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ON THE ACCURACY OF GRAVIMETRIC PROVISION OF ASTRONOMO-GEOMETRIC LEVELING ON GEODYNAMIC AND TECHNOGENIC POLYGONS

The purpose of this work is to prove the necessity and possibility of returning to the orthometric system of heights in Ukraine and to substantiate the ways of solving this problem. The method of achieving the goal is provided by theoretical studies of existing methods of astronomical and geodetic leveling, modern methods of forecasting neotectonic processes, GNSS accuracy and geometric leveling. The main results are: the requirements for the accuracy of gravimetric support of high-precision geometric leveling, both DGM of Ukraine and high-altitude network of geodynamic and man-made landfills. The theoretical possibility of determining orthometric heights for almost 90 % of the territory of Ukraine with an accuracy of even 0.2 mm per 1 km of double stroke has been established. Scientific novelty and practical significance: it has been proved that even at the maximum values of GPP anomalies it is possible to consider orthometric and normal heights as segments of normal to the reference ellipsoid, as well as geometrical heights; if at astronomical and geodetic leveling to define a deviation of a temple with accuracy mθ_sr = 0.2 “(accuracy of modern zenith systems even 0.08)”, it will bring an error in definition of a difference of orthometric heights of 0.2 mm on 1 km of the course if to determine this value from the available gravimetric maps of the deviation of the temple, this error will be 0.5–1 mm per 1 km of travel, which also corresponds to the leveling of even the first class; ; non-parallelism of equipotential surfaces should be taken into account when the difference between the force of gravity on the equipotential surface of the initial point of travel and at the point of intersection of this surface with the normal at the end point of travel exceeds 2 mGal; the force of gravity at the leveling station and on the force line of the field at the end of the course, at a height corresponding to the height of the corresponding leveling station, must be known at the sum of excesses during 10 m per 1 km with an accuracy of only 20 mGal. m per 1 km – 2 mGala, therefore, the modern model EIGEN-CG03C (accuracy is estimated within 8 Mgal) in most of the plains of Ukraine can provide gravimetric data for the creation of state leveling networks and high-precision leveling during engineering and geodetic works and works on geodynamic and man-made landfills.

Key words: deviation of steep lines; zenith systems; GNSS; geodetic and orthometric heights; astronomical leveling.

Introduction

The influence of GPP on the results of leveling, which is manifested in the non-parallelism of level surfaces, explains the impossibility of direct use of the measured excesses (only the hypsometric part) to calculate the heights. This problem is solved by using different height systems that meet certain requirements.

The first requirement is unconditional for the system – heights must be determined unambiguously, regardless of the leveling path. Only the value of the integral C_M corresponds to this property

$$C_M = W_0 - W_M = \int_0^M gdh = \sum_{i=1}^{n} g_i h_i, \qquad (1)$$

where in the right part the curvilinear integral calculated on any leveling line between any point on the equipotential surface W_0 (for example, point C, which is located on the field line passing through the point M (Fig. 1)) and the point M.

$g_i,\ h_i$ – measured at the leveling station values of gravity and excess.

The error of substituting the mean value of g measured at the station is neglected in all cases (see, for example, [Barlik, 2007]).

In 1954, Rome decided to call the integral $\int_0^M gdh$ geopotential value – C_M (geopotential number).

Geopotential values are widely used in the world to equalize high-precision leveling networks, for example, the Western European network – Réseau Européen Unifie de Nivellment (REUN) are connected to this network. [https://link.springer.com/article/10.1007] was equated by French and Danish surveyors J. Vignal and O. Simonsen. High-precision leveling networks of Central Europe (EUVN)]. The property that the sum of geopotential quantities in a closed loop should theoretically be equal to zero is also used. Obviously, the geopotential values should be equated in the first approximation and the network of high-precision leveling in Ukraine to reject errors of purely geodetic and gravimetric measurements.

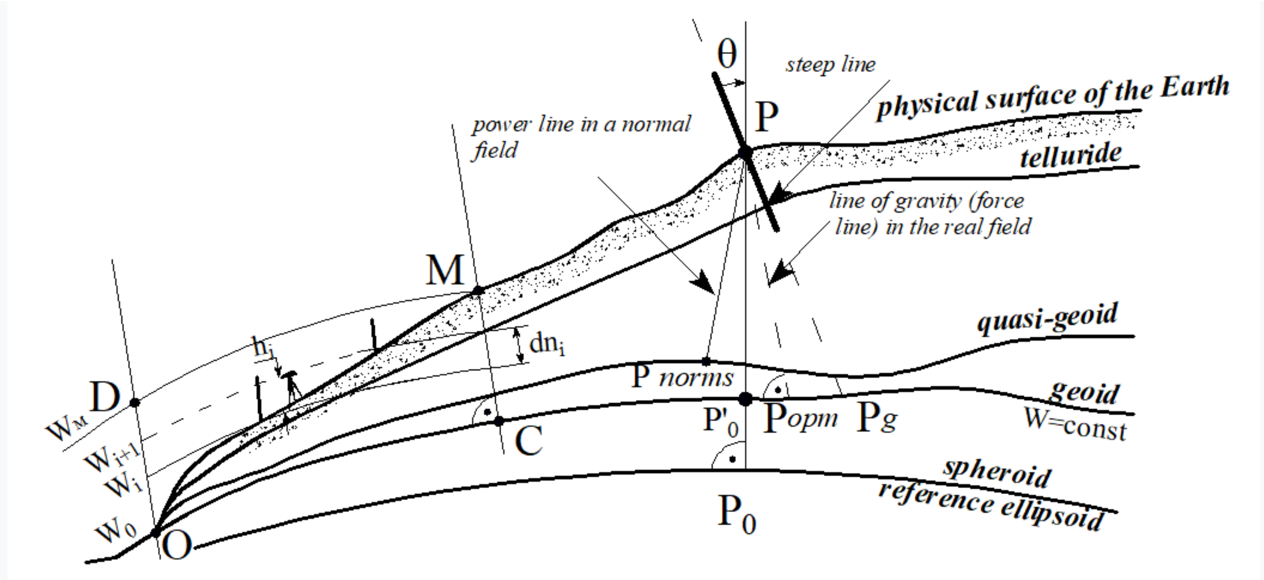

Fig. 1. Heights: geometric GNSS (geodetic), normal, orthometric

The value of C_M is used in all height systems without exception in order to obtain an unambiguous result. For example, the dynamic altitude system proposed F. Helmert [Helmert, 1884] in which the dynamic heights H_{dyn} are obtained by dividing geopotential quantities by some constant value of gravity, which for engineering and geodetic problems should be taken close enough to the real value – g in kg.

$$H^M_{dyn.} = \frac{W_M - W_0}{g\ (kgal)} = \frac{1}{g\ (kgal)}\int_0^M gdh. \qquad (2)$$

the obtained values have a dimension of length (meter) and are close in value to the sum of the measured exceedances. In 1955 the International Geodesy Association proposed one value of gravity for the whole Earth for GRS 80 $\gamma_0^{45^0}$=9.806199203 m/s^2. Accordingly, for dynamic height, only expression is often given instead of expression (2).

$$H^M_{dyn} = \frac{W_M - W_0}{\gamma_0^{45^0}} = \frac{1}{\gamma_0^{45^0}}\int_0^M gdh. \qquad (3)$$

But when using (3) it should be borne in mind that in Ukraine, with the difference in the latitudes of the leveling points from 44 ° to 52 °, the differences $g - \gamma_0^{45^0}$ can exceed 250 mGal. The calculation shows that the value of the dynamic correction will be 0.0002 Δh. Therefore, the difference between the values calculated by formulas (2) and (3) will practically manifest itself in excess of more than 5 m.

It is obvious that dynamic, dynamic regional (2) and geopotential systems are ideal for engineering and geodetic works, when it is necessary to bring into nature an equipotential surface, such as a reservoir support level, or a high-speed railway line to ensure minimum energy costs for travel.

Dynamic heights do not, unfortunately, have a clear geometric interpretation, so they cannot be used to determine the shape of the Earth, and, moreover, have no connection as orthometric with geometric heights, which are determined from GNSS measurements, although ideal for engineering solutions geodetic problems.

The second condition – altitudes should be determined only by measurements on the physical surface of the Earth, without the use of hypothetical data on its internal structure. Given that on geodynamic landfills, when executing engineering works on industrial sites, in mining areas, there is mostly detailed, real, rather than hypothetical information about the geological structure at depths to the geoid, and the distribution of gravity in height we emphasize that we need to know only in nodal points of the leveling network, and we emphasize once again, only to the geoid, this requirement does not seem so certain.

We will talk about this in more detail later, when presenting the material on orthometric heights. Here is just a quote from the work of the teacher's author Prof. M. K. Migal "…However, a deeper study of the issue shows… ..With the involvement of even scanty information about the densities of the upper layers of the Earth, the orthometric height in the same worst conditions is determined with an error of less than 1 m" [Migal, 1969].

The third condition – the accepted system of heights must meet a fairly strict method of determining the geoidal component of the geometric or geodetic height.

Regarding the last condition, the fulfillment, although not quite strict of the second condition, which is the basis of the normal system of heights by M. S. Molodensky, led to the fact that currently the geoidal component relative to even the theoretical surface - quasi-geoid, is determined with decimeter accuracy. [Dvulit, Golubinka 2009; Czarnecki, 2010], which in most cases does not meet the requirements of even hydrographic [Ostroumov, 2011] especially high-precision engineering and geodetic works. However, the differences in geoidal components of orthometric heights – the profile of the real physical surface of the geoid during astronomical-geometric leveling, (combination of astronomical and high-precision geometric leveling) can be determined on land theoretically with an accuracy of 1–2 mm [Hirt, & Bürki, 2005]. This even makes it possible to determine changes in the position and shape of the geoid based on the results of repeated astronomical and geometric leveling. Modern scientific hypotheses connect catastrophic deformations of the earth's surface – earthquakes – with these changes. It also, provided that the orthometric correction is determined with an accuracy that corresponds to the accuracy of the determination of the hypsometric part, allows us to control the results of GNSS leveling and vice versa, to control the results of determining orthometric heights, which, in our opinion, gives a significant advantage to the use of these heights instead of normal on geodynamic and man-made landfills. Although calculations show that in 90 % of the territory of Ukraine the differences in the sums of orthometric and normal heights are less than mm. Therefore, in the current situation, the use of normal heights for cartographic work is probably justified.

Presenting main material

Until the 1960s, orthometric heights were used in Ukraine, which, and only in all countries of the socialist camp, were replaced by normal heights, particularly in Ukraine in 1958. This change at that time could be explained by something other than ideological arguments, which, of course, were also present. However, with the development of GNSS leveling and achieving modern accuracy, this replacement, which is not essential for cartographic work, for high-precision, especially in geodynamic landfills, is unjustified. At least because the geodetic heights, which are determined by GNSS measurements, consist of two parts: the hypsometric part – orthometric height – $H_{orth.}$ – $\boldsymbol{PP'_0}$) (Fig. 1 – $\boldsymbol{PP'_0}$) and the geoidal part, the height of the geoid above the accepted reference ellipsoid – $\boldsymbol{P'_0 P_0}$, which can, we emphasize, independently determine with an accuracy of the order of mm [Hirt, et al., 2006], thus reliably and independently controlling the whole set of measurements.

Two reasons were put forward to justify the need for such a measure (replacement of orthometric heights with normal ones).

The first reason is that in the most general formula of orthometric height

$$H_{oet.} = \frac{C_M}{g_{av}} = \frac{\sum_{i=1}^{n} g_i h_i}{g_{av}} \tag{3}$$

includes the value of g_{av} – the average value of gravity on the segment $\boldsymbol{PP_{ort.}}$, which depends on the vertical gradient of gravity on the segment $\boldsymbol{PP_{ort.}}$. and which, indeed, by measurements only on the physical surface of the Earth, it is impossible to find exactly. It was noted that it depends in a difficult way on the distribution of density within the Earth, but for some reason did not take into account the requirements for the accuracy of determining -g_{av} and the fact that g_{av} we need to know only at the nodal points of leveling passages and only on the section from the reference to the geoid (99 % of the DGM of Ukraine is up to 300 m high). In the future we will show that in modern conditions, especially on geodynamic and man-made landfills there are several ways to find g_{av} and g_i– gravity at leveling stations with an accuracy that satisfies not only leveling of the I st class, but also high-precision short beam leveling with maximum currently achievable accuracy of 0.2 mm per 1 km of double stroke. That is, orthometric heights, which in contrast to normal have a clear geometric and physical meaning and therefore are given to determine the shape of the Earth – geoid, theoretically can be found throughout Ukraine with an accuracy determined mainly by the maximum current accuracy of the geometric leveling, even 0.2–0.3 mm per 1 km of double stroke. The accuracy of modern GNSS leveling corresponds to the accuracy of geometric leveling at least class II. This accuracy has been achieved in the last century not only in the United States but also in Poland [Cacon, 1999], at present it is definitely higher, although there are no other tools (besides GNSS) to confirm it on large distances. Therefore, even from formula (4) the difference in height of the geoid $(N_M - N_A)$, we can find, at least with centimeter accuracy, if known from GNSS leveling ΔH_{AM}^{geom} – the difference in geometric heights – heights relative to the geocentric ellipsoid and orthometric correction -p_{ort} with an accuracy corresponding to the accuracy of determining the amount of excesses – $[h]_{AM}$ from geometric leveling:

$$\Delta H_{AM}^{geom} = \Delta H_{AM}^{ort} + (N_M - N_A) = \\ = [h]_{AM} + p_{ort} - (N_M - N_A), \qquad (4)$$

ΔH_{AM}^{ort} – the difference of orthometric heights (hypsometric part, which is obtained according to the geometric leveling from p. A to p. M, as the sum of measured excesses $[h]_{AM}$, corrected by orthometric correction p_{ort}.

In addition, at least on geodynamic and man-made landfills it is possible to measure on specially created profiles ξ and η – components of deviations of the temple in the meridian and the first vertical and the results of astronomical leveling calculate $(N_M - N_A) = \frac{1}{\rho''}\int_A^M \theta dl$ – geoidal part, by integrating the components of the deviation of the temple – θ along the line of the profile, which in turn find the formula:

$$\theta_i = \frac{1}{2}\left[\!\left[(\xi_{i-1,i} - \xi_i)cos\alpha_{i-1,i} + (\eta_{i-1,i} - \eta_i)cos\alpha_{i-1,i}\right]\!\right]. \qquad (5)$$

Achieved accuracy of determination of ξ and η, using zenith systems and GNSS measurements, 0,1" is achieved in 20 minutes of observations. This ensures the accuracy of the geoid profile with an accuracy of 1–2 mm, which, in turn, allows using formula (4) to assess the accuracy of both the determination of orthometric corrections and GNSS altitudes at relatively short distances.

Recognizing that orthometric heights have a clear, unambiguous geometric meaning, are ideal for determining the shape of the Earth – geoid, most scientific sources in the former Soviet Union, even such an authoritative as [Pellinen, 1978] noted "…geometrically clearthe notion of orthometric height actually turns out to be strictly unrealizable ...". This is justified by the same impossibility of determining with sufficient accuracy the value of g_{av}. Although the question of what should be this accuracy, as we noted above, is not discussed. Moreover, there are even modern works in which it is proposed to abandon the concept of geoid for the same reason, replacing it with a quasi-geoid, which is not an equipotential surface.

The heights associated with the quasi-geoid are called normal – $\boldsymbol{PP}_{\text{норм}}$ (Fig. 1). They are officially used in the territories, and only, of the former socialist camp. Quasi-geoid, a surface that is almost identical to the surface of the geoid in the seas and oceans, slightly different from it (according to the literature) up to 2 cm in the plains and up to 2 m in the mountains, although this is mainly determined by the accepted height of the initial footstock. on the sum plain orthometric and normal exceedances in the course, as we will show below, they differ by millimeters and even smaller, which cannot be said about the oceans and seas. A quasi-geoid is as complex surface as a geoid, and it cannot be described by precisely known mathematical functions as a geoid, but unlike a geoid, a quasi-geoid is not an equipotential surface, so normal altitudes are unsuitable for engineering when the highest accuracy is required. Moreover, a number of authoritative researchers (see, for example, [Czarnecki, 2010]) do not consider normal heights suitable for studying the shape of the Earth. As we have already noted, the accuracy of determining the heights of the quasi-geoid above the reference ellipsoid, again according to available scientific publications, is now about a decimeter. It should be noted that the GNSS measurements on the plain (up to altitudes less than 1 km) can calculate geometric heights and, using modern existing digital maps of geoid heights, as we will show below, calculate orthometric rather than normal heights.

Although, it should be noted once again that very authoritative researchers not only in the USSR but, for example, in Poland [W. Szpunar, 1962], from modern [Dvulit, 2014] agreed with the possibility of using normal heights, which is quite true when it comes to work not with millimeter accuracy. The author understands, although disagrees with Prof. W. Szpunar, he wrote on the contrary that the normal heights according to the accepted method of calculating them are actually closer to orthometric, his work, in the totalitarian conditions of Poland at the time, most likely not published, this is even more true of L.P. Pellinen [Pellinen, 1978] and and prof. M. K Migal [Migal, 1969]. Given the controversial issue of choosing a height system, we will dwell on it in more detail. Let's start with the fact that according to accepted definitions, geometric heights are, unambiguously, the distance normal to the reference ellipsoid -PP_0 (see Fig. 1), at the same time in the scientific literature there are ambiguous definitions of normal and orthometric heights: orthometric heights – distances along the vertical line – PP_g, or along the power line of the real GPP – PP_{ort}, normal – along the power line of the normal field – PP_{norm} or normal PP_0', etc. This issue is discussed in more detail in the article [Kurenyov, 2010], although there is no final solution in this work. Of course, this complicates the presentation of the material, so we show that with the existing accuracy of determining heights, even with high-precision geometric leveling, replacement of the force line or vertical, normal to the reference ellipsoid does not lead to error to be taken into account, which greatly simplifies further material.

First, it is easy to determine by elementary calculation that even with the deviation of the vertical line – θ equal to 40" and the height of the point 8 km, the difference between the distance normal PP_0' and the vertical line $P\,P_g$ to the geoid (see Fig. 1) will be only 1 micron, there is no difference. Indeed, the difference can be expressed by the formula:

$$\Delta H = 2Hsin^2\frac{\theta}{2} =$$

$$= 2 \times 8\text{ km } \times \sin 20'' \leq 0.001\text{ mm}. \qquad (6)$$

Note that the key in the definition is "on the physical surface of the Earth", although if the object is in, even near space, and in this case, the calculation of (6) shows that the difference is only 10^{-9} H.

To be convinced of the possibility of replacing the lines of force of the gravimetric field PP_{ort} by the direction of the normal PP_0' to the reference ellipsoid when calculating heights, consider one idea [Migal, 1969], which is relatively simple and very clear in geometric terms allows to solve the problem, leading to formulas that are obtained purely mathematically [Moritz., 1979]. Note that the obtained solution allows to take into account the influence of possible anomalies of the GPP and the fact that Fig. 2, in contrast to that given in [Czarnecki, 2010], corresponds to our eastern hemisphere and takes into account the fact of decreasing distances between equipotential surfaces with latitude.

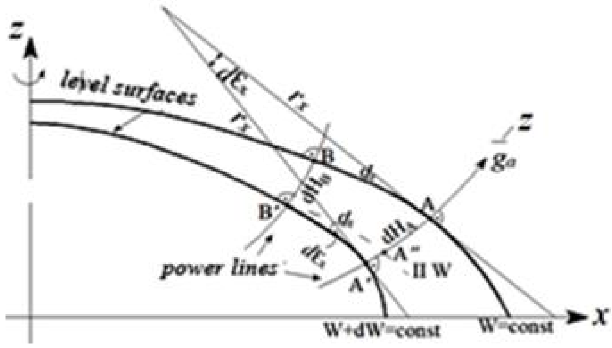

Fig. 2. The curvature of the level surfaces of the GPP

It is easy to understand if you remember that the equipotential surface (geoid) is a three-dimensional figure. In Fig. 2 shows a section of two close equipotential surfaces (with different potential values), the plane of the meridian. In Fig. 2 A and B are points on one equipotential surface W and A′ and B′ on another surface W + dW. For the curvature of the power line – K (detailed explanations are given in [Migal, 1969; Czarnecki, 2010] known formula:

$$K = \sqrt{K_x^2 + K_y^2} = \frac{\sqrt{W_{xz}^2 + W_{yz}^2}}{g} = \frac{\partial g}{g \partial s}, \quad (7)$$

where W_{xz}, W_{yz} – the corresponding horizontal gravity gradients.

Determine the maximum possible curvature of the force line by formula (7). The value of g, taking into account the fact that it can be in the denominator, without losing the accuracy of calculations, which can be ignored, can be taken equal to 9.8×105 mGal. With maximum changes in the horizontal gradient from 1 mGal per 1 km on the plain to 10 mGal per 1 km in the mountains, the radius of curvature of the power line according to (7) will vary within a minimum of 9000 km on the plain to 900 km in the mountains. To solve the problem, in this case, we approximate the force line by a circular curve, and the normal by a chord that converges a circular curve of length S and has length l:

$$l = 2r_x \sin\left(\frac{S}{2R}\right). \quad (8)$$

The calculation according to formula (8) at the radii of the power line GPZ -r_x from 900 to 9000 km (maximum possible values, which we substantiated above), shows that replacing the length of the circular curve (power line) with the chord length (normal) can lead to a change height cranges from 0.2 mm on the plain to 2 mm in the mountains at the maximum possible values of the horizontal gradient per 1 km from 1 mGal in the plain to 10 mGal in the mountains at altitudes of 2 km). That is, when leveling even the first class curvature of the force line can be neglected and consider the height (normal and orthometric) distance normal to the reference ellipsoid, as well as the geometric height.

We also pay attention to the fact that in other literature sources we can find the results of determining the curvature of the power line, which differ from the above by orders of magnitude. For example, in the already mentioned work [Czarnecki, 2010] to determine the curvature of the power line also provides a theoretical formula

$$k = -\frac{e'^2}{2\nu}\sin 2\phi, \quad (9)$$

where e'^2 =6,7394967755× 10^{-3};

$$\nu = \sqrt{x^2 + y^2 + (1 - e'^2)z^2};$$

$$\phi = \operatorname{arctg}\frac{\tau z}{\sqrt{x^2 + y^2}};$$

$$\tau = 1 + e'^2.$$

Assuming for the middle point of Ukraine coordinates: x = 3500000 m, y = 2100 000 m, z = 4700000 m, calculations by formula (9), we establish that r_x =1/k =3.8E+15 m. For such a significant value of the radius of curvature the difference between the length of the power line even 8 km and the chord corresponding to it is only 0.002 mm.

In the work [Brovar, et al., 1961] for the difference between the heights of the normal and the power line -ΔH gives the formula:

$$\Delta H = \frac{\beta^2 H^3}{6R^2}\sin^2 2B, \quad (10)$$

where β = 0.005302. This value – ΔH in medium latitudes, even at a height of 10 km does not exceed 0.01 mm.

The difference between the results obtained by formula (10) and (9) and (8) is explained, in our opinion, by the fact that (8) corresponds to the maximum (anomalous) values of the horizontal gravity gradient, and formulas (9) and (8) to normal GPZ.

That is, we can conclude that the replacement of the power line segment by the normal segment does not lead to unacceptable errors in determining the height, at the same time greatly simplifies the presentation of the material because the GNSS results we obtain planned coordinates and GRS-80 (WGS-84). Let's establish requirements to gravimetric maintenance of works on definition of orthometric heights.

Let’s write down the general formula of astronomical-geometric leveling, which is obtained by integrating along the line AM (Fig. 4) [Pellinen, 1978]:

$$\Delta H_{AM} = \sum_1^n dh - \int_1^n \frac{\theta dl}{\rho} = \sum_1^n dh - \frac{dl}{\rho}\int_1^n \theta, \quad (11)$$

where *n* is the number of stations in progress; ***dl*** – flight length at the leveling station; θ is the projection of the deviation of the temple on the direction of the leveling line, which is determined from (5). Note that by formula (11) we find ΔH_{AM} – excess between points A and M. To move to the geodetic heights of heights, we must also take into account the non-parallelism of level surfaces on the lines of force at points A and M.

From formula (11) we find why the accuracy of the definition of $\theta\ - m\theta_{\text{cp}}$ should be equal. To achieve the maximum possible accuracy provided by the level, for practical reasons, the value must, of course, be less than the accuracy of the level compensator, with which we bring the sight axis to the operating position, currently 0.5".

For the priori calculation of the accuracy of the UPC, with which we can determine $\sum_1^n dh$ per 1 km of double stroke, we take the UPC equal to 0.2 mm. Again, for the priori assessment of accuracy, we assume that along the line AM the deviation of the temple does not change and is equal to θ_{cp}, in this case:

$$\Delta\xi\ =\ \frac{dl}{\rho}\int_1^n\theta\ =\frac{dl}{\rho}n\,\theta_{\text{cp}}\ . \qquad (12)$$

Turning to the UPC, we find:

$$m\theta_{\text{cp}}\ =\frac{m\Delta\xi\rho}{dl\sqrt{n}}. \qquad (13)$$

Equating the UPC of the hypsometric part -0.2 mm and geoidal and, taking for calculation dl = 50 m and n = 20, we obtain $m\theta_{\text{cp}}$ = 0.2". Such and even greater accuracy can be achieved (and achieved) only with the use of modern anti-aircraft cameras [[Hirt, et al., 2006; Hirt, & Bürki, 2005].

If you use the available maps of deviations of the temple, which provide accuracy, for example, in Russia [Serapinas, 2012] definition of $m\theta_{\text{cp}}$ – 0.5–1", then from the same formula (13) we find that mΔξ and, accordingly, the accuracy of the geoidal part of astronomical-geometric leveling, we provide 0.5–1 mm per 1 km the same definition of the hypsometric part (taking into account the orthometric correction), this already corresponds to the leveling of class I. If you do not take into account the deviation of the temple, then, even on the plain, when the deviation of the temple is only θ = 5–10", the geoidal part will be 5–10 mm per 1 km, which may not satisfy even leveling of III class

Thus, if the astronomical-geometric leveling determines the deviation of the temple with an accuracy of $m\theta_{\text{cp}}$ = 0.2", it will make an error in determining the difference of geoidal heights of 0.2 mm per 1 km. , this error will be 0.5–1 mm per 1 km of travel, which corresponds to the leveling of the first class.

Astronomical-geometric leveling was widely used in Western Europe and the United States even before the advent of GNSS and inertial gravimetric method to determine the difference in deviation of the temple at neighboring points and zenith instruments. Astronomical leveling should, in our opinion, be used to develop a height base on geodynamic and technological landfills, as an independent method together with GNSS leveling.

Orthometric heights were already known in 1878, the name was proposed in 1887 (Goullier). Recall that our increased interest in them is dictated by the fact that knowing the geoidal component of geometric heights (and nowadays it is possible to determine it with an accuracy of at least hypsometric) can move from heights measured by GNSS to orthometric and vice versa. This allows us to control a fairly long leveling moves, even the first class according to the results of GNSS leveling, as we have already pointed out.

With this in mind, we consider the theory of orthometric heights in great detail, trying to obtain the most theoretically rigorous formula that will assess the impact on the accuracy of the results of simplifications allowed in the known methods of calculating orthometric heights. This is necessary because the accuracy of geometric leveling since the time these formulas were proposed has increased significantly, even to 0.2 mm per 1 km of double stroke.

Directly from Fig. 1 given the independence of the potential growth from the path of integration, we can write

$$C_M\ =\int_O^M g_i dh_i\ = W_0 - W_m\ =\int_O^C g_i dh_{\text{вим}}\ + \\ +\int\limits_C^M g_n dn_i\ =\int\limits_C^M g_n dn_i \qquad (14)$$

(t. C lies on the geoid and on the same power line with t. M).

g_n is the average value of gravity between adjacent i-th equipotential surfaces on the CM power line

The last integral in formula (14) is represented as follows: $\int_C^M g_n dn_i\ =\ g_{\text{cp}}H_{\text{орт.}}$. Note that the last equality is theoretically strictly true only when the values of dn from t. C to t. M (Fig. 1) are equal.

Hence, taking into account (14) the known expression for determining the orthometric height:

$$H_{\text{орт.}}\ =\frac{C_M}{g_{\text{cp}}}=\frac{\sum_{i=1}^n g_i h_i}{g_{\text{cp}}}. \qquad (15)$$

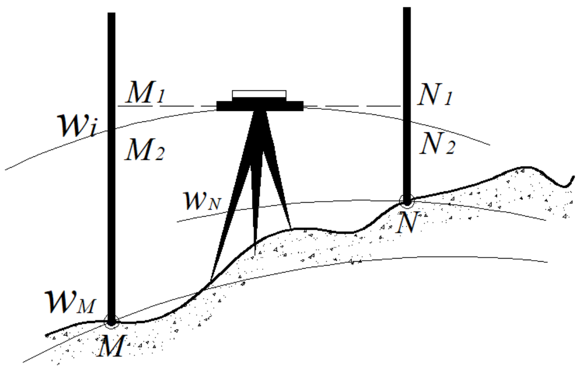

Fig. 3. Geometric leveling from the middle

Taking into account the debatable issue of choosing a system of heights, consider in great detail the process of high-precision geometric leveling from the middle at the station (see Fig. 3), where the following notations are adopted:

M1, N1 – readings on the rails installed at points M and N;

W_M, W_N, $W_i = W_{M_2} = W_{N_2}$ – the value of the gravitational potential, respectively, at points M, N, M_2, N_2.

Due to the dependences $dW = gdh$; $W_M - W_{M_2} = g_M MM_2$; $W_N - W_{N_2} = g_N NN_2$, $g = 0.5(g_M + g_N)$ and $\delta_g = (g_M - g_N)$, $g_M = g - 0.5\delta_g$, $g_N = g + 0.5\delta_g$.

Assuming $W_i = W_{M_2} = W_{N_2}$, we obtain

$$W_M - W_N = \mathrm{g}(MM_2 - NN_2) + $$
$$+ \delta_g 0.5(MM_2 + NN_2) = $$
$$= g\Big((MM_2 - NN_2) + \big(\delta_g 0.5(MM_2 + NN_2)\big)/\mathrm{g}\Big).$$

In the formula $MM_2 - NN_2 = \Delta h = MM_1 - NN_1$ – measured at the station excess. Neglecting $\delta_g 0.5(MM_2 + NN_2)/\mathrm{g}$, we obtain the known fundamental formula:

$$W_M - W_N = \mathrm{g}\,\Delta h, \qquad (16)$$

where g is the average value of gravity from the values at the points where the rails are installed.

Since, when calculating the difference in orthometric heights in the course, we use formula (16), we estimate what is the maximum error from neglecting the last member of formula (16) we can make. Take into account that the maximum length of rails -3 m. Accordingly δ_g change in gravity when changing height by 3 m. $0.5(MM_2 + NN_2)$ is also less than 3 m. From the expression for Fay reductio

$$\delta_g \approx \delta g_F(\text{mGal}) = 0.3086\Delta h\ (\text{km}) =$$
$$= 0.001\ \text{mGal}. \qquad (17)$$

Therefore, the value of the neglected term of formula (16) is equal to 0.001 mGal × 3000 mm / 1000 0 mGal = 3×10^{-4} mm, which has little effect on the accuracy of leveling during.

Let's evaluate in more detail the question of finding the average value of gravity on a vertical line in the final reference of the course, on the segment from the geoid to the reference. First, let's ask the question, what should be the accuracy of determining g_sr, so that the error does not exceed the accuracy of determining the hypsometric component in the course. To do this, differentiate (15) and moving to the UPC we obtain in the most general case:

$$m_H = \frac{C_M}{g_{\text{cp}}^2} m_g,$$

where, taking into account (17) we obtain, after minor transformations:

$$m_g = \frac{m_H}{H} g_{\text{cp.}} \qquad (18)$$

Assuming a neglected value of m_H = 0,2 mm and $g_{av.} = 10^6$ mGal, we find that at heights of 10m, 100 m and 1000 m we need to ensure the accuracy of determining $g_{av} - m_g$ equal to only 20, 2 and 0.2 mGal, respectively.

Now consider in great detail at what points we must determine the magnitude of gravity. This will help us to obtain the general formula of orthometric correction – the difference between the measured excess at the station and the difference of orthometric heights, and to evaluate the known methods of its determination. We turn to Fig. 4

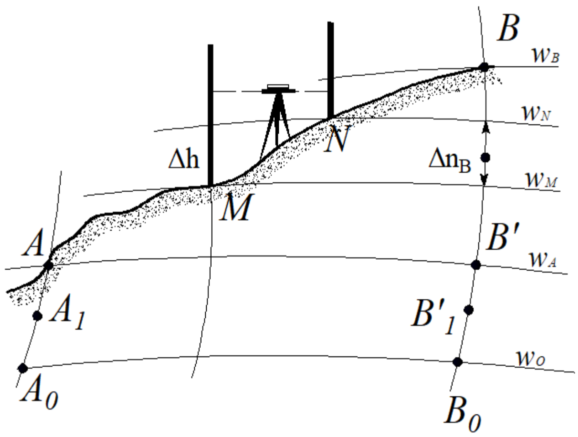

Fig. 4. Before deriving the formula of orthometric correction

On the power line at point B, at the end of the course $W_M - W_B = g_{B'} \times \Delta n_B$, where $g_{B'}$ – value of acceleration at the midpoint of the segment Δn_B. Accordingly, on the power line of the rear point of the station $W_M - W_N = g_{\text{M}} \times \Delta h$. From the equality $g_{B'} \times \Delta n_B = g_{\text{M}} \times \Delta h$, it follows:

$$\frac{\Delta n_B}{\Delta h} = \frac{g_{\text{M}}}{g_{B'}},$$

from here

$$\Delta n_B = \frac{g_{\text{M}}}{g_{B'}} \Delta h = \Delta h + \frac{g_{\text{M}} - g_{B'}}{g_{B'}} \Delta h.$$

The orthometric excess between points A and B will be represented as:

$$\sum_A^B \Delta h_B = \sum_A^B \Delta h + \sum_A^B \frac{g_{\text{M}} - g_{B'}}{g_{B'}} \Delta h. \quad (19)$$

Expression

$$p_{\text{орт.}} = \sum_A^B \frac{g_{\text{M}} - g_{B'}}{g_{B'}} \Delta h. \quad (20)$$

is orthometric correction in the measured amount of excesses in the course from point A to the end point of the course in the most accurate form. As we can see, to determine it accurately, we need to measure gravity at each point where the rails $-g_M$ are installed, and at their corresponding points at the intersection of the equipotential surface of point M and the force line of the end point – $g_{B'}$.

Turning to the orthometric heights of Fig. 4 we write down:

$$H_B = H_A + \sum_A^B \Delta h_B + (B_0B' - A_0A). \quad (21),$$

that is, the non-parallelism of equipotential surfaces at point A with a known height at the beginning of the course and at point B at the end of the course should be taken into account.

Assuming, with the admissible assumption that

$$W_0 - W_A = g_{A_1} A_0A = g_{B_1'} B_0B',$$

where A_1, B_1' – the middle points A_0A and B_0B', we obtain:

$$\frac{B_0B'}{A_0A} = \frac{g_{A_1}}{g_{B_1'}},$$

from where, after the obvious transformations

$$(B_0B' - A_0A) = (\frac{g_{A_1}}{g_{B_1'}} - 1)H_A. \quad (22)$$

Then the most general formula of orthometric heights will look like:

$$H_B = H_A + \sum_A^B \Delta h + \sum_A^B \frac{g_{\text{M}} - g_{B'}}{g_{B'}} \Delta h + \left(\frac{g_{A_1}}{g_{B_1'}} - -1\right) H_A. \quad (23)$$

Let us analyze the influence of the last term of expression (23) on the result of determining the orthometric height.

Having accepted, $g_{B_1'} = g_{A_1} + \Delta g$, where $\Delta g = g_{A_1} - g_{B_1'}$ get after the obvious transformations:

$$p_{\text{орт.}}^{\Delta g} = \left(\frac{g_{A_1}}{g_{B_1'}} - 1\right) H_A = \frac{\Delta g}{g_{B_1'}} H. \quad (24)$$

Accepting $\frac{p_{\text{орт.}}^{\Delta g}}{H} = \frac{0{,}2 \text{ mm}}{1 \text{ km}} = 2 \times 10^{-6}$ and $g_{B_1'} = 1000$ Gal, according to (24) we get that the last expression in formula (23) should already be taken into account if Δg is more than 2 mGal.

Determine the effect on the value of UPC orthometric correction – $m_{p_{\text{орт.}}}$ UPC errors determining the gravitational force at the leveling station $-m_{g_M}$ and again from the most general formula (23) determining the gravitational force on the power line at the end point of the stroke at a height corresponding to the height of the station m_{g_B}.

After simple transformations we present the third term in (23) in the form:

$$\sum_A^B (\frac{g_M}{g_{B'}} \Delta h - \Delta h).$$

Assuming to estimate the accuracy of the constant values of $\frac{g_M}{g_{B'}}$ and Δh and Δh, replace the sum in the expression by the product:

$$\frac{g_M}{g_{B'}} \Delta h \times n - \Delta h \times n = \frac{g_M}{g_{B'}} H - H.$$

Differentiating the last expression and moving to the UPC, we obtain after the obvious transformations:

$$m_{p_{\text{орт.}}}^{g_M} = \frac{m_{g_M}}{g_{B'}} H, \quad (25)$$

$$m_{p_{\text{орт.}}}^{g_{B'}} = \frac{g_M H m_{g_B}}{g_B^2}. \quad (26)$$

Assuming $m_{p_{\text{орт.}}}^{g_M}$=0.2 mm, we obtain from (25) that if we ensure the accuracy of determining the gravitational force at the station m_{g_M} equal to 2 mGala at a difference of elevations of 100 m, then at with a stroke length of 1 km, the resulting error in the sum of the measured exceedances will be equal to the instrumental error of the level measurements. If the course is laid on a plain and the difference in height does not exceed 10 m, the corresponding value is 20 mGal. Calculations by formula (18) give similar results. That is, when changing the excess between the starting point and the end in the interval 10–100 m per 1 km of travel, so that the errors introduced by gravimetric measurements of gravity do not exceed the instrumental accuracy of leveling, force gravity at the leveling station and on the field line at the end of the course, at a height corresponding to the height of the corresponding leveling station, must be known with accuracy, respectively, in the range of 20–2 mGal.

Given that the errors in determining the anomalies of gravity of the modern model EIGEN-CG03C are estimated within 8 Mgal [Bihter Erol, 2012], in most of the plain territory of Ukraine it (model) can provide gravimetric data for the creation of state leveling networks, and high-precision leveling during engineering and geodetic works and works on man-made landfills.

Given the requirements for the method and accuracy of determining $g_{B'}$, consider the existing methods for determining orthometric heights.

First of all, it is possible to solve this problem almost strictly if there are wells near the nodal points of the leveling network, for which graphs of gravity change on the segment from the geoid (equipotential surface of the initial reference) to the nodal point. Gravity meters for such measurements are available in the world, in the USSR developed in the 60' s. As of January 1, 2019, there were 2.233 explored deposits of only combustible minerals in Ukraine. If you look at the map of their location, the choice of wells in which you can perform gravimetric observations in the area to the geoid (mostly the entire territory of Ukraine is located at altitudes up to 300 m) is possible in most areas even less than 25 km.

The second possibility of a strict theoretical solution of the problem, using the value of the distribution of sedimentary rock densities to calculate the value of gravity at the points we need (with heights corresponding to the heights of the connecting points of the leveling) of the line of force. In gravimetry, this problem is called a direct gravimetric problem and is sufficiently theoretically substantiated and experimentally tested. In geodetic gravimetry, this problem was first solved by F. Helmert Helmert, 1884] and T. Nethammer [Niethammer, 1939, 1947] using the well-known Poincare-Praya reduction. Given the limitations of the size of the article, we note that achieving the required accuracy should not be particularly difficult, as the largest difference in the density of the earth's surface to the geoid is characterized by up to 0.6 g/cm^3, which according to [Czarnecki, 2010] can bring at altitudes of 10 m, 100 m and 1000 m to the maximum errors in determining the force of gravity at the respective points of the power line 0.2 mGal, 2 mGal and 20 mGal, respectively, which again coincides with the calculations of formulas (18) and (25).

The third implemented possibility to determine the orthometric correction, taking g $\cong \Upsilon$, for example, based on the Clero hypothesis of the spheroidal distribution of values of normal gravity -Υ with latitude and altitude, which is described by the known formula:

$$g \cong \Upsilon = \Upsilon_0^{45°}(1 - 0{,}5\beta \cos 2B)\left(1 - \frac{2H}{R}\right). \quad (27)$$

That is, in this case we can do without gravimetric measurements at all. We show that this is possible on the plain. Indeed, the maximum errors that can occur can be approximately estimated by differentiating expression (24) and taking the accuracy of replacing the value of gravity with a normal value of m_g, equal to the maximum value of anomalies. We will accept them for the plain of 20 mGal, and for mountainous areas – 150 mGal.

Calculation by the formula $m_{max} = m_g * H$ will give the following results. For mountainous areas m_max at the sum of excesses in the course from 1 m to 100 will be in the range of 0.15–15 mm, and on the plain 0.02–2 mm. That is, on the plain methods in which instead of the actual values of gravity use the values of normal gravity, meet the requirements for leveling even the first class.

Although the methods do not provide for the need to perform gravimetric work, but, in our opinion, they are of interest only as a control, because the real values of gravity we can get even, as we noted, with an error of 8 mGal, using a publicly available model EIGEN-CG03C.

In conclusion, we consider normal heights to make sure that the excesses calculated from normal heights can be used to calculate orthometric heights, and on the plain without any changes.

If in equation (15) we take g_{av} equal to $\gamma_{av.}$ – the normal acceleration calculated for the mean point on the power line at the end of the stroke, we obtain the equation proposed by M. Molodensky for the normal height of point B at the end of the level stroke:

$$H_{norm}^{B} = \frac{1}{\gamma_{\text{cp.}}} \int_{0}^{B} g_i h_i, \qquad (28)$$

where

$$\gamma_{av.} = \gamma_0 - \frac{H_B}{2}\left(2\frac{g}{R}\right) = \gamma_0 - 0.1543 H_B,$$

γ_0 – value of normal gravity on the geoid at point B.

We first find the maximum possible differences between the orthometric heights and the existing normal heights in the plains of Ukraine from comparison (28) and (15) follows:

$$H_{norm.}^{B} - H_{ort.}^{B} = \frac{g_{\text{cp}} - \gamma_{\text{cp.}}}{\gamma_{\text{cp.}}} H_{ort.}^{B} \qquad (29)$$

Assume $g_{\text{cp}} - \gamma_{av.}$ =30 mGal, $H_{ort.}^{B}$ =250 m , $\gamma_{av.}$ = 10^6 mGal, substituting in (29) we obtain:

$$H_{norm.}^{B} - H_{ort.}^{B} = \frac{30\text{ mGal}\times 250000\text{ mm}}{1000000\text{ mGal}} = 6\text{ mm}.$$

A similar calculation for the mountainous regions of Ukraine, at $g_{av} - \gamma_{av.}$ =130 mGal, $H_{ort.}^{B}$ =2000 m, will show that $H_{norm.}^{B} - H_{ort.}^{B}$ = = 250 mm.

Since $P.N._{\Delta h}$ – the normal correction is exceeded during AB is by the formula:

$$P.N._{\Delta h} = \frac{1}{\gamma_{av.}}\left(\gamma_0^B - \gamma_0^A\right) H_m + \\ + \frac{1}{\gamma_{av.}}(g_0 - \gamma_0)_{AB}\Delta h. \qquad (30)$$

then the analysis of expression (30) shows that in order to move from normal to orthometric heights in Ukraine, you can use the expression when calculating the orthometric correction:

$$P_{ort.} = P.N._{\Delta h} - \frac{1}{\gamma_{av.}}(g_0 - \gamma_0)_{AB}\Delta h. \qquad (31)$$

using existing Faye anomaly maps.

For preliminary calculations we use formula (17) for Fay reduction, assuming γ = 1000000 mGal, we obtain for Δh = 10 m, 100 m, 1 km, 2 km the difference between orthometric and normal elevations, respectively 3×10^{-5} mm, 3×10^{-3} mm, 0.3 mm, 1.2 mm. That is, with the correct calculation of exceedances of normal heights in the passages on the territory of Ukraine according to formulas (28) (31), except for areas with altitudes greater than 1 km, these excesses can be used to calculate orthometric heights.

Conclusions

1. At maximum changes of the horizontal gradient from 1 mGal per 1 km on the plain to 10 mGal per 1 km in the mountains, the radius of curvature of the GPP power line will vary from 9000 km on the plain to 900 km in the mountains. At such radii of the power line, when replacing the length of the circular curve (power line) with its chord (normal) can lead to a maximum change in height in the range from 0.2 mm on the plain to 2 mm in the mountains. Therefore, we can consider the height (normal and orthometric) of the normal distance to the reference ellipsoid, as well as the geometric height. The difference between these results and those given in other sources is explained by the fact that they correspond to the maximum (anomalous) values of the horizontal gradient of gravity, and not the normal GPP.

2. If the astronomical-geometric leveling determines the deviation of the temple with an accuracy of $m\theta_{\text{cp}}$ = 0.2" (the accuracy of modern zenith systems is even 0.08"), it will make an error in determining the difference in geometric heights of 0.2 mm per 1 km. If we determine this value from the available gravimetric deviation maps of the temple, this error will be 0.5–1 mm per 1 km of travel, which also corresponds to the leveling of even the first class.

When determining heights, the non-parallelism of equipotential surfaces at the point at the beginning of the course and at the end point of the course should be taken into account when the difference between gravity on the equipotential surface of the starting point and at the point of intersection of this surface with normal at the end point exceeds 2 mGal.

3. Theoretically, to accurately determine the orthometric heights, it is necessary to know the value of gravity at each point where the rails are installed and at their corresponding points at the

intersection of the equipotential surfaces of these points with the force line at the end point.

4. In order that the errors made by gravimetric measurements and calculated values of gravity do not exceed the instrumental accuracy of leveling, the force of gravity at the leveling station and the field line at the end of the course, at a height corresponding to the height of the leveling station. the sum of excesses in the course of up to 10 m per 1 km of travel with an accuracy of only 20 mGal, respectively, for the amount of excesses of 100 m per 1 km – 2 mGal. If we take into account that the errors in determining the anomalies of gravity of the modern model EIGEN-CG03C are estimated within 8 Mgal, then in most of the plains of Ukraine it (model) can provide gravimetric data for high-precision leveling during engineering and geodetic works landfills.

5. The first way to determine the value of gravity at any point of the power line at the end point of the leveling is to use measurements in wells near these points, a special gravimeter, which was developed in the 60s. Analysis of the location of canned wells only in the deposits of combustible minerals in Ukraine shows that this is a very real way. The second possibility is by solving a direct geophysical problem. Since the layers of the earth's crust to the geoid are characterized by maximum density differences up to 0.6 g/cm^3, even if we take this value as an error in determining g_{av}, it can lead to altitudes at 10 m, 100 m and 1000 m errors in determining gravity at the corresponding points of the power line are 0.2 mGal, 2 mGal and 20 mGal respectively, so this is also a very real way.

6. Two centuries ago, spherical-orthometric or normal methods were known in the plains of Ukraine, which instead of the actual values of gravity use the values of normal gravity, meet the requirements for leveling even class I, for mountainous areas are the most accurate known formulas Nethammer.

7. At correct calculation of excesses of normal heights in courses in the territory of Ukraine except for areas with heights more than 1 km these excesses can be used and for calculation of orthometric heights.

8. Joint implementation of astronomical-geometric and GNSS leveling, in compliance with the equirements for gravimetric support recommended in the article, will control both the results of GNSS leveling at short distances and geometric leveling by GNSS measurements at significant distances, which should be taken into account observations at geodynamic and man-made landfills.

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ДО ПИТАННЯ ТОЧНОСТІ ГРАВІМЕТРИЧНОГО ЗАБЕЗПЕЧЕННЯ АСТРОНОМО-ГЕОМЕТРИЧНОГО НІВЕЛЮВАННЯ НА ГЕОДИНАМІЧНИХ І ТЕХНОГЕННИХ ПОЛІГОНАХ

Мета цієї роботи – теоретично обгрунтувати вимоги до точності гравіметричного забезпечення астрономічного і астрономо-геометричного нівелювання на геодинамічних і техногенних полігонах, з врахуванням точності сучасного високоточного геометричного нівелювання. Методику досягнення мети забезпечено теоретичними дослідженнями існуючих способів астрономо-геометричного нівелювання, сучасних методів прогнозу неотектонічних процесів, точності ГНСС та геометричного нівелювання. Основні результати – встановлено вимоги до точності гравіметричного забезпечення високоточного астрономо-геометричного нівелювання висотної мережі геодинамічних та техногенних полігонів. Встановлена теоретична можливість визначення ортометричних і нормально-ортометричних висот практично на 90 % території України з точністю порядку навіть 0,2 мм на 1км подвійного ходу. Наукова новизна і практична значущість: доведено, що навіть при максимальних значеннях аномалій гравіметричного поля Землі можна вважати ортометричні і нормальні висоти відрізками нормалі до референц-еліпсоїда, як і геометричні висоти; якщо при астрономічному нівелюванні визначати відхилення виска з точністю $m\theta_{\text{ср}}$ = 0,2" (точність сучасних зеніт- систем навіть 0,08"), то це внесе похибку в визначення різниці геоїдальних частин геодезичних висот 0,2 мм на 1 км ходу, якщо ж визначати це значення з наявних гравіметричних карт відхилення виска, то ця похибка складе 0,5–1 мм на 1 км ходу, що також відповідає нівелюванню навіть І-го класу; непаралельність еквіпотенціальних поверхонь при обчисленні висот слід враховувати вже тоді, коли різниця сили тяжіння на еквіпотенціальній поверхні початкової точки ходу і в точці перетину цієї поверхні з нормаллю в кінцевій точці ходу перевищує 2 мГал;

силу тяжіння на станції нівелювання і на силовій лінії поля в кінці ходу, на висоті, що відповідає висоті відповідної станції нівелювання, треба знати при сумі перевищень в ході до 10 м на 1 км ходу з точністю всього 20 мГал, відповідно, при сумі перевищень 100 м на 1 км – 2 мГал, тому навіть модель EIGEN-CG03C (точність оцінюються в межах 8 мГал) на більшій частині рівнинної території України може забезпечити гравіметричними даними високоточне нівелювання при проведенні інженерно-геодезичних робіт та робіт на геодинамічних і техногенних полігонах.

Ключові слова: відхилення прямовисних ліній; зеніт-системи; ГНСС; геодезичні та ортометричні висоти; астрономічне нівелювання.

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