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# ENGINEERING SOLUTIONS FOR INCREASING THE ACCURACY OF GEODESIC MEASUREMENTS BY TOTAL STATIONS

The study considers developments of the scientific and pedagogical staff of the Department of Engineering Geodesy at Lviv Polytechnic National University. The developments were aimed at increasing the accuracy of geodetic measurements in the construction, operation and repair of unique buildings, structures or separate technological equipment which is particularly important for their safe operation. Auxiliary equipment (improved light-reflecting mark, device for linear-angular measurements, spherical reflector with a stand, two-prism transducer-vector) was designed to increase the accuracy of measurement parameters of engineering structures by electronic total stations. On the basis of theoretical calculations, we proposed an optimal image of the geodetic mark for viewing at different distances. Additionally, a three-dimensional holder was designed to compensate for the non-perpendicular error. A technique was developed and implemented in a device for linear-angular measurements, which made it possible to determine the length of segments from 1 to 30 meters with an accuracy of 0.1–0.3 mm. A spherical reflector and a stand were developed in order to transfer the coordinates of the geodetic base from the reference network to the measurement points of building structures. In the process of their use, errors in centering, reduction and height measurement are compensated. The application of the developed spherical reflector was tested during the restoration of the design position of the large-sized equipment of the power complex facility with an accuracy of 0.5 mm. A two-prism encoder vector was theoretically justified and developed to determine the dimensions of irregularly shaped structures in order to minimize the angles of the prism (to directly define the coordinates of the prism tip). The accuracy of determining the spatial coordinates by the vector encoder was investigated using the final measure as a reference value. According to the results of research, the deviation of the distances determined with the help of the vector transducer from the reference value is 0.3 mm.

*Key words:* reflective mark; device for linear and angular measurements; spherical reflector; dual-prism vector; spatial method of electronic total station; increasing the accuracy of measurements by electronic total stations.

## Introduction

This article examines the developments of the department of engineering geodesy of Lviv Polytechnic National University in the direction of increasing the geodetic measurement accuracy in the construction, operation, and repair of unique buildings, structures, or individual technological equipment. The accuracy of such measurements is regulated by the State Building Standards (SBS). In these norms, geodetic control is performed during the manufacture of construction structures, marking on the site and installation of equipment [DBN, 2010]. SBS provides control of the following parameters: straightness, concentricity, horizontality, tilt, verticality, parallelism, perpendicularity, planarity, curvature [DSTU, 2011]. SBS regulate the use of geodetic methods: microtriangulation, trilateration, hydroleveling, surveying with the use of equipment: optical and digital theodolites, electronic total stations, laser surveying pointers, interferometers [DSTU-H, 2009]. At the same time, SBS does not sufficiently disclose the issue of the use of equipment utilised to measure electronic total stations (stamps, prisms, etc.).

## Previous research

Control over the preservation of geometric parameters is especially important during the repair or replacement of the main units of industrial equipment, namely the rotor and stator, turbine shafts. For example, the calibration of the position of individual components of the rotor and stator of a turbogenerator should ensure an accuracy of 0.5 mm. Classic methods of installing large-sized equipment use special center finders (NIITMASH, auto center finder KNUBA) [Baran, 2012]. Of course, ensuring the accuracy of alignment with the PPS-11 system is quite difficult and requires considerable time, especially when restoring the center of the two-meter diameter stator with an insulating winding. All the listed methods are mechanical or opto-mechanical and require considerable time, human resources, and most importantly do not allow automating the process of controlling physical parameters during repair or equipment replacement.

In world practice, such tasks are solved by hand-type coordinate machines [Petrakov, & Shuplietsov, 2018]. These machines, with a measurement accuracy of several microns, can determine the center of the required number of points. But the issue of fixing a certain center for its further reproduction still remains. In the work [Zobrist, et al., 2009], a high accuracy of several hundredths of a millimeter was obtained in the determination of the surface of a spherical mirror using laser trackers and special spherical reflectors. The laser tracker is a slightly improved electronic total station with a more accurate rangefinder and more stable setup. According to [Leica TS30 White Paper], the laser tracker angle reading system and the electronic total station are comparable in accuracy. The accuracy of electronic tachymeters for operational surveys was investigated in the work [Burak, 2011]. The possibility of increasing the accuracy of measurements at short distances is shown in [Litinsky, 2014]. This work proposed a method of linear-angular measurements to increase the accuracy of determining small segments. It also introduced a method of optimal planning of segment determination accuracy. In another article [Litynskyi et al., 2015], the method was tested on the model basis of the second class. This study investigated a possibility of increasing the determination accuracy of ten-meter segments by electronic total stations by three times. The work [Vivat et al., 2018] considers the possibility of using electronic total stations with spherical reflectors for determining the geometric parameters of engineering structures. This work also proposes the use of sighting targets on a reflective basis to increase the sighting accuracy. All these studies can become the foundation for the development of a technique for controlling the geometric parameters of large equipment.

## Aim

The purpose of the study is research and development of auxiliary equipment to increase the accuracy of measurements of parameters of engineering structures by electronic total station.

## Methodology and results

### 1. Development of reflective stamp and 3D-holder

To enter the point coordinates of the electronic total station location in the coordinate system of the engineering structure or large-sized equipment of the construction structure, it is necessary to perform sighting and measurement at least two points with known coordinates. In this regard, in order to determine the coordinates of the starting points, sighting is carried out on reflective marks [Vivat, et al., 2015]. In order to increase the accuracy of measurements on reflective marks, it is necessary to ensure:

- maximum laser radiation return;
- the required size of the sight target for a certain distance;
- optimum vision at different angles of inclination;
- the perpendicularity of the laser beam.

The first stage can be implemented by choosing a reflective film, for example, Oralite 6710.

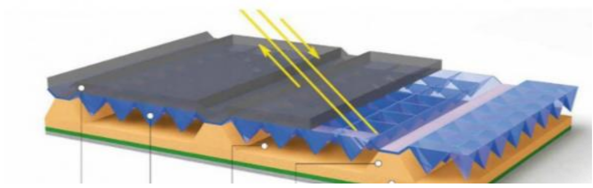


*Fig. 1. A reflective film*

The solution of the second stage depends on the laser beam scattering angle – $\chi$. The cross-sectional diameter of the laser beam d at a distance D can be calculated with the formula [Kostetskaya, 2000]:

$$d = \frac{\chi}{\rho''} D, \quad (1)$$

where the angle $\chi$ is an important quantity, but we did not find its value in any user manual for the electronic total station. Therefore, from (1), we determine $d$ = 10 mm for a distance of $D$ = 50 m and calculate $\chi$ = 40" for a certain device (Fig. 2).

Substituting in (1), $D$ = 100 and 200 m, we get $d$ = 19 and 40 mm, respectively. A distance of

200 m is considered the limit for highly accurate engineering measurements. Taking into account the problem of ensuring the perpendicularity of the plane of the mark to the beam, it is better to replace the intersection of lines with a point of a certain diameter. Let us look at this problem in more detail. For this purpose, we will use the formula of Rayleigh's resolution (Rayleigh’s criterion). This formula applies equally to all devices, as it is determined by the discriminating ability of the eye. We see two points separately if they are perceived by different light-sensitive cells on the retina of the eye. This occurs when the center of the diffraction disc of one cell coincides with the minimum in the diffraction pattern of the second one. In other words, the angular half-width of the first diffraction minimum from the slit became the condition or limit of the distinguishing ability (the ability to see separately). Linear resolution is the minimum distance between two separate point objects, for which they are perceived as separate objects, rather than merging into one point object; and angular – the minimum angle between point objects when they are still perceived as separate objects The resolution of optical devices is limited both by fundamental physical laws (e. g, light diffraction) and by device imperfections.

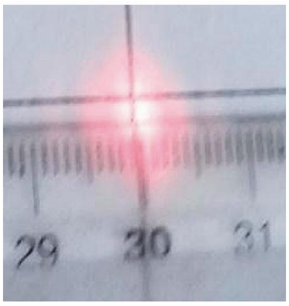



*Fig. 2. Leica TCRP1201R300 total station laser beam*

The optical resolution condition is written as:

$$\frac{d}{D} = 1{,}22\frac{\lambda}{a}, \tag{2}$$

where $\lambda$ = 570 nm is the average wavelength of the optical range, $a$ = 2 mm is the average diameter of the pupil of the human eye, $d$ is the distance between two points at which they are observed as separate ones, $D$ is the distance from the observer to the sighting target. By entering in (2) the number of seconds in radians and $v$ – magnification of the eyepiece, we obtain the formulas for determining the maximum resolution when viewing by eye and eyepiece:

$$\frac{d}{D} = 1.22\frac{\lambda \cdot \rho}{a}, \tag{3a}$$

$$\frac{d}{D} = 1.22\frac{\lambda \cdot \rho}{a \cdot v}. \tag{3b}$$

Substituting the values in the given formulas, we reduce them to the same units, for example, meters:

$$\left(\frac{d}{D}\right)_{Eye} = 1.22\frac{0.00000057 \cdot 206265}{0.002} = 71.71,$$

$$\left(\frac{d}{D}\right)_{\text{Optical tube}} = 1.22\frac{0.00000057 \cdot 206265}{0.002 \cdot 30} = 2.4.$$

Let us calculate the maximum resolution $d$ for different distances $D$. The obtained results are shown in Table 1. These calculations are confirmed by the results of Golovin – Sivtsev’s visual acuity test [Chizh, et al., 2013]. Vision is considered to be 100 % if a person can distinguish a line from 5 m with a distance of 3 mm between the elements.

*Table 1*

**Limiting linear resolution at various distances for the eye and tube with 30 magnification**

| *D*, m | 5 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| *d*, mm *Eye* | 1.7 | 3.5 | 7 | 10.5 | 14 | 17.5 |
| *d*, mm *O. T.* | 0.06 | 0.1 | 0.2 | 0.4 | 0.5 | 0.6 |

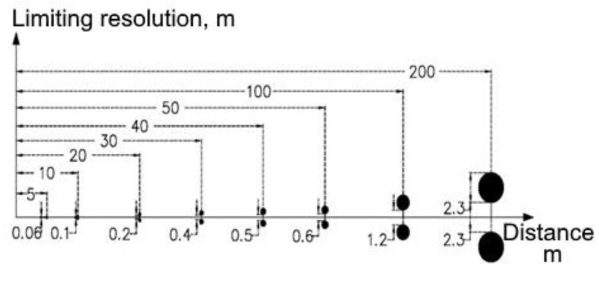



*Fig. 3. Limiting linear resolution at different distances for a tube with 30* magnification*

Fig. 3 graphically shows values of Table 2 for *tube with 30* magnification*

Based on theoretical calculations, we have proposed an optimal image of a geodetic mark for viewing at different distances, which is presented in Fig. 4.

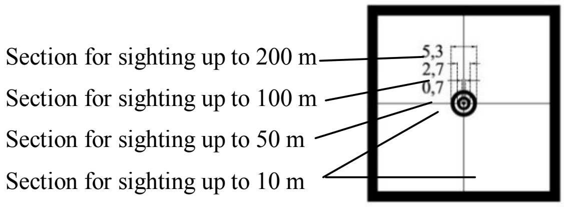



*Fig. 4. Image of the mark for accurate sighting at different distances*

To ensure high accuracy of measurements and to compensate for non-perpendicularity error, it is recommended to start surveying geodetic work with planning in three-dimensional space. To implement this goal, a three-dimensional holder was developed, Fig. 5. We will derive the formulas for determining the angle β between the faces of the 3D stamp and the angle of inclination of the stamp – ν:

$$\beta^{\circ} = \alpha_{Fall} + 180^{\circ} - \alpha_{Obstacle} - 90^{\circ}, \quad (3c)$$

$$\nu^{\circ} = \arctan\frac{h}{d}. \quad (3d)$$

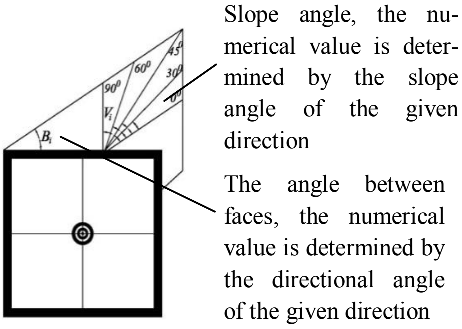



*Fig. 5. Three-dimensional holder*

The starting data for calculating the angles β and ν are the coordinates obtained from the project and reconnaissance of the engineering object. Fig. 6 provides an explanation of the calculation of angles β and ν.

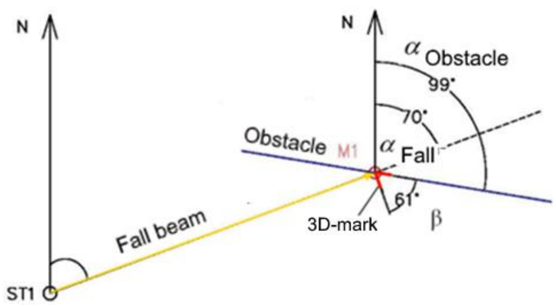


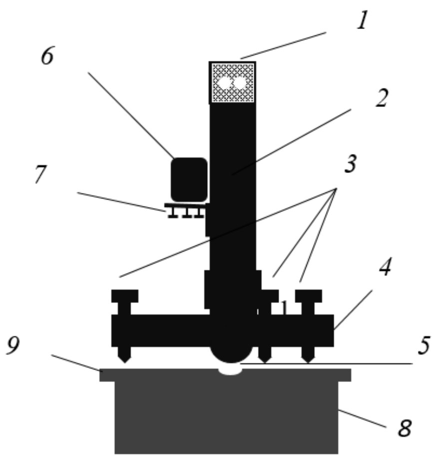

*Fig. 6. To calculate the angle between the faces of the 3D-mark*

## 2. Development of a device for linear and angular measurements

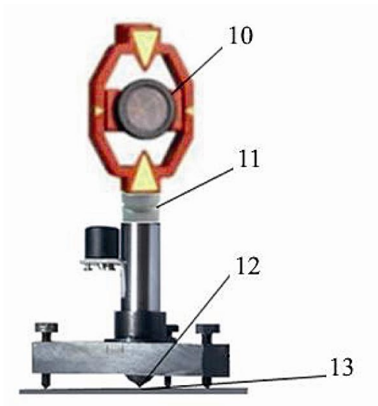
Fig. 7, *a* shows the proposed device on the geodetic center, the coordinate carrier of which is a 14 mm hole with a reflective film. Fig. 7, *b* demonstrates a standard mini reflector. The device consists of a rod approximately 130 mm long, the lower part of which is inserted into the hole of the stand. A spherical level with a division value of 5' is attached to the rod. The level has fixing screws. The bottom of the rod is spherical. The diameter of the ball is 14.2–14.5 mm, if the diameter of the center hole is 14 mm. The lower part of the ball ends with a cone-shaped projection, which is a continuation of the rod axis. This projection is used to install the rod on tapped or drilled centers of a small diameter. In the upper part of the rod there is a thread for installing reflectors and angle measuring marks on it. Fig. 7, *a* shows the brand we proposed for linear-angular measurements attached to a rod. The brand is made in the form of two symmetrically located holes relative to the axis of the rod. The distance between the holes is about 0.5 mm. So, a mark for measuring horizontal angles is two arcs that allow reducing the sighting error for different distances. A specially made reflective film is glued to the stamp. There are three lifting screws with a small thread pitch in the body of the stand. With the help of these lifting screws, the spherical level bubble is brought to the center.

The device consists of the following parts: 1 – a mark for linear and angular measurements; 2 – a rod; 3 – lifting screws; 4 – a stand; 5 – a spherical end of the rod, which is installed in the hole of the mark; 6 – a spherical level; 7 – correction screws of the level. The device is shown on the geodetic center 9 with a hole – 8. Also, a reflector 10 can be installed on the device, using an adapter 11. The device is installed on the sign so that the ball enters the hole of the sign. If the holes have a size of 1–3 mm, then a cone-shaped protrusion 12 is installed in the hole (Fig. 7, *b*).

In the proposed device, the distance from the mounting surface to the center of the mark to the top of the mark is approximately 140 mm. The scale value of the spherical level is 5'. It is assumed that the accuracy of setting the bubble of the spherical level to the middle is 0.15 of the value of the level division. Therefore, the error of centering the mark over the spherical head of the rod will be 0.03 mm. An increase in accuracy can be achieved by setting a more accurate level, for example a barrel-shaped level. The radial gap between the tangential surfaces of the rod and the support is 0.004 mm. Such wobble of the axis of the rod in the body of the stand, if the distance from the contact surface of the rod to the mark from the top of the stand is 50 mm, will be equal to 0.28'. This will lead to a centering error of the mark of 0.01 mm. The total error of centering the rod mark will be equal to 0.032 mm. For measurements, it is desirable to have at least 2 devices (better 3 or more, depending on the type of work).




*a*




*b*

*Fig. 7. Device for linear-angular measurements on the tubular center (a) and on the signs, the center carrier is a 1–3 mm hole (b)*

Before starting the measurements, the device is checked:

*Requirement 1: the axis of the spherical plane should be parallel to the axis of the rod.*

To check, the device is installed on a tubular sign whose center is the hole. With the help of lifting screws, the stands bring the bubble of the level to the middle. The rod is rotated by 180°. Permissible deviation is 0.2 division, which corresponds to 1'. Therefore, the rod axis will be set vertically with an accuracy of 0.5'.

*Requirement 2: the axis of the stamp should coincide with the axis of rotation of the rod.*

For example, at a distance of 5 m from the mark, a 1' electronic total station is installed. The membrane is applied to the stamp between the two holes. A horizontal circle is counted. The stamp is rotated by 180°. The horizontal circle is sighted and counted again. This is repeated several times. The accuracy of the measured angle between two positions of the mark will be approximately 1.4'. For such measurements, this will make it possible to determine the position of the intersection of the mark relative to the rod axis with an accuracy of 0.03 mm. All measurements are usually carried out for two positions of the brand, so this error is practically eliminated.

*Requirement 3: determine the instrument correction of the light range finder.*

The correction can be determined, if there is no base, by measuring the whole line $S$ and its two parts $S1$ and $S2$. The device correction K will be equal to $K = S - (S1 + S2) + 0.5\ t$. Here it is the distance between the reflective films (thickness of the mark).

In the work [Litynskyi et al., 2015], the developed device for linear-angular measurements on a linear basis of the II degree was tested. Comparison of the reference segments and those determined with the help of the device confirmed a high a priori estimate of accuracy. The differences in distances were within 0.1–0.3 mm of intervals of different lengths of 1–30 m, respectively. Such results confirm an increase in the accuracy of measuring segments by electronic total stations by almost one order of magnitude.

## 3. The development of a spherical reflector

To transfer the coordinates of the geodetic base from the initial base to the measurement points of building structures, a spherical reflector and a stand were also developed. Errors in centering, reduction and height measurement are compensated during their use.

For geometrical reasons, the effect of non-perpendicularity can be eliminated by combining the centers of the triple prism and the spherical frame. Since triple prisms are made of K8 glass with a refractive index $n = 1.518$, then according to [Rusinov, 1984] we write the formula for determining the distance from the front plane of the triple prism to the optical center:

$$\Delta = \frac{H}{n}, \quad (4)$$

where $H$ is the height of the triple prism and is a technical characteristic during production; $n$ is the refractive index, which depends on the type of glass.

The spherical reflector developed by us (Fig. 8) consists of the following elements: 1 – sphere of the required diameter depending on the diameter of the triple prism; 2 – spring plate; 3 – triple prism; 4 – pressure washer; 5 – fixing washer; 6 – sphere center; 7 – optical center of a triple prism.

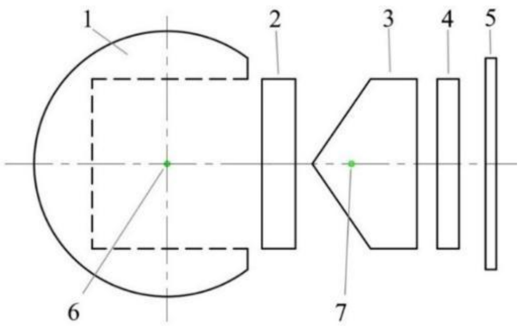
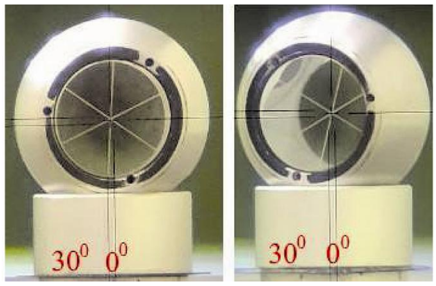



*Fig. 8. A spherical reflector was developed*

This design makes it possible to align the center of the sphere and the center of the triple prism with an accuracy of 0.1 mm, which will ensure the invariance of the coordinates of the triple prism center during measurement, even when the sphere is turned by an angle of 30°. We have manufactured a spherical reflector based on the dimensions of a triple prism with a diameter of 25.4 mm and a height of 19 mm. The diameter of the sphere is 42 mm with a 30 mm deep and 25.5 mm diameter hole, a 4 by 25 mm spring plate, a 1.5 mm by 25 mm thrust washer, and a 1 mm by 30 mm retaining washer. The check (Figs. 9, 10) confirmed the stability of the spatial position of the center of the triple prism when the sphere is turned. The measured distance to the spherical reflector in the position *a* and *b* was within 0.2 mm. According to [The method of determining… Paten, 2020], the absolute correction of the spherical reflector, which is –17.5 mm, is determined.




*a* *b*

*Fig. 9. Checking the triple prism centering in a developed spherical reflector*

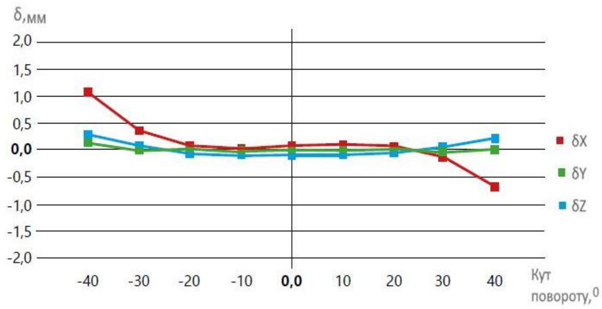



*Fig. 10. Study of the turning angle on the change of coordinates of the optical center*

The application of the developed spherical reflector was carried out during the restoration of the design position of the generator stator at Zaporizhskaya NPP [Vivat et al., 2022]. The reference network was created using the linear-angular measurement method. All distances and angles were measured by an electronic tachymeter using a developed spherical reflector. The lengths of the sides were within 10–30 meters, and the angles: horizontal 30–90°, vertical 0–5°.

### 4. Development of a double-prism vector

To determine the dimensions of irregularly shaped structures, it is necessary to develop a sensor that can be arbitrarily set and determine the coordinates of the tip. We have researched and developed a two-prism transducer vector. The paper [Bösemann, (2016)] outlines the need and feasibility of such a device, describes its advantages, and investigates the accuracy of the measurements.

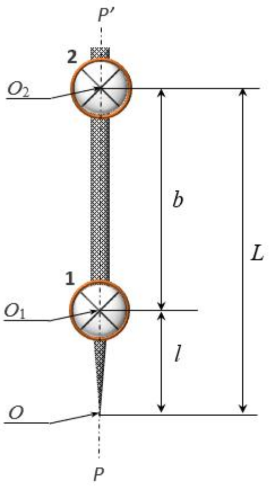



*Fig. 11. The main elements of the two-prism vector*

*PP'* – the main axis of the vector;

1, 2 – triple prisms;

*O* – is the pole of the vector;

*O*1, *O*2 – optical centers of reflecting prisms 1 and 2, respectively;

*L* – is the polar distance of point *O*2 (the distance between the pole of vector *O* and the optical center of reflector 2);

l – is the polar distance of point *O*1 (the distance between the pole of vector *O* and the optical center of reflector 1);

*b* – base (distance between optical centers of reflectors 1 (point *O*1) and 2 (point *O*2);

$r = b / L$ – the ratio of the base to the polar distance of point *O*2;

$k = 1/ L$ – the ratio of the polar distance of point *O*1 to the polar distance of point *O*2;

$q = 1/ b$ is the ratio of the polar distance of point *O*1 to the length of the base.

## The main geometric requirement

The pole of the vector *O*, the optical centers of the reflectors *O*1 and *O*2 must belong to the PP' axis. The fulfilment of the main geometric condition corresponds to the following mathematical ratio

$$L = l + b\,. \tag{5}$$

Consider the mathematical relations for calculating the coordinates of the pole point based on the known coordinates of the optical centers of the reflectors. Let us denote *x*1, *y*1, *z*1 – known coordinates of the optical center point *O*1; *x*2, *y*2, *z*2 – known coordinates of the optical center point *O*2; *x*0, *y*0, *z*0 – unknown coordinates of pole *O*.

We adapt the equation of a straight line in space for the conditions of our problem, then we obtain

$$\frac{x_1 - x_0}{x_2 - x_0} = \frac{y_1 - y_0}{y_2 - y_0} = \frac{z_1 - z_0}{z_2 - z_0} = \frac{l}{L} = k\,. \tag{6}$$

or

$$\frac{x_2 - x_2}{x_2 - x_0} = \frac{y_2 - y_1}{y_2 - y_0} = \frac{z_2 - z_1}{z_2 - z_0} = \frac{b}{L} = r\,, \tag{7}$$

or

$$\frac{x_1 - x_0}{x_2 - x_1} = \frac{y_1 - y_0}{y_2 - y_1} = \frac{z_1 - z_0}{z_2 - z_1} = \frac{l}{b} = q\,. \tag{8}$$

Let us write down the expressions for calculating the pole point coordinates from the last equation:

$$\left.\begin{aligned} x_0 &= x_1 - q(x_2 - x_1) \\ y_0 &= y_1 - q(y_2 - y_1) \\ z_0 &= z_1 - q(z_2 - z_1) \end{aligned}\right\} \rightarrow \left.\begin{aligned} x_0 &= (1+q)x_1 - qx_2 \\ y_0 &= (1+q)y_1 - qy_2 \\ z_0 &= (1+q)z_1 - qz_2 \end{aligned}\right\}. \tag{9}$$

Or in this form

$$\begin{aligned} x_0 &= p_1x_1 - p_2x_2 \\ y_0 &= p_1y_1 - p_2y_2\,, \\ z_0 &= p_1z_1 - p_2z_2 \end{aligned} \tag{10}$$

where $p_1$, $p_2$ are certain weighting factors, the values of which can be calculated, knowing the values of the ratios of the linear elements of the base l, *b*, *L*. In Table 2, we give expressions for calculating the weighting factors depending on the dimensionless characteristics of the base.

Analysing the results given in the table, one can note a peculiarity of the weighting coefficients, i. e., their difference $p_1 - p_2$ is always equal to one.

So, equations (9) can serve as working ones for calculating the coordinates of the pole points. To calculate the weighting coefficients in (9), we recommend using the ratios listed in the last column of Table 2.

*Table 2*

**Calculation of weighting factors depending on dimensionless characteristics of the basis**

| Weight / dimensionless characteristic of the basis | $k=\frac{1}{L}$ | $r=\frac{b}{L}$ | $q=\frac{1}{b}$ |
|---|---|---|---|
| $P_1$ | $1-(1-k)$ | $1/r$ | $1+q$ |
| $P_2$ | $k/(1-k)$ | $(1-r)/r$ | $q$ |
| $P_1-P_2$ | 1 | 1 | 1 |

**The accuracy of determining the coordinates of the pole point depends on the accuracy of the determination of the linear elements of the vector**

We will assume that the main geometric condition of the measurement vector is fulfilled (the pole point and the optical centers of the triple prisms lie on the same line). Let us examine how the accuracy of determining the linear elements of the vector affects the accuracy of determining the pole coordinates. For this, we will make the following substitution in equation (9) $q = 1/b$, then we will get:

$$\left.\begin{aligned} x_0 &= x_1 - \frac{l}{b}(x_2 - x_1) \\ y_0 &= y_1 - \frac{l}{b}(y_2 - y_1) \\ z_0 &= z_1 - \frac{l}{b}(z_2 - z_1) \end{aligned}\right\}. \quad (11)$$

Let us also assume that the coordinates of the optical centers $x1$, $y1$, $z1$ and $x2$, $y2$, $z2$ are determined without error. Let $mx0$, $my0$, $mz0$ be the root mean square error of determining the coordinates of the pole $x0$, $y0$, $z0$, respectively; $ml$, $mb$ are the mean squared errors of the linear elements of the measurement basis l and b.

Apply the rule for calculating the mean square error of the function of measured values to equation (11), and then we will have:

$$\left.\begin{aligned} m_{x_0}^2 &= \left(\frac{\partial x_0}{\partial l}\right)^2 m_l^2 + \left(\frac{\partial x_0}{\partial b}\right)^2 m_b^2 \\ m_{y_0}^2 &= \left(\frac{\partial y_0}{\partial l}\right)^2 m_l^2 + \left(\frac{\partial y_0}{\partial b}\right)^2 m_b^2 \\ m_{z_0}^2 &= \left(\frac{\partial z_0}{\partial l}\right)^2 m_l^2 + \left(\frac{\partial z_0}{\partial b}\right)^2 m_b^2 \end{aligned}\right\}. \quad (12)$$

Having calculated the values of the partial derivatives in (12), and assuming that the mean squared errors of determining the linear quantities are the same $ml = mb = m_{lin}$, we write down the final equation

$$\left.\begin{aligned} m_{x_0}^2 &= (x_2 - x_1)^2 \left(\frac{l^2 + b^2}{b^4}\right) m_{lin}^2 \\ \text{s}\, m_{y_0}^2 &= (y_2 - y_1)^2 \left(\frac{l^2 + b^2}{b^4}\right) m_{lin}^2 \\ m_{z_0}^2 &= (z_2 - z_1)^2 \left(\frac{l^2 + b^2}{b^4}\right) m_{lin}^2 \end{aligned}\right\}. \quad (13)$$

Let us add all equations in (13), taking into account that the square of the mean square error of the spatial location of the pole point, $M_0^2 = mx_0^2 + my_0^2 + mz_0^2$, we get:

$$M_0^2 = ((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2) \cdot \cdot \left(\frac{l^2 + b^2}{b^4}\right) m_{lin}^2 . \quad (14)$$

Given that

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = b^2,$$

write down the final dependence of the root mean square error of determining the spatial coordinates of the pole on the accuracy of determining the linear elements of the basis

$$M_0^2 = \sqrt{1 + q^2}\, m_{lin}^2, \quad (15)$$

where $q = 1/b$.

As you can see, in this case, the accuracy of determining the spatial location does not depend on the length of the base, but depends solely on the accuracy of measuring its linear elements and their ratio. At the same time $\sqrt{1+q^2}$, it serves as a scale multiplier of the linear error. The smaller $q$, the smaller the accuracy effect of linear measurements on the accuracy of determining the spatial location of the pole point.

**The determination accuracy of the pole point coordinates depends on the determination accuracy of the coordinates of the optical centers of the reflectors**

As in the previous case, we will assume that the main geometric condition of the measuring base is fulfilled (the pole point and the optical centers of the reflectors lie on the same line), and the dimensionless coefficient $q = l/b$ is determined without error. Let $m_{x1}$, $m_{y1}$, $m_{z1}$ be the mean squared errors of determining the $x_1$, $y_1$, $z_1$ coordinates of the optical center of the first reflector, and $m_{x1}$, $m_{y1}$, $m_{z1}$ be the mean squared errors of determining the $x_2$, $y_2$, $z_2$ coordinates of the optical center of the second reflector, respectively. Let us apply again the rule of determining the mean square error of the function of the measured values to equation (9). Let us write the result without intermediate statements

$$\left.\begin{aligned} m_{x_0}^2 &= (1-q)^2 m_{x_1}^2 + q^2 m_{x_2}^2 \\ m_{y_0}^2 &= (1-q)^2 m_{y_1}^2 + q^2 m_{y_2}^2 \\ m_{z_0}^2 &= (1-q)^2 m_{z_1}^2 + q^2 m_{z_2}^2 \end{aligned}\right\}. \quad (16)$$

As in the previous case, we denote by $M_0^2 = mx_0^2 + my_0^2 + mz_0^2$ and add all the equations in (16), then we get

$$\begin{aligned} M_0^2 &= (1+q)^2 (mx_1^2 + my_1^2 + mz_1^2) + \\ &+ q^2 (mx_2^2 + my_2^2 + mz_2^2). \end{aligned} \quad (17)$$

It is obvious that $mx_1^2 + my_1^2 + mz_1^2 = M_1^2$, $mx_2^2 + my_2^2 + mz_2^2 = M_2^2$ – the squares of the root mean square error of the spatial arrangement of the optical centers of reflectors 1 and 2, respectively. Then we will have it

$$M_0^2 = (1+q)^2 M_1^2 + q^2 M_2^2 . \quad (18)$$

We will assume that the accuracy of determining the coordinates of the optical centers of reflectors is the same, that is $M_1^2 + M_2^2 = M_P^2$, then we will write the final expression:

$$M_0^2 = \sqrt{(1+q)^2 + q^2} M_P \cdot \quad (19)$$

**The influence of temperature changes on the accuracy of the determination of the pole coordinates**

We will prove that the thermal expansion/contraction of the base of the transducer-vector rod will not affect the accuracy of determining coordinates. To do this, we write down the temperature equations for the change in the length of the linear elements of the basis

$$\begin{aligned} L &= L_0(1+\tau(t-t_0)), \\ l &= l_0(1+\tau(t-t_0)). \end{aligned} \quad (20)$$

Since $b = L - l$, then

$$\begin{aligned} &L_0(1+\tau(t-t_0)) - l_0(1+\tau(t-t_0)) = \\ &= b_0(1+\tau(t-t_0)), \end{aligned} \quad (21)$$

where $\tau$ is the thermal expansion coefficient of the material from which the base is made; $L_0$, $l_0$, $b_0$ – the values of the linear elements of the base determined at the temperature $t_0$; $L$, $l$, $b$ are the values of the linear elements of the basis defined at temperature $t$.

The coefficient $q = l/b$ is used in the working formulas (15). We present an expression for its calculation, taking into account the temperature change in the length of the linear elements of the basis

$$q = \frac{l}{b} = \frac{l_0(1+\tau(t-t_0))}{b_0(1+\tau(t-t_0))} = \frac{l_0}{b_0} . \quad (22)$$

As you can see, the temperature change does not affect the value of the dimensionless elements of the base. The coordinates of the optical centers 1 and 2 of the reflectors appear in working equations (9) and in practice they are calculated based on the results of measurements of the electronic total station. So, if the coefficient q does not depend on the change in temperature, and the coordinates are determined independently, then the thermal expansion/compression does not affect the results of determining the coordinates of the pole point.

The two-prism vector developed by us is presented in (Fig. 12) during the determination of linear elements on the displacement interferometer using six methods. Substituting the values of $m_{lin}$ = 0.01 mm and $MP$ = 0.05 mm into the formula (15), (19), we come to the conclusion that the a priori maximum error in determining the spatial position of the point should not exceed 0.11 mm.

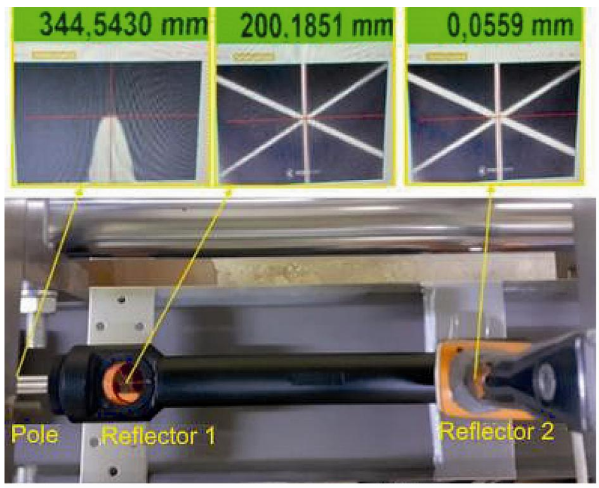



*Fig. 12. Determination of linear elements of a vector and constant correction*

The next step is to determine the constant correction of the encoder vector. This determination is made on a linear basis, which is fixed by two holes 9.9734 m long and determined by an invar-compared wire. An electronic tachymeter is installed at one end of the base, and a vector transducer at the other. In the vertical position, the transmitter-vector was set at the exact 5' level (Fig. 13). By measuring the distances to the two reflectors, we determined the permanent correction of the electronic Leica total station, which was +17.2 mm.

*Fig. 13. Determination of the instrument correction*

During the study of the accuracy of determining the spatial position of the tip of the vector encoder, we used a linear basis. An electronic tachymeter was installed at one end of the base, and a vector transducer at the other (Fig. 14). The transducer vector was not centered, that is, it can occupy an arbitrary position in space. Six measurements were made: in two measurements the transducer-vector was in an approximately vertical position, in the other four it was tilted by approximately 300. To fix the transducer-vector, a photo tripod was used. Distances and angles to two reflectors at two positions of the circle were measured using an electronic total station.

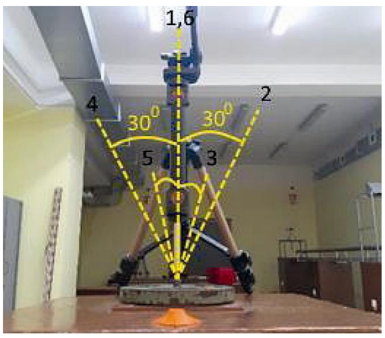



*Fig. 14. Investigation of the accuracy of the two-prism vector*

The coordinates of the point were determined by formulas (9). The results of the center determined independently six times are given in the Table 3. Since the diameter of the hole fixing the center of the point of the linear basis is about 1 mm, the value of the range in the measurements in the Table 3 could be due to this influence.

*Table 3*

**Coordinates of the control point on different settings of the double prism vector**

| Inclination angle Coordinate | X, m | Y, m | Z, m |
|---|---|---|---|
| $0^0$ | 989.9910 | 501.4731 | 99.2958 |
| $30^0$ | 989.9911 | 501.4727 | 99.2964 |
| $30^0$ | 989.9909 | 501.4729 | 99.2957 |
| $30^0$ | 989.9914 | 501.4734 | 99.2958 |
| $30^0$ | 989.9910 | 501.4733 | 99.2961 |
| $0^0$ | 989.9909 | 501.4735 | 99.2960 |
| Range, mm | 0.5 | 0.8 | 0.7 |
| STD, mm | 0.2 | 0.3 | 0.3 |

Let us examine the accuracy of determining the spatial coordinates by the vector encoder, using a finite measure. In three positions in space at distances of 6, 10, and 18 m from the electronic total station, the final measure was fixed in turn. The tip of the vector transducer was fixed at the beginning and end of the final measure (Fig. 15). An electronic total station was used to measure the distance and angles to the prisms of the transducer at two positions of the circle. The coordinates of the point were determined by formulas (9). A comparison of the reference distance of the final measure and the one determined by the vector encoder is given in Table 4.

*Table 4*

**Comparing distances in space defined by a vector**

| No. of measurement | Distance measured by vector, mm | Reference value, mm | Difference, mm |
|---|---|---|---|
| 1 (6 m) | 1000.27 | 1000.004 | 0.27 |
| 2 (10 m) | 1000.50 | 1000.004 | 0.50 |
| 3 (18 m) | 1000.21 | 1000.004 | 0.21 |
| Average | 0.32 | | |

*Fig. 15. Verification of the two-prism vector with a finite measure*

## Conclusions

The following conclusions can be drawn based on theoretical studies and practical implementation of the developed equipment:

1. An optimal sighting target in the form of a film reflector ensures the perpendicularity of the measuring beam, which increases the sighting accuracy and reduces the number of electronic tachymeter installation stations.
2. The optimal design of the spherical reflector was developed. It ensured high accuracy of sighting by the electronic total station. Using such a reflector, you can perform geodetic control of geometric parameters, for example, the horizontality of the surface.
3. With the help of the developed vector transducer and electronic total station, it is possible to perform almost all geodetic control tasks with an accuracy of up to 0.5 mm.

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## ІНЖЕНЕРНІ РІШЕННЯ ДЛЯ ПІДВИЩЕННЯ ТОЧНОСТІ ГЕОДЕЗИЧНИХ ВИМІРЮВАНЬ ЕЛЕКТРОННИМИ ТАХЕОМЕТРАМИ

Розглянуто розробки науково-педагогічних працівників кафедри інженерної геодезії Національного університету “Львівська політехніка” в напрямі підвищення точності геодезичних вимірювань у будівництві, експлуатації та ремонті унікальних будівель, споруд чи окремого технологічного обладнання, що особливо важливо для їх безпечної експлуатації. Розроблено допоміжне приладдя (удосконалену світловідбивну марку, пристрій для лінійно-кутових вимірювань, сферичний відбивач із підставкою, двопризмовий давач-вектор) для підвищення точності вимірювань параметрів інженерних споруд електронними тахеометрами. На основі теоретичних розрахунків запропоновано оптимальне зображення геодезичної марки для візування на різних віддалях, а для компенсації похибки неперпендикулярності – тривимірний тримач. Розроблено методику, яку реалізовано у пристрої для лінійно-кутових вимірювань, що дало змогу визначати довжини відрізків у межах від 1 до 30 м з точністю 0,1–0,3 мм. З метою передавання координат геодезичної основи від опорної мережі до точок виконання вимірювань будівельних конструкцій розроблено сферичний відбивач та підставку, завдяки використанню яких компенсуються похибки за центрування, редукцію та вимірювання висот. Розроблений сферичний відбивач апробовано під час відновлення проєктного положення великогабаритного обладнання об’єкта енергетичного комплексу з точністю 0,5 мм. Для визначення розмірів споруд неправильної форми з метою мінімізації кутів нахилу призми (безпосереднього визначення координат вістря призми) теоретично обґрунтовано та розроблено двопризмовий давач-вектор. Досліджено точність визначення просторових координат давачем-вектором із використанням кінцевої міри як еталонного значення. За результатами досліджень відхилення віддалей, визначених за допомогою давача-вектора, від еталонного значення становить 0,3 мм.

*Ключові слова:* світловідбивна марка; пристрій для лінійно-кутових вимірювань; сферичний відбивач; давач-вектор; просторовий метод електронної тахеометрії; підвищення точності вимірювань електронними тахеометрами.

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