ІНФОКОМУНІКАЦІЙНІ ТЕХНОЛОГІЇ ТА ЕЛЕКТРОННА ІНЖЕНЕРІЯ
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ALGORITHM FOR FORMATION OF A RANDOMIZED SYSTEM OF ITERATIVE FUNCTIONS BY KANTOR STRUCTURE

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This paper has been considered the results of the development of the randomized system of iterated functions (RSIF) formation algorithm from the existing fractal image of the “Fractal Dust” type (the Cantor set). The mathematical formulas and patterns for calculating the RSIF coefficients have been derived. This algorithm is to find the formulas of functions relative to the center of the first iteration of the fractal structure. This makes it possible to determine a randomized system of iterative functions from an existing fractal image. The construction algorithm does not use recursive functions and the entry of the loop into the loop, which allows without spending a lot of computing power, and is quite optimized. The algorithm will allow you to make direct and inverse transformations without involving additional software and hardware resources. The use of forward and inverse transformations will allow in the future to form a source data set for neural networks, which will form the basis of object recognition systems.

Key words: *fractal; Cantor set; randomized system of iterated functions (RSIF).*

1. Introduction

Construction of the fractals with the help of iterated functions system (IFS) includes two approaches: the determined and randomized approach. The determined system of iterated functions (DSIF) enables to receiving the image, but requires the processing of large arrays of zeros and ones. There is no need to store large arrays of data in memory in the case of the randomized algorithm. That is why it is convenient to use this algorithm on computers with limited resources, calculating one point on each step and immediately displaying it on the screen [1, p. 102–103]. The advantage of RSIF over DSIF also lies in the fact that the initial set amounts to only one point. The formation of RSIF for the construction of fractal images is a rather complicated process. That is why in the given paper we are offering a new approach to forming the RSIF structures of the Cantor set [2, p. 68–71].

2. Algorithm forming randomized system of iterative functions by based Cantor structure

Let’s form RSIF of the Cantor set (the method of Barnsley's iterated functions systems [3, p. 66–67]). For that purpose we should assign coordinate straight line OX and choose two points A and B with coordinates x_a and x_b, and point C with coordinate x, the coordinate of which will be changed halfway to the side of the point that will fall out (the randomized selection algorithm – point A or B):

$$\begin{cases} x_n = x_a - \dfrac{x_a - x_{n-1}}{k} \\ x_n = x_b - \dfrac{x_b - x_{n-1}}{k} \end{cases}, \qquad (1)$$

where x_{n-1} – means the original coordinate of point C; x_n – means the following coordinate of point C; x_a and x_b – mean the coordinates of points A and B; k – means the proportionality coefficient (the ratio of a segment length to the first iteration segment length).

The value of the proportionality coefficient will not be selected, but will be determined from formula (2):

$$k = \frac{L}{\Delta L_1}, \tag{2}$$

where L – means the total length of a segment; ΔL_1 – means the first iteration segment length.

Based on formulas (1) and (2), we show in Fig. 1 generalized flowchart of the algorithm for constructing the fractal "Cantor's set".

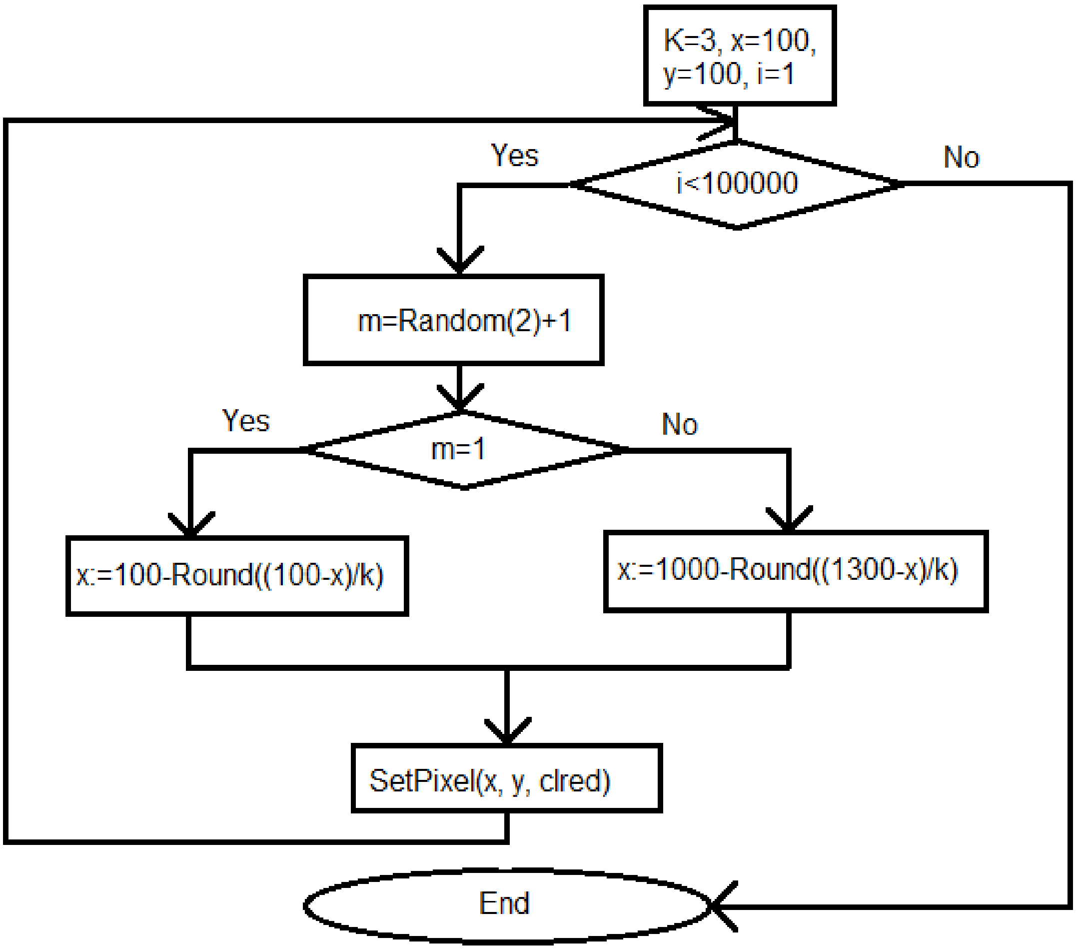

Fig. 1. Algorithm for constructing the fractal "Cantor's set"

Let's check the result of its work using PascalABC.NET [4]. The results are shown in Fig. 2 for difference value of the proportionality coefficient.

Analyzing the results of the algorithm presented in Fig. 1, under the condition $m = 1$, we derive a point of red color, which will correspond to the x_a coordinate. Similarly, for $m = 2$ we derive a blue point that corresponds to the x_b coordinate. The result is shown in Fig. 3.

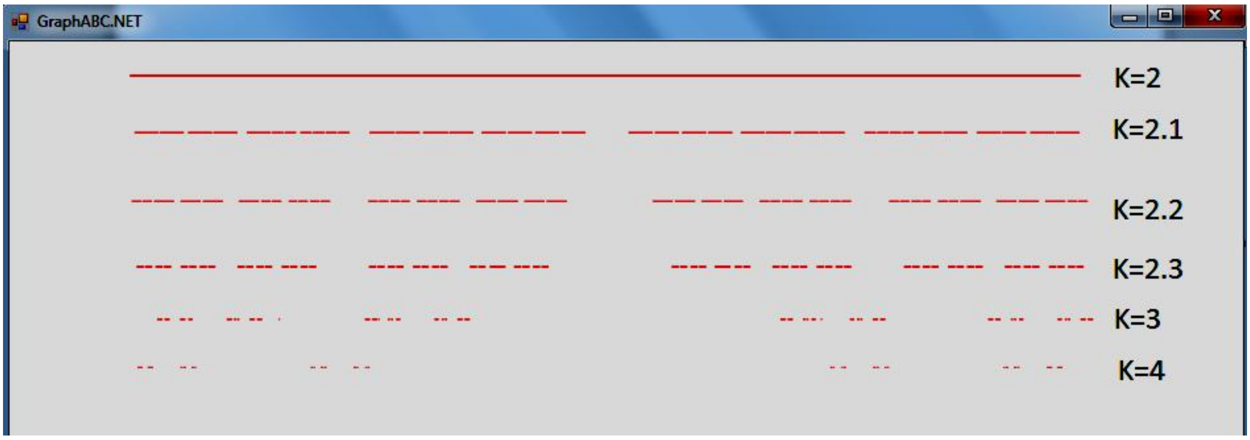

Fig. 2. The results of the algorithm for constructing the fractal "Cantor's set" at different values of proportionality coefficient

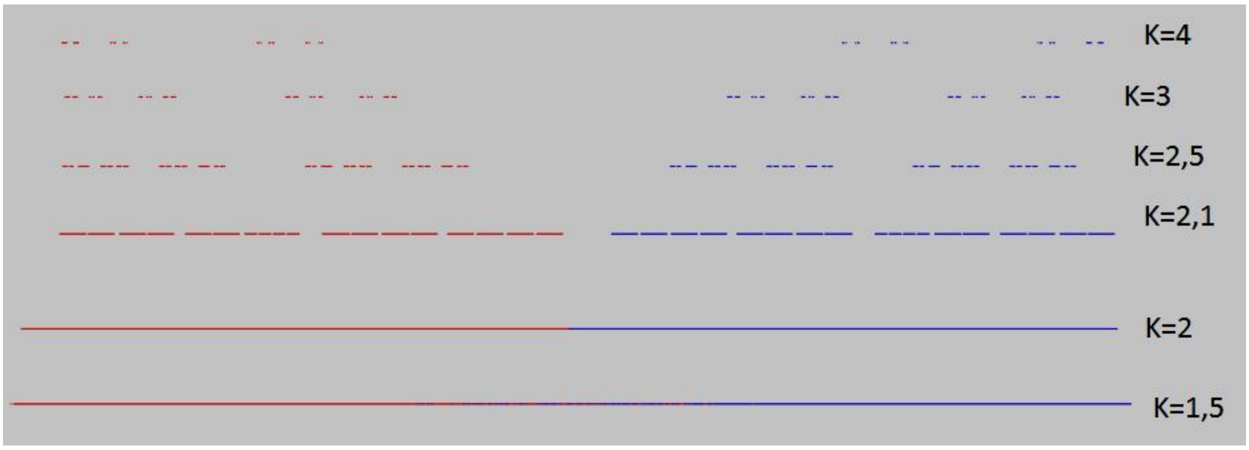

Fig. 3. The results of the algorithm for constructing the fractal "Cantor's set" at different values of proportionality coefficient

Let us put the third point inside of the segment and select the coefficient of similarity $k = 4$:

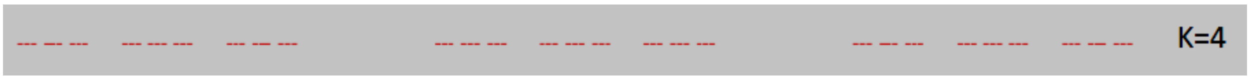

Fig. 4. The results of the algorithm for constructing the fractal "Cantor's set" at coefficient of similarity k=4

As we can see, that the point x_a or x_b, which is selected with the help of the randomized algorithm forms its own segment of the first iteration of the fractal. Let us change the location (coordinate) of a middle point of the segment. The result is shown in Fig. 5.

Fig. 5. The results of the algorithm for constructing the fractal "Cantor's set" after change the location (coordinate) of a middle point of the segment

From the results we can conclude that the length of the first iteration segment does not depend on the location of point $\Delta L_1 = L/k$.

Arising out of these two results we can draw a conclusion that there is a connection between a coordinate of the point, which is selected with the help of the randomized algorithm, and the coordinate of the middle of the first iteration segment, which it forms.

Let us deduct this dependence.

Let us take the segment lying on a coordinate straight line O_x with the length of L, the origin of which is located in point 0. Segment $[X_n, X_k]$ means the first iteration segment, which is formed is formed by the point with coordinate X_2. X_1 means the coordinate of the middle of segment $[X_n, X_k]$. L/k means the length of segment $[X_n, X_k]$.

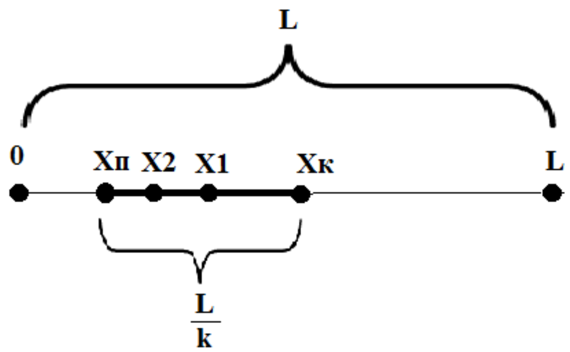

Fig. 6. First iteration segment

Based on the given figure we can deduce the following tendencies:

$$X_k - X_n = \frac{L}{k}, \tag{3}$$

$$X_1 - X_n = \frac{L}{2k} \Rightarrow X_n = X_1 - \frac{L}{2k}, \tag{4}$$

$$X_k - X_1 = \frac{L}{2k} \Rightarrow X_k = X_1 + \frac{L}{2k}. \tag{5}$$

Since point X_2 should preserve proportionality in relation to segment $[0, L]$ and segment $[X_n, X_k]$, the following tendency emerges:

$$\frac{X_2 - X_n}{X_2} = \frac{X_k - X_2}{L - X_2}, \tag{6}$$

$$\mathrm{X}_2\left(L - \left(X_k - X_n\right)\right) = L\mathrm{X}_\mathrm{n}\mathrm{X}_\mathrm{k}. \tag{7}$$

We substitute (3) and (4) in (7) and receive:

$$\mathrm{X}_2\left(L - \frac{L}{k}\right) = L\left(X_1 - \frac{L}{2k}\right). \tag{8}$$

We receive the dependence between the coordinate of the point that is selected using randomized algorithm X_2 and the coordinate of the middle of first iteration segment X_1:

$$\mathrm{X}_2 = \frac{kX_1 - \frac{L}{2}}{k - 1}. \tag{9}$$

3. Practical implementation of the RSIF recording algorithm for constructing fractals and deriving RSIF from finished fractals

In this paragraph we will describe the algorithm of writing RSIF for the construction of fractals and derivation of RSIF from finished fractals using the formula (9).

Using the example we will try to transform the constructed fractal to the system of functions, with the help of which it can be reproduced later.

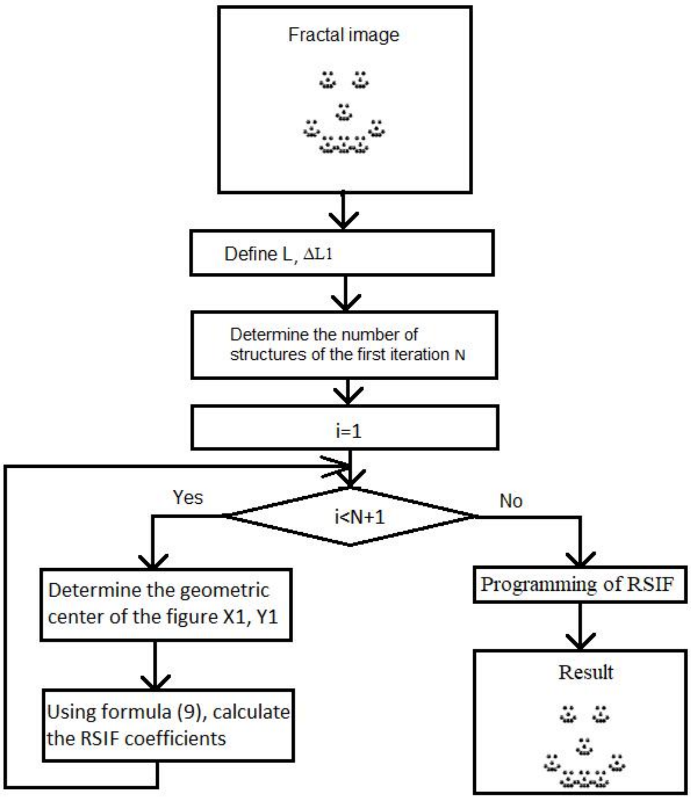

Fig. 7. Algorithm for constructing the fractal

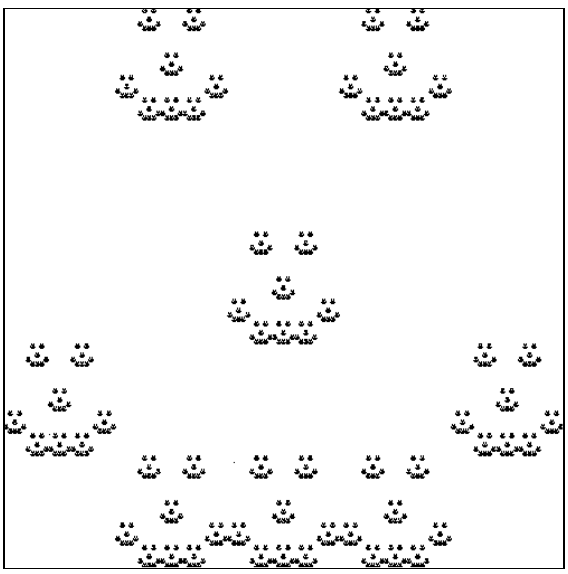

Fig. 8. Fractal

Step 1. Replace the first iteration figures into the form of equal rectangles (there will be their own k coefficients along X and Y accordingly):

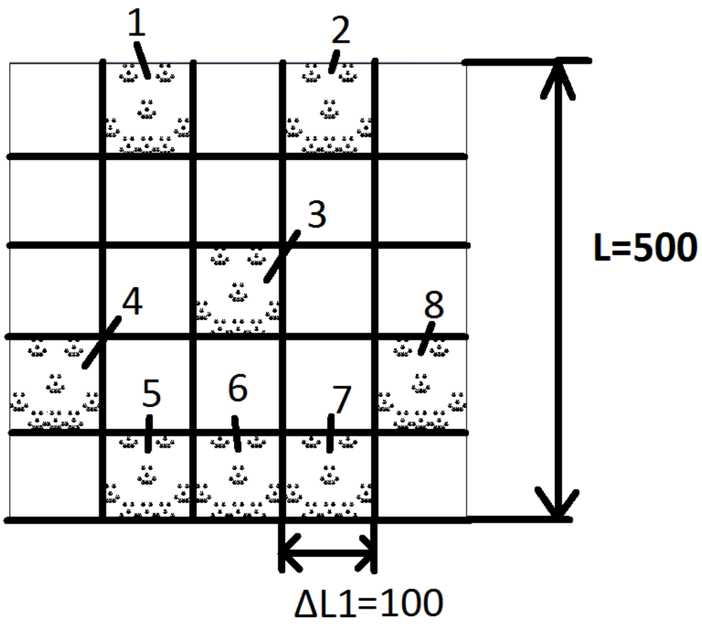

Fig. 9. Determine k *according to formula (2)* $k = \frac{L}{\Delta L_1} = \frac{500}{100} = 5$

Step 2. Determine k according to formula (2) I_A (Fig. 9).

Step 3. Determine the coordinates of the midpoints of figures X_1.

Step 4. Using formula (9) we calculate their corresponding coordinates of the point, which are selected with the help of randomized algorithm X_2.

Step 5. Write RSIF according to formulas (1).

Step 6. Programming of RSIF.

The results of steps 3–5 are presented in the Table 1.

Table 1

Results of the algorithm for constructing the fractal

Number of the first iteration figure	Coordinates of the figure midpoint	Coordinates of the point, which are selected using the randomized algorithm	RSIF
1	2	3	4
1	(150;0)	(125;0)	$x = 125 - \frac{(125 - x)}{5}$ $y = 0 - \frac{(0 - y)}{5}$
2	(350;0)	(375;0)	$x = 375 - \frac{(375 - x)}{5}$ $y = 0 - \frac{(0 - y)}{5}$

Continuation of Table 1

1	2	3	4
3	(250;250)	(250;250)	$x=250-\frac{(250-x)}{5}$ $y=250-\frac{(250-y)}{5}$
4	(50;350)	(0;375)	$x=0-\frac{(0-x)}{5}$ $y=375-\frac{(375-y)}{5}$
5	(150;500)	(125;500)	$x=125-\frac{(125-x)}{5}$ $y=500-\frac{(500-y)}{5}$
6	(250;500)	(250;500)	$x=250-\frac{(250-x)}{5}$ $y=500-\frac{(500-y)}{5}$
7	(350;500)	(375;500)	$x=375-\frac{(375-x)}{5}$ $y=500-\frac{(500-y)}{5}$
8	(450;350)	(500;375)	$x=500-\frac{(500-x)}{5}$ $y=375-\frac{(375-y)}{5}$

in a similar way:

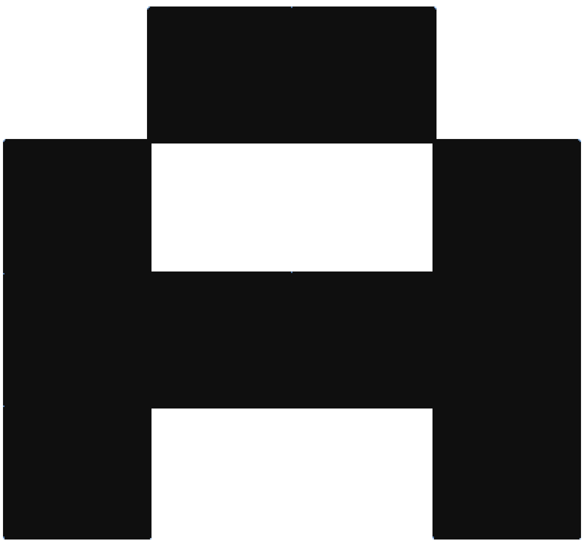

Fig. 10. The image can be transformed to the fractal

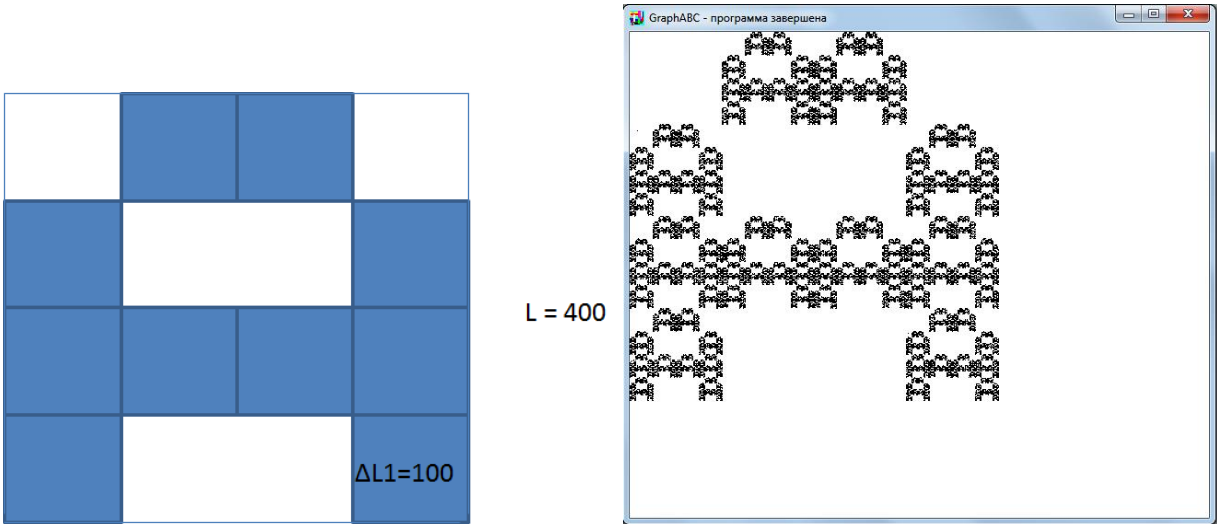

Fig. 11. The result of the algorithm with letter conversion

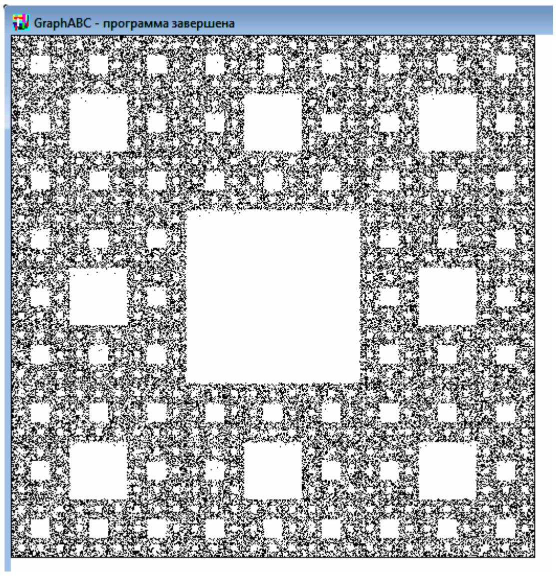

Fig. 12. Sierpinski's square (the result of the algorithm).

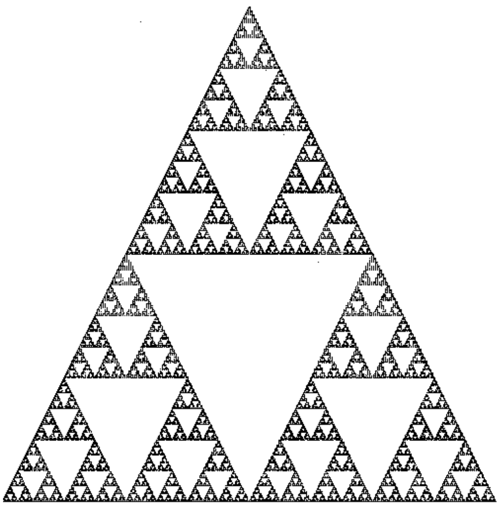

Fig. 13. Sierpinski's triangle (the result of the algorithm)

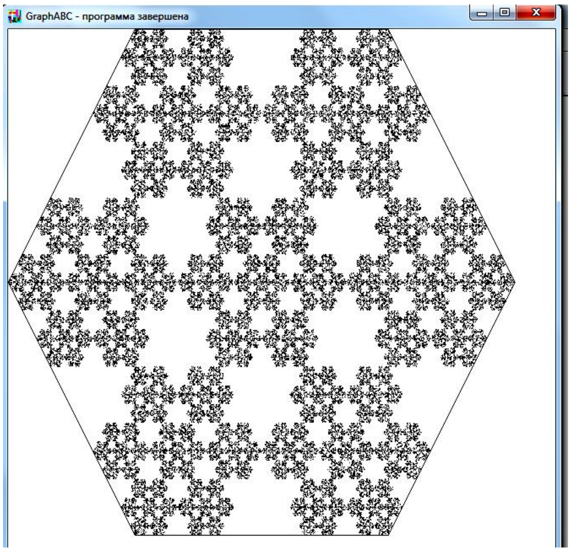

Fig. 14. Koch's Snowball (the result of the algorithm)

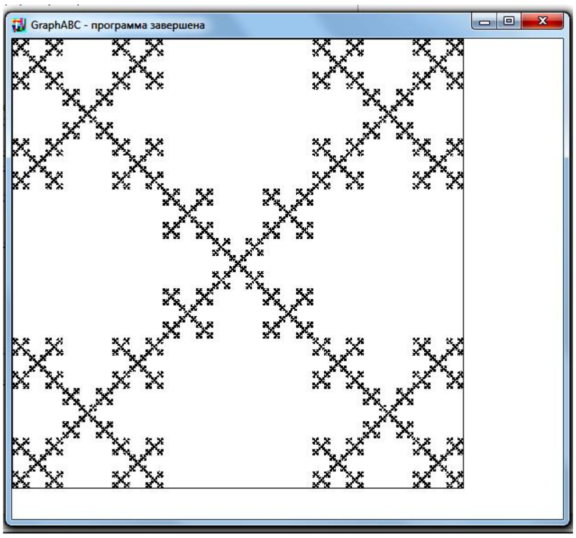

Fig. 15. Fractal Cross (the result of the algorithm)

Conclusion

Today, fractal structures are very common in various fields of science and technology, but there are very few tools for modeling fractal structures. In most cases, deterministic systems of iterative functions are used, which require significant mathematical calculations. Algorithms for RSIF simplify this task, but the calculation of coefficients for RSIF functions is quite complex and uncertain. Therefore, we proposed a new algorithm for constructing fractal structures using RSIF, based on finding the centers of the first iteration segments, with subsequent finding of RSIF coefficients, which will improve the accuracy of fractal images. It will allow you to make direct and inverse transformations without involving additional software and hardware resources. The use of forward and inverse transformations will allow in the future to form a source data set for neural networks, which will form the basis of object recognition. As a result of the research, the proposed algorithm and simulation simulation managed to reproduce fractal structures with an accuracy of 99.9 %, the following fractal structures were reproduced: Cantor's set, Serpinsky's triangle and square (Fig. 13 and Fig. 12), Koch's Snowball (Fig. 14), Fractal Cross (Fig. 15).

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АЛГОРИТМ ФОРМУВАННЯ РАНДОМІЗОВАНОЇ СИСТЕМИ ІТЕРАЦІЙНИХ ФУНКЦІЙ ЗА СТРУКТУРОЮ КАНТОРА

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У статті розглянуто результати розроблення алгоритму формування рандомізованої системи повторюваних функцій (RSIF) з наявного фрактального зображення типу "Фрактальний пил" (набір Кантора). Виведено математичні формули та схеми для розрахунку коефіцієнтів RSIF. Цей алгоритм полягає у знаходженні формул функцій відносно центра першої ітерації фрактальної структури. Це дає можливість визначити рандомізовану систему ітераційних функцій із наявного фрактального зображення. Алгоритм побудови не використовує рекурсивних функцій та входження циклу в цикл, що дає змогу не витрачати великих обчислювальних потужностей, і є доволі оптимізованим. Алгоритм дасть змогу виконувати прямі та зворотні перетворення без залучення додаткових програмно-апаратних ресурсів. Використання прямих і зворотних перетворень дасть змогу в майбутньому сформувати вихідний набір даних для нейронних мереж, що буде покладено в основу розпізнавання об'єктів.

Ключові слова: *фрактал; набір Кантора; рандомізована система повторюваних функцій (RSIF).*