MEASUREMENT OF NON-ELECTRIC QUANTITIES

IMPACT OF THE INTERACTION OF MOVING PLANETS ON THEIR ORBITS

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Abstract. The research raises the problem of determining and measuring the interaction of planets on the trajectory of their orbits around the Sun. Its solution becomes possible only based on an adequate mathematical model. For this purpose, Newton's law of universal gravitation was adapted to the case of moving masses in a wide range of velocities in flat Euclidean space and physical time. The finite speed of propagating the gravitational field is considered. Differential equations of planets' motion are obtained. Transients in the three-mass system of a star and two planets close to the Sun-Mercury-Venus system are simulated. To reveal deeply the essence of physical interaction, a hyperbolized transition process is simulated under artificially close moving masses.

Key words: Newton's law of gravitation, Differential equations of motion of celestial bodies, Three-mass system.

1. Introduction

The problem of measuring the interaction of gravitational forces of particular planets on their orbital trajectories has long been given much attention. One of the most famous measurement problems on this topic is the justification for the precession of the perihelion of Mercury's orbit. Studies [1–3] have convincingly shown that the mechanical motion of elliptical orbits in the field of electric and mechanical gravity in the mega- and macro- worlds are undoubtedly accompanied by a precession of the trajectory in the direction of rotation.

Thus, nowadays the precession of the orbit of Mercury is measured in the heliocentric coordinates $(574.1 \pm 0.65)\mathcal{C}$ per century. U. Le Verrier [4-5] has explained the phenomenon of planetary precession solely by the influence of other planets in the solar system. The obtained result was 526.7". The following theories, instead of searching for the physical essence of the dissimilarity, began to substantiate these 43" of difference of results. According to modern updated data, this difference is slightly larger and is equal to $(47,3 \pm 0,65)\mathcal{C}$ per century. To substantiate the above-mentioned 43", hypotheses were put forward mainly of two types/ First, the precession is caused by the impact of unknown matter near the Sun. Second, the new models of gravity, different from the Newtonian one, may exist. But the real cause of the phenomenon has not yet been established.

2. The Current State of the Problem

Modern attempts to solve the problem more precisely were complicated due to complex mathematical models describing the cosmological processes in celestial bodies [6-8], even involving evolutionary processes [8]. In addition, the emphasis is on the study of two-mass systems, which significantly narrows the problem.

3. The Aim of the Work

Current work aims to develop nonlinear differential equations of motion of celestial bodies considering the finite speed of propagation of the gravitational field, the results of the integration of which consist in the determination of the criteria for the impact of planets' interaction on their trajectories by simulating in dynamics.

4. Possible causes and mechanisms of orbits' deviations

Attempts to improve Newton's law of universal gravitation began in the late 19th century. Models with and without dependence on the speed of movement were offered. Let's name an example of two types of them [4,5].

4.1. Model-independent of velocity

Enhancement of Newton's law was performed by S. Newcomb, 1895:

$$F = G\frac{mM}{R^{2+d}}, \qquad d j = \frac{2p}{R^{2+d}} = 2p\left(1 + \frac{d}{2}\right),\qquad (1)$$

here F is the gravity of the masses m, M; R is the distance between the centers of mass; G is the gravitational constant; d is the correcting factor; dj is the precession of the perihelion by one revolution. Soon

H. Seeliger and K. Neumann have proposed another modification of the law of universal gravitation:

$$F = G \frac{mM}{R^2} e^{-1R}, \qquad (2)$$

In it, an additional multiplier provides faster than Newton's law, the decrease in gravity with distance.

4.2. Models with speed dependence

1. P. Gerber, 1898 have proposed a formula for the gravitational potential:

$$V = \frac{4p^2 A^3}{RT^2 \left(1 - \frac{1}{c} \frac{dR}{dt} \right)^2}, \qquad (3)$$

here T is the period of rotation; A is the major half-axis of the orbital ellipse; c is the speed of light.

2. A. Einstein in 1915 have calculated this deviation and obtained an almost exact coincidence with the observed at that time 43" per century:

$$dj = \frac{24p^3 A^2}{c^2 T^2 \left(1 - e^2 \right)}, \qquad (4)$$

here ε is the eccentricity of the ellipse of the trajectory. For Mercury, this formula gives 42.98" per century. All these recommendations are obtained as a result of an approximation of the trajectory of a certain planet in a particular quasi-stationary closed orbit. It can be named 2 objective reasons for the failures of that time consisting of the precession of the studied elliptical orbits:

1. The lack of computer technology did not allow the solution of the nonlinear differential equations of motion;

2. Mathematical methods of classical electricity and mechanics of moving bodies in space-time form did not apply to the analysis of chaotic motion in the complex fields of electric and gravitational forces.

4.3. The equation of motion of the planets

Successful mathematical modeling of transients of the orbital motion of planets in Euclidean space and physical time is possible only based on the equations of celestial mechanics, in which Newton's adapted law for moving masses is involved [2-3]. The inertial equations of moving interconnected by gravity n masses are obvious:

$$\frac{d\mathbf{v}_i}{dt} = \frac{1}{m_i} \sum_{k=1}^{n} \mathbf{F}_{ik}; \quad \frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i, \quad i, k = 1, 2, ..., n,$$

$$(5)$$

here $\mathbf{r}_i, \mathbf{v}_i$ are the radius vector of the trajectory and the vector of the velocity of the i-th mass m_i; $\mathbf{F}_{ik}$ is the vector of the force of gravitational interaction of the i-th and k-th masses. Equations (1) need to be explained, as it may be almost the speed of light. Functional dependence $m = m(v)$ becomes one of the annoying misunderstandings of possible physical interpretation. The fact is that the Lorentz coefficient refers to the force interaction of the masses, not the masses themselves! This is crystallized in the process of considering the finite speed of gravity propagation.

The force vector can be written in the general form [3]:

$$\mathbf{F}_{i,k} = G \frac{m_i m_k}{r_{ik}^2} \left(1 + \frac{v_{ik}^2}{c^2} + 2 \frac{v_{ik}}{c} \mathbf{r}_{ik0} \times \mathbf{v}_{ik0} \div \mathbf{r}_{ik0} \right), \qquad (6)$$

here r_{ik} is the radius of the distance between the masses; v_{ik} is the mutual instantaneous speed of movement; G is the gravitational constant; $\mathbf{r}_{ik0}, \mathbf{v}_{ik0}$ are the unit vectors of distance and speed of movement. The modulus of the force vector (6) is described by components:

$$F_{Nik} = G \frac{m_i m_k}{r_{ik}^2}, \qquad (7)$$

$$F_{Lik} = G \frac{m_i m_k}{r_{ik}^2} \frac{v_{ik}^2}{c^2}, \qquad (8)$$

$$F_{Tik} = 2G \frac{m_i m_k}{r_{ik}^2} \left(\frac{v_{ik}}{c} \mathbf{r}_0 \times \mathbf{v}_0 \div \right), \qquad (9)$$

here F_{Nik} is Newton's gravitational force; F_{Lik}, F_{Tik} which is the high-speed tangential and radial components of gravitational force. It is clear that at $v_{ik} \circledR 0$ the modulus of force interaction (6) degenerates into (7). The marginal share participation in the force interaction of components (8) and (9), based on the speed and orientation characteristics, is obvious:

$$\mathbf{F}_L = (0, 1)\mathbf{F}_N; \qquad \mathbf{F}_T = ((-2), (+2))\mathbf{F}_N. \qquad (10)$$

In [3] it was proved that the force component (8) is due to the tangential velocity component. In the electric field, it completely coincides with the Lorentz force, which in classical electrodynamics presents the force of the magnetic field or the so-called relativistic effect in the electric field. Being prolonged to mechanical interaction, it presents the corresponding gravitational-magnetic force [9-10].

The functional dependence (9) on the motion speed is more powerful compared to the similar of (8), because under the condition $v \pounds c$ the multiplier v/c in (8) is elevated to the second degree, and in (9) - to the first one. It is component (9) that closes the hitherto unknown triune essence of gravitational forces and makes it possible to solve the problem on a strictly mathematical basis.

4.4. Applied part

For certainty, let's consider the transient process of interaction of three cosmic masses: the Sun m_1, Mercury m_2, and Venus m_3. The balance of forces (5) under the condition of a relatively the fixed Sun is written as:

$$\frac{d\mathbf{v}_2}{dt} = \frac{1}{m_2}(\mathbf{F}_{21} + \mathbf{F}_{23}); \quad \frac{d\mathbf{r}_2}{dt} = \mathbf{v}_2;$$

$$\frac{d\mathbf{v}_3}{dt} = \frac{1}{m_3}(\mathbf{F}_{31} + \mathbf{F}_{32}); \quad \frac{d\mathbf{r}_3}{dt} = \mathbf{v}_3. \quad (11)$$

The vectors of the distance between the planets and their mutual velocity are found integrating (11):

$$\mathbf{v}_{23} = \mathbf{v}_2 - \mathbf{v}_3; \quad \mathbf{r}_{23} = \mathbf{r}_2 - \mathbf{r}_3. \quad (12)$$

To simplify the analysis, we solve the problem in 2D space due to the logical orientation of the Cartesian coordinate system with the center coinciding with the center of the star:

$$\frac{dv_{2x}}{dt} = \frac{1}{m_2}(F_{21x} + F_{23x}); \quad \frac{dr_{2x}}{dt} = v_{2x};$$

$$\frac{dv_{3x}}{dt} = \frac{1}{m_3}(F_{31x} + F_{32x}); \quad \frac{dr_{3x}}{dt} = v_{3x};$$

$$\frac{dv_{2y}}{dt} = \frac{1}{m_2}(F_{21y} + F_{23y}); \quad \frac{dr_{2y}}{dt} = v_{2y};$$

$$\frac{dv_{3y}}{dt} = \frac{1}{m_3}(F_{31y} + F_{32y}); \quad \frac{dr_{3y}}{dt} = v_{3y}. \quad (13)$$

Projections of gravitational forces are written under (6):

$$F_{21k} = -\frac{Gm_1 m_2 r_{21k}}{r_{21}^3}\left(1 + \frac{v_{2k}^2}{c^2} + 2\frac{r_{21x}v_{2x} + r_{21y}v_{2y}}{cr_{21}^2}\right);$$

$$F_{31k} = -\frac{Gm_1 m_3 r_{31k}}{r_{31}^3}\left(1 + \frac{v_{3k}^2}{c^2} + 2\frac{r_{31x}v_{3x} + r_{31y}v_{3y}}{cr_{31}^2}\right); \quad k = x, y; \quad (14)$$

$$F_{23k} = -\frac{Gm_2 m_3 r_{23k}}{r_{23}^3}\left(1 + \frac{v_{23k}^2}{c^2} + 2\frac{r_{23x}v_{23x} + r_{23y}v_{23y}}{cr_{23}^2}\right); \quad F_{32k} = -F_{23k},$$

here

$$r_{23k} = r_{21k} - r_{31k}; \quad v_{23k} = v_{21k} - v_{31k}, \quad k = x, y. \quad (15)$$

$$r_k = \sqrt{r_{kx}^2 + r_{ky}^2}, \quad k = 21, 31, 23; \quad v_k = \sqrt{v_{kx}^2 + v_{ky}^2}, \quad k = 2, 3, 23. \quad (16)$$

Expressions (13) - (16) form a complete system of algebraic-differential equations for the analysis of transients in the cosmic system: *star - 2 planets*. To obtain the desired unambiguous solution, it is necessary to set constant parameters G, m_1, m_2, m_3 as well as the space-velocity initial conditions:

$$r_{2k}(0), r_{3k}(0); \quad v_{2k}(0), v_{3k}(0), \quad k = x, y. \quad (17)$$

Under the condition $F_{23} = F_{32} = 0$, the differential equations (13) describe the independent physical processes of particular planets.

4.5. Simulation results

The results of the joint implementation of (13) - (16) by the numerical method are shown in Fig. 1–7 for constant parameters $Gm_1 = 13,27128 \times 10^{19}$,

$Gm_2 = 2,19185 \times 10^{13}$, $Gm_3 = 32,47926 \times 10^{13}$ (m³s⁻²) corresponding to the Sun, Mercury, and Venus. All dimensions in the simulation results are reduced to SI units.

Figure 1 shows the time dependence of the hodograph of spatial radius $r_{21(t)}$ obtained under the initial conditions:

$v_{2x}(0) = 59000; v_{2y}(0) = 0; r_{2x}(0) = 0; r_{2y}(0) = 0,4600 \times 10^{11};$

$v_{3x}(0) = 35020; v_{3y}(0) = 0; r_{3x}(0) = 0; r_{3y}(0) = 1,0821 \times 10^{11}$

The duration of the transition process is 5.108 s, which corresponds to approximately 15.844 Earth years. The time dependence of the spatial radius itself is shown in Fig.2. It testifies that the transition process continues and is far from the established value.
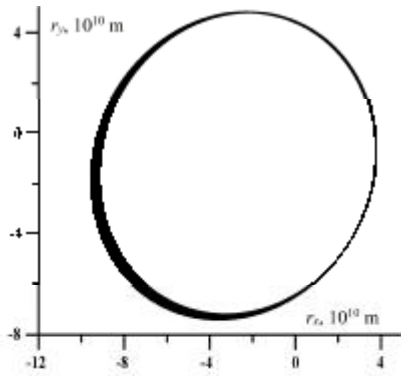
Fig. 1. Hodograph of the distance
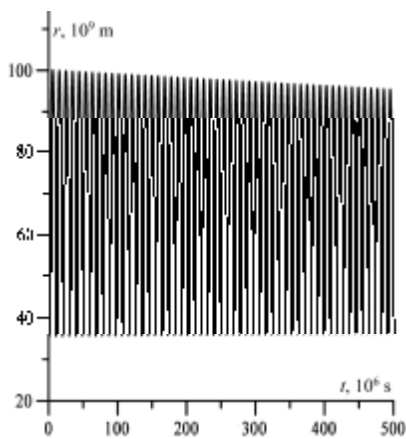*of the trajectory of Mercury*

Fig. 2. The time dependence of the radius
in the transition process shown in Fig. 1

For comparison, Fig.3 shows the same transient process, under the same initial conditions, within the same time limits, as in Fig.1, but data are calculated by classical equations involving only one component of the force, namely Newton's force (7). From the comparison of the results is evident that both processes differ not only quantitatively but also qualitatively. If the first of them is transient, then the second is stable, because it lacks a radial component of the force that would correct the orbit, including its precession. Accordingly, the thickness of the hodograph line is thinner in the latter case.

Analysis of digital data of the transient process of Fig.1 envisages that the gravitational interaction of planets on their orbits is rather insignificant, including the precession of these orbits. Thus, for a fixed transition time: $t = 3.80675 \cdot 10^6$ s maximum radius $r_{21\max} =$ 69941273166 m, minimum speed $v_{2\min} = 38803.91$ m/s ($F_{23} = F_{32}$ ¹ 0) ($r_{21\max} = 69941584666$ m; $v_{2\min} = 38803.91$ m/s. That is, the influence of all planets on the

precession of the orbit Mercury becomes similar, and this is how U. Le Verrier explained the reason for the similar influence of the planets on Venus: he has estimated it as 53% of the total impact. Based on the graphical and temporal resolutions, it is not possible to judge the course of the transient process in Fig.1, which is close to the existing conditions.
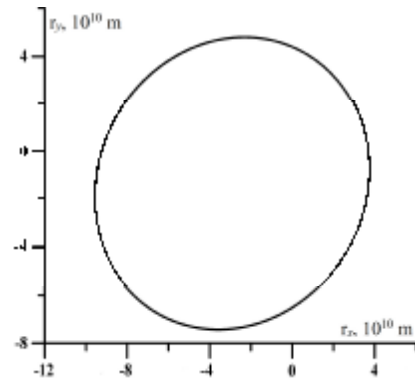

Fig. 3. Hodograph of the distance $r_{21}(t)$ corresponding
to the transient process of Fig.1, computed in the presence of
only Newton's gravitational force

Figure 4 shows the transition process in which the planets are close to the gravitational mass of the Sun under the initial conditions:

$$v_{2x}(0) = 25000; \; v_{2y}(0) = 0; \; r_{2x}(0) = -0,8000; \; r_{2y}(0) = 0,4000 \times 10^{11};$$

$$v_{3x}(0) = 42140; \; v_{3y}(0) = 0; \; r_{3x}(0) = 0; \; r_{3y}(0) = -0,9400 \times 10^{11};$$

In this case, not only the precession of the perihelion in the direction of the planet's rotation is visible, but also the gradual convergence of the planet with the star (rupture of the orbit).
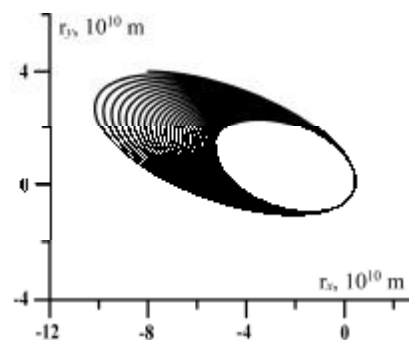

Fig. 4. Hodograph of distance $r_{21}(t)$
in the case of planets artificially close to the Sun

The duration of the transition process is $1 \cdot 10^8$ s. Although, according to the curve of Fig.2 (here the transition process continues), it was possible to get close enough to the observed results on the planet. From the

analysis of digital data of the last turn shown in Fig.1 (results of supervision in brackets) eccentricity makes:

$$e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} \qquad (18)$$

experimental data 0.2056), the maximum distance between the centers of gravitational masses r_{21max} = 68.79.109 (69.82.109) m, and the minimum distance between the centers of gravitational masses r_{21min} = 46.51.109 (46.00.109) m. It is interesting to note that as the planet "falls" on the star, the eccentricity of the quasi-elliptical trajectory decreases. Thus, in the transition process of Fig.1 in $5 \cdot 10^8$ s, it decreased from 0.2064 to 0.1932. To establish quantitative characteristics of the precession of the object, this is not enough: it needs a fragmentary frozen in time the state of the transition process; then it is necessary to evaluate the steady process. This is due to the need to define the initial conditions that exclude the transient reaction. To fulfill this, a closed condition is imposed on the established orbit, which according to Fig. 4, is absent.

Thus, the third force (9) is responsible for the precession of the perihelion of the elliptical orbits of the planets. For a circular orbit, this force is identically equal to zero. In circular orbits, the phenomenon of trajectory precession is impossible. The force (8) is due to the tangential component of the velocity, and the force (9) is due to the normal one. Therefore, the equations of classical physics operate successfully only for stationary trajectories, where the normal component of the force is absent, i.e. on ideal closed circular orbits. This is the main reason why the methods of classical physics were hastily pushed to the backyard of the microworld [10-11] and damaged the unity of the Universe.

To obtain a metrologically substantiated result of estimating the precession of the elliptical trajectory of Mercury based on differential equations of motion (Fig.5) it is necessary to know the exact initial space-velocity conditions laid down by Nature in the process of the Solar system's evolution, or the corresponding initial conditions, which exclude a transient reaction if a steady process exists.
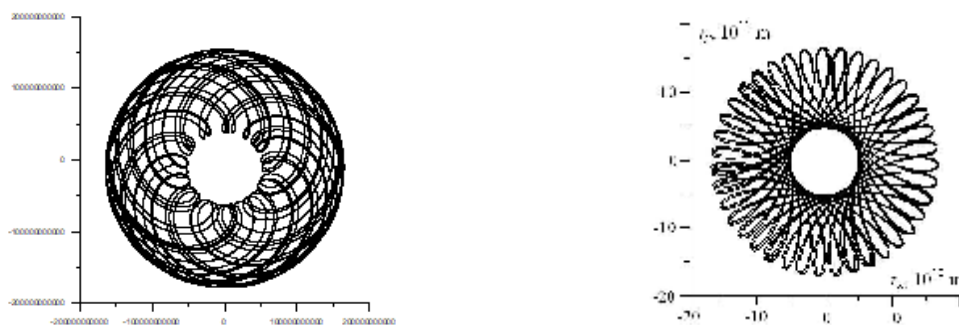

Fig. 5. Hodograph interplanetary (Mercury - Venus) distance $r_{23}(t)$ in the transition process shown in Fig. 1 with coordinated (left) and counter (right) mutual rotation around the Sun at a time of $1 \cdot 10^8 s$

This consideration makes us think anew about the words of great H. Poincare: "Objective reality, after all, is what is common to think beings. This common side can only be harmony, expressed by mathematical laws. It is this harmony is the only objective reality, the only truth we can achieve, and if I add that the universal harmony of the world is the source of all beauty, it will be clear how we must appreciate the slow and difficult steps forward that gradually open it to us".

5. Summary and Outlook

The simulation of the transitions of the star-planets system demonstrates the possibilities of a new approach to improving the accuracy of estimating the precession of the orbital elliptical orbits of the planets. The method is based on considering the action of an additional third component of the gravitational field force, which depends not only on the velocity of the mass but also on the spatial orientation of its trajectory. These results are presented holographically since the determination of quantitative characteristics based on differential equations of motion is complicated by the impossibility of establishing the initial spatial-velocity conditions laid down in the evolution of the stellar system.

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