UKRAINIAN JOURNAL OF MECHANICAL ENGINEERING AND MATERIALS SCIENCE
Vol. 7, No. 3–4, 2021

Nadiia Maherus[1]**, Yuriy Sholoviy**[2]

[1] Department of Robotics and Integrated Technologies of Mechanical Engineering, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: nadiia.i.maherus@lpnu.ua, ORCID 0000-0003-1946-8503,
[2] Department of Robotics and Integrated Technologies of Mechanical Engineering, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: yurii.p.sholovii@lpnu.ua,ORCID 0000-0003-0154-7983

JUSTIFICATION OF VIBRATING HOPPER HOLE PARAMETERS DURING FINE GRAINED BULK MATERIAL UNLOADING

Received: June 12, 2021 / Revised: August 8, 2021 / Accepted: December 26, 2021

https://doi.org/10.23939/ujmems2021.03-04.020

Abstract. *Purpose.* The leakage process of the fine grained bulk material (BM) from the unloading hopper is considered. As known the BM behaviour during the unloading process is directly related to the material stress state (active or passive) inside the hopper, which depends on the physical and mechanical bulk material properties, as well as hopper geometric parameters. At the boundary of the transition from the cylindrical part to the conical part of the unloading hopper hole, the horizontal stresses that precede the leakage of the material increase sharply. The use of vibration reduces horizontal stresses in the hopper and thus improves the bulk product fluidity. Also, the use of vibration reduces the probability of the free-fall arch formation over the outlet. Therefore, the present paper is aimed at developing the mathematical model of the fine grained bulk material flow under the vibration action from the conical hopper hole. *Methodology*. It is used one of the methods of studying the BM behavior, namely the analysis of the stress state of the conditionally stationary (pseudo-stationary) layer of the product and the force balance equation in the free-fall arch is made for further study of the geometric parameters of the unloading hole. *Findings*. In this paper, the mathematical model of the BM behavior in the conical hopper hole is developed. Since the most determining factor influencing the hopper productivity is the outlet diameter, so based on the developed model, it is established relationships that allow determining all parameters that affect the diameter. Graphical dependences are obtained, which allow to estimate the influence of the angle inclination of the hopper hole walls on the value of the hopper outlet size at active and passive stress states. The dependence of the diameter of the unloading hopper hole on the BM properties is also established. The BM properties directly affect the initial shear resistance of the material. The influence of vibration on the unloading hole geometry is presented. *Originality*. The pseudo-immobile layer of bulk product is considered in the developed mathematical model, and it is investigated how vibration affects the BM behavior in this layer. *Practical value*. The practical value of this paper lies in the possibility of further mathematical modeling of the influence of the BM properties, hopper geometric dimensions and vibration on the product behavior in the material leakage process from the hopper. The results of the studies, presented in the form of graphs, can be used during the vibrating hopper design and the selection of rational modes of hopper operation.

Keywords: unloading hole, fine grained bulk material, vibration, flow stress, hole parameters, free-fall arch.

Introduction and Literature Review

It is widely used hoppers and accumulative capacities which have a lot of tasks from the storage of considerable stocks of fine grained BMs to the continuous loading of equipment working positions in technological systems of the bulk product packaging. Regardless of the purpose of hoppers, their contents must be unloaded periodically or continuously through an outlet, which is located at the unloading hole bottom, whose shape often depends on the main hopper cross section. For the study of the BM behaviour during leakage from the hopper, the free-fall arch formation and the continuous flow conditions of material are important. In the paper [1], dense granular flow in hoppers was studied by numerical simulations, in attempts to explore the free-fall arch region and its boundary. A lot of problems appear during the unloading process of bulk products, especially when it means fine grained BMs. In the research [2] the motion conditions of cohesive and cohesionless BMs, the speed of material movement on the conveyor are investigated. These studies [2] consider the product movement only on the belt conveyor, which doesn't allow using them to predict the BM outflow from the vibrating hopper hole, where a possibility of an accumulation of more material, and an ability to influence its movement by changing vibration parameters is there.

Many studies have shown that the BM behaviour during the unloading process is directly related to its stress state (active and passive) inside the bunker, which depends on the physical and mechanical BM properties, as well as geometric parameters of the hopper. The change in the bulk material stress state causes various forms of the product movement [3] during the unloading process. It is established [4] that conditions of the BM movement in the unloading conical hopper hole are more unfavourable, than they are in its cylindrical part. Study and analysis of BM stress state allows predicting the product behaviour in the BM unloading process from hoppers and ensuring their reliable operation. The most common method of the BM behaviour study is its formalization in a form of a bulk discrete medium. This assumption can be used if element dimensions, where changes in the kinematic and dynamic parameters are determined in, is an order of magnitude bigger than a maximum particle size of the bulk medium. Under such conditions, it can be also used methods of bulk discrete media mechanic, which allow considering a deformation of the discrete material as a mutual slippage of individual particles.

The use of vibration can influence the BM movement. The main parameter determining the BM behaviour under the vibration action is a vibration intensity $a\omega^2$ [5]. The greatest compaction is achieved at vibration accelerations closed to gravitational accelerations. Therefore, it is very important for each BM to set an oscillation amplitude and an oscillation frequency, which correspond to the limit value of the vibration intensity. Using vibration with an intensity higher than a limit value leads to destruction of a connection between bulk medium particles and their circulation. An influence of vibration on the granular BM behaviour is investigated in the research [6]. In this study, it was discovered the degree of fluidization of the granular BM within a reservoir vibrated vertically with different frequencies and amplitudes. In the research [7] it was found that the average velocity of particles inside the granular flow decreased as the vibration frequency increased. It is established vibration modes for an occurrence of the vibration boiling in a granular material. However, these studies are limited with a consideration of a granular material layer with thickness up to 15 mm. Another research [8] allows generalizing conditions of the discharge of granular materials from the cylindrical hopper under vertical vibration. But only the hemispherical bottom hopper with five different angles of outlets was proposed there.

In the research [9] the mathematical model was developed, that allows effectively investigating fine grained BMs in the vibration boiling state. But, to stabilize the vibration boiling state of fine grained BMs, it is required the significant vibration intensity. However, designing a hopper for fine grained BMs, the main task is to ensure the continuous flow of BMs from the outlets during the unloading process but the implementation of vibration boiling of the product is not required.

Problem Statement

An application of vibration, which destroys contacts between the product particles, changes the material stress state and, thus, improves a flow rate, is one of effective methods to improve the BM unloading process. It is established that the BM behaviour can be determined by basic laws of a gravitational unloading process, provided there is a vibration-liquefied layer of the product [4]. Vibration is considered as a factor that prevents a formation of the free-fall arch and provides the continuous flow of the bulk product. This fact allows modelling the vibrational flow of the fine grained BM by using a gravitational flow model with certain physical and mechanical product properties. Therefore, the present paper is aimed at developing of the model of the unloading process of bulk material during the vibration action.

Main Material Presentation

Using one of the methods to study the bulk material behaviour, namely an analysis of a stress state of a conditionally immobile (pseudo immobile) product layer, a force balance equation in the free-fall arch is made. It has the next form for the unloading conical hopper hole (Fig. 1) in the projection on the vertical axis:

$$dG + dF_{\upsilon} - dF_{c} - dF_{odh} = 0\,, \tag{1}$$

where dG is the weight gain of the material; dF_{υ} is the inertial force gain, that occurs in the BM under the vibration action; dF_{odh} is the adhesion force gain (the adhesion between the product particles); dF_{c} is the resistance force gain.

To model the bulk material behaviour in the unloading conical hopper hole, it was investigated all force factors acting on the selected elementary volume $V_{\text{в}.v}$ of the free-fall arch with height dh (Fig. 1). The main force, that promotes the free gravitational flow through the outlet, is the material weight. The weight gain of the material is determined by:

$$dG = \rho \cdot g \cdot V_{\text{в}.v}\,, \tag{2}$$

where ρ is the bulk density of the BM; g is the gravitational acceleration; $V_{\text{в}.v} = S_h \cdot dh$; S_h - is the area of the horizontal section of the elementary volume in the unloading conical hopper hole.

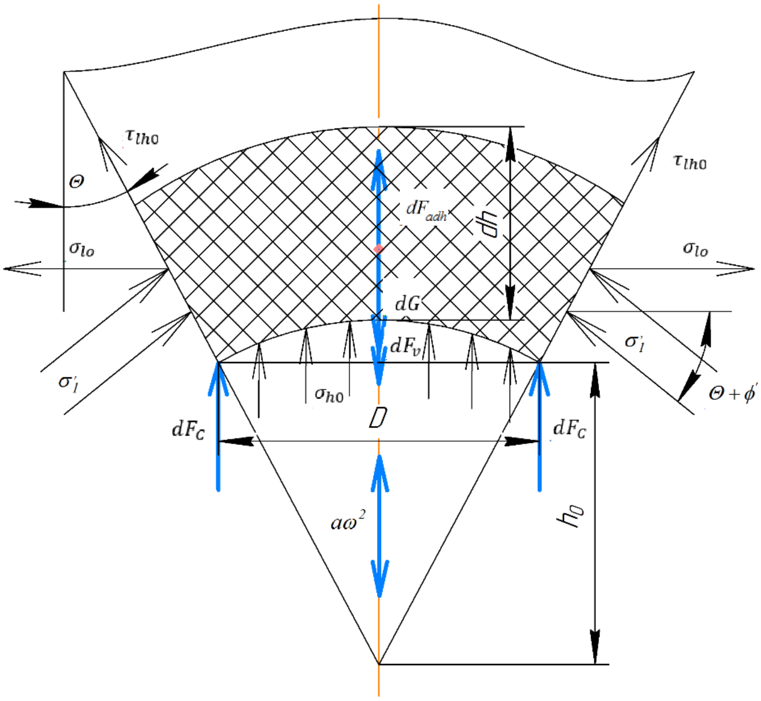


Fig. 1. The scheme of forces acting on the BM elementary volume in the unloading conical hopper hole

To simplify the calculations, the area of the elementary volume S_h in the unloading conical hopper hole (Fig. 1) is approximated as an area of a circle with a diameter D:

$$S_h = \pi \cdot \frac{D^2}{4}, \tag{3}$$

where D is the diameter of the hole at the place of the free-fall formation in the unloading hole.

Then taking into account the dependence (3), the equation (2) will take the form:

$$dG = \frac{\rho \cdot g \cdot d \cdot h \cdot \pi \cdot D^2}{4}. \tag{4}$$

In this mathematical model it is considered that the hopper is elastically connected to the unloading hole, which performs harmonic vertical oscillations. Since the low frequency vibration velocity used for fine grained BMs is several orders of magnitude lower than the velocity of the forced oscillation in the material, it can be assumed that the vibration acceleration is instantaneously transmitted to the bulk product and has a quasi-stationary character. The material vertical stress in this case can be represented as a sum of an inertial component σ_{vib} arising from the vibration action and a gravitational component discovered above.

The inertial component of the vibration action is determined by a magnitude of an applied force transmitted to the bulk material layer, and equal to the formula [10]:

$$\sigma_{vib} = \frac{4F(t)}{\pi (D_v)^2}, \tag{5}$$

where $F(t) = F_0^{'} \cdot \cos(\omega t + \psi_0)$, $F_0^{'}$ is the amplitude of the generalized forcing force; $\omega = 2 \cdot \pi \cdot v$ is the angular frequency; v is the ordinary frequency, Hz; ψ_0 is the initial phase of oscillation; D_v is the diameter of the vibrating hole at the place of the vibrator attachment.

The amplitude of the generalized forcing force can be written as:

$$F_0^{'} = m_s \cdot A \cdot \omega^2, \tag{6}$$

where m_s is the mass of the bulk material layer; A is the vibration amplitude.

Thus, the increase in the inertia force caused by the vibration action in the BM is determined by:

$$dF_\upsilon = \sigma_{vib} \cdot \pi \cdot D^2 / 4. \tag{7}$$

Taking into account expressions (5) and (6), the increase in the inertia force can be written as follows:

$$dF_\upsilon = \frac{m_s \cdot A \cdot \omega^2 \cdot \cos(\omega t + \psi_0) D^2}{D_v^2}. \tag{8}$$

The mass of the bulk material layer is determined by the formula:

$$m_s = \rho \cdot V_{в.о} = \frac{\rho \cdot d \cdot h \cdot \pi \cdot D^2}{4}. \tag{9}$$

As it is seen in the equation (8), an increase in the inertia force has a harmonic character, but to study the conditions of the free-fall arch destruction in the BM and ensure the material flow, it is necessary to investigate the maximum amplitude value of the inertia force. Since this mathematical model considers a vibrating hole, whose oscillation is realized by an elastic suspension and a vibrating exciter, it is assumed that $D_v \approx D$. Therefore, providing these conditions and using the equation (9), the inertia force gain will be equal to:

$$dF_v = \frac{A \cdot \omega^2 \cdot \pi \cdot D^2 \cdot \rho \cdot d \cdot h}{4}. \tag{10}$$

The adhesion forces caused by the presence of moisture in the material pores, as well as molecular forces have a significant impact on behaviour of fine grained BMs in the hopper. The material compaction, which is accompanied by an increase in material stress, is also facilitated by hopper hole parameters and the bulk material weight loaded in the hopper. To ensure efficient operation of the hopper, which is determined by the continuous bulk product flow from the outlet, the degree of a product compaction should not exceed a critical value of the adhesion force gain, which is determined by the dependence [10]:

$$dF_{odh} = \frac{dF_{\varpi}}{f}, \tag{11}$$

where $dF_{\varpi} = \tau_0 \cdot S_v$ is the separation force gain; τ_0 is the initial shear resistance of the BM under action of a tangential load; S_v is the area of the vertical section of the elementary volume; $f = \tan(\phi)$ is the internal friction coefficient of the BM; ϕ is the internal friction angle of the BM.

The area of the vertical section of the elementary volume in the unloading conical hopper hole (Fig. 1) is determined by:

$$S_v = dh \cdot l_{gl}, \tag{12}$$

where $l_{gl} = \frac{\delta \cdot D}{\sin(\delta)}$ is the length of the free-fall arch; $\delta = \theta + \phi^{'}$, θ is the angle inclination of the hole walls to the vertical; $\phi^{'}$ is the friction angle of BM to hole walls.

Substituting the expression (12) in the equation (11), it is obtained the value of the adhesion force gain for the unloading conical hopper hole:

$$dF_{adh} = \frac{\tau_0 \cdot d \cdot h \cdot \delta \cdot D}{f \cdot \sin(\delta)}. \tag{13}$$

During study the conditionally stationary layer of the BM, it is important to take into account the resistance of the bulk product that occurs during the BM movement toward the hopper outlet. Thus, it is added the resistance force in this mathematical model. The resistance force gain is determined by the formula [11]:

$$dF_c = \sigma_1^{'} \cdot \sin(\delta) \cdot dh \cdot \cos(\delta) \cdot P, \tag{14}$$

where $\sigma_1^{'}$ is the free flow stress; P is the perimeter of the hopper hole outlet.

Taking into account $P = \pi \cdot D$, the resistance force gain for the unloading hopper hole was found by the formula:

$$dF_c = \sigma_1^{'} \cdot \sin(\delta) \cdot d \cdot h \cdot \cos(\delta) \cdot \pi \cdot D. \tag{15}$$

Using the expressions for all force gains (4), (10), (13), (15) acting on the elementary volume in the unloading conical hopper hole, and substituting them in the force balance equation (1), it is obtained:

$$D = \frac{4 \cdot (\tau_0 \delta + 0{,}5 \cdot \sigma_1^{'} \cdot \sin(2\delta) \cdot \sin(\delta) \cdot \pi \cdot f)}{f \cdot \sin(\delta) \cdot \rho \cdot \pi (g + A \cdot \omega^2)}. \tag{16}$$

An important indicator, that determines the geometric parameters of the unloading hopper hole, is the magnitude of the BM free flow stress [10]. The relationship between the compaction stress and the BM free flow stress is determined by the leakage factor ff. As already mentioned, there are a number of reasons that block the flow of fine grained BMs from the hopper hole. In order that there were no conditions which interfere with BM free outflow from the hopper hole, it is necessary to prevent product compaction to an occurrence of the significant shear resistance. Accordingly, the higher the compaction stress σ_1 and the lower the free flow stress $\sigma_1^{'}$, the worse the product's ability to leak through the outlet. This ability is determined by the leakage factor [10]:

$$ff = \frac{\sigma_1}{\sigma_1^{'}} \ . \tag{17}$$

It is established that the leakage factor is in the range of 1.2-1.6 for the conical hopper [11]. The lower value of the leakage factor, the better is the unloading hole design. Therefore, to determine the free flow stress, it is important to set the main compaction stress [10]:

$$\sigma_1 = \frac{\sigma_{h0} + \sigma_{t0}}{2} + \sqrt{\left(\frac{\sigma_{h0} + \sigma_{t0}}{2}\right)^2 + \tau_{th0}} \ , \tag{18}$$

where σ_{h0} is the vertical stress of the BM in the considered section; σ_{t0} is the horizontal stress of the BM in the considered section; $\tau_{th0} = \sigma_{t0} \cdot f_w$ is the tangential stress in the bulk material layer.

$$\sigma_{h0} = \frac{\rho \cdot g \cdot h_0}{C_{1,2}(a_{1,2} - 1)}\left[1 - \left(\frac{h_0}{h_k + h_0}\right)^{a_{1,2}-1}\right], \tag{19}$$

where h_0 is the distance between the unloading hole outlet and an imaginary cone vertex; h_k is the height of the unloading hopper hole; $C_{1,2}$, $a_{1,2}$ are the coefficients of active and passive stress states, respectively.

The coefficients of active and passive stress states can be written as [10]:

$$C_{1,2} = 1 + 2f^2 \pm \frac{\sqrt{\left[\left(1+2f^2\right)^2 - 1\right]^{\varepsilon}} - \sqrt{\left[\left(1+2f^2\right)^2 - 1 - 4fw^2(1+f^2)\right]^{\varepsilon}}}{6(fw)^2(1+f^2)};$$

$$a_{1,2} = \frac{2\cos(\theta)(fw + \sin(\theta))}{C_{1,2} \cdot \tan(\theta)} \ . \tag{20}$$

Using the hypothesis of an ultimate balance of the product in the selected elementary volume with height dh the maximum vertical stress of the BM is written as [10]:

$$\sigma_{h0} = \sigma_{t0} \cdot \left(1 + 2f^2 \pm \sqrt{(1+2f^2)^2 - 1 - 4fw^2 \cdot \cdot (1+f^2)}\right). \tag{21}$$

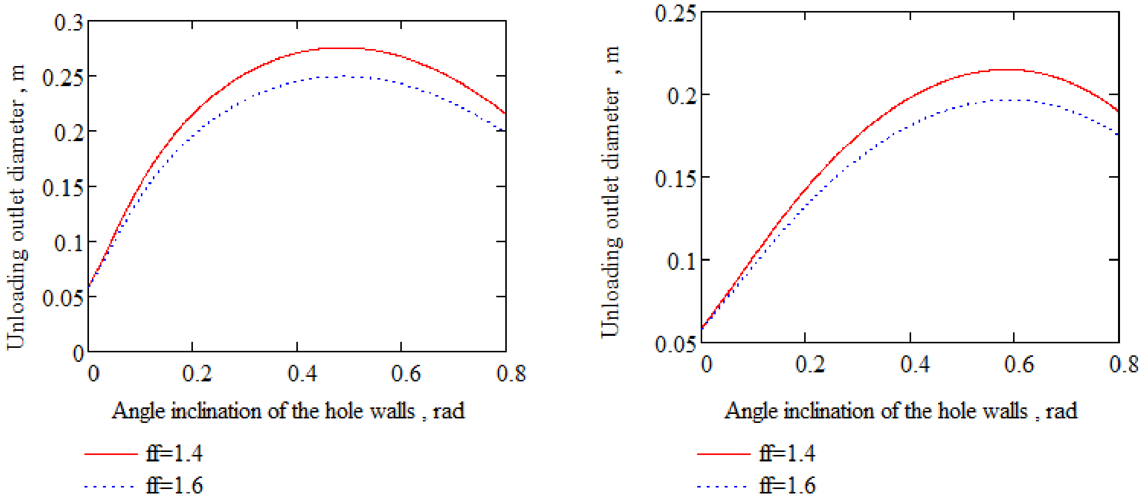
Thus, using the above equations, it is possible to determine the stress of BMs for the active and passive stress states. This allows investigating the basic geometric parameters of the unloading conical hopper hole, as well as to establish the diameter of the unloading hole, which the BM goes from the passive to the active state at.

Results and Discussion

During setting the optimal parameters of the unloading hopper hole, both its geometry and the properties of the fine grained BM are important. To model the behaviour of the fine grained BM during its leakage, a flour with the next properties was chosen: $\phi = 35^o$; $\phi^{'} = 30^o$; $\rho = 0{,}5 \cdot 10^3$ kg/m^3. Also, for the study of the hopper hole geometry it is used the following factors: $h_0 = 0{,}25\,\text{m}$; $h_0 + h_k = 0{,}45\,\text{m}$. The initial shear resistance of the flour varies within $\tau_0 = 50...150$ Pa [11].

Since the hopper hole oscillation will lead to the destruction of the free-fall arch over the outlet and, thus, prevent an occurrence of the passive stress state, so to obtain these graphs (Fig. 2) it is assumed that vibration is absent, and therefore the oscillation amplitude and oscillation frequency are equal to 0.

In the obtained graphical dependences (Fig. 2) it is seen that the unloading outlet diameter, which provides a particular stress state, increases with an increase in the angle inclination of the hole walls. These graphical dependences confirm the adequacy of the developed mathematical model. It can be seen that for realization of the active stress state (Fig. 2, *a*) the outlet diameter is bigger than for similar conditions of the passive state (Fig. 2, b). The character of the curves of the hopper outlet diameter in fig. 2,*a* is similar to Fig. 2, *b*, but the numerical values are slightly different. For example, for the angle inclination of the hole walls $\Theta = 0{,}4rad \approx 23^o$, the active stress state is realized at the unloading outlet diameter $D = 0{,}23m$ at the leakage factor $ff = 1{,}6$, and for $ff = 1{,}4$ there is $D = 0{,}25m$. A passive stress state for the same angle inclination is realized at the outlet diameter $D = 0{,}17m$. for the leakage factor $ff = 1{,}6$ and for $ff = 1{,}4$ there is $D = 0{,}19m$.



a b

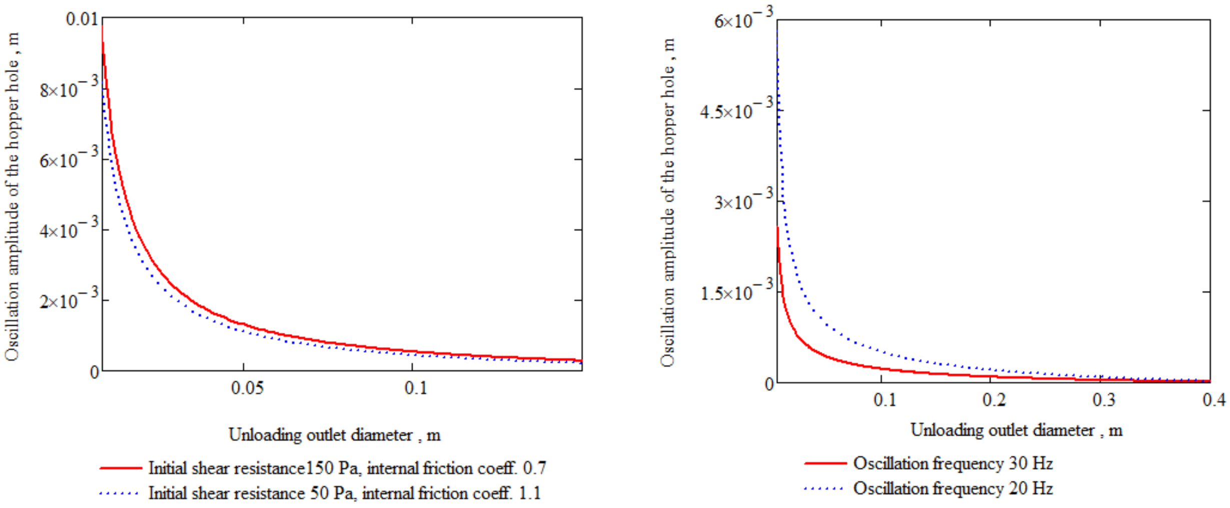
Fig. 2. The dependence of the unloading outlet diameter on the angle inclination of the hole walls: a – for the active stress state of the BM; b – for the passive stress state of the BM

The analysis of these graphical dependences allows drawing a conclusion that an increase in the angle inclination of the hole walls affects the bulk material behaviour more in the active stress state (Fig. 2, *a*). As it is seen, the outlet diameter increases at the angle inclination of the hole walls within $0 \le \Theta = 0{,}5rad \approx 30^o$. This fact can be explained by increasing in bulk material density and increase in adhesion forces. Further increase in the angle inclination of the hole walls does not lead to increase in the outlet diameter, as well as to partial decrease, because it is the maximum compaction. For the passive stress state (Fig. 2, *b*), the maximum compaction of the BM is realized at the angle $\Theta = 0{,}6rad \approx 35^o$, and a dependence of the unloading outlet diameter on the angle inclination of the hole walls is slightly less than in the active stress state.

Thus, it is possible to conclude that it is necessary to increase the outlet diameter to maintain the active stress state in case not to use additional devices those stimulate the outflow of BMs.

The next stage of the study was an establishment of the vibration parameters, which provide activation of the active stress state. It was studied an influence of physical and mechanical properties of the BM on the oscillation amplitude and the oscillation frequency (Fig. 3).

In fig. 3,*a* it is graphically presented the modelling results of an influence of the outlet size on the oscillation amplitude value. The following factors were used to obtain these dependencies: $\phi = 35^o$; $\phi^{'} = 30^o$; $\rho = 0{,}5 \cdot 10^3$ kg/m^3; $h_0 = D/\tan(\theta)$ m; $h_k = 0{,}45$ m; $v = 20Hz$.



a b

Fig. 3. The dependence of the oscillation amplitude on the outlet diameter:
a – for different properties of BM; b – at different oscillation frequencies of the hopper

Analysis of graphical dependences (Fig. 3, *a*) confirms that with an increase in the unloading outlet diameter at constant other hopper parameters it is necessary to provide a smaller amplitude oscillation to ensure the material leakage. Thus, at very small values of the unloading hole diameter of $D \leq 0.03$ m, the required amplitude reaches its critically high value a ϕ 4 mm due to a significant compaction of the BM in the unloading zone. Such modes are not recommended due to a significant dynamic loading on equipment.

The magnitude of the unloading hole amplitude is also influenced by adhesion forces in BM. Increase in adhesion forces leads to the higher required value of the amplitude with the constant outlet diameter. Based on the developed mathematical model, it is confirmed (Fig 3, *b*) that an increase in the outlet diameter D of the unloading hopper hole causes significantly a decrease in the value of the vibration acceleration $a\omega^2$. At the angle inclination of the hole walls $\theta = 45^0$ in the range $D \leq 0{,}4$ m, the oscillation amplitude is being decreased from 2.5 mm to several tenths of a millimetre at the oscillation frequency of $v = 20Hz$. At small values of the diameter D the oscillation amplitude can reach 6 mm. Therefore, analysing the graphical dependences (Fig. 3, b), it can be concluded that an increase in the outlet diameter leads to an increase in the area of a vibration propagation. This fact reduces the parameter $a\omega^2$ required to ensure the leakage of fine grained BMs.

As already noted in this study, the initial shear resistance affects the behaviour of the bulk product. Using the developed mathematical model, it is investigated an influence of the initial resistance of the BM on the unloading outlet diameter (Fig. 4).

The following parameters were used to obtain these dependencies: $\phi = 35^o$; $\phi^{'} = 30^o$; $\rho = 0{,}5 \cdot 10^3$ kg/m^3; $h_0 = D/\tan(\theta)$ m; $h_k = 0{,}45$ m.

In Fig. 3, *a* it is graphically presented the modelling results of an influence of the outlet size on the oscillation amplitude value. The following factors were used to obtain these dependencies: $\phi = 35^o$; $\phi^{'} = 30^o$; $\rho = 0{,}5 \cdot 10^3$ kg/m^3; $h_0 = D/\tan(\theta)$ m; $h_k = 0{,}45$ m; $v = 20Hz$.

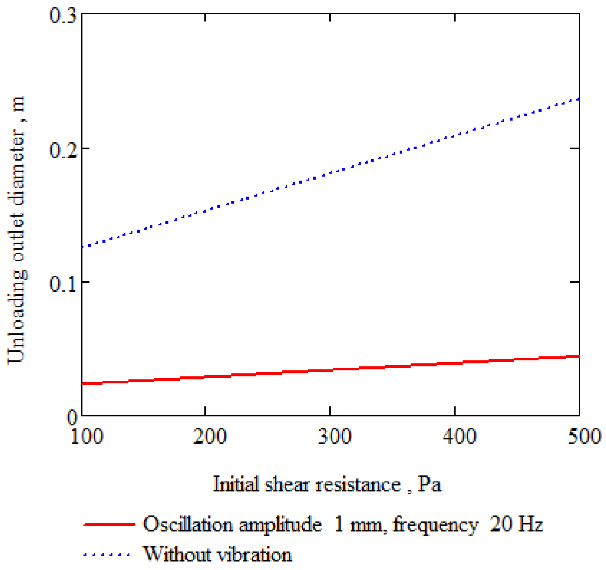


Fig. 4. Dependence of the outlet diameter on the initial shear resistance of the BM

In Fig. 4 it is established a dependence of the unloading outlet diameter on the initial shear resistance for two options: without using vibration and with vibration. It is investigated that in the absence of vibration the diameter increases from 0.12 m up to 0.23 m with increasing initial shear resistance within $100 \le \tau_o \le 500 Pa$. And the diameter changes only in the range $0{,}03 \le D \le 0{,}05$ m with using vibration with the amplitude 1 mm and the frequency 20 Hz. Thus, it is seen that the use of vibration significantly improves the BM flow and allows implementing the active stress state at lower values of the outlet diameter.

Conclusions

1. An absence of unloading hole vibration and an increase in the angle inclination of the walls from 0^o to 35^o causes the compaction of the BM in the leakage zone and an increase in the outlet diameter for both stress states of the BMs. Further an increase in the angle inclination of the walls does not affect the value of the hopper outlet size. However, in the passive stress state of the BM, the dependence of the outlet diameter on the angle inclination of walls is slightly less than in the active stress state.
2. It is necessary to increase the unloading outlet diameter in case not to use additional devices to improve conditions of BM leakage from the hopper, to ensure the active stress state, and, accordingly, to ensure the bulk product leakage.
3. An increase in the outlet diameter significantly reduces the intensity of vibration to ensure continuous leakage of the fine grained BM from the hopper.
4. The dependence of the unloading outlet diameter on the initial shear resistance of the fine grained BM is 4… 5 times higher without using vibration than with the vibrating hole.
5. The use of vibration significantly improves conditions of the fine grained BM leakage from the hopper and allows ensuring the continuous leakage at lower values of the unloading outlet diameter.

References

[1] P. Lin S. Zhang J. Qi et al., “ Numerical study of free-fall arches in hopper flows ”, *Physica A: Statistical Mechanics and its Applications*, vol. 417, pp. 29–40, 2015. https://doi.org/10.1016/j.physa.2014.09.032.

[2] A. Cyganiuk and P. Kurylo, “Modelling of the flow of streams of cohesionless and cohesive bulk materials in a conveyor discharge point with a flat conveyor belt”, *IJAME*, vol. 23, no. 1, pp. 24–35, 2018. https://doi.org/10.1515/ijame-2018-0002.

[3] G. Kache and J. Tomas, “Ausfliessen eines kohäsiven, hochdispersen pulvers” [“Outflow of the cohesive fine grained powder”], *Süttgut [Loses Material]*, vol. 6, pp. 246–252, 2010.

[4] T. Kollmann, “Schwingungsinduziertes Fließen feinstkörniger, kohäsiver Pulver” [“Vibration induced flow of fine grained, cohesive powders”], Sc.D. dissertation, Otto Von Guericke University Magdeburg, Magdeburg, 2002. [in German].

[5] N. I. Maherus, “Vplyv parametriv vibratsii ta heometrii lunky na rukh dribnodyspersnoho sypkoho materialu u konichnii luntsi dozatora” [“Influence of vibration parameters and hopper geometry on the movement of fine bulk material in the conical hopper hole”], *Vibratsii v tekhnitsi ta tekhnolohiiakh [Vibrations in technique and technologies]*, vol. 2, pp. 71–78, 2014. [in Ukrainian].

[6] K. Hashemnia and S. Pourandi, “Study the effect of vibration frequency and amplitude on the quality of fluidization of a vibrated granular flow using discrete element method”, *Powder Technology*, vol. 327, pp. 335–345, March 2018, https://doi.org/10.1016/j.powtec.2017.12.097.

[7] S. V. Vladimirov, “Protsesy fasuvannia krupiv kharchovykh produktiv i rozrobka konstruktsii obladnannia” [“Packing processes of food groats and development of the equipment designs”], Ph.D. dissertation, Mykhailo Tuhan-Baranovsky Donetsk National University of Economics and Trade, Donetsk, Ukraine, 2008. [in Ukrainian].

[8] J. Du, C. Liu et al., “Discharge of granular materials in a hemispherical bottom silo under vertical vibration”, *Powder Technology*, vol. 372, pp. 128–135, July 2020, https://doi.org/10.1016/j.powtec.2020.06.006.

[9] Y. Sholoviy, N. Maherus, et al., “Modeling of the finely-dispersed no coherent material flow from the loading hopper under vibration”, in *Proc. 18th Conf. on Optical Fibers and Their Applications*, Naleczow, 2019, pp. 1104508.

[10] A. V. Katalymov and V. A. Liubartovych, *Dozuvannia viazkykh ta sypkykh materialiv [Dosage of viscous and bulk materials]*. Lviv, Ukraine: Khimiia Publ., 1990. [in Ukrainian].

[11] G. Kache, “Verbesserung des Schwerkraftflusses kohäsiver Pulver durch Schwingungseintrag” [“Improving the gravity flow of cohesive powders through vibration input”], Sc.D. dissertation, Otto Von Guericke University Magdeburg, Magdeburg, Germany, 2009. [in German].