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A new mathematical model for contrast enhancement in digital images

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The aim of this work is to propose a new mathematical model for optimal contrast enhancement of a digital image. The main idea is to combine the Divide-and-Conquer strategy, and a reaction diffusion mathematical model to enhance the contrast, and highlight the information and details of the image, based on a new conception of the Sine-Cosine optimization algorithm. The Divide-and-Conquer technique is a suitable method for contrast enhancement with an efficiency that directly depends on the choice of weights in the decomposition subspaces.
Methods: in this paper, a new algorithm has been used for the optimal selection of the weights considering the optimization of the enhancement measure (EME).
Results: in order to evaluate the effectiveness of the proposed algorithm, experimental results are presented which show that the proposed hybridization technique is robustly effective and produces clear and high contrast images.

Keywords: *reaction diffusion, contrast enhancement, Divide-and-Conquer technique, Sine-Cosine algorithm, optimization algorithm, FitzHugh–Nagumo mathematical model.*

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1. Introduction

Image processing is a branch of computer science and applied mathematics, which is of a great interest in various real applications. It deals with digital images and their transformations to improve their quality or extract informations from them. Image processing aims to apply a set of techniques to improve the content and the quality of an image, using an automatic process. Also, it is used in manufacturing and computer vision. For military and security purposes, image processing and computer vision are used for detecting a target tracking, missile guidance and recognizing objects.

In addition, image processing methods are used to enhance, clean, combine and cut an image in photographic and film industry. Such methods are also used in the analysis of biomedical images [1]. Indeed, to diagnose a disease, it is either necessary to differentiate between normal and abnormal tissues, or identify the different human organs.

One of the most important tasks in image processing is contrast enhancement of digital images, which consists in recovering an image from a degraded one, usually a blurred or low-contrast image. Several methods have been proposed to improve the contrast of images. The first category of enhancement methods is based on histogram equalization involving global models [2, 3]. This method is used to increase the global contrast of many images, more specifically when the usable image data are represented by close contrast values. Through this adjustment, the intensities can be better distributed on the histogram. This allows for areas of lower local contrast to gain a higher contrast. Histogram equalization accomplishes this by effectively spreading out the most frequent intensity values.

Modifications of Histogram equalization method use multiple histograms, called sub-histograms, to emphasize local contrast, rather than global contrast. These include the CLAHE, MPHE and MBOBHE methods. For more details, we refer the reader to [4].

Digital image quality enhancement is a fundamental technique for several applications, including remote sensing [5,6], image processing, computer vision [4,7], biomedicine [8], atmospheric science [9], and video surveillance [10]. In the literature, there are many image enhancement techniques that have been successfully developed, and can be classified mainly in the following categories:

- Retinex-based algorithms [11] decompose an image into illumination and reflectance, and compute both components simultaneously with different illumination regularization constraints [12,13]. The multi-scale Retinex algorithm with color restoration, use Gaussian filtering to estimate and remove illumination and improve image contrast and color consistency. However, methods based on Retinex theory suffer from graying of uniform scenes and are not effective in unnatural images.
- Fuzzy masking algorithm is effective in improving the contrast and sharpness of images [14,15]. It is based on the use of filters to decompose an image into a base layer and a detail layer, and process the two parts respectively using an edge-preserving smoothing operator [16]. These blurred masking algorithms fail in the tradeoff between detail and naturalness.
- Histogram equalization is a technique that has been widely used to improve the contrast of images [17]. The specificity of this algorithm guarantees the improvement of global and local contrast, which subjects the image histogram to a process of modification and maximizes the measure of the increase in information. This method is easy to implement but produces poor performance in preserving the naturalness of the images.
- Divide and Conquer method: the enhancement methods just mentioned tend to neglect the difference between the low-frequency and high-frequency components of the image, and even these components cannot be effectively exploited for image enhancement. In particular, Divide-and-Conquer algorithm was applied to achieve parallel computers [18], cellular automata [19] and image enhancement [20]. For these reasons a new method of image enhancement has been developed which uses the "divide and conquer" strategy. First, series of efficient linear filters are designed to decompose the observed image into four subspaces using linear filters which extract exlusively low frequency and high frequency information from the whole image. Secondly, the enhancement result of each subspace of the image is obtained by the Gradient Distribution Specification method on each of these subspaces [20], which is better for image naturalization. Finally, the full image is reconstructed by weighted fusion of the enhanced subspace images for detail enhancement. The major problem of this technique is the choice of these weights. One of the objectives of our work is to propose an original algorithm to compute the optimal weights by introducing a meta-heuristic optimization method of a cost function based on the computation of the image enhancement at each iteration [21,22].
- Partial Differential Equations is a field that has attracted the attention of many researchers in recent years. This is mainly due to the mathematical formulation that frames any approach based on partial differential equations (PDE), and its good justification and interpretation of the obtained results, using these traditional and heuristic methods in image processing [23]. One of the most famous mathematical models that has been widely used in image processing is that of FitzHugh–Nagumo [24,25]. This model simulates the temporal response of a nerve axon to a stimulus, which is described by a pair of ordinary differential equations evolving in time, with an activating variable $u(t)$ and an inhibiting variable $v(t)$, as follows:

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{\tau}(u(u-a)(1-u)-v), \\ \dfrac{dv}{dt} = u - bv, \\ u(0) = u_0, \quad v(0) = v_0, \end{cases} \tag{1}$$

where τ is a small positive constant and a represents the threshold value, b is a positive constant, u_0 represents the initial image. Ref. [26] applied this system to the formation of patterns. Recently this model has been used by [27] to increase the contrast in an image.

In the next paragraph, we will see an improvement of this method by applying on each subspace the Fizhugh–Nagumo model instead of the method proposed by [20]. Despite this, the choice of balancing weights remains a major inconvenience. The main objective of this work is to determine the optimal balancing weights so that the resulting image is better contrasted.

In this study, in order to improve the efficiency of image enhancement, we introduce a cost function defined on the balancing weights, which is optimized using the improved Sine-Cosine algorithm. In Section 2, a modified version of the Divide-and-Conquer method and the Sine-Cosine meta-heuristic optimization method are presented. The third section is devoted to the general presentation of the hybrid model, the modified Divide-and-Conquer with an updated version of the Sine-Cosine Optimization (SCA) method for contrast enhancement of digital images. Experimental results and performance analysis are given in Section 4, and the conclusion is given in Section 5.

2. Background and materials

2.1. Divide-and-Conquer Technique

Another way to increase the contrast in a digital image is the Divide-and-Conquer strategy [28, 29]. Previous literatures on the decomposition models [30, 31] had shown the difference between low (cartoon) and high-frequency (edges and texture) components of an image; the smoothing areas represent low-frequency portions, while edges and details are contained in high-frequency parts. Afterwords, each component will be implemented separately. To this end, an observed image U is decomposed into different low and high-frequency, using the convolution of U with a predefined linear filters h_i:

$$U_i = U \otimes h_i \quad \text{for} \quad i = 1, 2, 3, 4, \tag{2}$$

where $h_3 = [-1, 1]$ and $h_4 = [-1, 1]^T$ are two high frequency filters and the symbol $\otimes$ is the two-dimensional convolution operator. The sub-images U_3 and U_4 contain the high-frequency component of the entire image U whereas, U_1 and U_2 forms regions with low frequency. Then, we define frequency responses H_1 and H_2 of low-frequency filters h_1 and h_2 as follow:

$$H_1 = I - H_3, \quad H_2 = I - H_4. \tag{3}$$

Where H_3 and H_4 are respectively the Fourier transform of h_3 and h_4. I is a matrix where each of its elements, is equal to one.

Once the decomposition process of the image U is obtained, the Fitzugh–Nagumo model (1) is applied to naturalize each observed subspace image U_i separately to get the image U^i_{FN}, then the enhanced subspace image U_i is reconstructed from a linear remapping approximation:

$$U_{dc} = \sum_{i=1}^{4} w_i U^i_{FN}, \tag{4}$$

where $\{w_i\}_{i=1}^4$ are the weights of balancing different sub-images. The algorithm that allows us to generate the new image is given below.

Algorithm 1 Outline of Modified Divide-and-Conquer method **(MDC)**

1: Input: Observed image U, complete filters $\{h_i\}_{i=1}^4$, balancing weights $\{w_i\}_{i=1}^4$.
2: Generate observed subspaces $\{H_i\}_{i=1}^4$ via (3).
3: Obtain enhanced subspaces $\left\{U^i_{FN}\right\}_{i=1}^4$ via (1).
4: Output: Final image $U_{dc} = \sum_{i=1}^4 w_i U^i_{FN}$.

2.2. Sine Cosine Algorithm

Sine-Cosine (SCA) is a metaheuristic algorithm developed through sine cosine mathematical functions, in order to be able to use it in the resolution of optimization problems. This optimization method was first designed by Mirjalili in 2016 (see [32]).

The modification of each search agent using the sine cosine algorithm is performed through the following two equations:

$$X_i^{t+1} = \begin{cases} X_i^t + r_1 \times \sin(r_2) \times \left|r_3 P_i^t - X_i^t\right|, & r_4 < 0.5, \\ X_i^t + r_1 \times \cos(r_2) \times \left|r_3 P_i^t - X_i^t\right|, & r_4 \geqslant 0.5. \end{cases} \tag{5}$$

Where r_2 is the random variable such that $r_2 \in [0, 2\pi]$, and r_3 represents a random variable. r_4 is used for the choice of search paths, sine or cosine, based on the random values of the first equation, and let P_i be the objective solution.

r_1 is chosen in such a way to be between 0 and m for balancing the iteration. This latter is defined by the following sense:

$$r_1(i) = m\left(1 - \frac{i}{i_{max}}\right),$$

where i_{max} is the total number of iteration, m is a nonnegative constant and i is the present iteration.

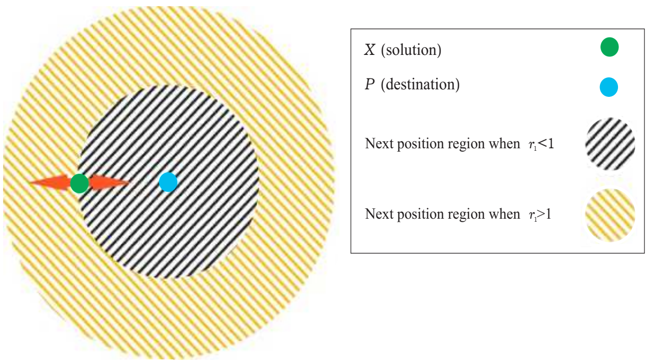

Fig. 1. Basic Sine-Cosine Algorithm.

SCA is an optimization algorithm which refers to a population already generated by sine and cosine mathematical functions. Among the similar algorithms, we find GA, PSO, GSA, etc. which generate the start set of random solutions.

Subsequently, its different solutions are evaluated thanks to the cost function. After the different resolutions are evaluated, the best among them is chosen. The optimal solution is stored and represented as the next destination point. The different results are updated to generate new ones, through the sine and cosine functions. The algorithm ends when the number of iterations of the algorithm is achieved.

The algorithm 2 demonstrates the different stages of this process.

Algorithm 2 Steps of Sine-Cosine Algorithm **(SCA)**

1: Initialize the solutions.
2: Evaluate each solution by the cost function.
3: Update the best obtained solution.
4: Update the parameters r_1, r_2, r_3, and r_4.
5: Update the solution using the equation (5) defined before, until reaching the maximum number of iterations.
6: Return the optimal solution.

3. The proposed contrast enhancement mathematical model

In the present work, in order to improve the image quality, the effective measure of enhancement EME is used. It is obtained by splitting the image U k_1k_2 blocks $w_{k,l}(i,j)$ of sizes l_1l_2, using the following equation

$$\mathrm{EME}(U) = \frac{1}{k_1k_2}\sum_{l=1}^{k_1}\sum_{k=1}^{k_2} 20\log\frac{u^w_{\max;k,l}}{u^w_{\min;k,l}}. \tag{6}$$

Where $u^w_{max;k,l}$ and $u^w_{min;k,l}$ are respectively maximum and minimum values of the image U inside the block $w_{k,l}$. A higher value of EME indicates that the image is enhanced very well.

At this stage of our study, we are going to proceed to the hybridization between both methods, Modified Divide-and-Conquer and SCA algorithms.

- **Initialization**
  a) Input: Observed image U, complete filters $\{h_i\}_{i=1}^4$.
  b) Generate observed subspaces $\{H_i\}_{i=1}^4$ via (3).
  c) Obtain enhanced subspaces $\left\{U^i_{FN}\right\}_{i=1}^4$ via (1).
  d) Generate randomly by using the uniform low, N individuals : $[W^1, W^2, \dots, W^N]$, where $W^j = (w^j_1, w^j_2, w^j_3, w^j_4)$ represent the balancing weights in Modified Divide-and-Conquer Algorithm (MDCA).
- **Application of MDC Algorithm**
  a) For each individual $W^j = (w^j_1, w^j_2, w^j_3, w^j_4)$, obtain a new image $U^j_{dc} = \sum_{i=1}^4 w^j_i U^i_{FN}$ using Algorithm 1.
  b) Calculate the cost function generated by the EME (6):

$$F(W^j) = \frac{1}{1+\mathrm{EME}(U^j_{dc})^2}. \tag{7}$$

- **Application of Sine-Cosine Algorithm**
  a) Optimize the cost function F using Sine-Cosine Algorithm (SCA) to obtain w^*_i.
  b) Output the final image: $U^* = \sum_{i=1}^4 w^*_i U^i_{FN}$.

Remark 1. The choice of the meta-heuristic method Sine-Cosine is justified since the function to minimize denoted by F is not differential.

4. Results and discussion

In this section, all experimental simulations are performed in Matlab R2018 on a Dell Precision Tower 7910 Workstation with a processor: Intel® Xeon® Processor E5 v4 Family with number E5-2603V4. This machine is caracterized by it's robustness in the professional segment and was tuned for best performance. The software application was written by the authors in Matlab programming language. Here, we consider different test images that are poorly contrasted (see Figs. 1, 2). The contrast enhancement results of the different methods on these images, are shown in Figs. 3–5. In order to evaluate the enhancement result, the EME contrast factor of the image is evaluated.

In the following figures 4, 5, classical FitzHugh–Nagumo model is applied for gray scale and RGB image.

Now we make use of the classical Divide-and-Conquer Algorithm (see Figs. 6, 7).

Finally, we apply the proposed Modified Divide-and-Conquer together with Sine-Cosine Algorithm (see Figs. 8, 9).

Fig. 2. Gray scale image with low contrast.

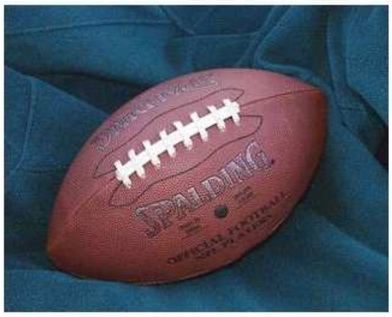

Fig. 3. Color image with low contrast.

Fig. 4. Classical FitzHugh–Nagumo model applied to gray scale image.

Fig. 5. Classical FitzHugh–Nagumo model applied to color image.

Fig. 6. Classical Divide-and-Conquer Algorithm applied to gray scale image.

Fig. 7. Classical Divide-and-Conquer Algorithm applied to color images.

Fig. 8. The proposed model applied to gray scale image.

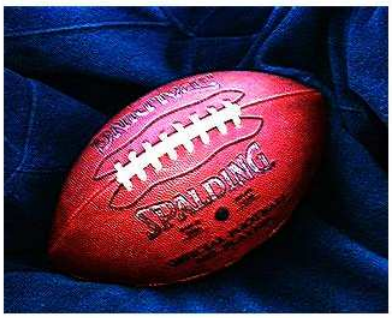

Fig. 9. The proposed model applied to color image.

The results obtained by the proposed model are more satisfactory than those obtained by classical FitzHugh–Nagumo model and classical Divide-and-Conquer Strategy.

The following table 1 shows the value of measures enhancement of three models: the classical FitzHugh–Nagumo model, the classical Divide-and-Conquer Technique and the proposed model.

Table 1. EME values of each image by three models.

	Original image	FitzHugh–Nagumo	Divide-and-Conquer	Proposed model
Gray scale image	2.2797	7.503	10.1172	10.7092
Color image	13.2673	34.6616	42.3244	50.4548

After optimizing by our proposed model for the gray scale image, the best solution obtained is: $w_1^* = 0.2$, $w_2^* = 1.5$, $w_3^* = 0.63831$ and $w_4^* = 0.71263$. The effective measure enchancement is $\text{EME} = 10.6941$. Concerning the color image, the best solution obtained is: $w_1^* = 1.1588$, $w_2^* = 0.2$, $w_3^* = 0.24495$ and $w_4^* = 0.49454$. The effective measure enchancement is $\text{EME} = 50.4548$.

5. Conclusion

Through this result, the proposed model has always a high value of enhancement measure EME than classical FitzHugh–Nagumo system and classical Divide-and-Conquer for different images, which explains the good quality of the image enhanced by the proposed method.

As a next step of this work, it remains to use other meta-heuristic algorithms rather than the Sine-Cosine algorithm and to evaluate its performance against other algorithms, including ABC (artificial bee colony), BOA (Butterfly optimization algorithm), CSA (crow search algorithm), HS (harmony search) and finally SSA (Salp swarm optimization).

The second main objective is the treatment of blurred images through the introduction of a partial differential equation, making use of image restoration and contrast enhancement.

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Нова математична модель для підвищення контрастності цифрових зображень

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Метою даної роботи є запропонування нової математичної моделі для оптимального підвищення контрастності цифрового зображення. Основна ідея полягає в поєднанні стратегії "розділяй і володарюй" та реакційно–дифузійної математичної моделі для підвищення контрасту і виділення інформації та деталей зображення на основі нової концепції алгоритму синусно-косинусної оптимізації. Техніка "розділяй і володарюй" є підходящим методом для посилення контрасту з ефективністю, яка безпосередньо залежить від вибору ваг у підпросторах декомпозиції.
Методи: у цій роботі використано новий алгоритм для оптимального вибору ваг з урахуванням оптимізації міри покращення (EME).
Результати: для оцінки ефективності запропонованого алгоритму подано експериментальні результати, які показують, що запропонована методика гібридизації є надійно ефективною і дає чіткі та висококонтрастні зображення.

Ключові слова: *реакція, дифузія, посилення контрасту, техніка "розділяй і володарюй", алгоритм синус-косинус, алгоритм оптимізації, математична модель ФітцХью–Нагумо.*