

DIFFERENTIAL EQUATIONS OF A NONLINEAR MULTIPOLAR ELEMENT

Vasil Tchaban

Lviv Polytechnic National University, Ukraine

vasyl.y.chaban@lpnu.ua

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Abstract. A method for forming nonlinear differential equations of a multipolar element, which connect its independent pole currents and independent polar voltages, is proposed. The difficulty of the analysis is that some of the internal and external unknowns may be under the symbol of differentiation. The starting information for this formation is the common differential equations of the system of internal and external currents and voltages. The method is demonstrated on the case of formation of the corresponding differential equations of the system as such that is formed by bipolar elements. The analysis is significantly simplified in the presence of internal D -degenerations of the system or resistive circuits.

Keywords: nonlinear differential equations, multipolar element, independent pole currents and polar voltages.

1. Introduction

Any multipolar (n -pole) element is characterized in relation to the outer part of the circuit by a system of equations that express the relationships between its independent voltages and currents due to the nature of the element itself. These equations do not depend on the nature of the outer circuit. They must be satisfied both in the idle state of the element (when it is not included in the circuit at all), and in the state of short circuit (when all its poles are shorted together). The number of such equations for the selected system of independent side voltages and pole currents is in principle unlimited.

The equations of the element and the outer part of the circuit must provide the definition of all n - s independent voltages of the parties and n - s independent pole currents, where s is the number of electrically separated parts or channels in the case of the passable element. Independent side voltages and pole currents of the element and the outer part of the circuit will be chosen so that they can be common to both the element and the outer part of the circuit. Therefore, it is necessary that the equations of the element and the outer part of the circuit taken together constitute a system of $2'(n-s)$ independent equations. This is possible only when the element and the outer part of the circuit are characterized by n - s independent equations. This is at the same time the largest number of equations of the element.

If the number of independent element equations were greater than n - s , then, based on arbitrarily selected n - s equations, the calculation of the circuit would obtain different expressions for the side voltages and pole currents of the element, and this would contradict the experimentally established fact of unambiguous state of the circuit.

A multipolar element can have an arbitrary complex and unpredictable structure, the most complex physical processes can occur in it, therefore, the establishment of independent equations sometimes requires extraordinary ability and skill of the researcher. It is not always possible to describe a physical process with sufficient accuracy; in many cases it is necessary to be limited to approximate methods or to use experimental dependences. Sometimes deep knowledge of the physical process from related fields (physics, mechanics, heat engineering, chemistry, etc.) is required to describe the element. It is almost impossible to describe all unforeseen situations. But then we will consider a fairly universal approach, based on the description of the element as a calculation scheme built of bipolar elements.

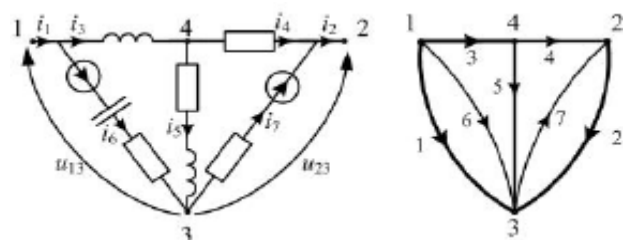


Fig. 1. Calculation scheme of the electric circuit of a tripole three-pole and its graph

Fig. 1 shows a calculation scheme of a three-pole element. It is characterized by a system of independent polar currents $I = (i_1, i_2)$, and a system of independent polar voltages $U_{\Pi} = (u_{13}, u_{23})$. These currents and voltages belong to the external characteristics of the element, they represent the physical state of the element in the outer circuit. The remaining currents and voltages are internal and are not considered if possible.

Matrix equation (5) represents a system of output equations of the output electric circuit formed by bipolar elements. If, according to (3), (4), voltages u_1, u_2, \dots, u_7

and current i_6 , are excluded from this system of equations, we obtain the corresponding system of hybrid equations of this tripole. We also write this system in the matrix form

$$\begin{array}{|c|c|c|c|c|c|c|} \hline 1 & & & 1 & C_6 \frac{d}{dt} & 1 & \\ \hline & 1 & & & & -1 & -1 \\ \hline & & L_3 \frac{d}{dt} & & & & \\ \hline & & L_3 \frac{d}{dt} & L_5 \frac{d}{dt} + r_5 & & & \\ \hline & & 1 & -1 & & -1 & \\ \hline & & & & & & r_7 \\ \hline -r_6 & & -r_6 & & 1 & & \\ \hline \end{array} \cdot \begin{array}{|c|} \hline i_1 \\ \hline i_2 \\ \hline i_3 \\ \hline i_5 \\ \hline u_{C6} \\ \hline i_4 \\ \hline i_7 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \\ \hline u_{13} - u_{23} \\ \hline u_{13} \\ \hline \\ \hline e_7 - u_{23} \\ \hline u_{13} + e_6 \\ \hline \end{array}. \quad (6)$$

Having excluded currents: $i_4 = i_3 - i_5$; $i_7 = (e_7 - u_{23})/r_7$, $i_7 = (e_7 - u_{23})/r_7$ from the hybrid equations (6), we obtain

$$\begin{array}{|c|c|c|c|c|} \hline 1 & & 1 & & C_6 \frac{d}{dt} \\ \hline & 1 & -1 & 1 & \\ \hline & & L_5 \frac{d}{dt} + r_4 & -r_4 & \\ \hline & & L_3 \frac{d}{dt} & L_5 \frac{d}{dt} + r_5 & \\ \hline -r_6 & & -r_6 & & 1 \\ \hline \end{array} \cdot \begin{array}{|c|} \hline i_1 \\ \hline i_2 \\ \hline i_3 \\ \hline i_5 \\ \hline u_{C6} \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}. \quad (7)$$

$$\begin{array}{|c|} \hline \\ \hline (e_7 - u_{23})/r_7 \\ \hline u_{13} - u_{23} \\ \hline u_{13} \\ \hline u_{13} + e_6 \\ \hline \end{array}.$$

Equations (7) connect the pole currents i_1, i_2 with the polar voltages u_{13}, u_{23} . They contain, in addition, three internal quantities i_3, i_5, u_{C6} , which cannot be excluded in the usual way, since they contain unknowns under the symbol of differentiation. But it is from this moment that the actual formation of the equations of a nonlinear multipole begins. That is why we offer an original approach, the essence of which is given below.

Differential equations that are subject to exclusion are translated into the status of voltage or current sources $u_C \textcircled{R} e_C, i_L \textcircled{R} j_L$. Since these values are obtained as a result of integrating separate equations, they should be transferred to the right-hand side of (7). Then their time derivatives $du_C/dt, di_L/dt$ are subject to exclusion. Therefore, we give equation (7) the corresponding form stated

$$\begin{array}{|c|c|c|c|c|} \hline -r_6 & & & & \\ \hline & 1 & & & \\ \hline & & L_3 & & \\ \hline & & L_3 & L_5 & \\ \hline 1 & & & & C_6 \\ \hline \end{array} \cdot \begin{array}{|c|} \hline i_1 \\ \hline i_2 \\ \hline di_3/dt \\ \hline di_5/dt \\ \hline du_{C6}/dt \\ \hline \end{array} = \begin{array}{|c|} \hline u_{13} + e_6 - e_{C6} + r_6 j_3 \\ \hline (e_7 - u_{23})/r_7 + j_3 - j_5 \\ \hline u_{13} - u_{23} - r_4(j_3 - j_5) \\ \hline u_{13} - r_5 j_5 \\ \hline j_3 \\ \hline \end{array}. \quad (8)$$

To generalize, we write system (8) in the matrix-block form

$$\begin{array}{|c|c|} \hline B_1 & B_2 \\ \hline B_3 & B_4 \\ \hline \end{array} \times \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline \end{array} = \begin{array}{|c|} \hline y_1 \\ \hline y_2 \\ \hline \end{array}, \quad (9)$$

where x_1, x_2 are respectively, the subcolumns of the pole currents of the element and derivatives to be excluded; y_1, y_2 are the subcolumns of the corresponding right parts.

Based on (9) you can write

$$\begin{aligned} (B_1 - B_2 B_4^{-1} B_3)x_1 &= y_1 - B_2 B_4^{-1} y_2; \\ x_2 &= B_4^{-1} (y_2 - B_3 x_1). \end{aligned} \quad (10)$$

According to (8)–(10), we obtain the final equations of the multipole. After arranging the unknowns we will have

$$U = AI + E, \quad (11)$$

where

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}; I = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}; A = \begin{bmatrix} r_6 & \\ & -r_7 \end{bmatrix}; \quad (12)$$

$$E = \frac{e_{C6} - e_6 - r_6 j_3}{e_7 + r_7 (j_3 - j_5)}.$$

The differential equations of the excluded derivatives will be

$$\frac{d}{dt} \begin{bmatrix} u_{C6} \\ i_3 \\ i_5 \end{bmatrix} = \begin{bmatrix} (u_{13} - u_{23} - r_4(i_3 - i_5)) / L_3 \\ (u_{23} - (r_4 - r_5)i_5 - r_5 i_5) / L_5 \\ -(i_1 + i_3) / C_6 \end{bmatrix}. \quad (13)$$

In the general case, the matrix A in (11) also contains differentiation symbols, if reactive elements are present in the pole current branches.

Equations (13) in the case of multipole operation in the system are integrated as independent. The required pole voltages are obtained from the equations of the system, which absorb the matrix-coefficients (12).

Note that in the presence of EJ -degeneracy, matrix B_4 in (10) can be special, for example, at $r_6 = 0$, in our example! In this case, you need to turn to the methods of solving degenerations of ideal energy sources [3].

In the case of a resistive nonlinear circuit, as already mentioned, there is no such problem of excluding internal electromagnetic quantities. It is also simplified in D -degenerate circuits, where the derivatives of all unknowns are present in the original equations of the multipole. In this case, in (9), it is sufficient to make a simple replacement $x_i \rightarrow dx_i / dt, i = 1, 2$ and further analysis becomes obvious [4]. Based on this, the problem of forming nonlinear differential equations of multipolar elements with totally D -degenerate internal electromagnetic circuits [4], which are electric machines and transformers of electromachine and electric power systems, was solved [5].

2. Conclusions

1. A method for forming nonlinear differential equations of a multipolar element, which connect its independent pole currents and independent polar voltages, based on the differential equations of a nondegenerate system of its external and internal voltages and currents is proposed.

2. The analysis is significantly simplified in cases of total D -degenerations of the system or resistive circuits.

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ДИФЕРЕНЦІАЛЬНІ РІВНЯННЯ НЕЛІНІЙНОГО БАГАТОПОЛЮСНОГО ЕЛЕМЕНТА

Чабан Василь

Запропоновано метод формування нелінійних диференціальних рівнянь багатополісного елемента, які пов'язують між собою його незалежні полюсні струми і незалежні полярні напруги. Трудність аналізу полягає в тому, що частина як внутрішніх, так і зовнішніх невідомих можуть перебувати під символом диференціювання. Стартовою інформацією для даного формування виступають спільні диференціальні рівняння системи внутрішніх і зовнішніх струмів і напруг. Метод продемонстровано на випадку формування відповідних диференціальних рівнянь системи як такої, що утворена двополюсними елементами. Аналіз суттєво спрощується у випадку наявності внутрішніх D -вироджень системи або резистивних кіл.



Vasil Tchaban: D.Sc, full professor at Lviv Polytechnic National University (Ukraine), as well as Lviv Agrarian National University, Editor-in-Chief of Technical News journal. His Doctor-eng. habil degree in Electrical Engineering he obtained at Moscow Energetic University (Russia) in 1987. His research interests are in the areas of mathematical

modelling of electromechanical processes in electric and gravity fields theories, surrealistic short story writer. He is the author of 550 scientific publications and 800 surrealistic short stories including 49 books (12 monographs, 17 didactic, 10 humanistic and 10 of the arts), 1500 aphorisms. The total number of publications is over 1500.

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