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A fractional-order model for drinking alcohol behaviour leading to road accidents and violence

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In this paper, we propose a new fractional-order model of alcohol drinking involving the Caputo derivative and six groups of individuals. We introduce road accidents and violence related to alcohol consumption as separate classes to highlight the role of alcoholism in the aggressive and risky behaviour of heavy drinkers. We show the existence and uniqueness of the non-negative solutions, and we determine the basic reproduction number R_0. The sensitivity analysis of the model parameters is performed to characterize the important parameters that have the most effects on the reproduction number. Furthermore, the stability analysis of the model shows that the system is locally and globally asymptotically stable at drinking-free equilibrium E^0 when $R_0 < 1$, and the drinking present equilibrium E^* exists. The system is locally and globally asymptotically stable at E^* when $R_0 > 1$. Finally, numerical simulations are carried out to illustrate the theoretical results for different values of the order of the fractional derivative.

Keywords: *fraction-order model, drinking alcohol behaviour, road accident, epidemiological approach, stability analysis, sensitivity analysis.*

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1. Introduction

Drinking alcohol is a human act that leads very often to several harmful consequences on drinkers themselves and society. Alcohol use is causally related to serious health problems, including an increased risk of strokes, certain cancers, chronic diseases and cirrhosis of the liver. Alcohol abuse also contributes to death and disability through road accidents and injuries, violence, crime and suicide, particularly among young people. In the global status report on alcohol and health, published in 2018, the World Health Organization reports that, in 2016, the mortality resulting from alcohol consumption was higher than that caused by diseases such as tuberculosis, HIV/AIDS and diabetes. Among the 3 million deaths caused by harmful use of alcohol (5.3% of all deaths worldwide), 28% of deaths were attributed to road traffic accidents, violence and suicide, 21% of them were attributed to pathologies affecting the digestive system, 19% are attributable to cardiovascular diseases, and 32% are attributed to infectious diseases, cancers, mental disorders or other conditions [1].

Epidemiological models have become important tools that can accurately predict the dynamics of infectious diseases and provide useful measures to analyse and control their spread, see for instance [2–4]. Recently, new applications of the epidemiological approach have emerged, in particular for social phenomena such as drug use [5, 6], smoking [7], elections [8–10] and the spread of rumours [11]. As for alcoholism, it is considered as a kind of disease that is characterized by alcohol craving and persistent consumption, which depends on social interactions and can be viewed as a socially contagious process. Numerous studies use the epidemiological approach to investigate the dynamics of alcohol drinking, analyse the behaviour of drinkers and propose some solutions to reduce the harm on the drinkers and society as well as minimizing the number of addicted drinkers. For example, Sharma et al. [12] developed a mathematical model of alcohol abuse and discussed the existence, local, global stability of drinking-free, endemic equilibrium and sensitivity analysis of a basic reproduction number R_0. They demonstrated that backward bifurcation can occur when $R_0 = 1$. Ma et al. [13] modeled alcoholism as a contagious disease and analysed the impact of awareness programs and time delay on the alcohol drinking distribution. Their results show that the awareness programs are effective measures in controlling heavy drinking and the stability of their model will change when the time delay is increased. Wang et al. [14] proposed and analysed a non-linear alcoholism model and used optimal control to hinder interaction between susceptible individuals and infected individuals. Huo et al. [15] proposed a new social epidemic model to depict alcoholism with media coverage which was proven to be an effective way of pushing people to quit drinking. Xiang et al. [16] were interested in the global property of a drinking model with public health educational campaigns. They conclude that educational campaigns have a positive effect on controlling drinking dynamics. Giacobbe et al. [17] considered a mathematical model that describes the dynamics of a population divided into three categories and used a supplementary variable that represents an external influence. They investigate the existence of endemic equilibrium and analysed the stability of the equilibrium. Authors in [18] used a non-linear $SHTR$ mathematical model to study the dynamics of the drinking epidemic. They divided the population into four classes: non-drinkers (S), heavy drinkers (H), drinkers receiving treatment (T) and recovered drinkers (R). They discussed the existence and stability of drinking-free and endemic equilibrium. Bonyah et al. [19] formulated a deterministic alcohol model and carried out the stability analysis of the equilibrium. The effect of time-dependent control is examined to establish the best strategy in controlling alcohol consumption and gonorrhoea dynamics. Khajji et al. [20] proposed a discrete alcohol model that divided the population into six compartments. They formulated an optimal control problem with three controls: the first one represents awareness programs for potential drinkers, the second one is the effort to encourage the rich people to go to the private treatment center, and the third one is follow-up and psychological support for temporary quitters of drinking. The authors used Prontryagin's maximum principle to find optimal strategies that minimize the number of drinkers and maximize the number of heavy drinkers who join an addiction treatment center. (See also [21, 22] and the references contained therein).

The concept of fractional calculus was firstly proposed by Guillaume de l'Hôpital in 1695. It is currently used in several fields, such as finance, chemistry, physics, mathematical biology and many others (see [23–25]). Techniques of fractional calculus have been employed at the modeling of many processes that carry information about their present as well as past states; in particular, in epidemiological modeling (refer to [26–29]). Derivatives and integrals of fractional order consider the system memory, hereditary properties and non-local distributed effects. These effects are essential for portraying the real-world problems [30].

Motivated by the fact that the fractional-order models have been proven valuable in understanding the dynamics of phenomenon characterised by the memory effect, we propose and analyse a new fractional-order model that describes drinking behaviour as a contagious disease that can spread through social interaction, and includes new compartments representing two major social problems related to addiction to drinking alcohol, namely road accidents and violence. Recall that driving under the influence of alcohol has a high probability of leading to serious traffic accidents. In many high-income countries about 20% of fatally injured drivers have excess alcohol in their blood. Studies in low-income countries have shown alcohol to be present in between 33% and 69% of fatally injured drivers [31]. Furthermore, ample evidence shows that drinking is often accompanied by deliberate violence. An interesting study on the question of whether those individuals who consume alcohol have an increased probability of violent behaviour can be found in [32]. To the best of our knowledge, this work is the first that proposes a fractional-order model for drinking alcohol behaviour leading to road accidents and violence.

The rest of the paper is organized as follows. In Section 2, we present our fractional order mathematical model and the main dynamical system. Section 3 presents some results and definitions on fractional calculus to be used in the next. In Section 4, we discuss basic properties and positivity of solutions. In Section 5, we analyse the local and global stability and the problem of parameters sensitivity. Some numerical simulations are discussed in Section 6. Finally, we conclude the paper in Section 7.

2. Model formulation

In this section, we propose a fractional order continuous-time model that describes the population dynamics and the interactions between the drinker's classes. We followed the methods of [33]. We consider that the total population N is divided into six compartments:

(a) **The potential drinkers** (P) refer to the individuals whose age is over adolescence and adulthood and are potential drinkers of alcohol. The total number of individuals in this compartment increases by the recruitment rate denoted by b and decreases by an effective contact with the moderate drinkers at β rate and natural death μ. The contact that occurs between potential drinkers and moderate drinkers in some social occasions like weddings, celebrating graduation ceremonies, weekend parties and the end of the year celebration, can make potential drinkers acquire drinking behaviour and become moderate drinkers. In other words, it is assumed that the acquisition of a drinking behaviour is analogous to acquiring disease infection.

(b) **The compartment** (M) represents moderate drinkers. These individuals can control their consumption of alcohol during some social events or their intake of alcohol is unapparent to their social environment. These drinkers do not fall into any social problems or negative consequences due to alcohol consumption. Friends or family do not complain about their consumption of alcohol. One of the characteristics of these drinkers is that they do not think about drinking very often or often feel a need to drink. Alcohol does not dominate their thoughts and they do not need to set limits when they drink. They are not prone to extreme mood swings, fighting or being violent. The number of the individuals of this compartment increases by potential drinkers who transfer into moderate drinkers at rate β. It is decreased when moderate drinkers become heavy drinkers at a rate σ and also by natural death at rate μ.

(c) **The compartment** (H) consists of heavy drinkers who have addiction to alcohol. An alcoholic person finds it difficult to control or set limits to their consumption. Most alcoholics move from the state of potential drinkers to moderate drinkers. Alcohol makes the life of an alcoholic seem uncontrollable. Everything surrounding an alcoholic becomes endangered including their job, their family, their social circle and their health. Even if such condition renders these negative consequences, the alcoholic is unable to give up drinking. A disclaim of not having any problem begins to appear in the behaviour of alcoholics, this disclaim can make it even more difficult for the person to get help. Alcohol addiction amounts to be a disease; it causes some changes in the chemicals of the addict's brain and turns alcohol to the most important thing in their life. People considered alcoholic will usually need to get help at a rehab to overcome their addiction. This compartment becomes larger as the number of heavy drinkers increases by the rate σ and decreases by the rates α_1 (a rate the heavy drinkers individuals becomes heavy drinkers with violence), α_2 (a rate the heavy drinkers individuals they becomes heavy drinkers with accidents), α_3 (a rate the heavy drinkers individuals they becomes recovered and quitters of drinking) and δ_1 (the death rate induced by H) and also by natural death at rate μ.

(d) **Violent heavy drinkers** (V) have a prolonged and excessive consumption of alcohol and they commit various acts of violence. The compartment of the violent heavy drinkers is increased by the rate α_1 and decreased by the rates γ_1 (a rate of the violent heavy drinkers who recover and quit drinking), and δ_3 (the death rate induced by the violence of heavy drinkers), and also by natural death at rate μ.

(e) **The heavy drinkers who cause accidents** (A) have a prolonged and excessive alcohol consumption. The compartment of the heavy drinkers who cause accidents is increased by the rate α_2, decreased by the rates γ_2 (a rate the heavy drinkers who cause traffic accidents and becomes recovered and quitters of drinking), and δ_2 (the death rate induced by the traffic accidents of heavy drinkers), and also by natural death at rate μ.

(f) $Q(t)$ refers to the individuals who temporarily and permanently quit drinking. It is increased by the rates γ_1, γ_2 and α_3 and decreased by the rate μ.

The dynamics of the model is governed by the following fractional order differential equations:

$$\begin{cases} \dfrac{d^\alpha P(t)}{dt^\alpha} = b - \beta\dfrac{PM}{N} - \mu P, \\ \dfrac{d^\alpha M(t)}{dt^\alpha} = \dfrac{PM}{N} - (\sigma+\mu)\,M, \\ \dfrac{d^\alpha H(t)}{dt^\alpha} = \sigma M - (\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1)H, \\ \dfrac{d^\alpha V(t)}{dt^\alpha} = \alpha_1 H - (\mu+\gamma_1+\delta_3)\,V, \\ \dfrac{d^\alpha A(t)}{dt^\alpha} = \alpha_2 H - (\mu+\gamma_2+\delta_2)\,A, \\ \dfrac{d^\alpha Q(t)}{dt^\alpha} = \alpha_3 H + \gamma_1 V + \gamma_2 A - \mu Q, \end{cases} \tag{1}$$

where $\frac{d^\alpha X(t)}{dt^\alpha}$ are the derivative of $X(t)$ of order α $(0 < \alpha \leqslant 1)$ in the sense of Caputo. The total population size at time t is denoted by $N(t)$ with $N(t) = P(t) + M(t) + H(t) + V(t) + A(t) + Q(t)$. The definitions of all variables of the proposed model are given in Table 1. Also, the parameters and their interpretation are introduced in Table 2.

The graphical representation of the proposed model is shown in Figure 1.

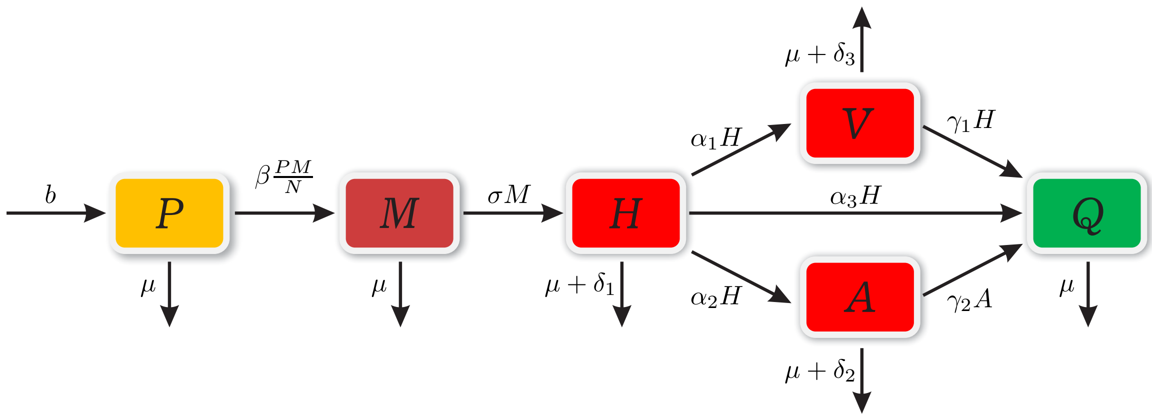

Fig. 1. Schematic diagram of the six drinking classes in the model.

Table 1. State variables and their meanings.

$P(t)$	Potential drinkers at time t
$M(t)$	Moderate drinkers at time t
$H(t)$	Heavy drinkers at time t
$V(t)$	Violent heavy drinkers at time t
$A(t)$	Heavy drinkers who cause traffic accidents at time t
$Q(t)$	Recovered and Quitters of drinking at time t

We consider system (1) with the following parameter values.

Table 2. Description of parameters of the model (1).

Parameter	Description	Value	Source
b	Recruitment rate of susceptible individuals	159.5	Assumed
β	Contact rate of susceptible individuals with moderate individuals	0.4	[34]
μ	The natural death rate	0.1595	[34]
σ	Proportion of exposed individuals that join addicted class	0.3	[35]
α_1	The rate of the H drinkers individuals becomes heavy drinkers with violence	0.03	Assumed
α_2	The rate of H drinkers individuals they becomes heavy drinkers with accidents	0.02	Assumed
α_3	The rate of the H individuals they becomes recovered and quitters of drinking	0.3	[36]
δ_1	The death rate induced by the heavy drinkers	0.035	[35]
δ_2	The death rate induced by traffic accidents due to drinking alcohol	0.002	Assumed
δ_3	The death rate induced by violence due to drinking alcohol	0.001	Assumed
γ_1	The rate of the violent heavy drinkers who recover and quit drinking	0.001	Assumed
γ_2	The rate of the H who cause traffic accidents and becomes recovered and quitters of drinking	0.001	Assumed

3. Preliminaries on the Caputo fractional calculus

In this section, we introduce some fundamental definitions and necessary lemmas about fractional calculus used in the next sections. There are many types of fractional derivatives but the most used ones in mathematical modeling and engineering applications are Riemann-Liouville derivative and Caputo derivative (see [37–41]). In this paper, we develop our model with Caputo fractional derivatives. Its main advantage is the initial values for fractional differential equations with the Caputo derivatives taking on the same form as for integer order differential equations.

1. The fractional integral of order $\alpha > 0$ of a function $f\colon \mathbb{R}^+ \to \mathbb{R}$ is defined as follows:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-x)^{\alpha-1} f(x)\, dx,$$

where Γ is the Euler Gamma function:

$$\Gamma(s) = \int_0^{+\infty} t^{s-1} e^{-t}\, dt.$$

2. The Caputo fractional order derivative can be defined as follows:

$$\begin{aligned} \frac{d^\alpha f(t)}{dt^\alpha} &= I^{n-\alpha} \frac{d^n}{d^n t} f(t) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n}{d^n \tau} f(\tau)\, d\tau, \end{aligned}$$

where Γ is the Euler gamma function, $f(t)$ is a time dependent function and α is the order of the derivative ($n-1 < \alpha \leqslant n$). In particular, when $0 < \alpha \leqslant 1$, we have

$$\frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} f'(\tau)\, d\tau.$$

3. The Laplace transform of the Caputo fractional derivative is given by

$$\mathcal{L}\left(\frac{d^\alpha f(t)}{dt^\alpha}\right) = \lambda^\alpha F(\lambda) - \sum_{k=0}^{n-1} f^{(k)}(0)\, \lambda^{\alpha-k-1}. \tag{2}$$

4. The Mittag–Leffler function $E_{\alpha,\beta}$, defined by

$$E_{\alpha,\beta}(k) = \sum_{n=0}^{+\infty} \frac{k^n}{\Gamma(\alpha n + \beta)}.$$

5. The Laplace transform of the Mittag–Leffler function

$$\mathcal{L}\big(t^{\beta-1} E_{\alpha,\beta}(\pm a t^{\alpha})\big) = \frac{\lambda^{\alpha-\beta}}{\lambda^{\alpha} \mp a}. \tag{3}$$

6. Let $\alpha, \beta > 0$ and $z \in \mathbb{C}$, then the Mittag–Leffler function satisfies the equality given by

$$E_{\alpha,\beta}(z) = z\, E_{\alpha,\alpha+\beta}(z) + \frac{1}{\Gamma(\beta)}. \tag{4}$$

7. Let $f\colon \mathbb{R}^+ \to \mathbb{R}$ with $n \geqslant 1$. Consider the following autonomous non-linear fractional-order system:

$$\begin{cases} \dfrac{d^{\alpha} X(t)}{dt^{\alpha}} = f(X), \\ X(t_0) = X_0, \end{cases} \tag{5}$$

with $0 < \alpha < 1$, $t_0 \in \mathbb{R}$ and $X_0 \in \mathbb{R}^n$. The equilibrium points of the above system are solutions to the equation $f(X) = 0$. An equilibrium is locally asymptotically stable if all eigenvalues (λ_i) of the Jacobian matrix $J = \frac{\partial f}{\partial X}$ evaluated at the equilibrium satisfy $|\arg(\lambda_i)| > \frac{\alpha\pi}{2}$, and for the global existence of a solution of system (5), we need the following result.

Lemma 1 (see [42]). *Assume that f satisfies the following conditions:*
(i) $f(X)$ and $\frac{df}{dX}(X)$ are continuous for all $X \in \mathbb{R}^n$;
(ii) $\|f(X)\| \leqslant \omega + \lambda \|X\|$ for all $X \in IR^n$, where ω and λ are two positive constants.
Then, system (5) has a unique solution on $[t_0, +\infty)$.

Lemma 2 (see [43]). *Assume that $f \in C^1[a,b]$. If $\frac{d^{\alpha} f(t)}{dt^{\alpha}} \geqslant 0$ for all $t \in [a,b]$ and all $\alpha \in (\alpha_0, 1)$ with some $\alpha_0 \in (0,1)$, then f is monotone increasing. Similarly, if $\frac{d^{\alpha} f(t)}{dt^{\alpha}} \leqslant 0$ for all $t \in [a,b]$ and all $\alpha \in (\alpha_0, 1)$ with some $\alpha_0 \in (0,1)$, then f is monotone decreasing.*

4. Basic properties of the model

4.1. Invariant region

It is necessary to prove that all solutions of system (1) with positive initial data will remain positive for all times $t > 0$. This will be established by the following theorem.

Theorem 1. *The feasible region Ω defined by*

$$\Omega = \left\{ (P, M, H, V, A, Q) \in \mathbb{R}_+^6 \colon P + M + H + V + A + Q \leqslant \frac{b}{\mu} \right\} \tag{6}$$

is positively invariant for the system (1).

Proof. We will accomplish the proof through two steps:

Step 1: we will prove that the solution of system (1) is always non-negative. According to system (1), we get easily

$$\left.\frac{d^{\alpha} P(t)}{dt^{\alpha}}\right|_{p=0} = b \geqslant 0,$$

$$\left.\frac{d^\alpha M(t)}{dt^\alpha}\right|_{M=0} = 0,$$
$$\left.\frac{d^\alpha H(t)}{dt^\alpha}\right|_{H=0} = \sigma M \geqslant 0,$$
$$\left.\frac{d^\alpha V(t)}{dt^\alpha}\right|_{V=0} = \alpha_1 H \geqslant 0,$$
$$\left.\frac{d^\alpha A(t)}{dt^\alpha}\right|_{A=0} = \alpha_2 H \geqslant 0,$$
$$\left.\frac{d^\alpha Q(t)}{dt^\alpha}\right|_{Q=0} = \alpha_3 H + \gamma_1 V + \gamma_2 A \geqslant 0.$$

From Lemma 2, we have $P(t)$, $M(t)$, $H(t)$, $V(t)$, $A(t)$ and $Q(t)$ are positive for all $t > 0$. Then, the solution of system (1) will remain in $\mathbb{R}^6_+$.

Step 2: By adding the equations of system (1), we get

$$\frac{d^\alpha N(t)}{dt^\alpha} = b - \mu N(t) - \delta_1 H - \delta_2 A - \delta_3 V,$$

implies that

$$\frac{d^\alpha N(t)}{dt^\alpha} \leqslant b - \mu N(t).$$

Applying the Laplace transform, we obtain

$$\lambda^\alpha \mathcal{L}(N(t)) - \lambda^{\alpha-1} N(0) \leqslant \frac{b}{\lambda} - \mu \mathcal{L}(N(t)).$$

It follows that

$$\mathcal{L}(N(t)) \leqslant \frac{\lambda^{\alpha-1}}{\lambda^\alpha + \mu} N(0) + \frac{\lambda^{\alpha-(\alpha+1)}}{\lambda^\alpha + \mu} b.$$

From (3) we deduce

$$N(t) \leqslant E_{\alpha,1}(-\mu t^\alpha) N(0) + t^\alpha E_{\alpha,\alpha+1}(-\mu t^\alpha) b.$$

According to (4), we have

$$N(t) \leqslant E_{\alpha,1}(-\mu t^\alpha) N(0) + \frac{b}{\mu}(1 - E_{\alpha,1}(-\mu t^\alpha));$$

$$N(t) \leqslant \left(N(0) - \frac{b}{\mu}\right) E_{\alpha,1}(-\mu t^\alpha) + \frac{b}{\mu}.$$

Since $E_{\alpha,1}(-\mu t^\alpha) \geqslant 0$ for any $t \geqslant 0$ and $N(0) \leqslant \frac{b}{\mu}$, then $N(t) \leqslant \frac{b}{\mu}$. Hence, for the analysis of the model (1), we only need to consider its dynamics on the set Ω given by Eq. (6). ■

4.2. Existence and uniqueness of global solution

In the following result we will prove the existence of a unique global solution of the system (1).

Theorem 2. *The fractional order initial value system* (1) *has a unique global solution on* Ω.

Proof. Let $X = (P, M, H, V, A, Q)$, then system (1) can be rewritten as follows:

$$\frac{d^\alpha X(t)}{dt^\alpha} = F(X),$$

where

$$F(X)=\begin{pmatrix} b-\beta\frac{PM}{N}-\mu P\\ \beta\frac{PM}{N}-(\sigma+\mu)M\\ \sigma M-(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1)H\\ \alpha_1 H-(\mu+\gamma_1+\delta_3)V\\ \alpha_2 H-(\mu+\gamma_2+\delta_2)A\\ \alpha_3 H+\gamma_1 V+\gamma_2 A-\mu Q \end{pmatrix}.$$

Firstly, it is easy to see that F satisfies the first condition of Lemma 1. For the second condition, let us rewrite the column matrix F as follows:

$$F(X)=\overline{b}+(MG_1+G_2)X,$$

where

$$\overline{b}=\begin{pmatrix} b\\0\\0\\0\\0\\0 \end{pmatrix},\qquad G_1=\begin{pmatrix} -\frac{\beta}{N} & 0 & 0 & 0 & 0 & 0\\ \frac{\beta}{N} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$G_2=\begin{pmatrix} -\mu & 0 & 0 & 0 & 0 & 0\\ 0 & (\mu+\sigma) & 0 & 0 & 0 & 0\\ 0 & \sigma & -(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1) & 0 & 0 & 0\\ 0 & 0 & \alpha_1 & -(\mu+\gamma_1+\delta_3) & 0 & 0\\ 0 & 0 & \alpha_2 & 0 & -(\mu+\gamma_2+\delta_2) & 0\\ 0 & 0 & \alpha_3 & \gamma_1 & \gamma_2 & -\mu \end{pmatrix}.$$

It follows that there exist $w=\|\overline{b}\|$ and $\theta=|M|\|G_1\|+\|G_2\|$ such that

$$\|F(X)\|\leqslant w+\theta\|X\|.$$

Then, system (1) has a unique global solution on Ω. ∎

Since the first three equations in system (1) are independent of the variables V, A and Q, it is sufficient to consider the following reduced system:

$$\begin{cases} \dfrac{d^\alpha P(t)}{dt^\alpha}=b-\beta\dfrac{PM}{N}-\mu P,\\ \dfrac{d^\alpha M(t)}{dt^\alpha}=\beta\dfrac{PM}{N}-(\sigma+\mu)M,\\ \dfrac{d^\alpha H(t)}{dt^\alpha}=\sigma M-(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1)H. \end{cases}\tag{7}$$

5. Equilibria and their stability analysis

In this section, we discuss the existence, the local and global stability of equilibria for system (7). The sensitivity analysis of the reproduction number R_0 is also discussed.

5.1. Equilibrium point

By using the next generation matrix method formulated in [44], the reproduction number R_0 can be given as follows:

$$R_0=\frac{\beta}{\mu+\sigma}.$$

It measures the average number of new drinkers generated by single drinker in a population of potential drinkers. The value of R_0 will indicate whether the epidemic could occur or not. Furthermore, it can be seen easily that system (7) has two equilibrium points:

(i) The drinking-free equilibrium $E^0\left(\frac{b}{\mu},0,0\right)$ is achieved in the absence of drinking ($M = H = 0$).
(ii) The drinking present equilibrium $E^*(P^*, M^*, H^*)$ is achieved when drinkers exist ($M \neq 0$ and $H \neq 0$), where:

$$\begin{cases} P^* = \dfrac{b}{\mu R_0}, \\ M^* = \dfrac{b(R_0-1)}{\beta}, \\ H^* = \dfrac{b\sigma(R_0-1)}{\beta(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1)}. \end{cases}$$

5.2. Local stability analysis

Now, we proceed to study the local stability behaviour of equilibria E^0 and E^*. Firstly, we analyze the local stability of the drinking-free equilibrium E^0.

Theorem 3. *The drinking-free equilibrium E^0 of the system (7) is asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. The Jacobian matrix for the drinking-free equilibrium is given by:

$$J(E^0) = \begin{pmatrix} -\mu & -\beta & 0 \\ 0 & \beta-\sigma-\mu & 0 \\ 0 & \sigma & -\mu-\alpha_1-\alpha_2-\alpha_3-\delta_1 \end{pmatrix}.$$

Its characteristic equation is given by $\det(J(E^0) - \lambda I_3) = 0$, where I_3 is a square identity matrix of order 3. Therefore, eigenvalues of the characteristic equation of $J(E^0)$ are

$$\begin{cases} \lambda_1 = -\mu, \\ \lambda_2 = -(\sigma+\mu-\beta) = -(\mu+\sigma)(1-R_0), \\ \lambda_3 = -(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1). \end{cases}$$

Therefore, all the eigenvalues of the characteristic equation are clearly negatives ($|\arg(\lambda_i)| > \frac{\alpha\pi}{2}$ for $i = 1,2,3$) if $R_0 < 1$. We conclude that drinking-free equilibrium is locally asymptotically stable for $\alpha \in (0,1]$ if $R_0 < 1$ and unstable if $R_0 > 1$. ■

Now, we analyze the local stability of the drinking present equilibrium.

Theorem 4. *The drinking present equilibrium E^* is locally asymptotically stable if $R_0 > 1$, and unstable otherwise.*

Proof. The Jacobian matrix at E^* is given by

$$J(E^*) = \begin{pmatrix} -\beta\frac{M^*}{N}-\mu & -\beta\frac{P^*}{N} & 0 \\ \beta\frac{M^*}{N} & \beta\frac{P^*}{N}-\sigma-\mu & 0 \\ 0 & \sigma & -\mu-\alpha_1-\alpha_2-\alpha_3-\delta_1 \end{pmatrix}, \qquad (8)$$

where

$$\begin{cases} P^* = \dfrac{b}{\mu R_0}, \\ M^* = \dfrac{b(R_0-1)}{\beta}, \\ H^* = \dfrac{b\sigma(R_0-1)}{\beta(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1)}. \end{cases}$$

From the Jacobian matrix at E^* in Eq. (8), one of the negative eigenvalues is $\lambda_1 = -(\mu+\alpha_1+\alpha_2+\alpha_3+\delta_1)$ whose real part is negative. Thus, we have $|\arg(\lambda_1)| > \frac{\alpha\pi}{2}$. The other eigenvalues are obtained from the quadratic equation:

$$P(\lambda) = \lambda^2 + a_1\lambda + a_2, \tag{9}$$

where

$$\begin{cases} a_1 = \beta\dfrac{M^*}{N} + \mu > 0, \\ a_2 = \beta^2\dfrac{P^*}{N}\dfrac{M^*}{N} > 0. \end{cases}$$

By Routh–Hurwitz stability criterion, the system (7) is locally asymptotically stable if $a_1 > 0$ and $a_2 > 0$. It is clear that all of the eigenvalues are negative ($|\arg(\lambda_i)| > \frac{\alpha\pi}{2}$ for $i = 1, 2, 3$). Thus, the drinking present equilibrium E^* of system (7) is locally asymptotically stable. ∎

5.3. Global stability

To show that the system (7) is globally asymptotically stable, we will use the Lyapunov function theory for both drinking-free equilibrium and drinking present equilibrium.

Theorem 5. *The drinking-free equilibrium E^0 is globally asymptotically stable if $R_0 < 1$, and unstable otherwise.*

Proof. Consider the following Lyapunov function

$$V(P,M,H,V,A,Q) = \frac{1}{2}\left[(P-P_0)+M\right]^2 + \frac{(2\mu+\beta_2)}{\beta}M.$$

The derivative of $V(P,M,H,V,A,Q)$ with respect to t gives

$$\begin{aligned} \frac{d^\alpha V}{dt^\alpha} &\leqslant \left[(P-P_0)+M\right]\left[\frac{d^\alpha P}{dt^\alpha} + \frac{d^\alpha M}{dt^\alpha}\right] + \frac{(2\mu+\sigma)}{\beta}\frac{d^\alpha M}{dt^\alpha}, \\ \frac{d^\alpha V}{dt^\alpha} &\leqslant -\mu(P-P_0)^2 - (\mu+\sigma)M^2 - \frac{(\mu+\sigma)(2\mu+\beta_2)N}{\beta}\left[1-R_0\right]M. \end{aligned}$$

So, $\frac{d^\alpha V}{dt^\alpha} \leqslant 0$ if $R_0 \leqslant 1$.

Furthermore $\frac{d^\alpha V}{dt^\alpha} = 0$ if $P = P_0$ and $M = 0$. Hence, by LaSalle's invariance principle [45], E^0 is globally asymptotically stable. ∎

Now, we will give the global stability of E^* in the following result:

Theorem 6. *The drinking present equilibrium point E^* is globally asymptotically stable if $R_0 > 1$.*

Proof. Consider the Lyapunov function V defined on $\Gamma = \{(P,M)/P > 0, M > 0\}$ by

$$\begin{aligned} &V : \Gamma \to \mathbb{R} \\ &V(P,M) = \left[P - P^*\ln\frac{P}{P^*}\right] + \left[M - M^*\ln\frac{M}{M^*}\right]. \end{aligned}$$

Its fractional time derivative along the solution of system (1) implies that

$$\frac{d^\alpha V}{dt^\alpha} \leqslant \left(1-\frac{P^*}{P}\right)\frac{d^\alpha P}{dt^\alpha}+\left(1-\frac{M^*}{M}\right)\frac{d^\alpha M}{dt^\alpha},$$

$$\frac{d^\alpha V}{dt^\alpha} \leqslant \left(1-\frac{P^*}{P}\right)\left[b-\beta\frac{PM}{N}-\mu P\right]+\left(1-\frac{M^*}{M}\right)\left[\beta\frac{PM}{N}-(\sigma+\mu)M\right],$$

$$\frac{d^\alpha V}{dt^\alpha} \leqslant -b\frac{[P-P^*]^2}{PP^*} \leqslant 0.$$

Also, we obtain

$$\frac{d^\alpha V}{dt^\alpha}=0 \Leftrightarrow P=P^*.$$

Hence, by LaSalle's invariance principle [45] the drinking present equilibrium point E^* is globally asymptotically stable. ■

5.4. Sensitivity Analysis of R_0

Sensitivity analysis is commonly used to determine the model robustness to parameter values, that is, to help us know the parameters that have a high impact on the reproduction number R_0. Using the approach in Chitnis et al. [46], we calculate the normalized forward sensitivity indices of R_0. Let

$$\Upsilon_m^{R_0}=\frac{m}{R_0}\times\frac{\partial R_0}{\partial m},$$

denote the sensitivity index of R_0 with respect to the parameter m. We get

$$\begin{cases} \Upsilon_\beta^{R_0}=1, \\ \Upsilon_\sigma^{R_0}=-\dfrac{\sigma}{\mu+\sigma}, \\ \Upsilon_\mu^{R_0}=-\dfrac{\mu}{\mu+\sigma}. \end{cases}$$

From the above discussion we observe that the basic reproduction number R_0 is most sensitive to changes in β. If β will increase R_0 will also increase with the same proportion and if β decreases in same the proportion, μ and σ will have an inversely proportional relationship with R_0. So, an increase in any of them will bring about a decrease in R_0, however, the size of the decrease will be proportionally smaller. Recall that μ is the natural death rate of the population. It is clear that the increase in either of these rates is neither ethical nor practical. Given R_0's sensitivity to β, it seems sensible to focus efforts on the reduction of β. In other words, this sensitivity analysis tells us that prevention is better than cure. Efforts to increase prevention are more effective in controlling the spread of habitual drinkers than efforts to increase the numbers of individuals accessing treatment.

In table 3, we present the sensitivity indices of all model parameters R_0. The parameters are arranged from the most sensitive to the least sensitive.

Table 3. Parameters and their sensitivity indices.

Parameter	Description	Sensitivity index
β	The effective contact rate	**+1**
μ	The natural death rate	−0.35
σ	Progression rate from M to H	−0.65

Hence, with sensitivity analysis, one can get insight on the appropriate intervention strategies to prevent and control the spread of drinking behaviour of the population on drinkers classes that described by model (1).

6. Numerical simulations and discussion

In this section, we illustrate some numerical solutions of model (1) for different values of the parameters. Generalized Euler method (GEM) is a generalization of the classical Euler's method. For more details, see [47]. The headlines of this method are given as follows: Let $[0,T]$ be the interval over which we want to find the solution of the system (1). We subdivide the interval $[0,T]$ into k subintervals $[t_j, t_{j+1}]$ of equal width $h = \frac{T}{k}$ by using the nodes $t_j = jh$ for $j = 0, 1, 2, \cdots, k-1$. Then, the general formula for GEM when $t_{j+1} = t_j + h$ is

$$\begin{aligned}
P_{j+1} &= P_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_1(P_j, M_j, H_j, V_j, A_j, Q_j),\\
M_{j+1} &= M_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_2(P_j, M_j, H_j, V_j, A_j, Q_j),\\
H_{j+1} &= H_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_3(P_j, M_j, H_j, V_j, A_j, Q_j),\\
V_{j+1} &= V_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_4(P_j, M_j, H_j, V_j, A_j, Q_j),\\
A_{j+1} &= A_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_5(P_j, M_j, H_j, V_j, A_j, Q_j),\\
Q_{j+1} &= Q_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_6(P_j, M_j, H_j, V_j, A_j, Q_j),
\end{aligned}$$

where the system (1) can be as follows:

$$\begin{aligned}
\frac{d^\alpha P(t)}{dt^\alpha} &= f_1(P, M, H, V, A, Q, t),\\
\frac{d^\alpha M(t)}{dt^\alpha} &= f_2(P, M, H, V, A, Q, t),\\
\frac{d^\alpha H(t)}{dt^\alpha} &= f_3(P, M, H, V, A, Q, t),\\
\frac{d^\alpha V(t)}{dt^\alpha} &= f_4(P, M, H, V, A, Q, t),\\
\frac{d^\alpha A(t)}{dt^\alpha} &= f_5(P, M, H, V, A, Q, t),\\
\frac{d^\alpha Q(t)}{dt^\alpha} &= f_6(P, M, H, V, A, Q, t).
\end{aligned}$$

Let present some numerical simulations in order to illustrate our theoretical results. We consider system (1) with parameter values given in Table 2.

We begin by a graphic representation of the drinking-free equilibrium E^0 and use the same parameters and different initial values, $R_0 = 0.87 < 1$.

From Figure 2, one can notice that the solutions converge to the equilibrium point $E^0(1000; 0; 0; 0; 0; 0)$ when $R_0 < 1$ for the different values of α. Hence, from the theorem 5, the model (1) is globally asymptotically stable. Moreover, we observe that the solutions with smaller order α have faster convergence speed compared to the higher.

Also, there is graphic representation of the drinking present equilibrium E^* for different values of α and $R_0 = 1.30 > 1$ with $\beta = 0.6$.

From Figure 3, we notice that the solutions converge to the equilibrium point

$$E^*(765.83; 81.28; 36.7; 45.44; 45.16; 23.57)$$

when $R_0 > 1$ for the different values of α. Hence, from the theorem 6, the model (1) is globally asymptotically stable. Similar to the drinking-free equilibrium condition, we see that as the order α decreases, the convergence of solutions is faster for the drinking present equilibrium.

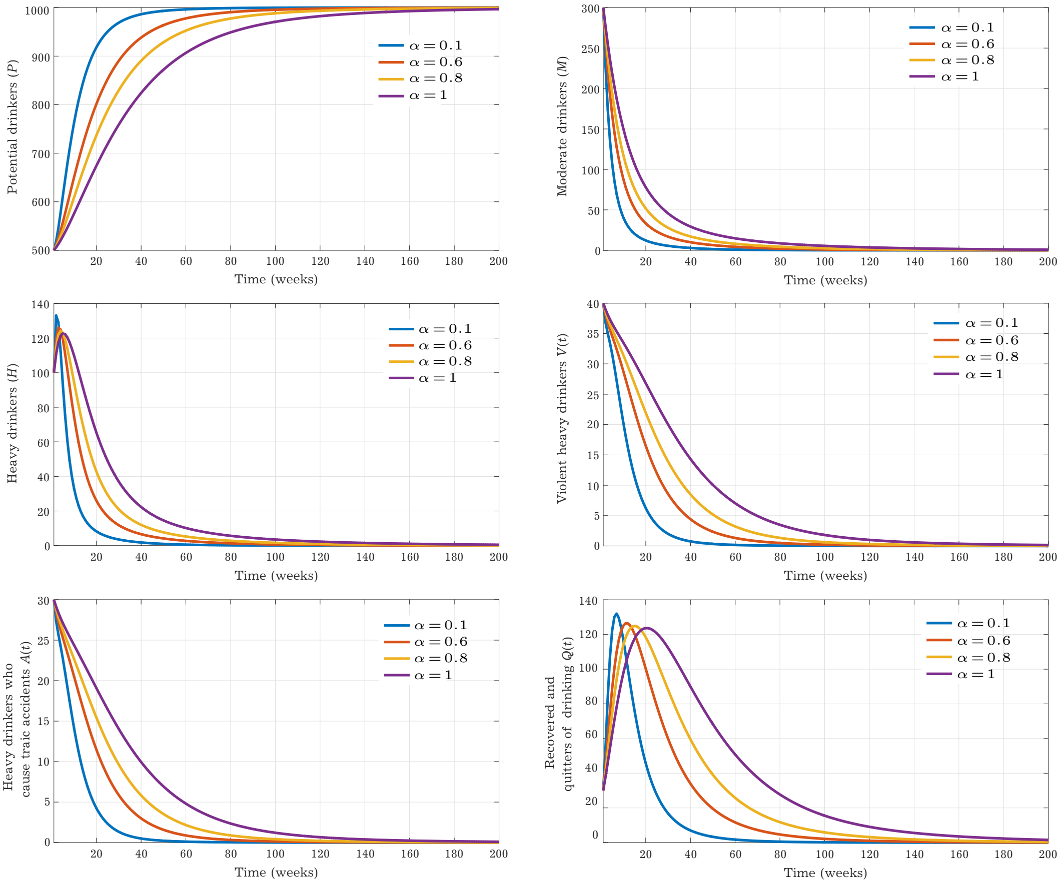

Fig. 2. Stability of drinking-free equilibrium E^0 for different values of α, and when $R_0 = 0.87 < 1$.

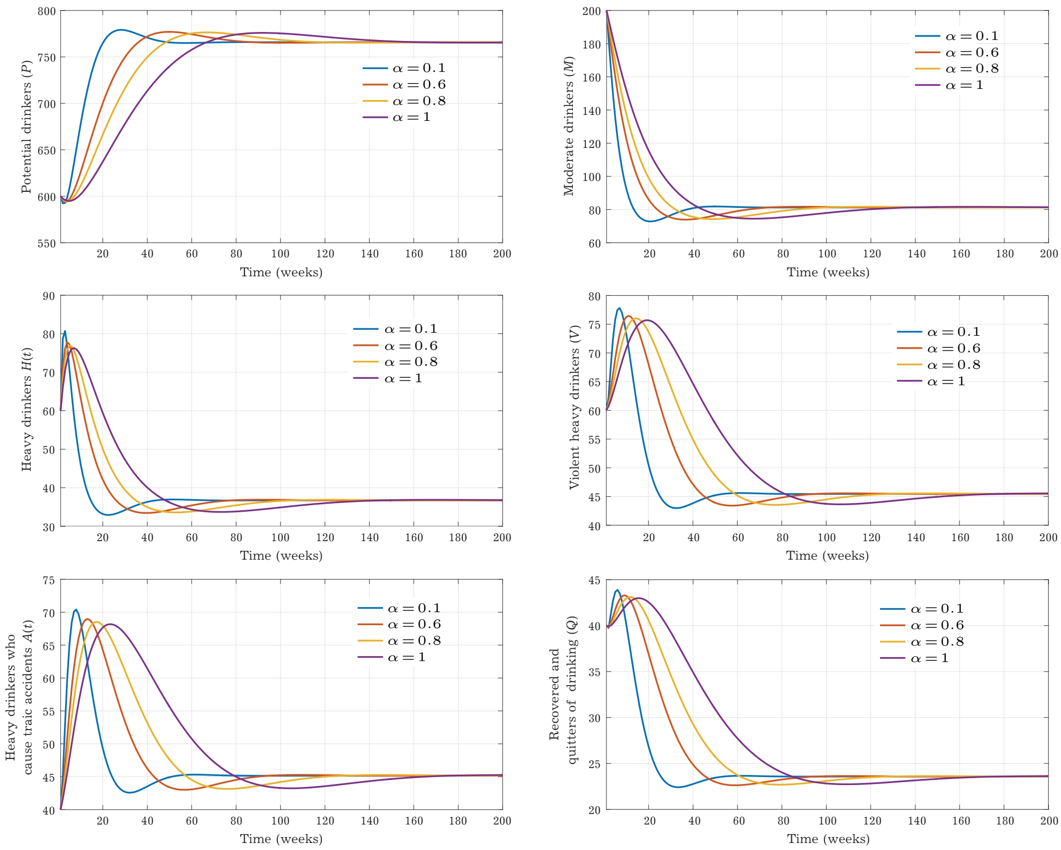

Fig. 3. Stability of alcohol present equilibrium E^* for different values of α, and when $R_0 = 1.30 > 1$.

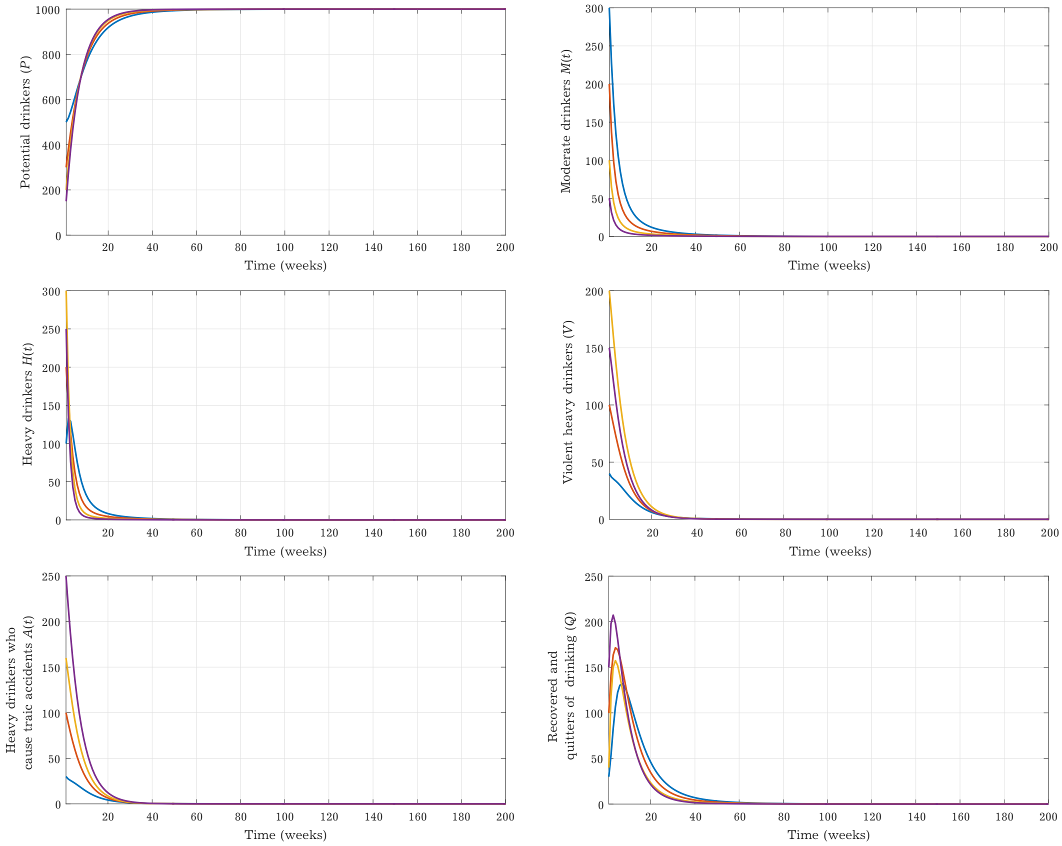

Fig. 4. Stability of drinking-free equilibrium E^0 for different initial values of each variable state, when $R_0 = 0.87 < 1$ and with fixed value of fractional derivative $\alpha = 0.1$.

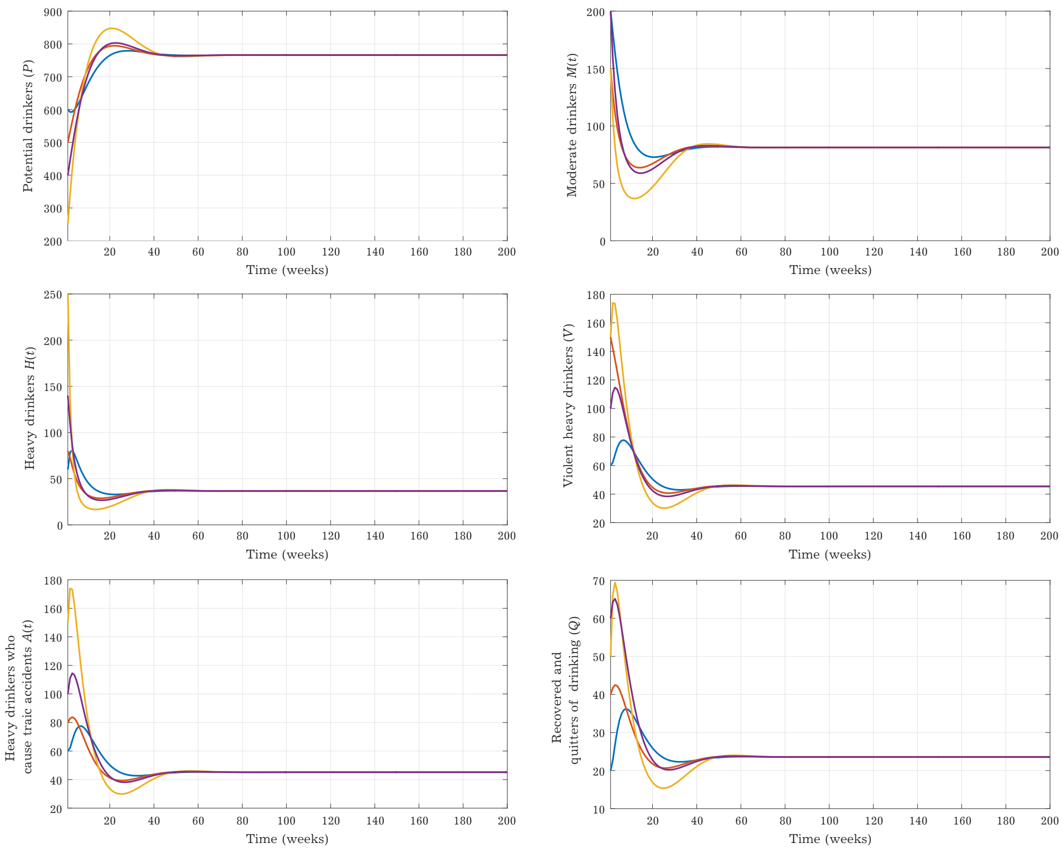

Fig. 5. Stability of alcohol present equilibrium E^* for different initial values of each variable state, when $R_0 = 1.30 > 1$ and with fixed value of fractional derivative $\alpha = 0.1$.

For a fixed value of α and for different initial values for each variable of state, we show in Figure 4 that the solutions converge to the drinking-free equilibrium E^0 when $R_0 = 0.87 < 1$, which implies that the drinking-free equilibrium E^0 of system (1) is globally asymptotically stable on Ω.

For a fixed value of α and for different initial values for each variable of state, we show in Figure 5 that the solutions converge to the alcohol present equilibrium E^* when $R_0 = 1.30 > 1$, which implies that the alcohol present equilibrium E^* of system (1) is globally asymptotically stable on Ω.

From all these figures, we show that the equilibrium points E^0 and E^* of system (1) are globally asymptotically stable on Ω if the conditions of theorems 5 and 6 are satisfied. Also, all solutions of model (1) converge to the equilibrium points E^0 and E^* for different values of α. In addition, the solutions converge rapidly to their steady state when the value of α is very small.

7. Conclusion

In this paper, we have formulated a new fractional-order model that describes the population dynamics of alcohol drinkers, and which includes two important classes, i.e., the class of the violent heavy drinkers, and the class of heavy drinkers who cause traffic accidents. This model takes also into consideration some new important characteristics, such as the death rate induced by heavy drinkers, the death rate induced by traffic accidents due to drinking alcohol and the death rate induced by violence due to drinking alcohol. We have calculated the basic reproduction number (R_0) and studied the sensitivity analysis of the model parameters to determine the parameters that have a high impact on the reproduction number. The stability analysis of both local and global behaviour of drinking dynamic shows that the local asymptotic stability for the drinking-free equilibrium E^0 can be obtained, if the threshold quantity $R_0 < 1$. While the present equilibrium E^* is locally asymptotically stable if $R_0 > 1$. By constructing a suitable Lyapunov function, we have proved that E^0 is globally asymptotically stable if $R_0 \leqslant 1$ and E^* is globally asymptotically stable if $R_0 > 1$. The numerical simulations are carried out, for different values of the order (α) of the fractional derivative, to illustrate the theoretical results. The simulations results show that the solutions with smaller order α have faster convergence speed to equilibrium point compared to the higher order α. Finally, a possible extension of this work consists of investigating the dynamics of our model when we consider the age and/or gender structure, especially knowing the harmful effect of binge drinking on adolescent and young people among both genders.

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Модель дробового порядку поведінки в стані алкогольного сп'яніння, що призводить до дорожньо-транспортних пригод і насильства

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У цій роботі пропонуємо нову модель вживання алкоголю дробового порядку за участю похідної Капуто та шести груп осіб. Вводимо дорожньо-транспортні пригоди та насильство, що пов'язані зі вживанням алкоголю, як окремі класи, щоб підкреслити роль алкоголізму в агресивній та ризикованій поведінці людей, що зловживають алкоголем. Показано існування та єдиність невід'ємних розв'язків і визначено основне число відтворення R_0. Проаналізовано чутливість параметрів моделі для характеристики важливих параметрів, які найбільше впливають на число відтворення. Крім того, аналіз стійкості моделі показує, що система локально та глобально асимптотично стійка в рівновазі без пиття E^0, якщо $R_0 < 1$, і рівновага з питтям E^* існує. Система локально та глобально асимптотично стійка для E^*, коли $R_0 > 1$. Насамкінець, проведено чисельне моделювання для ілюстрації теоретичних результатів для різних значень порядку дробової похідної.

Ключові слова: *модель дробового порядку, поведінка при вживанні алкоголю, дорожньо-транспортна пригода, епідеміологічний підхід, аналіз стійкості, аналіз чутливості.*