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ANALYSIS OF LYAPUNOV MATRICES' APPLICATION METHODS FOR OPTIMIZATION OF STATIONARY DYNAMIC SYSTEMS

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In this article there has been conducted analysis of Lyapunov matrix application in order to form control inputs under different dynamic systems' optimization methods oriented by quadratic integral criterion. For this purpose, the methods of finding the Lyapunov matrix and optimization based on the Bellman functional equation with subsequent application of the Riccati equation, optimization taking into account the initial values of state variables, optimization based on the Bellman equation using linear matrix inequalities and Lyapunov equation are considered. Despite the complexity of solving the Riccati equation, the problem of finding the Lyapunov matrix is unambiguous only in the case of application of optimization methods based on dynamic Bellman programming and representation of the Bellman function by the Lyapunov function. Optimization based on the application of the linear matrix inequality condition is not unambiguous, as it requires the choice of the inequality solution. The optimization of the system by the integral quadratic criterion and the initial values of the state variables is also ambiguous because there is a problem of solving nonlinear interconnected optimization equations.

Keywords: system optimization, Lyapunov function, feedback loop matrix, control input.

Problem description

Most of the system around us can be considered as dynamical systems from some point of view [1]. Usually we are interested in control of these systems. There are many different approaches that allow achieving a control system [2]. Some of them involve only structural synthesis (passivity based [3], intelligent [4], adaptive control system [5], etc.) and some also parametric synthesis (feedback linearization [6], modal [6], fuzzy regulation systems [4], etc.). Usually, parametric synthesis is a complex procedure that involves much mathematics. One of the most fundamental cores of optimal system synthesis is an energetic Lyapunov function, based on which many methods built [2]. Thus, it's important to review the issue of forming the control input based on the Lyapunov function and find the most efficient method.

Research relevance

Optimization of dynamic systems based on criteria derived from quadratic forms of state variables and control inputs requires feedback loop matrix to be found, and by all the state variables present in the system [7]. Including the control inputs in performance quality criterion of the systems limits the amplitude of the latter, thus minimizing the amounts of energy needed for control implementation. Optimized control law synthesis based upon the principle of linear quadratic regulator is impossible to conduct unless taking into consideration a condition of integral criterion quadratic form being (either during optimization by the starting conditions or by Bellman dynamic programming) a Lyapunov function with Lyapunov matrix included as well. Sometimes this matrix forms the control inputs ambiguously, however otherwise it is utilized as intermediate part of feedback loop gain matrix definition procedure.

The analysis of such optimization methods and the role of Lyapunov function during application of the said methods are what this article is dedicated for. In this research there have been analysed methods of finding the Lyapunov matrices based upon solutions of Riccati algebraic equation, upon solution of Lyapunov equation and also upon linear matrix inequalities. There are made conclusions about efficacy of the said methods from perspective of forming the control input task.

Definition of goals and tasks of article

The research described in this article aims to analyse the efficiency of control input formation for dynamic system as a feedback loop gain by state variables, finding which is based upon solving the Lyapunov function.

Analysis of recent researches and publications

In order to define the conditions of dynamic systems optimization one has to include for consideration a signed positive function with a signed negative derivative of one. This function passes the conditions to be defined as a Lyapunov function and is a basis for system optimization. This is because of the optimized system having to be stable while its overall positivity along with negativity of its derivative provides its diminishment over time, thus having a phase portrait of system trajectory converging into a coordinate zero point. An important component of a set Lyapunov function is the matrix **P** [7], through which the synthesized control input is formed in case of system optimization by the principles of Bellman's dynamic programming; in case of finding feedback loop matrix during processes optimization based upon starting values of state variables; in case of system optimization using both linear matrix inequality and the Lyapunov equation.

Main matter description

According to these three optimization methods let us analyse the ways of finding the matrix P and basing upon it the control inputs as a solution of dynamic system optimization task.

Let us assume a given dynamic system being described as a following vector-matrix equation

$$\dot{\mathbf{X}} = \mathbf{A} \cdot \mathbf{X} + \mathbf{B} \cdot \mathbf{U}, \tag{1}$$

where **A** and **B** – matrices of coefficients; **U** – could be a vector or a scalar; **X** – vector of state coordinates.

The quadratic integral optimization criterion for such system could be arranged into following view

$$J = \int_0^{\infty} (\mathbf{X}^{\mathbf{T}} \cdot \mathbf{Q} \cdot \mathbf{X} + \mathbf{U}^{\mathbf{T}} \cdot \mathbf{R}_2 \cdot \mathbf{U})dt \rightarrow \min. \tag{2}$$

It is necessary to find the optimal control with feedback loop that provides a transition from any given starting point X(0) into end point X(∞) = 0 simultaneously providing minimum of defined quality functional. This task is called the task of stationary linear state regulator synthesis and is typically solved using method of dynamic programming i.e. through application of Bellman's functional equation.

$$\min_{u \in W(u)} \left\{ F(x,u) + \sum_{i=1}^{n} \frac{\partial S(x_1, x_2, \ldots x_n)}{\partial x_i} f_i(x,u) \right\} = 0 \tag{3}$$

and the minimization equation

$$\frac{\partial F(x,u)}{\partial u}+\sum_{i=1}^{n}\frac{\partial S(x_1,x_2,...x_n)}{\partial x_i}\cdot\frac{\partial f_i(x,u)}{\partial u}=0\,, \tag{4}$$

where $\dot{X}=f_i(x,u)$ – equation of the object by the coordinate i, and there could be n of such equations; $\frac{\partial S(x_1,x_2,...x_n)}{\partial x_i}$ – partial derivative of Bellman's function by coordinate *i*; $\frac{\partial f_i(x,u)}{\partial u}$ – partial derivative by the control input.

Integrated expression for optimization criteria (2), being an expression for F(x,u), is assumed to be a positive defined expression in square form. Thus for systems with linear object we assumed F(x,u) to be a Lyapunov function. That is why a part S(x) of Bellman's equation (Bellman's function) also has to be considered being a Lyapunov function because $S(x_i)$ is exactly a minimal value from integral of F(x,u) by starting value x_i, while function S(x) is function of minimums of integral criterion dependent of what value of X we do consider being a starting one. Thus, both first and a second part of Bellman equation should be Lyapunov functions.

Taking into account (1,2,4) it is simple to show that

$$\mathbf{U^T}\cdot\mathbf{R}_2=-\frac{\partial S(x)}{\partial x}\cdot\mathbf{B}\,;$$

$$\mathbf{U_{opt}}=-\mathbf{R_2^{-1}B^T}\cdot\left(\frac{\partial S(x)}{\partial x}\right)^T.$$

Now setting the Bellman's function, viewed as a Lyapunov function $\mathbf{S}(x)=\mathbf{X^T}\cdot\mathbf{P}\cdot\mathbf{X}$, we shall find

$$\mathbf{U_{opt}}=-\mathbf{R_2^{-1}}\cdot\mathbf{B^T}\cdot\mathbf{P}\cdot\mathbf{X}\,,$$

where matrix P is found as a solution from Riccati's algebraic equation

$$\mathbf{A^T}\cdot\mathbf{P}+\mathbf{P}\cdot\mathbf{A}-\mathbf{P}\cdot\mathbf{B}\cdot\mathbf{R_2^{-1}}\cdot\mathbf{B^T}\cdot\mathbf{P}+\mathbf{Q}=\mathbf{0}\,.$$

The mathematic model of Riccati's equation solution in most cases allows to choose definite solution based upon Sylvester's criterion [8], meaning a view and pattern of matrix **P**, as shown in our previous research [9].

Another optimization method utilizing Lyapunov function along with matrix **P** is optimization based on starting conditions for systems being described in models of state variables.

In this case let us assume control input **U** being a linear combination of state variables meaning $\mathbf{U}=\mathbf{K}\cdot\mathbf{X}$. Substituting this expression for **U** inside of vector-matrix equation (1) we shall receive:

$$\dot{\mathbf{X}}=\mathbf{A}\cdot\mathbf{X}+\mathbf{B}\cdot\mathbf{U}=\mathbf{A}\cdot\mathbf{X}+\mathbf{B}\cdot\mathbf{K}\cdot\mathbf{X}=(\mathbf{A}+\mathbf{B}\cdot\mathbf{K})\cdot\mathbf{X}=\mathbf{H}\cdot\mathbf{X}\,. \tag{5}$$

Integral quadratic criterion in this case has to be expressed as a function from state vector.

$$J=\int_0^{\infty}(\mathbf{X^T}\cdot\mathbf{X})dt\rightarrow\min\,. \tag{6}$$

Now let us consider the existence of certain function $\mathbf{V}=\mathbf{X^T}\cdot\mathbf{P}\cdot\mathbf{X}$, derivative of which equals $\dot{\mathbf{V}}=-\mathbf{X^T}\cdot\mathbf{X}$, thus

$$\frac{d}{dt}(\mathbf{X^T}\cdot\mathbf{P}\cdot\mathbf{X})=-\mathbf{X^T}\cdot\mathbf{X}\,, \tag{7}$$

where function **V** is nothing else than Lyapunov function containing matrix **P** that has to be defined.

Once again to remind that the choice of function **V** in this case is dictated by a system stability condition meaning a finite value of integral quadratic quality index.

Simple mathematic transformations while considering (5–7) bring us to system of two equations with following view

$$J=\int_0^\infty (\mathbf{X}^T \cdot \mathbf{X})dt = \mathbf{X}^T(0)\cdot \mathbf{P}\cdot \mathbf{X}(0); \quad (8)$$

$$\mathbf{H}^T \cdot \mathbf{P} + \mathbf{P}\cdot \mathbf{H} = -I. \quad (9)$$

Thus the procedure of optimization by aforementioned method could be summarized into two steps to follow:

– considering matrix **H** known, the matrix **P** is found inside of function of feedback gain coefficients by state variables that also satisfies equation (9);

– out of minimization equations $\frac{dJ}{dK_i} = 0$ the feedback gain coefficients are to be found.

Here as we might notice matrix **P** to play intermediate role before forming a control input **U**=**K**·**X.** Contrary to previous task where found matrix **P** form control input for optimization, in this case to find a control input one has also to go through optimization equations' solution conditions which, as our research shoved [10], could be nonlinear, especially for cases where are different feedback channel sets being more numerous than two, and then the task of aforementioned synthesis tasks shall require additional study. At the same time finding the matrix **P** out of Riccati equation during synthesis of optimal system with the method described in the first task requires complete quadratic criterion to be present (2). Inside of it **Q** could be an identity matrix. System also could be with single input and single output (SISO), but energy restrictions at control input are compulsory.

These restrictions aren't considered through quality criterion expression

$$J = \int_0^\infty (\mathbf{X}^T \cdot \mathbf{Q} \cdot \mathbf{X})dt \rightarrow \min. \quad (10)$$

where matrix **Q** could be identity matrix leading to variation of functional (6). Lyapunov equation [11] at quality functional (10) shall look the following way

$$\mathbf{A}^T \cdot \mathbf{P} + \mathbf{P}\cdot \mathbf{A} = -\mathbf{Q}.$$

If we formed the matrix $\mathbf{H} = \mathbf{A} + \mathbf{B}\cdot\mathbf{K}$, then equation (9) during system optimization by starting values of state variables is certain to be Lyapunov equation [8].

Now viewing an optimization method that use a condition entered as a linear matrix inequality. As said before, let the control input of optimal dynamic system being found from functional Bellman's equation has the following view

$$\mathbf{U}_{opt} = -\mathbf{R}_2^{-1}\mathbf{B}^T \cdot \mathbf{P}\cdot \mathbf{X}.$$

Now for optimal closed-loop system

$$\dot{\mathbf{X}} = \mathbf{A}\cdot \mathbf{X} + \mathbf{B}\cdot \mathbf{U}_{opt}. \quad (11)$$

Writing $\mathbf{V}(x) = \mathbf{X}^T \cdot \mathbf{P}\cdot \mathbf{X}$ and value of this function on trajectory X(t) equals

$$\mathbf{V}(x) = \int_0^\infty (\mathbf{X}^T \cdot \mathbf{R}_1 \cdot \mathbf{X} + \mathbf{U}^T \cdot \mathbf{R}_2 \cdot \mathbf{U})\, dt.$$

This value decreases monotonically because the larger **t** the lesser integration partition is. Thus function $\mathbf{V}(x) = \mathbf{X}^T \cdot \mathbf{P}\cdot \mathbf{X}$ is a Lyapunov function for optimal system. Let us try form the conditions of finding these matrices **P**>0 without bounding **P** with Riccati equation. That is to provide system with the same feedback as previously written in equation of optimal control input with quadratic function $\mathbf{V}(x) = \mathbf{X}^T \cdot \mathbf{P}\cdot \mathbf{X}$ also being a Lyapunov function.

Then substituting in equation (11) the mentioned control input $\mathbf{U}_{opt}$, let us write

$$\dot{\mathbf{X}} = \mathbf{A}\cdot \mathbf{X} + \mathbf{B}\cdot \mathbf{U}_{opt} = (\mathbf{A} - \mathbf{B}\cdot \mathbf{R}_2^{-1}\cdot \mathbf{B}^T \cdot \mathbf{P})\cdot \mathbf{X} = \mathbf{A}_c \cdot \mathbf{X},$$

and remember that

$$\dot{\mathbf{V}}(x) = \frac{d}{dt}\mathbf{V}[x(t)] = \dot{\mathbf{X}}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{X} + \mathbf{X}^{\mathbf{T}} \cdot \mathbf{P} \cdot \dot{\mathbf{X}} = \mathbf{X}^{\mathbf{T}} \cdot \mathbf{A}_{\mathbf{c}}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{X} + \mathbf{X}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{A}_{\mathbf{c}} \cdot \mathbf{X} = \mathbf{X}^{\mathbf{T}}(\mathbf{A}_{\mathbf{c}}^{\mathbf{T}} \cdot \mathbf{P} + \mathbf{P} \cdot \mathbf{A}_{\mathbf{c}})\mathbf{X}.$$

The condition $\dot{\mathbf{V}}(x) < 0$ is met if

$$\mathbf{P} \cdot \mathbf{A}_{\mathbf{c}} + \mathbf{A}_{\mathbf{c}}^{\mathbf{T}} \cdot \mathbf{P} < 0.$$

By expanding this equation we will get

$$\mathbf{P} \cdot \mathbf{A} - \mathbf{P} \cdot \mathbf{B} \cdot \mathbf{R}_2^{-1} \cdot \mathbf{B}^{\mathbf{T}} \cdot \mathbf{P} + \mathbf{A}^{\mathbf{T}} \cdot \mathbf{P} - \mathbf{P} \cdot \mathbf{B} \cdot \mathbf{R}_2^{-1} \cdot \mathbf{B}^{\mathbf{T}} =$$
$$= \mathbf{P} \cdot \mathbf{A} + \mathbf{A}^{\mathbf{T}} \cdot \mathbf{P} - 2 \cdot \mathbf{P} \cdot \mathbf{B} \cdot \mathbf{R}_2^{-1} \cdot \mathbf{B}^{\mathbf{T}} < 0.$$

Afterwards let us multiply each element of expression at $\mathbf{Q} = \mathbf{P}^{-1}$ and get

$$\mathbf{Q} \cdot \mathbf{A} + \mathbf{Q} \cdot \mathbf{A}^{\mathbf{T}} - 2\mathbf{B} \cdot \mathbf{R}_2^{-1} \cdot \mathbf{B}^{\mathbf{T}} < 0 \qquad (12)$$

at $\mathbf{Q} > 0$.

So, to get any Lyapunov function for closed-loop system (11) it is necessary to find a matrix set that meet inequality (12) and to assume $\mathbf{P} = \mathbf{Q}^{-1}$.Expression (12) is called a linear matrix inequality.

Now let's evaluate which criterion value we will get by choosing a certain control input value. If we assume any solution $\mathbf{Q}$ of inequality (12) and choose feedback of view

$$\mathbf{U} = -\mathbf{R}_2^{-1} \cdot \mathbf{B}^{\mathbf{T}} \cdot \mathbf{Q}^{-1} \cdot \mathbf{X},$$

then in order to find value of quadratic criterion J let us use a statement [2] according to which if matrix **A** corresponds to a stable system; $\mathbf{X}(t)$ – system solution $\dot{\mathbf{X}} = \mathbf{A} \cdot \mathbf{X}$; $\mathbf{X}(0) = X_0$, then the value of functional $J = \int_0^{\infty} (\mathbf{X}^{\mathbf{T}} \cdot \mathbf{W} \cdot \mathbf{X})dt$ at $\mathbf{W} > 0$ is equal $\mathbf{X}_0^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{X}_0$, where **P** is a solution of Lyapunov equation

$$\mathbf{A}^{\mathbf{T}} \cdot \mathbf{P} + \mathbf{P} \cdot \mathbf{A} = -\mathbf{W}.$$

Analyzing three described optimization methods we can come to conclusion that the task of finding Lyapunov matrix is unambiguous only in case of using the optimization methods that base on the principles of Bellman's dynamic programming, forming the feedback gain matrix

$$\mathbf{K} = -\mathbf{R}_2^{-1} \cdot \mathbf{B}^{\mathbf{T}} \cdot \left(\frac{\partial S(x)}{\partial x}\right)^{T},$$

and i presentation of Bellman's unction as Lyapunov function

$$\mathbf{S}(x) = \mathbf{V}(x) = \mathbf{X}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{X},$$

where matrix **P** is found as the solution of aforementioned Riccati algebraic equation. Then optimal value of quality functional **J** is written as $\mathbf{J}_{min} = \mathbf{X}_0^T \cdot \mathbf{P} \cdot \mathbf{X}_0$ and this expression is a quadratic function of starting state values.

Of course the task of solving Riccati equation is uneasy and requires application of dedicated computing software. In the meantime the optimization task based upon application of linear matrix inequality isn't unambiguous. That is because at first we choose value of $\mathbf{Q} = \mathbf{P}^{-1}$ that corresponds to condition (12) and then seek **P** out of Lyapunov equation that defines criterion value. In this case unambiguity comes from the choice of inequality solution (12). It is possible that originated from Lyapunov equation matrix **P** would not meet the selected value of **Q** out of condition (12), and then it would be necessary to pick another value for **Q**.

It is also important to note that during optimization of system by integral quadratic criterion and starting conditions values there is none of that unambiguity as well, the one that is native to optimization task by Bellman: found the expression for control input **U**, found the Lyapunov matrix and substituted it inside of expression for **U**, formed a structure and parameters of feedback gain matrix by state condition values. In the latter case solving the Lyapunov equation

$$\mathbf{H}^{\mathbf{T}} \cdot \mathbf{P} + \mathbf{P} \cdot \mathbf{H} = -\mathbf{I}, \text{ or } \ -\mathbf{Q} \quad \text{for energy restriction}$$

find **P** as feedback coefficient function, meaning **P**(K). Afterwards substituting **P**(K) inside of expression for index J we get J(K) and by the equation set $\frac{\partial J}{\partial K_i} = 0$ seek feedback gain matrix elements by the state variables. There is no Riccati equation solving procedure described here, however there is detailed the topic of solving the nonlinear interconnected optimization equations, especially for systems of grade count beyond two.

Conclusions

The conducted research let us make a conclusion that the most efficient method of finding the control input for system optimization is forming one based upon Lyapunov matrix which is found from solution of Riccati equation. The next steps of these theoretical researches will be a practical comparison of these approaches for systems of different complexity.

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АНАЛІЗ МЕТОДІВ ЗАСТОСУВАННЯ МАТРИЦЬ ЛЯПУНОВА ДЛЯ ОПТИМІЗАЦІЇ СТАЦІОНАРНИХ ДИНАМІЧНИХ СИСТЕМ

Проаналізовано застосування матриці Ляпунова з метою формування керуючих входів за різними методами оптимізації динамічних систем, орієнтованими за квадратичним інтегральним критерієм. Для цього розглянуто методи знаходження матриці Ляпунова та оптимізації на основі функціонального рівняння Беллмана із подальшим застосуванням рівняння Ріккаті, оптимізації з урахуванням початкових значень змінних стану, оптимізацію на основі рівняння Беллмана з використанням лінійних матричних нерівностей та рівняння Ляпунова. Незважаючи на складність розв'язування рівняння Ріккаті, задача знаходження матриці Ляпунова є однозначною лише у разі застосування методів оптимізації на основі динамічного програмування Беллмана та подання функції Беллмана функцією Ляпунова.

Оптимізація на основі застосування умови лінійної матричної нерівності не є однозначною, оскільки потребує вибору розв'язку нерівності. Оптимізація системи за інтегральним квадратичним критерієм та початковими значеннями змінних стану також є неоднозначною, оскільки існує проблема розв'язування нелінійних взаємопов'язаних рівнянь оптимізації.

Ключові слова: оптимізація системи; функція Ляпунова; матриця контуру зворотного зв'язку; керуючий вхід.