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Numerical optimization of the likelihood function based on Kalman filter in the GARCH models

Benmoumen M.

LaMSD, Department of Mathematics, Faculty of Sciences, Mohammed the First University, Oujda, Morocco

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In this work, we propose a new estimate algorithm for the parameters of a GARCH(p, q) model. This algorithm turns out to be very reliable in estimating the true parameter's values of a given model. It combines maximum likelihood method, Kalman filter algorithm and the simulated annealing (SA) method, without any assumptions about initial values. Simulation results demonstrate that the algorithm is liable and promising.

Keywords: *GARCH models, maximum likelihood, Kalman filter, simulated annealing.*
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1. Introduction

State-space models and Kalman filtering have become important and powerful tools for the statistician and the econometrician. Together they provide researcher with a modeling framework and a computationally efficient way to compute parameter estimates over a wide range of situations. Problems involving stationary and non-stationary stochastic processes, systematically or stochastically varying parameters, and unobserved or latent variables (as signal extraction problems) all have been fruitfully approached with these tools. In addition, smoothing problems and time series with missing observations have been studied with methodologies based on this combination. The state-space model and Kalman filter recursions were first introduced in linear time series models, especially for estimation and prediction of autoregressive moving average (ARMA) processes (see Jones(1980) [1–3]). In each of these instances, the state-space formulation and Kalman filter have yielded a modeling and estimation methodology that is much less cumbersome than more traditional regression-based approach.

In this paper, we, in turn, mobilize the state-space representation and the Kalman filter to handle parameters estimation problem in GARCH(p, q) models case. The GARCH models, which stand for "generalized autoregressive conditionally heteroscedastic", were proposed by Bollerslev [4], as an extension of the ARCH models introduced by Engle [5]. Several authors have discussed estimation issues in ARCH and GARCH models, assuming that the fourth moment exists Weiss [6] established the asymptotic properties of the QMLE for ARCH models. Lumsdaine [7] treated the special case of the strictly stationary GARCH$(1, 1)$ model and established the asymptotic properties for the local QMLE (see also [8]). In [7], the conditions on the coefficients α_1 and β_1 allow to handle the IGARCH$(1, 1)$ model (see Definition 2.1 with $p = q = 1$ and $\alpha_1 + \beta_1 = 1$). They are, however, very restrictive with regard to the independent and identically distributed (i.i.d) process: it is assumed that $E|\eta_t|^{32} < \infty$ and that the density of η_t has a unique mode and is bounded in a neighborhood of 0. In [8], the consistency of the global estimator is obtained under the assumption of second-order stationarity.

Berkes, Horváth, and Kokoszka were the first to give a rigorous proof of the asymptotic properties of the QMLE in the GARCH(p, q) case under very weak assumptions; see [9–13]. The assumptions given in [9] were weakened slightly in [14]. The proofs presented here come from that paper. An extension to non-iid errors was proposed by Escanciano (2009).

Through this work, we aim to contribute to parameter estimation matters in GARCH(p,q) models by means of a new approach. The developed method relies on the fact that the log-likelihood function can be calculated across the Kalman filter algorithm (see [15,16]), providing an appropriate state-space representation of the concerned model. The optimization of log-likelihood function is carried out by SA method (see [17]), which is a global optimization algorithm for functions of continuous variables.

Note that the same idea has been used for parameters estimation in some non-linear time series. (see [18–22] and [23]).

The content of the rest of the document is organized as follows. Section 2 presents the GARCH(p,q) model and its main properties. In Section 3, we announce the central result of this study. First, we state the representation of the state space for GARCH(p,q); then we express the log-likelihood function via the Kalman filter; then we apply the SA method to obtain its minimum. In the next section, we drive some empirical experiments to illustrate the algorithm performances. And finally, we end the discussion with a conclusion in Section 5.

2. Preliminary notes

Definition 1 (Strong GARCH(p,q) process). *Let (η_t) be sequence of independent and identically distributed (i.i.d.) random variables ($E(\eta_t)=0$, $E(\eta_t^2)=1$). The process (ε_t) is called a strong GARCH(p,q) (with respect to the sequence (η_t)) if*

$$\begin{cases} \varepsilon_t = \sigma_t \eta_t, \\ \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \end{cases} \tag{1}$$

where the α_i and β_j are nonnegative constants and ω is a (strictly) positive constant.

The issue of stationarity for the GARCH(p,q) models was discussed by Bougerol and Picard [24]. The special case of GARCH$(1,1)$ model was studied by Nelson [25] under the assumption $E\log^+\eta_t^2 < \infty$ and extended by Klüppelberg and al. [26] to the case of $E\log^+\eta_t^2 = +\infty$. Bollerslev contributed as well in this matter by deriving conditions for GARCH(p,q) second-order stationarity, [4].

It is sometimes useful to consider the ARCH(∞) representation introduced by Robinson for GARCH(p,q) process, [27]; for more details see [28]. The strictly stationary of ARCH(∞) process was established by Robinson and Zaffaroni and Douc, Roueff and Soulier (2008) [29]. The second-order stationarity, as well as the positivity of the autocovariance of the squares, were obtained, on other hand, by Giraitis, Kokoszka and Leipus [28].

Milhoj, Karanasos and He and Teräsvirta were interested in the examination of the fourth-order moment structure and the autocovariance of the squares of GARCH processes [30–32], while Ling and McAleer [33] stated the necessary and sufficient condition for the existence of even-order moments, in addition they derived an existence condition for the moment of order s, with $s>0$; see [34]. Chen and An [35] also established a sufficient condition for the existence of even order moments.

Theorem 1 (Second-order stationarity). *If there exists a GARCH(p,q) process, in the sense of Definition 2.1, which is the second-order stationary and non-anticipative, and if $\omega > 0$, then*

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1. \tag{2}$$

Conversely, if condition (2) holds, the unique strictly stationary solution of model (1) is a weak white noise (and thus is second-order stationary). In addition, there exists no other second-order stationary solution.

3. Estimating algorithm of parameters of the GARCH(p,q) model

Let (ε_t) be a GARCH(p,q) model defined by (1). We suppose that $\sum_{i=1}^{q}\alpha_i+\sum_{j=1}^{p}\beta_j<1$ and (η_t) is an i.i.d $N(0,1)$.

Remark 1. Denote by $\theta=(\omega,\alpha_1,\dots,\alpha_q,\beta_1,\dots,\beta_p)'$ the GARCH(p,q) parameter and define the QMLE by minimizing:

$$\tilde{\ell_n}(\theta)=n^{-1}\sum_{t=1}^{n}\left\{\frac{\varepsilon_t^2}{\tilde{\sigma}_t^2(\theta)}+\log\tilde{\sigma}_t^2(\theta)\right\}. \tag{3}$$

Where

$$\tilde{\sigma_t}^2(\theta)=\omega+\sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2+\sum_{j=1}^{p}\beta_j\tilde{\sigma}_{t-j}^2 \quad \text{for} \quad t=1,\dots,n. \tag{4}$$

With initial values for ε_0^2, ε_1^2, …, ε_{1-q}^2 and $\tilde{\sigma}_0^2(\theta)$, $\tilde{\sigma}_1^2(\theta)$, …, $\tilde{\sigma}_{1-p}^2(\theta)$ (in practice the choice of the initial values is important in QMLE method).

Without any assumptions about the initial values $(\varepsilon_0^2,\ \varepsilon_1^2,\ \dots,\ \varepsilon_{1-q}^2)$ and $\tilde{\sigma}_0^2(\theta)$, $\tilde{\sigma}_1^2(\theta)$, …, $\tilde{\sigma}_{1-p}^2(\theta)$ which are not known in practice, we intend to produce $(\tilde{\sigma_t}^2(\theta))$.

Let $\theta=(\theta_1,\theta_2,\dots,\theta_{p+q+1})$, where $\theta_1=\omega$, $\theta_2=\alpha_1$, …, $\theta_{q+1}=\alpha_q$, $\theta_{q+2}=\beta_1\dots$, $\theta_{q+p+1}=\beta_p$ denote the vector of unknown parameters, $(\varepsilon_1,\varepsilon_2,\dots,\varepsilon_n)$ the observed data, and $F_t=(\varepsilon_1,\dots,\varepsilon_t)$ is the set of observations available at time $t=1,\dots,n$. In this study, we propose estimating θ by using quasi-maximum likelihood, given by minimizing:

$$\ell_n(\varepsilon_1,\dots,\varepsilon_n;\theta)=\frac{1}{n}\sum_{t=1}^{n}\left\{\frac{\varepsilon_t^2}{\hat{\sigma}_{t|t-1}^2(\theta)}+\log\hat{\sigma}_{t|t-1}^2(\theta)\right\}, \tag{5}$$

where the $\hat{\sigma}_{t|t-1}^2(\theta)$ is generated recursively, for $t\geqslant 1$, by the Kalman Filter, without any assumptions about pre-sample values which is essential in other methods of estimating the likelihood function. Our algorithm relies essentially on the state-space representation of our model, here we give the convenient one.

We pose $m=\max(p,q)$, $X_t=\varepsilon_t^2$ and $\nu_t=\varepsilon_t^2-\sigma_t^2$.

Then

$$\sigma_t^2=\omega+\sum_{i=1}^{m}(\alpha_i+\beta_i)\sigma_{t-i}^2+\sum_{i=1}^{m}\alpha_i\nu_{t-i}=\omega+\sum_{i=1}^{m}(\alpha_i+\beta_i)X_{t-i}+\sum_{i=1}^{m}-\beta_i\nu_{t-i}.$$

Or

$$X_t=\omega+\sum_{i=1}^{m}(\alpha_i+\beta_i)X_{t-i}+\nu_t+\sum_{i=1}^{m}-\beta_i\nu_{t-i}.$$

This representation is given by:

$$\begin{cases} Z_{t+1}=AZ_t+G\nu_t+\Omega\text{: state equation,}\\ X_t=HZ_t+\nu_t\text{: observation equation.}\end{cases}$$

Where

$$H=(1,0,\dots,0),\ \Omega=(\omega,\omega,\dots,\omega),\ G=\begin{pmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \vdots\\ \alpha_m\end{pmatrix},\ A=\begin{pmatrix}\alpha_1+\beta_1 & 1 & 0 & \dots & 0\\ \alpha_2+\beta_2 & 0 & 1 & \ddots & \vdots\\ \vdots & \vdots & \ddots & \ddots & 0\\ \vdots & 0 & \dots & 0 & 1\\ \alpha_m+\beta_m & 0 & \dots & \dots & 0\end{pmatrix}\in\mathcal{M}(m,m)(\mathbb{R}).$$

The j^{ith} component of the state vector Z_t is given by:

$$Z_{j,t+1} = \omega + \sum_{i=j}^{m} (\alpha_i + \beta_i) X_{t+j-i} + \sum_{i=j}^{m} -\beta_i \nu_{t+j-i} \quad j = 1, \dots, m.$$

Kalman filter recursively generates an optimal forecast $\hat{Z}_{t+1|t} = E[Z_{t+1}|F_t]$ of the state vector Z_{t+1}, and $\hat{\sigma}^2_{t+1|t} = H\hat{Z}_{t+1|t}$, with associated mean square error $P_{t+1|t} = V[Z_{t+1} - \hat{Z}_{t+1|t}]$, $t = 1, \dots, n$.

The log-likelihood function was constructed via the Kalman filter and optimized using the SA method. We chose this method because it doesn't involve likelihood derivatives besides SA algorithm is not limited to the local minima encountered in the first place but adopts an iterative random search procedure with adaptive moves along with the coordinate directions until a probabilistic control criterion is satisfied. Next, we provide an algorithms series that together will help to constitute our global estimating algorithm QMLKF (quasi-maximum likelihood and Kalman filter estimation).

Algorithm 1 Test(θ)

```
1: if ∑_{i=2}^{p+q+1} θ_i < 1 then
       Then go to next;
2: else
3:     return to the previous step and take the previous point as starting point.
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Algorithm 2 KF(θ)

1: Initialization of the state vector $\hat{Z}_{1|0}$ which denotes a forcast of Z_1. The forcast of σ^2_1 is given by $\hat{\sigma}^2_{1|0} = H\hat{Z}_{1|0}$;
2: Iterate on $\hat{Z}_{t+1|t}$ for $t = 2, \dots, n$;
3: The forcast of σ^2_{t+1} is given by $\hat{\sigma}^2_{t+1|t} = H\hat{Z}_{t+1|t}$;
4: Compute $\ell_n(\varepsilon_1, \dots, \varepsilon_n; \theta)$.

Algorithm 3 QMLKF)

1: Initialize: the vector parameters θ the step vector ν and the temperature T.
2: Starting from the point θ_i, generate a random point θ along the direction h: $\theta = \theta_i + r\nu_{m_h} e_h$,
where r is a random number generated in the range $[-1, 1]$ by a pseudorandom generator;
e_h is the vector of the hth coordinate direction; and ν_{m_h} is the component of the step vector ν; along the same direction.
3: Call sub algorithm Test(θ).
4: Call sub algorithm KF(θ)
Compute KF(θ_i) and KF(θ)
If KF(θ) $\leqslant$ KF(θ_i) accept the new point
Else accept or reject the new point with acceptance probability p:
$p = \exp\left(\frac{\mathrm{KF}(\theta_i) - \mathrm{KF}(\theta)}{T}\right)$
generate a uniformly distributed random number p' in the range $[0, 1]$
If $p' < p$, the point is accepted otherwise it is rejected.
5: Steps 1 to 3 are repeated for each coordinate direction i, $i = 1, \dots, m$
(m is the dimension of the vector parameter).
6: Steps 1 to 4 are repeated N_s times (N_s is the number of step variation)
and the step vector ν is adjusted.
7: Steps 1 to 5 are repeated N_T times (N_T is the number of temperature reduction)
the temperature is reduced following the rule: $T' = r_T T$ with $r_T \in [0, 1]$.
8: Steps 1 to 6 are repeated until a termination criterion is satisfied.

4. Simulation study

To evaluate the performance of our estimation algorithm, we run a comparative study that involves estimates obtained by the quasi-maximum likelihood method (QMLE) considered in the literature, to achieve this we consider two examples of model GARCH$(1,2)$ and GARCH$(2,1)$:

1. $\begin{cases} \varepsilon_t = \sigma_t \eta_t, & (\eta_t) \text{ iid } N(0,1), \\ \sigma_t^2 = 1 + 0.3\varepsilon_{t-1}^2 + 0.2\varepsilon_{t-2}^2 + 0.4\sigma_{t-1}^2. \end{cases}$
2. $\begin{cases} \varepsilon_t = \sigma_t \eta_t, & (\eta_t) \text{ iid } N(0,1), \\ \sigma_t^2 = 1 + 0.4\varepsilon_{t-1}^2 + 0.2\sigma_{t-1}^2 + 0.3\sigma_{t-2}^2 \end{cases}$

In order to achieve a Monte Carlo simulation for the models above, we generated 1000 replications of sample sizes $n = 50$, 100 and 150, and applied the QMLKF algorithm for each sample.

We summarize the results of this experiment in Table 1 and Table 2. We use notation QMLE for the quasi-maximum likelihood estimators, QMLKF for the estimation by our algorithm. The mean and MSE are calculated for each estimator.

Recall that the quasi-maximum likelihood estimator (QMLE) is obtained by the SA method.

Table 1. Mean and MSE of estimated parameters.

			QMLKF		QMLE	
		true	mean	MSE	mean	MSE
n=50	ω	1	1.0871	0.0512	1.1347	0.0596
	α_1	0.3	0.2468	0.0372	0.2207	0.0587
	α_2	0.2	0.2241	0.0563	0.2272	0.0651
	β_1	0.4	0.3175	0.0502	0.2880	0.0969
n=100	ω	1	1.0425	0.0143	1.0618	0.0297
	α_1	0.3	0.2569	0.0247	0.2315	0.0324
	α_2	0.2	0.2124	0.0291	0.2195	0.0307
	β_1	0.4	0.3347	0.0382	0.3292	0.0674
n=150	ω	1	1.0378	0.0113	1.0583	0.0213
	α_1	0.3	0.2847	0.0114	0.2715	0.0200
	α_2	0.2	0.2110	0.0213	0.2182	0.0261
	β_1	0.4	0.3605	0.0326	0.3431	0.0446

Table 2. Mean and MSE of estimated parameters.

			QMLKF		QMLE	
		true	mean	MSE	mean	MSE
n=50	ω	1	1.0953	0.0816	1.1490	0.1142
	α_1	0.4	0.4482	0.0614	0.4670	0.0791
	β_1	0.2	0.1574	0.0318	0.1543	0.0359
	β_2	0.3	0.3412	0.0627	0.3525	0.1078
n=100	ω	1	1.0472	0.0652	1.0664	0.1053
	α_1	0.4	0.4302	0.0527	0.4411	0.0431
	β_1	0.2	0.1803	0.0268	0.2418	0.0301
	β_2	0.3	0.3254	0.0439	0.3341	0.0834
n=150	ω	1	1.0347	0.0395	1.0590	0.0851
	α_1	0.4	0.4215	0.0197	0.3704	0.0289
	β_1	0.2	0.2102	0.0138	0.1832	0.0286
	β_2	0.3	0.3104	0.0268	0.3237	0.0736

The results of the empirical study presented in the table above showed that our algorithm is efficient, indeed, the samples mean square errors are generally smaller than those generated by the quasi-maximum likelihood estimators (QMLE), on the other hand, the bias of the QMLKF estimator is each time lower than the bias of the QMLE. We also find that by increasing the sample size, the bias and the MSE decrease considerably without any conditions on the initial values. We can therefore conclude that the performance of our estimation procedure is promising.

5. Conclusion

In this article, we designed a new algorithm to generate the quasi-maximum likelihood estimates of GRCH(p, q) model parameters. The construction of the log-likelihood function was based on the Kalman filter while the optimization part was realized by SA method. The simulations show that our estimation approach performed successfully and that it is more efficient in terms of bias and standard error than the competition, without assumptions on the initial values.

Data availability

Only computer-generated data have been used so all researchers can find our results from the application of our algorithms and computer-simulated data.

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Recently, the same idea has been developed for parameters estimation in the ARCH(p) models presented as a congress paper entitled "Numerical optimization of the likelihood function based on Kalman filter in the ARCH models" see [21].

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Чисельна оптимізація функції правдоподібності на основі фільтра Калмана в моделях GARCH

Бенмумен М.

LaMSD, кафедра математики, факультет природничих наук,
Університет Мохаммеда Першого, Уджда, Марокко

У цій роботі пропонується новий алгоритм оцінки параметрів моделі GARCH(p, q). Цей алгоритм виявляється дуже надійним в оцінці справжніх значень параметрів даної моделі. Він поєднує в собі метод максимальної правдоподібності, алгоритм фільтра Калмана та метод симуляції "відпалу" (СА) без будь-яких припущень щодо початкових значень. Результати моделювання демонструють, що алгоритм придатний і перспективний.

Ключові слова: *моделі GARCH, максимальна ймовірність, фільтр Калмана, моделювання "відпалу".*