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Volodymyr Zelenyak[1], Liubov Kolyasa[2], Myroslava Klapchuk[3]

[1] Department of Mathematics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: volodymyr.zelenyak@gmail.com; ORCID 0000-0002-6653-4326
[2] Department of Mathematics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: kolyasa.lubov@gmail.com ORCID 0000-0002-9690-8042
[3] Department of Mathematics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: m.klapchuk @gmail.com ORCID 0000-0003-4826-0824

MATHEMATICAL MODELING OF THERMOELASTIC STATE IN A THREE-COMPONENT PIECEWISE-HOMOGENEOUS PLATE CONTAINING A CRACK

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Abstract. *Purpose*. A two-dimensional mathematical model of the problem of thermoelasticity for three-component plate containing a crack has been built. The stress intensity coefficients in the vertices of the crack increase affecting strength of the body significantly. This leads to the growth of a crack and, as a result, to further local destruction of a material. Therefore, such a model reflects, to some extent, the destruction mechanism of the elements of engineering structures with cracks. Graphic dependences of stress intensity factors (SIFs) at the tops of the crack have been built. This would make it possible to obtain the critical values of constant temperature in the two joined dissimilar elastic half-planes containing an inclusion and a crack in order to prevent crack growth, which would not allow the local destruction of the body. *Methodology.* Based on the method of the function of a complex variable we have studied the two-dimensional thermoelastic state for body with crack as stress concentrators. As result, the problem of thermoelasticity was reduced to a system of two singular integral equations (SIE) of the first and second kind, a numerical solution of which was found by the method of mechanical quadratures. The two-dimensional mathematical model of the thermoelastic state has been built in order to determine the stress intensity coefficients at the top of the crack and inclusion. The systems of singular integral equations of the first and second kinds of the specified problem on closed (contour of inclusion) and open (crack) contours are constructed. Numerical solution of the integral equations in the case of constant temperature in the two joined dissimilar elastic half-planes containing the crack and an inclusion was obtained by the mechanical quadrature method. Influence of thermophysical and mechanical properties of an inclusion on the SIF sat the crack types was investigated. Graphic dependences of the stress intensity factors which characterize distribution of the intensity of stresses at the vertices of a crack have been built, as well as on its elastic and thermoelastic characteristics of inclusion. This would make it possible to analyze the intensity of stresses in the neighborhood of a crack vertices depending on the geometrical and mechanical factors. As a result, this allow to determine the critical values of temperature in the three-component plate containing a crack in order to prevent the growth of the crack, as well as to prevent the local destruction of the body. It was found that that the appropriate selection of mechanical and thermophysical characteristics of the components of a three-component plate containing a crack can be useful to achieve an improvement in body strength in terms of the mechanics of destruction by reducing stress intensity factors at the crack's vertices. *Originality*. The solutions of the new two-dimensional problem of thermoelasticity for a specified region (a two joined dissimilar elastic half-planes containing inclusion and a crack) due to the action of constant temperature is obtained. The studied model is the generalization of the previous models to determine the two-dimensional thermoelastic state in a piecewise-homogeneous plate weakened by internal cracks. *Practical value*. The practical application of this model is a more complete description of the stress-strain state in piecewise homogeneous structural elements with cracks operating under temperature loads. The results of numerical calculations obtained from the solution of systems of equations and presented in the form of graphs can be used in the design of rational modes of operation of structural elements. This takes into account the possibility of preventing the growth of cracks by appropriate selection of composite components with appropriate mechanical characteristics.

Keywords: crack, inclusion, thermoelasticity, stress intensity factor, singular integral equation.

Introduction

Many modern structures contain components that often have to work under the influence of thermal heating, as a result of which temperature stresses appear in them. This is especially typical for some tools and structures of the heating industry. The efficiency of those elements is often largely determined by the intensity of stresses in some areas and the level of their concentration, for example, in the vicinity of technological in homogeneities (cracks, inclusions). In this case, the destruction of materials is associated with the presence of sharp stress concentrators such as cracks. The study of the thermoelastic state near the crack is necessary in order to calculate the strength from the standpoint of fracture mechanics. This is especially important for structures made of high-strength as well as low-plastic materials that can be subjected to various types of thermal loads. Therefore, it is important as theoretically, as well as practically studying the distribution of stresses in the vicinity of stress concentrators such as cracks. The stress intensity at the crack vertices is expressed in terms of stress intensity coefficients (SICs). Analysis of these parameters allows determining the limit value of heat load at which the crack begins to grow, while the body begins to collapse locally.

Therefore, such studies are important for calculating the strength in terms of fracture mechanics. In particular, in the case of piecewise homogeneous bodies with a crack, the stress intensity coefficients can be reduced by selecting the appropriate mechanical and thermophysical characteristics of the composite components themselves.

Literature Review

Investigations of the problem of thermoelasticity in the two-dimensional case for piecewise homogeneous bodies with cracks by the method of singular integral equations have already been described in the literature before. In particular, we studied the thermoelastic state in a finite [1], semi-finite [2] and infinite [3] flat region with foreign inclusions and cracks, in a plate with a circular two-component compound inclusion and crack [4]. The plane thermoelastic state in the case of half-space, which is locally heated under the action of the heat flux of its free surface and contains inclusions and cracks, was also investigated by the method of singular integral equations (SIEs). Using the method of functions of the complex variable, the SIEs solved the problems of thermal conductivity and thermal elasticity for a plane with thermally insulated or thermally conductive cracks located in a circular foreign inclusion [6], and for bodies with thermal cylindrical inclusion [7] and cracks. Based on the boundary method [8] and the Fourier transform method [9], the thermoelasticity problem for a crack plane were solved.

The review of the main literature sources showed that the mathematical models used to study the thermoelastic interaction of a crack with a curvilinear line connecting two dissimilar media have remained little studied. Therefore, it is necessary to build mathematical models to determine the thermal loads when the growth of the crack would begin and the body starts to locally collapse. The study of such models will make it possible to offer one of the approaches to prevent the growth of cracks, for example by selecting the components of a piecewise-homogeneous plate with appropriate mechanical and thermophysical characteristics.

The model for a circular disk with an inclusion and a crack considered below is of great practical importance for the calculation of the thermoelastic state in composite materials, taking into account the different stress concentrators in them. Such materials are often used to make structural elements used in construction, mechanical engineering and other industries.

Problem Statement

Consider an infinite body (plane), which consists of two elastic heterogeneous bodies (half-planes) S^+ and S^- with a line of junction L_0 and shear modules G_+ and G_-. The lower half-plane contains a circular inclusion bounded by a contour L_1 with a shear modulus G_1 and a crack L_2. We assume that the contours L_0, L_1, L_2 have no common points and a positive direction by passing the contours, is such that the inclusion or the upper half-plane remains on the left (Fig. 1). Contours $L_n (n = \overline{1,2})$ are assigned to local coordinate systems $x_n O_n y_n$ whose axes $O_n x_n$ are parallel to the axis O_x that coincides with the contour L_o. The points O_n determine in the basic system of Cartesian coordinates xOy complex coordinates $z_n^0 = x_n^0 + i \cdot y_n^0$, and the relationship between the coordinates of points on the plane give the relationship $z = z_n + z_n^0$; $z = x + i \cdot y$; $z_n = x_n + i \cdot y_n$.

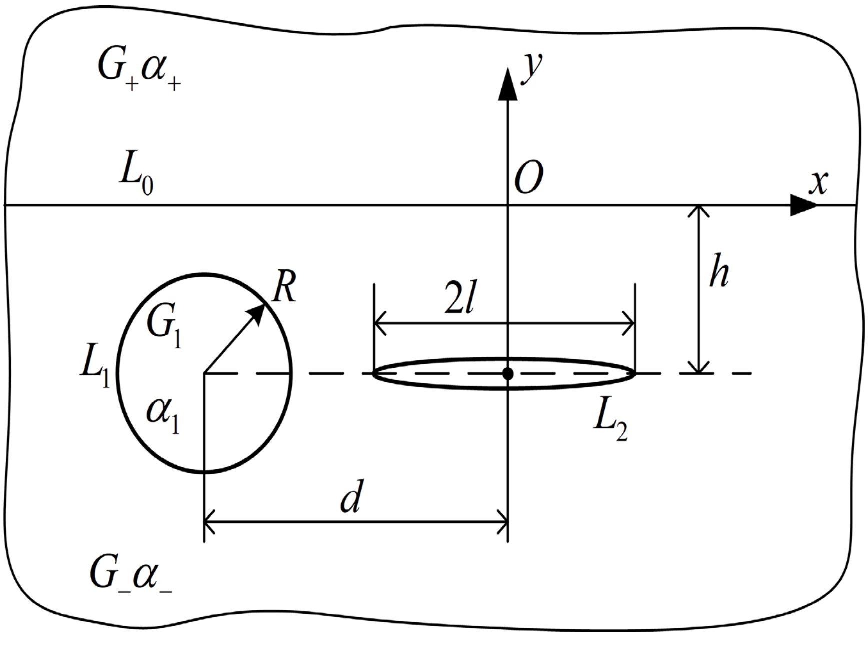

Fig. 1. Geometry of soldered heterogeneous half-planes with a circular inclusion and a crack

Let the folded plane with the inclusion and the crack be under a stationary temperature field $T(x, y) = T_c = const \neq 0$. Then let's suppose that the stationary temperature $T_c = 0$ corresponds to the state when the stresses in the whole piecewise-homogeneous plane with a crack are zero.

In research we assumed that the conditions of ideal mechanical contact (equality of stresses and displacements in the approach to the left and right to the contour) are set on the contour of inclusion L_1 and the contour of the junction of half-planes L_0.

$$[N(t_1)+iT(t_1)]^+ = [N(t_1)+iT(t_1)]^-,$$
$$[N(t_n)+iT(t_n)]^+ = [N(t_n)+iT(t_n)]^-, \quad n=\overline{0,1}, \tag{1}$$

$$(u_n+iv_n)^+ - (u_n+iv_n)^- = 0, \quad t_n \in L_n,\ n=\overline{0,1}. \tag{2}$$

The crack's contours L_2 do not come into touch during the process of deformation and there are no forced loads on them.

$$[N(t_2)+iT(t_2)]^{\pm} = 0 \ t_n \in L_n,\ n=2. \tag{3}$$

We also believe that the temperature coefficients of linear expansion (TCLE) of half-planes are equal to $(a_+^t = a_-^t)$, which allows, in the absence of force loads, to satisfy the conditions of ideal mechanical contact at the junction line of half-planes at infinity. If equality $(a_+^t = a_-^t)$ is not satisfied, then in this problem, in addition to the heat load, one should set such effortsat infinity that would allow to satisfy these conditions.

Main Material Presentation

Complex potentials can be chosen in the following form [10]:

$$\Phi(z) = \Phi_1(z)+\Phi_2(z);$$
$$\Psi(z) = \Psi_1(z)+\Psi_2(z), \tag{4}$$

where

$$\Phi_1(z) = \frac{1}{2\pi}\sum_{k=1}^{2}\int_{L_k}\frac{Q_k(t_k)dt_k}{\zeta_k - z};\ Q_k(t_k) = \begin{cases} g_1(t_1), & t_1 \epsilon L_1, \\ g_2'(t_2), & t_2 \epsilon L^- \end{cases};\ \zeta_k = t_k + z_k^0,$$

$$\Psi_1(z) = \frac{1}{2\pi}\sum_{k=1}^{2}\int_{L_k}\left[\frac{\overline{Q_k(t_k)dt_k}}{\zeta_k - z} - \frac{\overline{\zeta_k}Q_k(t_k)dt_k}{(\zeta_k - z)^2}\right],$$

$$\Phi_2(z) = \frac{1-\Gamma_0}{2\pi(1+\chi_-\Gamma_0)}\sum_{k=1}^{2}\int_{L_k}\left[\frac{Q_k(t_k)dt_k}{z-\overline{\zeta_k}} + \frac{(\zeta_k - \overline{\zeta_k})\overline{Q_k(t_k)dt_k}}{(\overline{\zeta_k}-z)^2}\right];$$

$$\Psi_2(z) = \frac{1-\Gamma_o}{2\pi(1+\chi_-\Gamma_o)}\sum_{k=1}^{2}\int_{L_k}\left\{\begin{aligned} &\frac{\overline{\varsigma_k}Q_k(t_k)dt_k}{(\overline{\varsigma_k}-z)^2} + \\ &+\left[\frac{(\overline{\varsigma_k}-\varsigma_k)(\overline{\varsigma_k}+z)}{(\overline{\varsigma_k}-z)^3} - \frac{1}{\overline{\varsigma_k}-z}\right]Q_k(t_k)dt_k \end{aligned}\right\}, z\in S^-, \varsigma_k \in S^-.$$

Here, $g_1(t_1)$ is unknown functionon the inclusion's contour L_1; $g_2'(t_2)$ – unknown function on the crack's contour L_2 (the derivative of unknown jump of displacements when crossing the crack line). The function $g_2'(t_2)$ must have integrative features at the ends of the crack. $\Gamma_0 = G_+/G_-$; G_+ (G_-) are the shear modules for of the upper (lower) half-plane. The complex potentials $\Phi_1(z)$, $\Psi_1(z)$, $\Phi_2(z)$, $\Psi_2(z)$ characterize the perturbed thermal stress state due to an inclusion and a crack.

Note that the choice of complex potentials in the form of (4) provides exact satisfaction of the boundary condition (1) on the contour L_0. As a result, the unknown function on the contour L_0 is excluded, and the order of the SIEs, which we obtain after satisfying the other boundary conditions, has been reduced from the third to the second order.

Satisfying using potentials (4) the second equality of the boundary condition (1) on the inclusion's contour L_1 and the boundary condition (2) on the crack's contour L_2 we obtain a system of two singular integral equations of the second and first kind with respect to two unknown functions $Q_1(t_1)$ and $Q_2(t_2)$ on the contours L_1 and L_2

$$\begin{aligned} &A_1 Q_1(\tau_1) + \frac{1}{2\pi}\int_{L_1}\left[R_{11}(t_1,\tau_1)\,Q_1(t_1)dt_1 + S_{11}(t_1,\tau_1)\overline{Q_1(t_1)}\ \overline{dt_1}\right] + \\ &+\frac{1}{2\pi}\int_{L_2}\left[R_{12}(t_2,\tau_1)\,Q_2(t_2)dt_2 + S_{12}(t_2,\tau_1)\overline{Q_2(t_2)}\ \overline{dt_2}\right] = P_1(\tau_1), \quad \tau_1 \in L_1 \quad ; \\ &\frac{1}{2\pi}\int_{L_1}\left[R_{21}(t_1,\tau_2)\,Q_1(t_1)dt_1 + S_{21}(t_1,\tau_2)\overline{Q_1(t_1)}\ \overline{dt_1}\right] + \\ &+\frac{1}{2\pi}\int_{L_2}\left[R_{22}(t_2,\tau_2)\,Q_2(t_2)dt_2 + S_{22}(t_2,\tau_2)\overline{Q_2(t_2)}\ \overline{dt_2}\right] = P_2(\tau_2), \quad \tau_2 \in L_2\,. \,, \end{aligned} \tag{5}$$

where

$$R_{nk}(t_k,\tau_n) = R^1_{nk}(t_k,\tau_n) - \frac{1-\Gamma_0}{1+\chi_-\Gamma_0}\left\{\frac{B_n}{\overline{\overline{T_{nk}}}} + \frac{C_n\left(\overline{\zeta_k} - \zeta_k\right)}{T^2_{nk}} + \right.$$

$$\left. + e^{-2i\alpha_n}\,\frac{\overline{d\tau_n}}{d\tau_n}C_n\cdot\frac{\left(2\eta_n - \zeta_k - \overline{\eta_n}\right)\left(\overline{\zeta_k} - \zeta_k\right)}{T^3_{nk}} - \frac{1}{T_{nk}}\right\};$$

$$S_{nk}(t_k,\tau_n) = S^1_{nk}(t_n,\tau_n) + \frac{1-\Gamma_0}{1+\chi_-\Gamma_0}\left[\frac{B_n(\zeta_k - \overline{\zeta_k})}{\overline{T^2_{nk}}} + \frac{C_n}{T_{nk}} - e^{-2i\alpha_n}\frac{\overline{d\tau_n}}{d\tau_n}C_n\cdot\frac{H_{kn}}{T^2_{nk}}\right],$$

$$R^1_{nk}(t_k,\tau_n) = \frac{B_n}{H_{kn}} - \frac{C_n}{\overline{\overline{H_{kn}}}}\cdot e^{-2i\alpha_n}\,\frac{\overline{d\tau_n}}{d\tau_n};$$

$$S^1_{nk}(t_k,\tau_n) = -C_n\left[\frac{1}{\overline{\overline{H_{kn}}}} - e^{-2i\alpha_n}\,\frac{\overline{d\tau_n}}{d\tau_n}\,\frac{H_{kn}}{\overline{H_{kn}}^2}\right];$$

$$H_{kn} = \zeta_k - \eta_n; \qquad T_{nk} = \zeta_k - \overline{\eta_n}\,,\ \eta_n = \tau_n e^{i\alpha_n} + z^0_n\,,\ (k=1,2;\ n=1,2);$$

$$A_1 = i\lfloor 1 + \chi_1 + \Gamma_1(1+\chi_-)\rfloor/2\,,$$

$$\Gamma_1 = G_1/G\_C_n = (2-\Gamma_1)\delta_n - 1\,;$$

$$B_n = (\chi_1 - \Gamma_1\chi_- - 1)\delta_n + 1\,;\ P(\tau_n) = \delta_n\left[\left(\Gamma_1\beta^t_- - \beta^t_1\right)T_c\right];\ \delta_n = \begin{cases}1, n=1\\ 0, n=\overline{2,N}\end{cases},$$

$$\beta^t_- = \alpha^t_- E_-/\left(1+\mu_-\right);\quad \beta^t_1 = \alpha^t_1 E_1/\left(1+\mu_1\right).$$

α^t_-, G_-, E, μ $\left(\alpha^t_1,\ G_1,\ E_1, \mu_1\right)$ are the temperature coefficient of linear expansion, the shear module, the elasticity Young module, the Poisson coefficient of the lower half-plane (respectively, inclusion).

The system of equations (4) has an unique solution for its arbitrary right-hand side, provided the following condition^

$$\int_{-1}^{1} g'_2\left(t_2\right)dt_2 = 0\,, \tag{6}$$

which provides un ambiguous movements when by-passing the contour of the crack.

Determining the unknown functions $Q_1(t_1)$ and $Q_2(t_2)$ from the system of equations (5) (6), we can further find the distribution of thermal stresses in the whole piecewise homogeneous plate with a crack. Therefore the stress intensity coefficients (SICs) K_I, K_{II} in the crack vertices are found by the formula [10]

$$K^{\pm}_I - iK^{\pm}_{II} = \mu \lim_{t_2\to l^{\pm}_2}\left[\sqrt{2\pi\left|t_2 - l^{\pm}_2\right|}\,Q_2\left(t_2\right)\right].$$

Here indexes "–" refer to the top of the crack ($t_k = l_k^-$), and"+" – to its end ($t_k = l_k^+$). The numerical values of the stress intensity coefficients K_I, K_{II} are real quantities that characterize the stressed-strained state in the neighborhood of the crack's vertices.

Results and Discussion

Let the unloaded crack of length $2l$ with the center at the point $(0,-h)$ be parallel to the junction line l_0 of the half-planes. The center of the circular inclusion of the radius R is on the crack line at a distance d from the center of the crack, the axes of local coordinate systems $O_n x_n$ are parallel to the axis Ox (Fig. 1). We believe that the conditions of ideal mechanical contact are fulfilled on the inclusion contour l_1. The solution of the system of equations (5) under condition (6) is found numerically by the method of mechanical quadratures [11].

Graphic dependences for the dimentionless stress intensity coefficients $F_1^- = K_I^- / K_T$ and $F_2^- = K_{II}^- / K_T = 0$ ($K_T = T_c \beta_-^{\,2} \sqrt{\pi l} / (1 + \chi_-)$) in the left crack top $x_2 = -l$ closer to the inclusionare obtained for different values of elastic, thermophysical and geometric parameters of the piecewise-homogeneous plane, when $\frac{l}{h} = 0{,}5$; $\frac{d}{h} = 2$; $\frac{a}{h} = 0{,}5$; $\chi_+ = \chi_- = 2$ Lines 1 correspondtothevalue $G_1 / G_- = 0{,}5$ lines 2 – $G_1 / G_- = 1$; lines 3 – $G_1 / G_- = 2$ (Fig. 2 - 5, 8) and, respectively, lines 1 correspond to the value $G_1 / G_- = 0{,}5$; lines 2 – $G_1 / G_- = 1$; lines 3 – $G_1 / G_- = 2$ (Fig. 6 - 7).

When the temperature coefficient of the linear expansion of the inclusion α_1^t increases, thentheSIC F_1^- increases linearly with increasing ratio α_1^t / α_-^t, and increasing the stiffness of the inclusion (shear modulus G_1) significantly enhances the growth of SIC F_1^-, and increasing the stiffness of the upper half-plane (shear modulus G_+) increases SIC F_1^- slightly. In this case, if $\alpha_1^t \; π \; \alpha_-^t$, then SIC $F_1^- \; π \; 0$, and when $\alpha_1^t \; ϕ \; \alpha_-^t$, then SIC $F_1^- \; ϕ \; 0$ (Fig. 2).

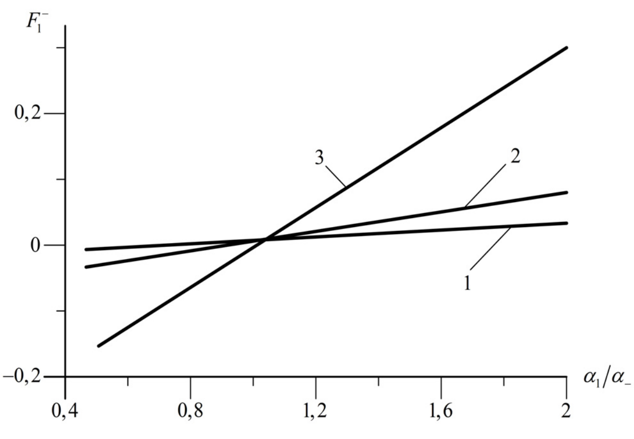

Fig. 2. Dependence of SIC F_1^- on α_1 / α_- at $G_+ / G_- = 0{,}5$

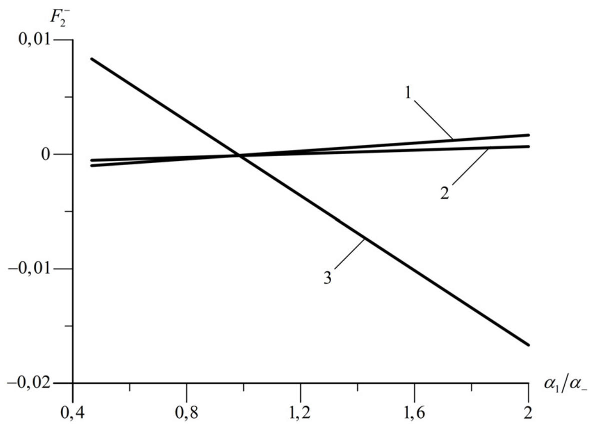

Fig. 3. Dependence of SIC F_2^- on α_1 / α_- at $G_+ / G_- = 0{,}5$

The values of SIC F_2^- are much smaller in absolute value than the SIC F_1^- for the same values of the parameters of the problem (Fig. 3,4,7,8). If the the temperature coefficient of the linear expansion of the inclusion is greater than the lower half-plane ($\alpha_1 \; ϕ \; \alpha_-$), then increasing the stiffness of the inclusion (shear modulus G_1) causes a nonlinear increase in SIC F_1^- (Fig. 6), and increasing the stiffness of the upper half-plane (shear modulus G_2) slightly increases (decreases) SIC F_1^- if the stiffness of inclusionis less than or equal to) than the stiffness of the lower half-plane (Fig. 5)

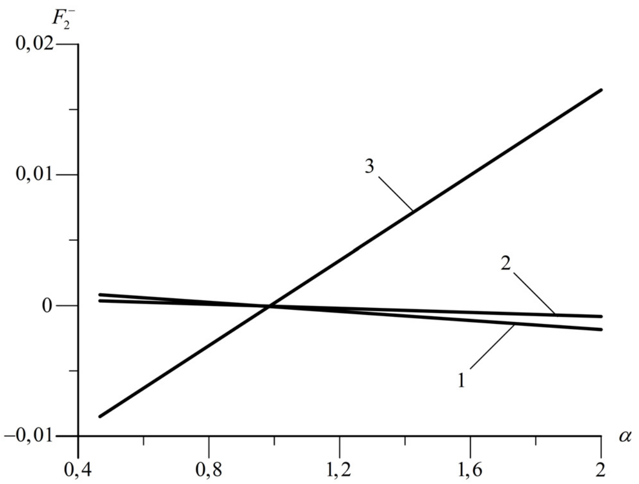

Fig. 4. Dependence of SIC F_2^- on α_1 / α_- at $G_+ / G_- = 2$

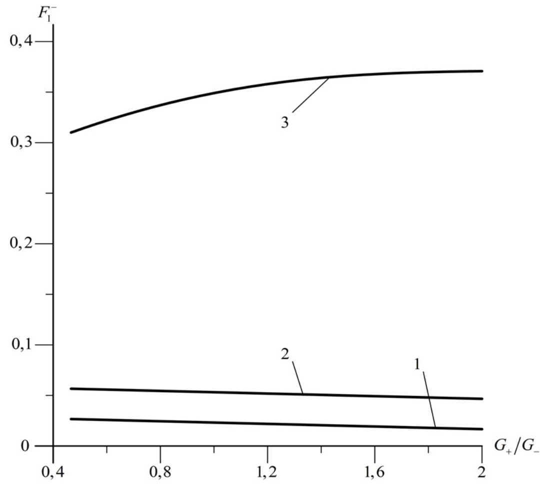

Fig. 5. Dependence of SIC F_1^- on G_+ / G_- at $\alpha_1 / \alpha_- = 2$

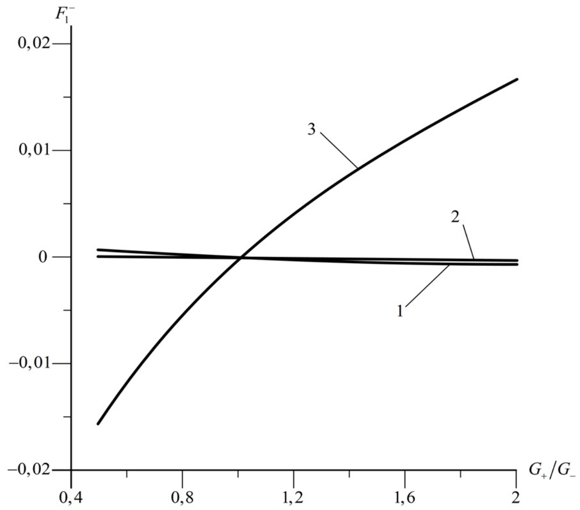

Fig. 6. Dependenceof SIC F_1^- on G_1 / G_- at $\alpha_1 / \alpha_- = 2$

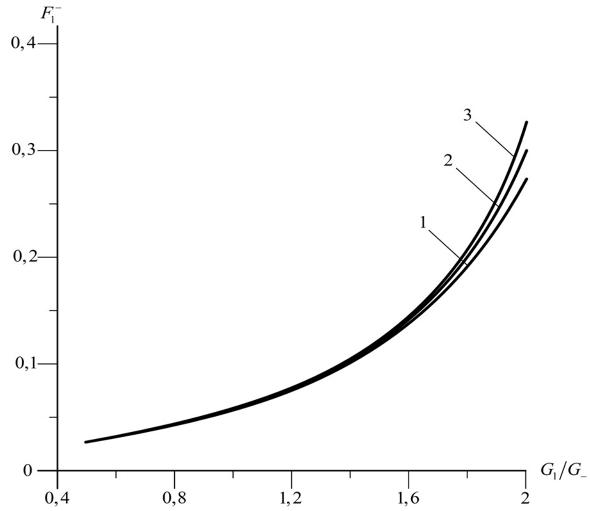

Fig. 7. Dependenceof SIC F_2^- on G_1 / G_- at $\alpha_1 / \alpha_- = 2$

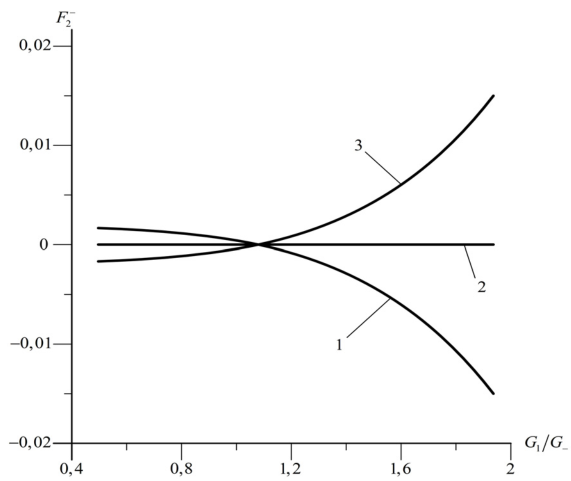

Fig. 8. Dependenceof SIC F_2^- on G_+ / G_- at $\alpha_1 / \alpha_- = 2$

Note that here it isn't taking into account the possible contact of the cracks. Therefore, in some cases, the SIC $F_I^{\pm}$ can acquirenegative values, which we do not take into account. But this result can also be used to obtain information about the compressive normal stresses in the vicinity of the crack vertices, which inhibits crack growth and local destruction of the body. If it is necessary to take into account the contact, the problem should be formulated as a mixed problem on the contours of the crack. Its solution is much more complicated, but nevertheless the solution can also be obtained by the method of singular integral equations.

In this problem, the lips of the crack do not touch each other. Then, according to the criterion, taking into account the hypothesis of the initial crack growth, from the boundary equilibrium [12] it is possible to find the critical values of the temperature T_{cr} when the growth of the crack and the local destruction of the body begin, according to the formula

$$T_{cr} = \frac{K_{1c}}{F_1^{\pm}}. \tag{7}$$

Here K_{1C} is a constant of the material that characterizes the resistance of the material to the destruction and is determined experimentally.

Based on the analysis of numerical results for SIC F_1^{-} from formula (7) the following conclusions follow. When the temperature coefficient of the linear expansion of the inclusionis greater than the lower half-plane ($\alpha_1 \phi \alpha_-$), then the increase in the stiffness of the inclusion (shear modulus G_1) causes a decrease in the critical temperature to start the crack growth from the left top (closest to the inclusion) (Fig. 6). A similar situation is observed with increasing stiffness of the upper half-plane in the case of more rigid inclusion than the lower half-plane (Fig. 5). An increase of the temperature coefficient of the linear expansion of the inclusion also causes a decrease in the critical temperature to begin crack growth.

Conclusions

1. A two-dimensional mathematical model has been constructed forthe problemof stationary thermoelasticity for a three-component piece-homogeneous plate with a crack,in the form ofSIEs of the first kind on the crack's contour and the second kind on the inclusion's contour. That allows to obtain the solution of the integral equation with high accuracy in comparison with the asymptotic method of a small parameter, which can be used only for certain types of integral equations.

2. Numerical solution of SIE systems is obtained in the partial case of soldered heterogeneous half-areas with a circular inclusion and a crack when we have a uniform temperature distribution in the whole piecewise homogeneous plate with crack $T(x, y) = T_c = const \neq 0$. Using this solution, stress intensity coefficients (SICs) at the crack vertices were calculated, which can be apply to determining the critical value of the constant temperature in the plate at which the crack begins to grow.

3. Graphic dependence of stress intensity coefficients on inclusion characteristics is obtained; in particular, it is found that the appropriate selection of mechanical and thermophysical characteristics of inclusion can determine the creation of compressive or tensile normal stresses around crack vertices, which can be used to develop rational modes of structural elements in terms of preventing crack growth.

References

[1] V. M. Zelenyak Thermoelastic equilibrium of a three-layer circular hollow cylinder weakened by a crack. *MaterialsScience*. 52, no. 2. pp. 253–260, 2016. https://doi.org/10.1007/s11003-016-9952-y.

[2] H. Sekine Thermal stress around a ribbon like inclusion in a semi–infinite medium under uniform heat flow. *Journal of Elasticity Applied Mathematics and Mechanics*, 8(1), pp. 81–95, 1978. https://doi.org/10.1007/s11003-016-9952-y.

[3] N. Hasebe, X. Wang, T. Saito, W. Sheng Interaction between a rigid inclusion and a line crack under uniform heat flux. *Int. j. Solids Struct*. 44. pp. 2426–2441, 2007. https://doi.org/10.1016/j.ijsolstr.2006.07.016.

[4] V. M. Zelenyak Thermoelastic interaction of a two-component circular inclusion with a crack intheplate. *Materials Science*.48, no. 3, pp. 301–307, 2012. https://doi.org/10.1007/s11003-012-9506-x.

[5] S. Matysiak, O. O. Evtushenko, V. M. Zeleniak Heating of a half-space containing an inclusion and a crack. *Materials Science*. 40, no. 4, pp. 467–474, 2004. https://doi.org/10.1007/s11003-005-0063-4.

[6] G. S. Kit, M.G. Krivtsun Plane thermoelasticity problem for bodies with cracks.*Naukova dumka*, 1983. [in Russian].

[7] H. S. Kit, M. S. Chernyak Stressed state of bodies with thermal cylindrical inclusions and cracks plane deformation. *Materials Science.* 46(3), pp. 315–324, 2010. https://doi.org/10.1007/s11003-010-9292-2.

[8] H. Chen, Q. Wang, G. Liu, J. Sun Simulation of thermoelastic crack problems using singularedge-based smoothed finite element method. *Int. J. of Mech. Sci.*, 115, 116, pp. 23–134, 2016. https://doi.org/ 10.1016/j.ijmecsci.2016.06.012.

[9] H. J. Choi Thermoelastic interaction of two offset interfacial crack sin bonded dissimilar half-planes with a functionally graded interlayer. *Acta Mechanica,* 225(7), pp. 2111–2131, 2014. https://doi.org/10.1007/s00707-013-1080-2.

[10] M. P. Savruk, *Dvumernye zadachi uprugosti dlya tel s treshchinami* [*Two-dimensional elasticity problems for bodies with cracks*],Kyiv: Naukova dumka, 1981, 324 p. [in Russian].

[11] F. Erdogan, G. D. Gupta, T. S. Cook The numerical solutions of singular integral equations Methods of analysis and solutions of crack problems. – Leyden: *Noordhoff Intern. Publ*.(*Mechanics of fracture; 1).*, pp. 368–425, 1973.

[12] V. V. Panasyuk, M. P. Savruk, and A. P. Datsyshin, *Raspredelenie napryazheniy okolo treshchin v plastinah I obolochkah* [*Distribution of stresses near cracks in plates and shells*], Kyiv: Naukova dumka, 1976, 444 p. [in Russian]. https://doi.org/10.1007/978-94-017-2260-5_7.