformed configurations of the plate for $\lambda = 309870$ using a distribution of 21×21 points.

conditioning of the system matrix. It is necessary to avoid a small shape parameter to ensure that the matrix of the system is well conditioned. However, obtaining a good accuracy for the RBF method requires a small shape parameter. Note that this choice leads to ill-conditioned matrix. Therefore, the solution is to find a compromise to get the best results in terms of accuracy and conditioning. This is called the Uncertainty Principle [49]. The stability result of the proposed method can be determined by the condition number, which is defined as follows:

$$\kappa(R) = \|R\|\|R^{-1}\|, \tag{19}$$

where the R is the momentum matrix assigned to the MQ RBF interpolation. The resulting system matrix must have a condition number in the range $[10^{13}, 10^{15}]$ [49]. We present in Fig. 2 the condition number for the MQ RBF interpolation in each point of the domain. This figure shows that the condition number is in the range mentioned before for all steps.

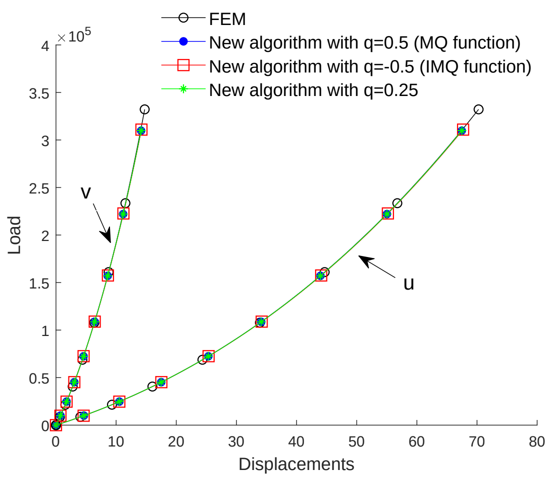

Fig. 4. Evolution of load λ versus displacements u and v in x direction for the new algorithm with different values of q and FEM at point of coordinate $(x = 0, y = 100)$.

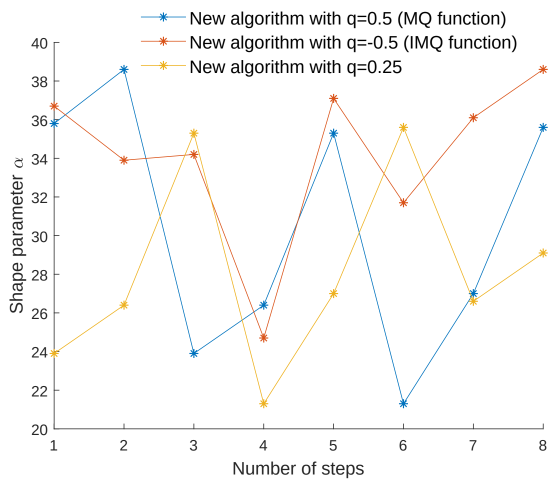

Fig. 5. Evolution of the optimal shape parameter α_{optimal} versus number of steps for the new algorithm.

Figure 4 shows the evolution of load λF versus displacements u and v at point of coordinate $(x = 0, y = 100)$. These results are obtained by the proposed algorithm, using the GA for finding the good shape parameter and the two functions MQ and IMQ, and compared by FEM.

Figure 5 presents the evolution of the optimal shape parameter α_{optimal} with respect to number of steps for the new algorithm which shows that $\alpha_{optimal}$ varies versus the number of steps. Therefore the parameter $\alpha_{optimal}$ can vary slightly with respect to the increase in loading or deformation.

6.2. Bi-dimensional structure in bending

In this second example, we consider a bi-dimensional elastic plate without contact in bending clamped at the left end, and subjected at the right end to an imposed load λF; with $F = 1\,\text{MPa}$. The used mechanical and geometrical characteristics of the considered structure are: Young's modulus $E = 210\,\text{GPa}$, Poisson's ratio $\nu = 0.3$, length $L = 200\,\text{mm}$ and height $l = 10\,\text{mm}$. These data are chosen according to the same study as in the previous example. The obtained result is compared with the ones obtained by the FEM. Regarding the high order algorithm, we use the same parameters of the previous example with the distributed points number 606. Figure 6 shows the initial and deformed configurations of plate in bending, where the representation of the initial state in black color and the deformed configuration in red color for $\lambda = 9458.9$.

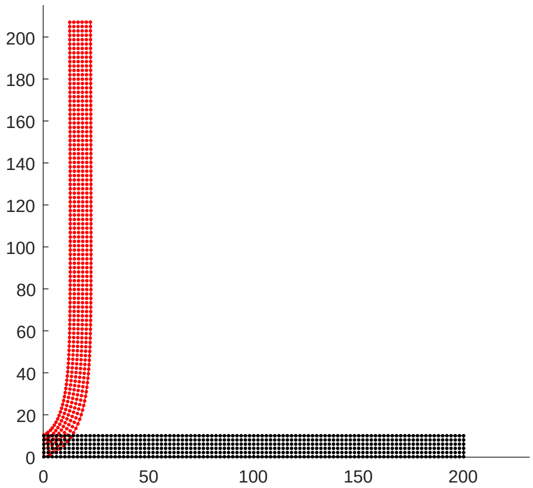

Fig. 6. Initial and deformed configurations of the plate for $\lambda = 9458.9$ using a distribution of 101×6 points.

Figure 7 represents the evolution of displacements u and v versus loading parameter λ at point of coordinate $(x = 0, y = 200)$ using the proposed algorithm with the MQ function where $q = 0.5$ and $q = 0.25$. We can see that the calculation by the high order algorithm based on FEM stops because the convergence radius of this high order algorithm tends to zero which shows the advantage of the high order RBF algorithm based on genetic algorithm to simulate this kind of problems.

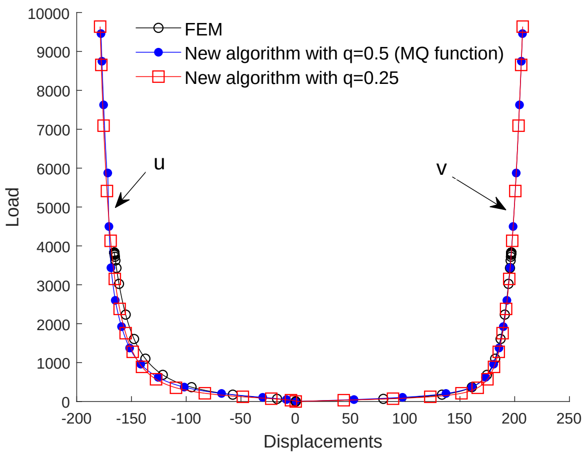

Fig. 7. Evolution of load λ versus displacements u and v in x direction for the new algorithm, with different values of q using a distribution of 101×6 points, and FEM at point of coordinate $(x = 0, y = 100)$.

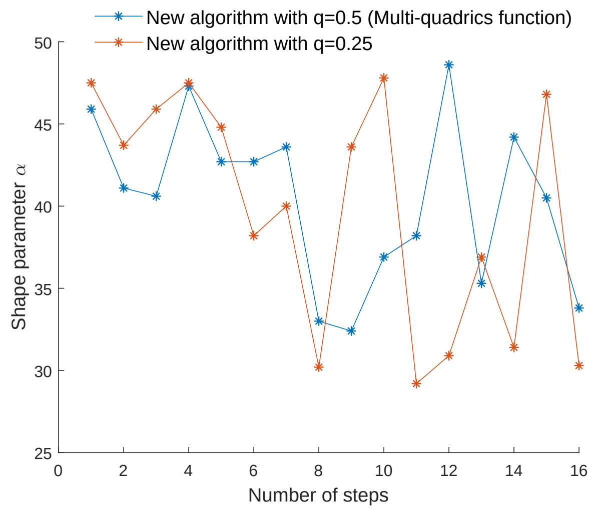

Fig. 8. Evolution of the optimal shape parameter α_{optimal} versus number of steps for the new algorithm using a distribution of 101×6 points.

Figure 8 presents the evolution of the optimal shape parameter $\alpha_{optimal}$ with respect to number of steps for the new algorithm in the case of bending for the both values $q = 0.5$ and $q = 0.25$.

Figure 9 represents the evolution of displacements u and v versus loading parameter λ at point of coordinate $(x = 0, y = 200)$ using the proposed algorithm with the MQ function where $q = 0.5$ and different distributions of point 101×6 and 141×8. We can observe that the calculation by the high order algorithm converges better towards the FEM solution using the distribution 141×8, that we can consider it the optimal distribution to obtain good results.

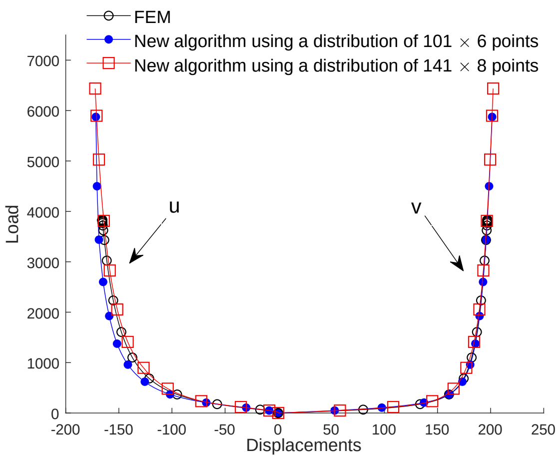

Fig. 9. Evolution of load λ versus displacements u and v in x direction for the new algorithm with $q = 0.5$, and FEM at point of coordinate $(x = 0, y = 100)$.

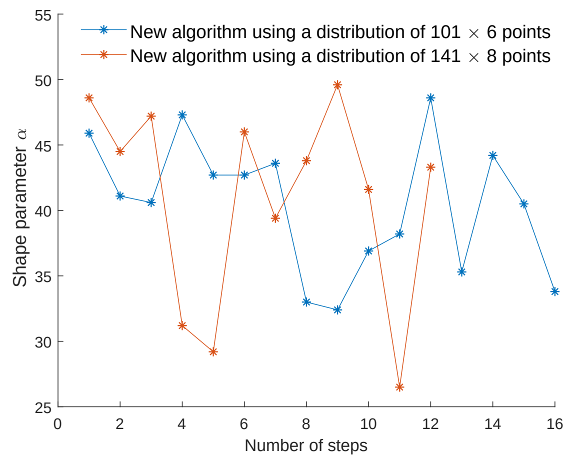

Fig. 10. Evolution of the optimal shape parameter α_{optimal} versus number of steps for the new algorithm with $q = 0.5$.

Figure 10 presents the evolution of the optimal shape parameter α_{optimal} with respect to number of steps for the new algorithm for the both distributions of point 101×6 and 141×8. So this figure shows that the values of α_{optimal} depend on distribution of point. This is the advantage of the proposed algorithm which adapts with the distribution of points to find better results.

7. Conclusion

In the present new algorithm based on RBF functions, the accuracy depending on the value of the shape parameter is satisfied by a strategy for selecting optimal shape parameter. The determination of good shapes parameters is always the subject of exceptional researches. The shape parameter can be computed efficiently by using the optimization technique of the relative error of the displacement at first order based on GA. In this context, we propose a new adaptive algorithm, based on the strong form RBF approximation with automatic selection of the shape parameter, for solving nonlinear elasticity problem with large deformation. In this adaptive algorithm, we are coupled a collocation-path following method with a strategy based on genetic algorithm for determining optimal choice of the shape parameter. Numerical results show that the proposed adaptive algorithm with automatic selection of the shape parameter can produce more accuracy compared to the same algorithm with a fixed and given shape parameter. In addition, for structure modeling in large deformation, this new adaptive algorithm gives good results with a good precision compared to FEM.

[1] Buhmann M. Radial basis functions: theory and implementations. Cambridge Monographs on Applied and Computational Mathematics. Cambridge: Cambridge University Press (2003).

[2] Lucy L. B. A numerical approach to the testing of the fission hypothesis. Astronomical Journal. **82**, 1013–1024 (1977).

[3] Gingold R. A., Monaghan J. J. Smoothed particle hydrodynamics: theory and application to non-spherical stars. Monthly Notices of the Royal Astronomical Society. **181**, 375–389 (1977).

[4] Melenk J. M., Babuska I. The partition of unity finite element method: Basic theory and applications. Computer Methods in Applied Mechanics and Engineering. **139** (1–4), 289–314 (1996).

[5] Belytschko T., Lu Y.Y., Gu L. Element free Galerkin Methods. International Journal for Numerical Methods in Engineering. **37**, 229–256 (1994).

[6] Kansa E. J. Multiquadrics – a scattered data approximation scheme with applications to computational fluid dynamics – I surface approximations and partial derivative estimates. Computers & Mathematics with Applications. **19** (8–9), 127–145 (1990).

[7] Timesli A., Braikat B., Lahmam H., Zahrouni H. A new algorithm based on Moving Least Square method to simulate material mixing in friction stir welding. Engineering Analysis with Boundary Elements. **50**, 372–380 (2015).

[8] Timesli A. Optimized radius of influence domain in meshless approach for modeling of large deformation problems. Iranian Journal of Science and Technology-Transactions of Mechanical Engineering (2021).

[9] Nayroles B., Touzot G., Villon P. Generalizing the finite element method : diffuse approximation and diffuse elements. Computational Mechanics. **10**, 307–318 (1992).

[10] Kansa E. J. Multiquadrics – a scattered data approximation scheme with applications to computational fluid dynamics – II solutions to parabolic, hyperbolic and elliptic partial differential equations. Computers & Mathematics with Applications. **19** (8–9), 147–161 (1990).

[11] Hardy R. L. Multiquadric equations of topography and other irregular surfaces. Journal of Geophysical Research. **76** (8), 1905–1915 (1971).

[12] Yoon J. Spectral approximation orders of radial basis function interpolation on the Sobolev space. SIAM Journal on Mathematical Analysis. **33** (4), 946–958 (2001).

[13] Madych W. R. Miscellaneous error bounds for multiquadric and related interpolators. Computers and Mathematics with Applications. **24** (12), 121–138 (1992).

[14] Cheng A. H. D., Golberg M. A., Kansa E. J., Zammito G. Exponential convergence and $H-c$ multiquadric collocation method for partial differential equations. Numerical Methods of Partial Differential Equations. **19**, 571-594 (2003).

[15] Buhmann M., Dyn N. Spectral convergence of multiquadric interpolation. Proceedings of the Edinburgh Mathematical Society. **36** (2), 319–333 (1993).

[16] Powell M. J. D. The theory of radial basis function approximation in 1990. Advances in numerical Analysis. **2**, 105–209 (1990).

[17] Hon Y. C. A quasi-radial basis functions method for American options pricing. Computers & Mathematics with Applications. **43** (3–5), 513–524 (2002).

[18] Esmaeilbeigi M., Hosseini M. M., Syed Tauseef M. D. A new approach of the radial basis functions method for telegraph equations. International Journal of Physical Sciences. **6** (6), 1517–1527 (2011).

[19] Bhatia G. S., Arora G. Radial Basis Function Methods for Solving Partial Differential Equations-A Review. Indian Journal of Science and Technology. **9** (45), 1–18 (2016).

[20] Ferreira A. J. M. Thick Composite Beam Analysis Using a Global Meshless Approximation Based on Radial Basis Functions. Mechanics of Advanced Materials and Structures. **10** (3), 271–284 (2003).

[21] Ferreira A. J. M, Carrera E., Cinefra M., Roque C. M. C. Analysis of laminated doubly-curved shells by a layerwise theory and radial basis functions collocation, accounting for through-the-thickness deformations. Computational Mechanics. **48**, 13–25 (2011).

[22] Chen C. S., Fan C. M, Wen P. H. The method of particular solutions for solving elliptic problems with variable coefficients. The International Journal for Numerical Methods in Biomedical Engineering. **8** (3), 545–559 (2011).

[23] Chen C. S., Fan C. M, Wen P. H. The method of particular solutions for solving certain partial differential equations. Numerical Methods for Partial Differential Equations. **28**, 506–522 (2012).

[24] Fornberg B., Wright G. Stable computation of multiquadric interpolants for all values of the shape parameter. Computers & Mathematics with Applications. **48** (5–6), 853–867 (2004).

[25] Fornberg B., Zuev J. The Runge phenomenon and spatially variable shape parameters in RBF interpolation. Computers & Mathematics with Applications. **54** (3), 379–398 (2007).

[26] Larsson E., Fornberg B. Theoretical and computational aspects of multivariate interpolation with increasingly flat radial basis functions. Computers & Mathematics with Applications. **49** (1), 103–130 (2005).

[27] Carlson R., Foley T. The parameter R^2 in multiquadric interpolation. Computers and Mathematics with Applications. **21** (9), 29–42 (1991).

[28] Madych W. R., Nelson S. A. Multivariate Interpolation and Conditionally Positive Definite Functions. II. Mathematics of Computation. **54**, 211–230 (1990).

[29] Madych W. R., Nelson S. A. Multivariate interpolation and conditionally positive definite functions. Approximation Theory and its Applications. **4**, 77–89 (1988).

[30] Wendland H. Scattered Data Approximation (Cambridge Monographs on Applied and Computational Mathematics). Cambridge: Cambridge University Press (2004).

[31] Madych W. R. Error estimates for interpolation by generalized splines. Curves and Surfaces. 297–306 (1991).

[32] Kansa E. J., Carlson R. E. Improved accuracy of multiquadric interpolation using variable shape parameters. Computers & Mathematics with Applications. **24** (12), 99–120 (1992).

[33] Franke R. Scattered data interpolation: tests of some methods. Mathematics of Computation. **38** (157), 181–200 (1982).

[34] Fasshauer G. E., McCourt M. Stable evaluation of gaussian radial basis function interpolants. SIAM Journal on Scientific Computing. **34** (2), A737–A762 (2012).

[35] Foley T. A. Near optimal parameter selection for multiquadric interpolation. Journal of Applied Science and Computation. **1**, 54–69 (1994).

[36] Rippa S. An algorithm for selecting a good value for the parameter c in radial basis function interpolation. Advances in Computational Mathematics. **11**, 193–210 (1999).

[37] Chen W., Hong Y., Lin J. The sample solution approach for determination of the optimal shape parameter in the Multiquadric function of the Kansa method. Computers & Mathematics with Applications. **75** (8), 2942–2954 (2018).

[38] Zheng S., Feng R., Huang A. The Optimal Shape Parameter for the Least Squares Approximation Based on the Radial Basis Function. Mathematics. **8** (11), 1923 (2020).

[39] Afiatdoust F., Esmaeilbeigi M. Optimal variable shape parameters using genetic algorithm for radial basis function approximation. Ain Shams Engineering Journal. **6** (2), 639–647 (2015).

[40] Esmaeilbeigi M., Hosseini M. A new approach based on the genetic algorithm for finding a good shape parameter in solving partial differential equations by Kansa's method. Applied Mathematics and Computation. **249**, 419–428 (2014).

[41] Biazar J., Hosami M. Selection of an Interval for Variable Shape Parameter in Approximation by Radial Basis Functions. Advances in Numerical Analysis. **2016**, Article ID: 1397849 (2016).

[42] Weikuan J., Dean Z., Tian S., Chunyang S., Chanli H., Yuyan Z. A New Optimized GA-RBF Neural Network Algorithm. Computational Intelligence and Neuroscience. **2014**, Article ID: 982045 (2014).

[43] Saffah Z., Timesli A., Lahmam H., Azouani A., Amdi M. New collocation path-following approach for the optimal shape parameter using Kernel method. SN Applied Sciences. **3**, 249 (2021).

[44] Timesli A. Buckling analysis of double walled carbon nanotubes embedded in Kerr elastic medium under axial compression using the nonlocal Donnell shell theory. Advances in Nano Research. **9** (2), 69–82 (2020).

[45] Timesli A. An efficient approach for prediction of the nonlocal critical buckling load of double-walled carbon nanotubes using the nonlocal Donnell shell theory. SN Applied Sciences. **2**, Article number: 407 (2020).

[46] Cochelin B. A path-following technique via an asymptotic-numerical method. Computer and Structures. **53** (5), 1181–1192 (1994).

[47] Mitchell M. An Introduction to Genetic Algorithms. Cambridge, MA: MIT Press (1996).

[48] Hassouna S., Timesli A. Optimal variable support size for mesh-free approaches using genetic algorithm. Mathematical Modeling and Computing. **8** (4), 678–690 (2021).

[49] Schaback R. Error estimates and condition numbers for radial basis function interpolation. Advances in Computational Mathematics. **3**, 251–264 (1995).

РБФ колокаційний підхід на основі генетичного алгоритму для оптимального вибору параметра форми

Саффаг З.[1], Хассауна С.[2], Таймслі А.[2], Азоуані А.[3,4], Лахмам Г.[1]

[1] *Університет Касабланки Хасан II, LIMAT лабораторія, Факультет наук Бен М'сік, LIMAT лабораторія, Марокко*
[2] *Університет Касабланки Хасан II, Національна вища школа мистецтв та ремесел (Ensam Касабланка), 20670 Касабланка, Марокко, SEISEE лабораторія, Марокко*
[3] *Султан Мулай Слиманський університет, Національна школа прикладних наук Хоурібга, LIPIM лабораторія, Марокко*
[4] *Вільний університет Берліна, Інститут математики I, Арнімалее 7, 14195, Берлін, Німеччина*

Стаття презентує новий метод для розв'язання складної проблеми та обговорення поточних досліджень, а саме: вибір оптимальних параметрів форми для радіальної базисної функції (РБФ) метода колокації, як інтерполяції, так і нелінійних диференціальних рівнянь у частинних похідних. Для цього потрібно досягти компромісу між точністю та стабільністю, що називається принципом компромісу або невизначеності. Використання генетичного алгоритму та продовження шляху дозволяє нам, з одного боку, уникнути локальної оптимальної проблеми, яка пов'язана з інтерполяційними матрицями РБФ, а з іншого боку, — відобразити оригінальну проблему оптимізації визначення параметра форми у проблему пошуку кореня. Наші обчислювальні експерименти, що застосовуються до нелінійних задач у структурних розрахунках, використовуючи запропонований адаптивний алгоритм на основі генетичної оптимізації з автоматичним вибором параметра форми, можуть давати більшу точність порівняно з арт-алгоритмом з літератури з фіксованим і даним параметром форми та методом скінченних елементів.

Ключові слова: *великі деформації, сильна форма, метод колокації РБФ, генетичний алгоритм, автоматичний вибір параметра форми.*