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Investigation of drying the porous wood of a cylindrical shape

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In the presented study, the mathematical model for drying the porous timber beam of a circular cross-section under the action of a convective-heat nonstationary flow of the drying agent is constructed. When solving the problem, a capillary-porous structure of the beam is described in terms of a quasi-homogeneous medium with effective coefficients, which are chosen so that the solution in a homogeneous medium coincides with the solution in the porous medium. The influence of the porous structure is taken into account by introducing into the Stefan–Maxwell equation the effective binary interaction coefficients. The problem of mutual phase distribution is solved using the principle of local phase equilibrium. The given properties of the material (heat capacity, density, thermal conductivity) are considered to be functions of the porosity of the material as well as densities and heat capacities of body components. The solution is obtained for determining the temperature in the beam at an arbitrary time of drying at any coordinate point of the radius, thermomechanical characteristics of the material, and the parameters of the drying agent.

Keywords: *plane problem of heat conduction, cylindrical beam, drying agent, porous medium, quasi-homogeneous approximation, integral transform, Bessel functions of the first and second kind, Kontorovich–Lebedev transform, Steklov's theorem, Green's function, moving boundary.*

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1. Introduction

The processes of hydrothermal treatment of dehumidified materials are energy-intensive, and the use of heat and moisture treatment is the main means to create energy-saving technology for their drying. The application of enhanced systems increases drying efficiency by 26.5% time reduction [1]. Therefore, the search for such enhanced techniques is topical to a nowadays science of drying. The purpose of this study is to construct a mathematical model of the process of drying the cylindrical porous beam under the action of the convective-thermal non-stationary flow of the drying agent in order to minimize the energy costs of this process.

To control the process of drying we need to know the mechanism of moisture transfer inside the material. In [2], different models of moisture migration during the drying process of porous media as well as the numerical simulation used for their description, its restriction and applications are discussed. The collection of papers joined in [3] offers a comprehensive review of mechanisms in drying porous materials at the pore scale and macro scale levels, including various drying technologies, and discussions of the drying dynamics of fibrous porous material. In the case of wood, it is important to take into account both the outer shape of a body and the capillary-porous texture of its inner structure. The transfer of liquid occurs in the micropores of its cell membranes, where it condenses due to the fact that unsaturated vapor in the macropores becomes oversaturated in the micropores. In the cavities of cells, moisture moves in the form of steam [4].

During heat and moisture treatment of wood, its ability to take or give off moisture in the form of steam according to the state of the environment is used. If the water steam pressure in the drying agent is lower than the pressure in the wood, desorption occurs, i.e., the wood is dried. Otherwise, the wood absorbs water in the form of steam until the pressure is equalized and the balance of moisture in the wood and the environment sets in. In the process of drying, a zone consisting of dry pores emerges and those saturated with liquid, which are separated by the surface of the phase transition [5].

When the body is heated, moisture molecules near its surface acquire a greater speed and, overcoming the forces of molecular bonds, fly into the environment. At the beginning of oven-drying, the surface layer of the body comes into contact with the hot environment of the chamber and the moisture molecules attain a higher temperature than it is in the center of wood. A temperature gradient appears, which causes the flow of moisture towards low temperatures, and its place is substituted by steam. Therefore, when the wood is dried in a high temperature environment, it has an excess pressure of the steam-air mixture, which causes a steady movement of water vapor directed from the center to the surface of the material [5]. However, the issue of the wood shape influence its drying rate raises: whether the drying time of a wooden plate differs from that one of a cylindrical beam [6].

When solving the problem of drying objects with a capillary-porous structure, in particular wood, they can be described in terms of a quasi-homogeneous medium with effective coefficients, which are selected so that the solution in a homogeneous medium coincides with the solution in the porous medium [7, 8]. The finite-difference approximations for the implementation of mathematical models, which provides accounting for the eridarity and selforganization of the material also can be used [9, 10]. Researcers often use non-integer integro-differentiation to model systems, which are characterized by "memory" effects, structural heterogeneity, spatial non-locality, deterministic chaos, and self-organization [11,12]. We take into account the influence of the porous structure by introducing the effective binary interaction coefficients into the Stefan–Maxwell equation [13]. To solve the problem of mutual phase distribution we use the principle of local phase equilibrium. The given properties of the material, namely: heat capacity C, density ρ, thermal conductivity coefficients λ, we introduce into our calculations as functions of the porosity of the material as well as functions of densities and heat capacities of body components.

We assume the initial temperature of the cylinder does not depend on its length and changes within its cross-section only. Since the length of a wood column is much larger than the cross-sectional size and the coefficient of moisture conductivity is much greater than the coefficient across the fibers, and due to the great complexity of the structure of wood material, we limit ourselves to considering the plane average heat transfer problem. As a tool for describing heat conduction, we use differential equations in modeling nonstationary processes [14,15]. We use the method of integral transformations to find the solutions [16].

2. Problem formulation

Consider a long cylinder of the radius R ($0 \leqslant r \leqslant R$). Given the symmetry of the boundary conditions of this problem, we introduce a polar coordinate system (r, φ), the polar axis of which is directed along the axis of the cylinder. The cylinder is under the action of convective-thermal non-stationary flow of the drying steam-air agent of the velocity υ. We assume that the drying agent regime is three-stage, non-stationary, and includes heating, keeping, and cooling.

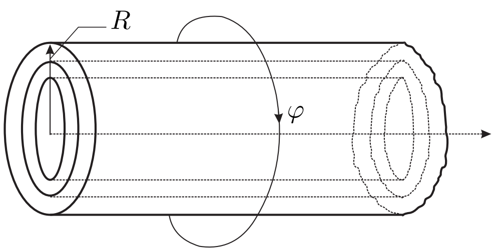

Fig. 1. Schematic representation of the wooden cylindrical beam.

The control parameter in this process is the temperature of the drying agent T_a. In convective drying, the heat supplied by the gas is used to evaporate the liquid, heat the material, and overcome the energy of moisture bonds with the material. We assume that the moisture in the dried area is removed and in the rest of the volume it is preserved, known and its density is ρ_L. The moisture content W retained in the body is calculated by the formula $W = \rho_L\left(\frac{V-V_m}{V}\right)$, where V is the body volume; V_m is the volume of the dried area. Note that when hot air contacts with moisture particles, the latter break down into steam and smaller liquid particles.

The process of heat conduction is described by the equation:

$$\left[\Pi(C_v\rho_v+C_a\rho_a)+(1-\Pi)C_s\rho_s\right]\frac{\partial T}{\partial\tau}+\gamma_1^2T = \lambda\left[r^2\frac{d^2T}{dr^2} + (2\alpha+1)r\frac{dT}{dr} + (\alpha^2-\lambda^2r^2)T\right],\ 2\alpha+1>0. \quad (1)$$

Here τ is time; r is the radius of running point$(0 \leqslant r \leqslant R)$; γ_1^2 is the particle decomposition coefficient.

Equation (1), using the Bessel differential operator, takes the form

$$B_\alpha[T] = \left[r^2\frac{d^2T}{dr^2} + (2\alpha+1)r\frac{dT}{dr} + (\alpha^2-\lambda^2r^2)T\right]$$

for the given volumetric heat capacity $c\rho$ and averaged thermal conductivity λ in the quasi-homogeneous approximation, which can be used in wood drying problems with acceptable temperature gradients, has the form [16]:

$$\frac{\partial T}{\partial\tau} + \gamma^2T = a^2B_\alpha[T,r], \quad \gamma^2 = \frac{\gamma_1^2}{c\rho}, \quad \alpha>0, \quad (2)$$

where $a^2 = \frac{\lambda}{\left[\Pi(C_v\rho_v+C_a\rho_a)+(1-\Pi)C_s\rho_s\right]}$ is the averaged thermal diffusivity coefficient.

Let us construct the solution of Eq. (2) under the following boundary conditions:

$$T(\tau,r)\big|_{\tau=0} = g(r), \quad r\in(0,R), \quad (3)$$

$$\lim_{r\to0}\frac{\partial}{\partial r}(r^\alpha T) = 0, \quad \left(\alpha_{11}^1\frac{\partial}{\partial r}+\beta_{11}^1\right)T\big|_{r\to R} = T_a(\tau). \quad (4)$$

Here T_a is the temperature of the drying agent; γ^2 is responsible for the multiplication of particles of the steam-air mixture (averaged coefficient of decomposition) in the porous material under the action of the drying agent; indices v, a, s indicate the components of steam, air, and skeleton, respectively; Π, C_v, C_a, C_s, ρ_v, ρ_a, ρ_s are porosity, heat capacity, and density of steam, air, and skeleton, respectively; λ is the averaged coefficient of thermal conductivity; α_{11}^1, β_{11}^1 are coefficients of thermal conductivity and heat transfer on the outer surface of the cylinder.

The scheme of $T_a(\tau)$ behavior is shown in Fig. 2

The temperature of the drying agent $T_a(\tau)$ is as follows:

$$T_a(\tau) = \begin{cases} T_0 + \dfrac{T_{\max}-T_0}{\tau_1}\tau, & 0\leqslant\tau\leqslant\tau_1, \\ T_{\max}, & \tau_1\leqslant\tau\leqslant\tau_2, \\ \dfrac{T_{\max}\tau_3 - T_1\tau_2}{\tau_3-\tau_2} - \dfrac{T_{\max}-T_1}{\tau_3-\tau_2}\tau, & \tau_2\leqslant\tau\leqslant\tau_3. \end{cases} \quad (5)$$

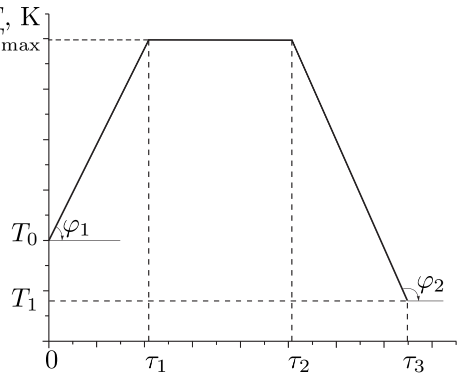

Fig. 2. Control function $T_a(\tau)$.

Here T_0 is the initial temperature of the drying agent; cooling is carried out until an equilibrium temperature is reached.

3. Solving the problem

Let us find a solution for the boundary value problem (2)–(4).

Consider the governing function $T_a(\tau)$.

We expand this function into a trigonometric Fourier series with respect to cosine:

$$T_a(\tau) = \alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos\frac{n\pi}{\tau_3}\tau, \quad \nu_n^2 = \frac{n\pi}{\tau_3};$$

$$\alpha_0 = \frac{2}{\tau_3}\left[T_{\max}\left(-\frac{\tau_1}{2} + \frac{\tau_2}{2} + \frac{\tau_3}{2}\right) + T_0\frac{\tau_1}{2} + T_1\left(-\frac{\tau_2}{2} + \frac{\tau_3}{2}\right)\right],$$

$$\alpha_n = \frac{2}{\tau_3}\left[\frac{(T_{\max} - T_0)}{\tau_1\nu_n^4}\left(\cos\nu_n^2\tau_1 - 1\right) + \frac{T_{\max}}{\nu_n^2}\left(\sin\nu_n^2\tau_2 - \sin\nu_n^2\tau_1\right)\right]$$
$$+ \frac{2}{\tau_3}\left\{-\frac{T_{\max}\tau_3 - T_1\tau_2}{\tau_3 - \tau_2}\frac{\tau_2}{n\pi}\sin\nu_n^2\tau_2 - \frac{T_{\max} - T_1}{\tau_3 - \tau_2}\frac{1}{\nu_n^4}\left[(-1)^n - \cos\nu_n^2\tau_2 - \frac{n\pi\tau_2}{\tau_3}\sin\nu_n^2\tau_2\right]\right\}.$$

Let $T^*(p,r)$ be the image of the Laplace transform of the temperature $T(\tau,r)$:

$$L\left[T(\tau,r)\right] = \int_0^{\infty} T(\tau,r)e^{-p\tau}d\tau = T^*(p,r).$$

Then, in accordance with the problem (1)–(4), we obtain the following boundary value problem with respect to the function $T^*(p,r)$:

$$\left(B_{\nu,\alpha} - \lambda^2\right)T^* = \frac{d^2T^*}{dr^2} + \frac{2\alpha+1}{r}\frac{dT*}{dr} - \left(\lambda^2 + \frac{\nu^2 - \alpha^2}{r^2}\right)T^* = -\tilde{g}(r), \tag{6}$$

$$\lim_{r\to 0}\frac{\partial}{\partial r}\left(r^{\alpha-\nu}T^*(p,r)\right) = 0, \quad \left(\alpha_{11}^1\frac{d}{dr} + \beta_{11}^1\right)T^*\big|_{r=R} = T_a^*(p), \tag{7}$$

$$\tilde{g}(r) = a^{-2}r^{-2}g(r), \quad \nu^2 = a^{-2}\left(p + \gamma^2\right), \quad p = \sigma + i\tau, \quad i^2 = -1.$$

Let us fix $\operatorname{Re}\nu = \operatorname{Re}\left[a^{-1}(p+\gamma^2)^{1/2}\right] > 0$. Construct a Cauchy function for Eq. (6) to satisfy homogeneous boundary conditions [14]. A fundamental function $\varepsilon_\alpha^*(p,r,\rho)$ satisfying the homogeneous equation corresponding to Eq. (6) and the homogeneous conditions corresponding to the conditions (7) is the Cauchy function. The solution of Eq. (6) satisfying the homogeneous conditions corresponding to the conditions (7), has the form:

$$T^*(p,r) = \int_0^R \varepsilon_\alpha^*(p,r,\rho)\tilde{g}(\rho)\rho^{2\alpha+1}d\rho,$$

where $\varepsilon_\alpha^*\left(p,r,\rho\right)$ is a fundamental function of the boundary value problem (6)–(7) with the following properties:

1. The function $\varepsilon_\alpha^*(p,r,\rho)$ satisfies the homogeneous equation corresponding to Eq. (6) and the following boundary conditions [16]:
$$\lim_{r\to 0}\frac{\partial}{\partial r}\left(r^{\alpha-\nu}\varepsilon^*\right) = 0, \quad \left(\alpha_{11}^1\frac{d}{dr} + \beta_{11}^1\right)\varepsilon^*\big|_{r=R} = 0.$$
With this, $\varepsilon_\alpha^*(p,r,\rho)\big|_{r=\rho+0} - \varepsilon_\alpha^*(p,r,\rho)\big|_{r=\rho-0} = 0$.
2. The following holds $\frac{d}{dr}\varepsilon_\alpha^*(p,r,\rho)\big|_{r=\rho+0} - \frac{d}{dr}\varepsilon_\alpha^*(p,r,\rho)\big|_{r=\rho-0} = \rho^{-(2\alpha+1)}$.

Let us put $\varepsilon_\alpha^*(p,r,\rho) = \begin{cases} A_1 I_{\nu,\alpha}(\lambda\rho), & 0<r<\rho<R; \\ A_2 I_{\nu,\alpha}(\lambda r) + B_2 K_{\nu,\alpha}(\lambda r), & 0<\rho<r<R. \end{cases}$

Here $I_{\nu,\alpha}(\lambda r)$ and $K_{\nu,\alpha}(\lambda r)$ are modified Bessel functions of the first and second kind [17]; $\nu = ia^{-1}\beta$, $\operatorname{Re}\nu \geqslant \alpha \geqslant -\frac{1}{2}$ write down in the form:

$$p = -\left(\beta^2 + \gamma^2\right) = \left(\beta^2 + \gamma^2\right)e^{\pi i},$$

$$dp = -2\beta d\beta, \quad b(\beta) = a^{-1}\beta.$$

Returning to the original, we obtain

$$T(\tau, r) = \frac{a^{-2}}{2\pi i} \int_0^R \left(\int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \varepsilon_\alpha^*(p, r, \rho) e^{p\tau} dp \right) g(\rho) \rho^{2\alpha - 1} d\rho,$$

where a^{-2} is a weight function [16]. The special points of the Cauchy function $\varepsilon^*(p, r, \rho)$ are the branching points $p = -\gamma^2 \leqslant 0$ and the point $p = \infty$.

Let us define the real-valued functions:

$$C_\alpha(\lambda r, a^{-1}\beta) = I_{ia^{-1}\beta,\alpha}(\lambda r) + i\pi^{-1} \sinh \pi\beta \, K_{ia^{-1}\beta,\alpha}(\lambda r),$$
$$D_\alpha(\lambda r, a^{-1}\beta) = \pi^{-1} \sinh a^{-1}\pi\beta \, K_{ia^{-1}\beta,\alpha}(\lambda r).$$

For $r \approx 0$, the asymptotic equations are known [16]:

$$C_\alpha(r, \beta) = \frac{r^{-\alpha} g_1(\beta, r)}{|\Gamma(1 + i\beta)|^2}, \quad D_\alpha(r, \beta) = \frac{r^{-\alpha} g_2(\beta, r)}{|\Gamma(1 + i\beta)|^2}, \quad \Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad \operatorname{Re} z > 0,$$

$$C_\alpha(r, \beta) = \frac{r^{-\alpha} \left[\beta \, g_2(\beta, r) - \alpha \, g_1(\beta, r)\right]}{|\Gamma(1 + i\beta)|^2}, \quad D_\alpha(r, \beta) = -\frac{r^{-\alpha-1} \left[\beta \, g_1(\beta, r) + \alpha \, g_2(\beta, r)\right]}{|\Gamma(1 + i\beta)|^2},$$

$$g_1(\beta, r) = \Gamma_1(\beta) \cos\left(\beta \ln \frac{r}{2}\right) + \Gamma_2(\beta) \sin\left(\beta \ln \frac{r}{2}\right),$$

$$g_2(\beta, r) = \Gamma_2(\beta) \cos\left(\beta \ln \frac{r}{2}\right) - \Gamma_1(\beta) \sin\left(\beta \ln \frac{r}{2}\right),$$

where $\Gamma_i(\beta)$ are real gamma functions in the expansion of gamma functions of the complex argument in terms of the real and imaginary parts.

For modified Bessel functions, we have

$$I'_{\nu,\alpha}(z) = (\nu - \alpha) I_{\nu,\alpha}(z) + z \, I_{\nu+1,\alpha+1}(z).$$

For $r \approx 0$, $I_{\nu,\alpha}(z) \approx \frac{z^{\nu-\alpha}}{2^\nu \Gamma(\nu+1)}$, $I'_{\nu,\alpha}(z) \approx \frac{(\nu-\alpha) z^{\nu-\alpha-1}}{2^\nu \Gamma(\nu+1)}$.

$$I_{\nu,\alpha}(ir) = \exp\left[(\nu - \alpha)\frac{\pi i}{2}\right] J_{\nu,\alpha}(r); \quad K_{\nu,\alpha}(ir) = -\frac{\pi i}{2} \exp\left[-(\nu + \alpha)\frac{\pi i}{2}\right] \left[J_{\nu,\alpha}(r) - iN_{\nu,\alpha}(r)\right].$$

The asymptotics of Bessel functions for great argument values are as follows:

$$J_{\nu,\alpha}(r) = \sqrt{\frac{2}{\pi}} r^{-(\alpha+\frac{1}{2})} \cos\left(r - \frac{\pi\nu}{2} - \frac{\pi}{4}\right), \qquad N_{\nu,\alpha}(r) = \sqrt{\frac{2}{\pi}} r^{-(\alpha+\frac{1}{2})} \sin\left(r - \frac{\pi\nu}{2} - \frac{\pi}{4}\right),$$

$$I_{\nu+1,\alpha+1}(r) = \sqrt{\frac{2}{\pi}} r^{-(\alpha+\frac{3}{2})} \sin\left(r - \frac{\pi\nu}{2} - \frac{\pi}{4}\right), \quad N_{\nu+1,\alpha+1}(r) = -\sqrt{\frac{2}{\pi}} r^{-(\alpha+\frac{3}{2})} \cos\left(r - \frac{\pi\nu}{2} - \frac{\pi}{4}\right).$$

Let us introduce the following variables:

$$\begin{aligned}
X^{11}_{\alpha;11}(\lambda R, b) &= \left(\alpha^1_{11} \frac{d}{dr} + \beta^1_{11}\right) C_\alpha(\lambda r, b)\Big|_{r=R} \\
&= \alpha^1_{11} \lambda \left[\frac{dI_{i\beta,\alpha}(\lambda R)}{dr} + \frac{i}{\pi} \sinh \pi b \, \frac{dK_{i\beta,\alpha}(\lambda R)}{dr}\right] + \beta^1_{11} \left[I_{i\beta,\alpha}(\lambda R) + i\pi^{-1} \sinh \pi b \, K_{i\beta,\alpha}(\lambda R)\right] \\
&= \tilde{X}^{11}_{\alpha;11}(\lambda R) + i \frac{\sinh \pi b}{\pi} \tilde{X}^{12}_{\alpha;11}(\lambda R); \\
X^{12}_{\alpha;11}(\lambda R, b) &= \left(\alpha^1_{11} \frac{d}{dr} + \beta^1_{11}\right) D_\alpha(\lambda r, b)\Big|_{r=R} \\
&= \alpha^1_{11} \lambda \left[\frac{1}{\pi} \sinh \pi b \, \frac{dK_{i\beta,\alpha}(\lambda R)}{dr}\right] + \beta^1_{11} \pi^{-1} \sinh \pi b \, K_{i\beta,\alpha}(\lambda R) \\
&= \pi^{-1} \sinh \pi b \left[\alpha^1_{11} \lambda \frac{d}{dr} K_{i\beta,\alpha}(\lambda R) + \beta^1_{11} K_{i\beta,\alpha}(\lambda R)\right] = \tilde{X}^{12}_{\alpha;11}(\lambda R) \frac{\sinh \pi b}{\pi},
\end{aligned} \tag{8}$$

$$\tilde{X}^{11}_{\alpha;11}(\lambda R)=\alpha^1_{11}\lambda\frac{dI_{i\beta,\alpha}(\lambda R)}{dr}+\beta^1_{11}I_{i\beta,\alpha}(\lambda R),\quad \tilde{X}^{12}_{\alpha;11}(\lambda R)=\alpha^1_{11}\lambda\frac{dK_{i\beta,\alpha}(\lambda R)}{dr}+\beta^1_{11}K_{i\beta,\alpha}(\lambda R).$$

If we pass to the Bessel functions of a real argument $J_{\nu,\alpha}(\lambda R,b)$ and $N_{\nu,\alpha}(\lambda R,b)$, then, given their properties [14,17] we obtain:

$$X^{11}_{\alpha;11}(\lambda R,b)=\left(\alpha^1_{11}\frac{\nu-\alpha}{R}+\beta^1_{11}\right)J_{\nu,\alpha}(\lambda R,b)-\alpha^1_{11}R\lambda^2J_{\nu+1,\alpha+1}(\lambda R,b),\quad b=a^{-1}\beta;$$
$$X^{12}_{\alpha;11}(\lambda R,b)=\left(\alpha^1_{11}\frac{\nu-\alpha}{R}+\beta^1_{11}\right)N_{\nu,\alpha}(\lambda R,b)-\alpha^1_{11}R\lambda^2N_{\nu+1,\alpha+1}(\lambda R,b).$$

Let us determine the functions

$$\begin{aligned}U^{11}_{\nu,\alpha;11}(\lambda R,b)&=X^{11}_{\alpha;11}(\lambda R,b)-i\,X^{12}_{\alpha;11}(\lambda R,b)=\tilde{X}^{11}_{\alpha;11}(\lambda R)=\alpha^1_{11}\lambda\frac{dI_{i\beta,\alpha}(\lambda R)}{dr}+\beta^1_{11}I_{i\beta,\alpha}(\lambda R)\\&=\left(\alpha^1_{11}\frac{\nu-\alpha}{R}+\beta^1_{11}\right)I_{\nu,\alpha}(\lambda R,b)+\alpha^1_{11}R\lambda^2I_{\nu+1,\alpha+1}(\lambda R,b);\quad \nu=a^{-1}\beta;\\U^{12}_{\nu,\alpha;11}(\lambda R)&=\pi\,(\sinh b\pi)^{-1}\,X^{12}_{\alpha,11}(\lambda R,b)=\left(\alpha^1_{11}\frac{d}{dr}+\beta^1_{11}\right)K_{\nu,\alpha}(\lambda r)\Big|_{r=R}=\tilde{X}^{12}_{\alpha,11}(\lambda R,b)\\&=\left(\alpha^1_{11}\frac{\nu-\alpha}{R}+\beta^1_{11}\right)K_{\nu,\alpha}(\lambda R)+\alpha^1_{11}R\lambda^2K_{\nu+1,\alpha+1}(\lambda R).\end{aligned}\tag{9}$$

Satisfying the condition (7), we obtain the algebraic system of equations for determining the coefficients A_1, A_2, B_2:

$$(A_2-A_1)I_{\nu,\alpha}(\lambda\rho)+B_2K_{\nu,\alpha}(\lambda\rho)=0,$$
$$(A_2-A_1)I'_{\nu,\alpha}(\lambda\rho)+B_2K'_{\nu,\alpha}(\lambda\rho)=-\frac{1}{\lambda\rho^{2\alpha+1}}.$$

Given the relation

$$I_{\nu,\alpha}(\lambda\rho)K'_{\nu,\alpha}(\lambda\rho)-I'_{\nu,\alpha}(\lambda\rho)K_{\nu,\alpha}(\lambda\rho)=-(\lambda\rho)^{-(2\alpha+1)},$$

we obtain

$$(A_2-A_1)=-\lambda^{2\alpha}K_{\nu,\alpha}(\lambda\rho),\quad B_2=\lambda^{2\alpha}I_{\nu,\alpha}(\lambda\rho).\tag{10}$$

Satisfying the boundary condition for $r=R$, we obtain:

$$A_2U^{11}_{\nu,\alpha;11}(\lambda R)+B_2U^{12}_{\nu,\alpha;11}(\lambda R)=0,\tag{11}$$

$$\begin{aligned}A_2&=\lambda^{2\alpha}\frac{\Psi^{1*}_{\nu,\alpha,11}(\lambda R,\lambda\rho)}{U^{11}_{\nu,\alpha,11}(\lambda R)}-\lambda^{2\alpha}K_{\nu,\alpha}(\lambda\rho)=\lambda^{2\alpha}\left\{\frac{\psi^{1*}_{\nu,\alpha,11}(\lambda R,\lambda\rho)-U^{11}_{\nu,\alpha,11}(\lambda R)\,K_{\nu,\alpha}(\lambda\rho)}{U^{11}_{\nu,\alpha,11}(\lambda R)}\right\}\\&=-\lambda^{2\alpha}\frac{U^{12}_{\nu,\alpha,11}(\lambda R)}{U^{11}_{\nu,\alpha,11}(\lambda R)}I_{\nu,\alpha}(\lambda\rho)=-B_2\frac{U^{12}_{\nu,\alpha,11}(\lambda R)}{U^{11}_{\nu,\alpha,11}(\lambda R)},\qquad A_1=\frac{B_2K_{\nu,\alpha}(\lambda\rho)}{I_{\nu,\alpha}(\lambda\rho)}+A_2.\end{aligned}$$

Here $A_1=\lambda^{2\alpha}\left(U^{11}_{\nu,\alpha;11}(\lambda R)\right)^{-1}\Psi^{1*}_{\nu,\alpha;11}(\lambda R,\lambda\rho)$; $\left(U^{12}_{\nu,\alpha;11}(\lambda R)\right)=\frac{A_2\left(U^{11}_{\nu,\alpha;11}(\lambda R)\right)}{\lambda^{2\alpha}I_{\nu,\alpha}(\lambda r)}$;

$$\Psi^{1*}_{\nu,\alpha;11}(\lambda R,\lambda r)=U^{11}_{\nu,\alpha;11}(\lambda R)\,K_{\nu,\alpha}(\lambda r)-U^{12}_{\nu,\alpha;11}(\lambda R)\,I_{\nu,\alpha}(\lambda r).$$

Then the function $\varepsilon^*_\alpha(p,r,\rho)$, due to the symmetry with respect to the diagonal $r=\rho$, has the form

$$\varepsilon^*_\alpha(p,r,\rho)=\frac{\lambda^{2\alpha}}{U^{11}_{\nu,\alpha;11}(\lambda R,b)}\begin{cases}I_{\nu,\alpha}(\lambda r)\Psi^{1*}_{\nu,\alpha;11}(\lambda R,\lambda\rho), & 0<r<\rho<R,\\ I_{\nu,\alpha}(\lambda\rho)\Psi^{1*}_{\nu,\alpha;11}(\lambda R,\lambda r), & 0<\rho<r<R.\end{cases}\tag{12}$$

The roots $p_n=-(\beta_n^2+\gamma^2)$ of the transcendental equation $U^{11}_{\nu,\alpha;11}(\lambda R,b)=0$ are simple poles of $\varepsilon^*_\alpha(p,r,\rho)$.

Consider the transcendental equation

$$\left(\alpha_{11}^1 \frac{\nu-\alpha}{R} + \beta_{11}^1\right) I_{\nu,\alpha}(\lambda R, b) - \alpha_{11}^1 \lambda^2 R I_{\nu+1,\alpha+1}(\lambda R, b) = 0,$$

where $p = -(\beta^2+\gamma^2) = (\beta^2+\gamma^2)e^{\pi i}$, $b\,(\beta) = a^{-1}\beta$ form a discrete spectrum, $\{b_n\}_{n=1}^{\infty}$.

Let us denote: $\Psi_{\nu,\alpha;11}^1(\lambda R, \lambda r, b) = \pi^{-1}(\sinh \pi b) \times \Psi_{\nu,\alpha;11}^{1*}(\lambda R, \lambda r, b)$.

Here $\Psi_{\nu,\alpha;11}^1(\lambda R, \lambda r, b)$ is the eigenfunction of the problem that satisfies Eq. (2) and homogeneous boundary conditions. We use it to construct a solution of the problem that satisfies the inhomogeneous condition at the outer surface of the cylinder, i.e., reflects the effect of the drying agent.

The original of the fundamental function

$$\begin{aligned}\varepsilon_\alpha(t,r,\rho) &= \int_0^\infty e^{-(\beta^2+\gamma^2)t}\Psi_{\nu,\alpha;11}^1(\lambda R, \lambda r, b)\Psi_{\nu,\alpha;11}^1(\lambda R, \lambda \rho, b)\frac{2\beta\lambda^{2\alpha}d\beta}{\left(X_{\alpha;11}^{11}\right)^2+\left(X_{\alpha;11}^{12}\right)^2} \\ &= \int_0^\infty e^{(\beta^2+\gamma^2)t}V_\alpha(r,\beta)\,V_\alpha(\rho,\beta)\,\Omega_\alpha(\beta)\,d\beta; \qquad (13)\end{aligned}$$

$$\Omega_\alpha(\beta) = \frac{2\beta\lambda^{2\alpha}}{\left(X_{\alpha;11}^{11}(\lambda R,\beta)\right)^2+\left(X_{\alpha;11}^{12}(\lambda R,\beta)\right)^2}.$$

By the generalized convolution theorem

$$\varepsilon_\alpha(t,r,\rho) = \sum_{n=1}^{\infty} e^{-(\beta_n^2+\gamma^2)t}\frac{V_\alpha(b_n r)\,V_\alpha(b_n\rho)}{\|V_\alpha(b_n r)\|_1^2},$$

where $\|V_\alpha(b_n r)\|_1^2$ is the squared norm of its own function, b_n are roots of the function $U_{\nu,\alpha;11}^{11}(\lambda R, b)$.

$$\Psi_{\nu,\alpha;11}^{1*}(i\lambda R, i\lambda\rho) = -\frac{\pi}{2}e^{-\pi i\alpha}\Psi_{\nu,\alpha;11}^1(\lambda R, \lambda\rho),$$

$$V_\alpha(r,\beta) = \Psi_{\alpha;11}^1(\lambda R, \lambda r, \beta) = X_{\alpha;11}^{11}(\lambda R,\beta)D_\alpha(\lambda r,\beta) - X_{\alpha;11}^{12}(\lambda R,\beta)C_\alpha(\lambda r,\beta), \qquad (14)$$

$$\Psi_{\nu,\alpha;11}^{*1}(i\lambda R, i\lambda r) = -\frac{\pi}{2}e^{-\pi i\alpha}\Psi_{\nu,\alpha;11}^1(\lambda R, \lambda r).$$

Here $V_\alpha(r,\beta) = \Psi_{\alpha;11}^1(\lambda R, \lambda r, \beta)$ is eigenfunction (spectral function) of the problem (6), $\Omega_\alpha(\beta)$ is a spectral density.

Returning in Eq. (13) to the original, we obtain a solution $T_{odn}(t,r)$ of the homogeneous parabolic Cauchy problem (2)–(3):

$$\begin{aligned}T_{odn}(t,r) &= \int_0^R \varepsilon_\alpha(t,r,\rho)\,g(\rho)\,\rho^{2\alpha-1}a^{-2}d\rho \\ &= \int_0^\infty e^{-(\beta^2+\gamma^2)t}V_\alpha(r,\beta)\int_0^R g(\rho)\,V_\alpha(\rho,\beta)\,\sigma\,\rho^{2\alpha-1}d\rho\,\Omega_\alpha(\beta)\,d\beta, \quad \sigma = a^{-2}. \qquad (15)\end{aligned}$$

From Eq. (15) for $t=0$, we obtain the integral image:

$$g(r) = \int_0^\infty V_\alpha(r,\beta)\int_0^R g(\rho)V_\alpha(\rho,\beta)\,\sigma\,\rho^{2\alpha-1}d\rho\,\Omega_\alpha(\beta)\,d\beta. \qquad (16)$$

From Eq. (16), it follows that the function $\varepsilon_\alpha(t,r,\rho)$ defined by Eq. (13) is a delta-shaped sequence with respect to t for $t \to 0+$.

The integral image (16) defines a direct

$$H_\alpha\left[g(r)\right] = \int_0^R g(r)\,V_\alpha(r,\beta)\,\sigma\,r^{2\alpha-1}dr \equiv \tilde{g}(\beta) \qquad (17)$$

and inverse

$$H_\alpha^{-1}\left[\tilde{g}(r)\right] = \int_0^\infty \tilde{g}(\beta)\,V_\alpha(r,\beta)\,\Omega_\alpha(\beta)\,d\beta \equiv g(r) \qquad (18)$$

Kontorovich–Lebedev transform over the interval $[0,R]$.

Given the theorem on the basic identity of the integral transform [16] of a differential operator B_α, i.e., if the function $g(r)$ is such that the function $f(r) = B_\alpha[g(R)]$ is continuous on the set $(0, R)$ and the boundary conditions hold:

$$\lim_{r\to 0} r^{2\alpha+1}\left(\frac{dg}{dr}V_\alpha(r,\beta) - g(r)\frac{dV_\alpha}{dr}\right) = 0, \quad \left(\alpha_{11}^1 \frac{d}{dr} + \beta_{11}^1\right) g(r)\Big|_{r=R} = g_R(\tau), \tag{19}$$

then for any $\lambda \in (0,\infty)$, the following equality holds:

$$H_\alpha\left[a^2 B_\alpha[g(r)]\right] = -\beta^2 \tilde{g}(\beta) + \frac{\sinh \pi\beta}{\pi \lambda^{2\alpha}} g_R(\tau). \tag{20}$$

Therefore, based on the relation (17), it follows:

$$H_\alpha\left[a^2 B_\alpha[g(r)]\right] = -\beta^2 \int_0^R g(r)\, V_\alpha(r,\beta)\,\sigma\, r^{2\alpha-1} dr + \frac{\sinh \pi\beta}{\pi \lambda^{2\alpha}} T_a(R,\tau). \tag{21}$$

From the properties of the eigenfunction $V_\alpha(r,\beta)$, it follows that

$$\left(\alpha_{11}^1 \frac{dV_\alpha(r,\beta)}{dr} + \beta_{11}^1 V_\alpha(r,\beta)\right)\Big|_{r=R} = 0, \qquad (a^2 B_\alpha + \beta^2) V_\alpha(r,\beta) = 0.$$

From Eq. (21), taking into account Eq. (19), we obtain

$$\begin{aligned}
H_\alpha\left[a^2 B_\alpha[g(r)]\right] &= a^2 \int_0^R B_\alpha[g(r)]\; V_\alpha(r,\beta)\,\sigma\, r^{2\alpha-1} dr \\
&= \int_0^R \left[r^2 \frac{d^2 g}{dr^2} + (2\alpha+1)\, r\, \frac{dg}{dr} - \lambda^2 r^2 g(r) + \alpha^2 g(r)\right] V_\alpha(r,\beta) r^{2\alpha-1} dr \\
&= a^2 \sigma\, r^{2\alpha+1}\left[g'(r) V_\alpha(r,\beta) - g(r)\, V_\alpha'(r,\beta)\right]\Big|_0^R + \int_0^R g(r)\, a^2 \sigma\, B_\alpha\left[V_\alpha(r,\beta)\right] r^{2\alpha-1} dr \\
&= R^{2\alpha+1}\left[g'(r)\, V_\alpha(r,\beta) - g(r)\, V_\alpha'(r,\beta)\right]\Big|_{r=R} - \beta^2 H_\alpha[g(r)],
\end{aligned}$$

where $H_\alpha[g(r)]$ is defined by Eq. (17); $g_R = T_{aR}(R,t)$ is the temperature of the drying agent.

Then from Eq. (21), we obtain:

$$\begin{aligned}
\frac{g_R}{\alpha_{11}^1} V_\alpha(R,\beta) &= \frac{g_R}{\alpha_{11}^1}\left[X_{\alpha;11}^{11}(\lambda R, b) D_\alpha(\lambda R, b) - X_{\alpha;11}^{12}(\lambda R, b) C_\alpha(\lambda R, b)\right] \\
&= g_R\left[C_{r\alpha}'(\lambda R, b) D_\alpha(\lambda R, b) - D_{r\alpha}'(\lambda R, b) C_\alpha(\lambda R, b)\right] = \frac{\sinh \pi b}{\pi \lambda^{2\alpha} R^{2\alpha+1}} T_{aR}.
\end{aligned} \tag{22}$$

The equations of heat conduction and boundary conditions have the following form:

$$\frac{\partial T}{\partial \tau} + \left(\beta^2 + \gamma^2\right) T = 0; \quad T(\tau,\beta)\big|_{\tau=0} = g(\beta), \quad \left(\alpha_{11}^1 \frac{d}{dr} + \beta_{11}^1\right) T(r)\Big|_{r=R} = T_{aR}(\tau).$$

As a result of Eq. (21), we obtain

$$\frac{\partial \tilde{T}}{\partial \tau} + \left(\beta^2 + \gamma^2\right) \tilde{T} = \frac{\sinh \pi b}{\pi \lambda^{2\alpha}} T_{aR}(\tau); \quad \tilde{T}(\tau,\beta)\big|_{\tau=0} = \tilde{g}(\beta). \tag{23}$$

The solution of the Cauchy problem (23) is the function

$$\tilde{T}(\tau,\beta) = e^{-(\beta^2+\gamma^2)\tau} \tilde{g}(\beta) + \int_0^\tau e^{-(\beta^2+\gamma^2)(\tau-t)} \left[\frac{\sinh \pi b}{\pi \lambda^{2\alpha}}\Big(\alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos \nu_n^2 t\Big)\right] dt. \tag{24}$$

Let us apply the integral operator H_α^{-1} (18) to $\tilde{T}(\tau,\beta)$, we obtain the solution of the problem (24):

$$T(t,r)=\int_0^t\int_0^R\int_0^\infty e^{-(\beta^2+\gamma^2)(t-\tau)}V_\alpha(r,\beta)\,V_\alpha(\rho,\beta)\,\Omega_\alpha(\beta)\,d\beta\big[\delta_+(\tau)g(\rho)\big]\sigma\rho^{2\alpha-1}d\rho\,d\tau$$
$$+\int_0^\infty\int_0^t e^{-(\beta^2+\gamma^2)(t-\tau)}\frac{\sinh\pi b}{\pi\lambda^{2\alpha}}T_a(\tau)\,V_\alpha(r,\beta)\,\Omega_\alpha(\beta)\,d\beta\,d\tau. \quad (25)$$

From Eqs. (17)–(18) and Steklov's theorem, any vector-function $f(r)=B_\alpha\big[g(r)\big]$ being continuous on $(0,R)$ and satisfying zero boundary conditions can be decomposed in terms of a system of eigenfunctions $V_\alpha(r,\beta_j)_{j=1}^\infty$ into an absolutely and uniformly convergent Fourier series.

It is known that one eigenvector-function $V_\alpha(r,\beta_j)$ corresponds to each eigenvalue β_j and the system of spectral functions $V_\alpha(r,\beta_j)_{j=1}^\infty$ is complete and closed. The squared norm of the eigenfunction $\|V_\alpha(r,\beta_j)\|^2=\int_0^R\left[V_\alpha(r,\beta_j)\right]^2\sigma\,r^{2\alpha-1}dr$.

Thus, given Eq. (17), the inverse integral operator (18) can be written down as follows

$$H_\alpha^{-1}\left[\tilde{g}(r)\right]=\sum_{j=0}^\infty\tilde{g}(\beta_j)V_\alpha(r,\beta_j)\big(\|V_\alpha(r,\beta_j)\|^2\big)^{-1}\equiv g(r),$$

and the function

$$G_\alpha(t,r,\rho)=\int_0^\infty e^{-(\beta^2+\gamma^2)t}V_\alpha(r,\beta)\,V_\alpha(\rho,\beta)\,\Omega_\alpha(\beta)\,d\beta, \quad (26)$$

by taking into account the initial temperature state of the body, according to the theory of surpluses can be represented as calculated integral in the form:

$$G_\alpha(t,r,\rho)=\sum_{j=1}^\infty e^{-(\beta_j^2+\gamma^2)t}\frac{V_\alpha(r,\beta_j)V_\alpha(\rho,\beta_j)}{\|V_\alpha(r,\beta_j)\|^2}\,\sigma a^2,$$

where

$$V_\alpha(r,\beta_j)=\Psi^1_{\alpha;11}(\lambda R,\lambda r,\beta_j)=\frac{\sinh\pi\beta_j}{\pi}\left[X^{11}_{\alpha;11}(\lambda R,\beta_j)D_\alpha(\lambda r,\beta_j)-X^{12}_{\alpha;11}(\lambda R,\beta_j)C_\alpha(\lambda r,\beta_j)\right];$$

as well as the Green's function generated by the thermal regime at the boundary $r=R$

$$W_\alpha(t,r)=\int_0^\infty e^{-(\beta^2+\gamma^2)t}V_\alpha(r,\beta)\frac{\sinh\pi\beta}{\pi\lambda^{2\alpha}}\Omega_\alpha(\beta)\,d\beta,\quad b=a^{-1}\beta, \quad (27)$$
$$W_\alpha(t,r)=\sum_{j=1}^\infty e^{-(\beta_j^2+\gamma^2)t}\frac{\sinh\pi\beta_j}{\pi\lambda^{2\alpha}}\frac{V_\alpha(r,\beta_j)}{\|V_\alpha(r,\beta_j)\|^2}\,\sigma\,a^2.$$

Then the solution will take the form:

$$T(t,r)=\int_0^t\int_0^R G_\alpha(t-\tau,r,\rho)\left[\delta_+(\tau)g(\rho)\right]\sigma\rho^{2\alpha-1}d\rho\,d\tau+\int_0^t W_\alpha(t-\tau,r)\,T_a(\tau)\,d\tau. \quad (28)$$

Here $\delta_+(t)$ is a delta-function concentrated at the point $0+$.

According to Eq. (28), taking into account the properties of delta-function and Eq. (17), we obtain:

$$T(t,r)=\int_0^\infty e^{-(\beta^2+\gamma^2)t}\tilde{g}(\beta)\,V_\alpha(r,\beta)\,\Omega_\alpha(\beta)\,d\beta+\int_0^\infty\int_0^t e^{-(\beta^2+\gamma^2)(t-\tau)}\frac{\sinh\pi\beta}{\pi\lambda^{2\alpha}}T_{aR}(\tau)\,V_\alpha(r,\beta)\,\Omega_\alpha(\beta)\,d\beta.$$

Let us denote $\int_0^t e^{-(\beta^2+\gamma^2)(t-\tau)}\frac{\sinh\pi b}{\pi\lambda^{2\alpha}}\,T_{aR}(\tau)\,d\tau=T_{wa}(t,\beta)$. We transit to improper integrals

$$T(t,r)=\int_0^\infty e^{-(\beta^2+\gamma^2)t}\tilde{g}(\beta)\,V_\alpha(r,\beta)\,\Omega_\alpha(\beta)\,d\beta+\int_0^\infty T_{wa}(\tau,\beta)\,V_\alpha(r,\beta)\,\Omega_\alpha(\beta)\,d\beta.$$

Taking into account Eqs. (26), (27), we obtain

$$T(t,r)=\sum_{j=1}^{\infty}e^{-(\beta_j^2+\gamma^2)t}\tilde{g}(\beta_j)\frac{V_\alpha(r,\beta_j)}{\|V_\alpha(r,\beta_j)\|^2}\,\sigma\,a^2+\sum_{j=1}^{\infty}T_{wa}(\tau,\beta_j)\frac{V_\alpha(r,\beta_j)}{\|V_\alpha(r,\beta_j)\|^2}\,\sigma\,a^2.$$

Determine the effect of initial conditions and temperature of the drying agent on the drying process. Given Eq. (16), the initial condition ($g(r)=\sum_{j=0}^{2}g_{j0}r^j$) is chosen. Then

$$\tilde{g}(\beta)=\frac{1}{a^2}\int_0^R\sum_{j=0}^{2}\left(g_{j0}r^{j+2\alpha-1}\right)$$
$$\times\left[X^{11}_{\alpha;11}(\lambda R,\beta)\left(\pi^{-1}\sinh\pi\beta\,K_{i\beta,\alpha}(\lambda r)\right)-X^{12}_{\alpha;11}(\lambda R,\beta)\left(I_{i\beta,\alpha}(\lambda r)+i\left(\pi^{-1}\sinh\pi\beta\,K_{i\beta,\alpha}(\lambda r)\right)\right)\right]dr.$$

Given the expressions for the Bessel functions

$$I_{\nu,\alpha}(\lambda\rho)=(\lambda\rho)^{-\alpha}I_\nu(\lambda\rho),\qquad K_{\nu,\alpha}(\lambda\rho)=(\lambda\rho)^{-\alpha}K_\nu(\lambda\rho),$$

let us determine

$$\tilde{g}(\beta)=\frac{1}{a^2}\Bigg\{X^{11}_{\alpha;11}(\lambda R,\beta)\pi^{-1}\sinh\pi\beta\int_0^R\sum_{j=0}^{2}\left(g_{j0}r^{j+\alpha-1}\right)K_{i\beta}(\lambda r)\,dr-X^{12}_{\alpha;11}(\lambda R,\beta)$$
$$\times\int_0^R\sum_{j=0}^{2}\left(g_{j0}r^{j+\alpha-1}\right)I_{i\beta}(\lambda r)\,dr-iX^{12}_{\alpha;11}(\lambda R,\beta)\pi^{-1}\sinh\pi\beta\int_0^R\sum_{j=0}^{2}\left(g_{j0}r^{j+\alpha-1}\right)K_{i\beta}(\lambda r)\,dr\Bigg\}$$
$$=\frac{1}{a^2}\Bigg\{\left[X^{11}_{\alpha;11}(\lambda R,\beta)-iX^{12}_{\alpha;11}(\lambda R,\beta)\right]\pi^{-1}\sinh\pi\beta\int_0^{\lambda R}\sum_{j=0}^{2}\lambda^{-(2\alpha+j)}\left(g_{j0}r^{j+\alpha-1}\right)K_{i\beta}(\lambda r)\,dr$$
$$-X^{12}_{\alpha;11}(\lambda R,\beta)\int_0^{\lambda R}\sum_{j=0}^{2}\lambda^{-(2\alpha+j)}\left(g_{j0}r^{j+\alpha-1}\right)I_{i\beta}(r)\,dr\Bigg\};$$

$$\tilde{T}_{go}(\tau,\beta)=e^{-(\beta^2+\gamma^2)\tau}\Bigg\langle\frac{1}{a^2}U^{11}_{\alpha,11}(\lambda R)\pi^{-1}\sinh\pi\beta\,\lambda^{-\alpha}\sum_{j=0}^{2}g_{j0}\frac{1}{\lambda^{j+\alpha}}$$
$$\times\left\{\tfrac{2^{i\beta-1}\Gamma(i\beta)}{j+\alpha-i\beta}(\lambda R)^{j+\alpha-i\beta}{}_1F_2\left(\tfrac{j+\alpha-i\beta}{2};1-i\beta;\tfrac{j+\alpha-i\beta+2}{2},\tfrac{(\lambda R)^2}{4}\right)\right.$$
$$\left.+\tfrac{2^{-i\beta-1}\Gamma(-i\beta)}{j+\alpha+i\beta}(\lambda R)^{j+\alpha+i\beta}{}_1F_2\left(\tfrac{j+\alpha+i\beta}{2};1+i\beta;\tfrac{j+\alpha+i\beta+2}{2},\tfrac{(\lambda R)^2}{4}\right)\right\}$$
$$-X^{12}_{\alpha;11}(\lambda R,\beta)\tfrac{\lambda^{-\alpha}}{a^2}\sum_{j=0}^{2}g_{j0}\frac{1}{\lambda^{j+\alpha}}\left\{\tfrac{1}{2^{i\beta}(j+\alpha+i\beta)\Gamma(1+i\beta)}(\lambda R)^{j+\alpha+i\beta}{}_1F_2\left(\tfrac{j+\alpha+i\beta}{2};\tfrac{j+\alpha+i\beta+2}{2},1+i\beta;\tfrac{(\lambda R)^2}{4}\right)\right\}\Bigg\rangle.$$

Here ${}_1F_2\left((a_1)_k;(b_1)_k,(b_2)_k;z\right)=\sum_{k=0}^{\infty}\frac{(a_1)_k}{(b_1)_k(b_2)_k}\frac{z^k}{k!}$ are generalized hypergeometric function, where $(a_1)_k=\frac{j+\alpha\pm i\beta}{2}$; $(b_1)_k=\frac{j+\alpha\pm i\beta+2}{2}$; $(b_2)_k=1\pm i\beta$; $z=\frac{(\lambda R)^2}{4}$ are Pochhammer's polynomials [18]. Let us write these functions.

Let us write the first of these functions

$$\Phi_1={}_1F_2\left(\tfrac{j+\alpha-i\beta}{2};1-i\beta;\tfrac{j+\alpha-i\beta+2}{2},\tfrac{(\lambda R)^2}{4}\right)$$
$$=1+\frac{\left(\frac{j+\alpha-i\beta}{2}\right)\frac{(\lambda R)^2}{4}}{(1-i\beta)\left(\frac{j+\alpha-i\beta+2}{2}\right)1!}+\frac{\left(\frac{j+\alpha-i\beta}{2}\right)\left(\frac{j+\alpha-i\beta}{2}+1\right)\left(\frac{(\lambda R)^2}{4}\right)^2}{(1-i\beta)(1-i\beta+1)\left(\frac{j+\alpha-i\beta+2}{2}\right)\left(\frac{j+\alpha-i\beta+2}{2}+1\right)2!}$$
$$+\frac{\left(\frac{j+\alpha-i\beta}{2}\right)\left(\frac{j+\alpha-i\beta}{2}+1\right)\left(\frac{j+\alpha-i\beta}{2}+2\right)\left(\frac{(\lambda R)^2}{4}\right)^3}{(1-i\beta)(1-i\beta+1)(1-i\beta+2)\left(\frac{j+\alpha-i\beta+2}{2}\right)\left(\frac{j+\alpha-i\beta+2}{2}+1\right)\left(\frac{j+\alpha-i\beta+2}{2}+2\right)3!}+\ldots,$$

$$(a_1)_k=\tfrac{j+\alpha-i\beta}{2};\quad(b_1)_k=1-i\beta,\quad(b_2)_k=\tfrac{j+\alpha-i\beta+2}{2};\quad z=\tfrac{(\lambda R)^2}{4}.$$

The second function:

$$(a_1)_k = \tfrac{j+\alpha+i\beta}{2}; \quad (b_1)_k = 1+i\beta, \quad (b_2)_k = \tfrac{j+\alpha+i\beta+2}{2}; \quad z = \tfrac{(\lambda R)^2}{4};$$

$$\begin{aligned}\Phi_2 &= {}_1F_2\left(\tfrac{j+\alpha+i\beta}{2}; 1+i\beta, \tfrac{j+\alpha+i\beta+2}{2}; \tfrac{(\lambda R)^2}{4}\right)\\ &= 1 + \frac{\left(\frac{j+\alpha+i\beta}{2}\right)\frac{(\lambda R)^2}{4}}{(1+i\beta)\left(\frac{j+\alpha+i\beta+2}{2}\right)1!} + \frac{\left(\frac{j+\alpha+i\beta}{2}\right)\left(\frac{j+\alpha+i\beta}{2}+1\right)\left(\frac{(\lambda R)^2}{4}\right)^2}{(1+i\beta)(1+i\beta+1)\left(\frac{j+\alpha+i\beta+2}{2}\right)\left(\frac{j+\alpha+i\beta+2}{2}+1\right)2!}\\ &\quad + \frac{\left(\frac{j+\alpha+i\beta}{2}\right)\left(\frac{j+\alpha+i\beta}{2}+1\right)\left(\frac{j+\alpha+i\beta}{2}+2\right)\left(\frac{(\lambda R)^2}{4}\right)^3}{(1+i\beta)(1+i\beta+1)(1+i\beta+2)\left(\frac{j+\alpha+i\beta+2}{2}\right)\left(\frac{j+\alpha+i\beta+2}{2}+1\right)\left(\frac{j+\alpha+i\beta+2}{2}+2\right)3!} + \dots .\end{aligned}$$

The third function [18]:

$$(a_1)_k = \tfrac{j+\alpha+i\beta}{2}; \quad (b_1)_k = \tfrac{j+\alpha+i\beta+2}{2}, \quad (b_2)_k = 1+i\beta; \quad z = \tfrac{(\lambda R)^2}{4},$$

$$\begin{aligned}\Phi_3 &= {}_1F_2\left(\tfrac{j+\alpha+i\beta}{2}; \tfrac{j+\alpha+i\beta+2}{2}, 1+i\beta; \tfrac{(\lambda R)^2}{4}\right)\\ &= 1 + \frac{\left(\frac{j+\alpha+i\beta}{2}\right)\frac{(\lambda R)^2}{4}}{\left(\frac{j+\alpha+i\beta+2}{2}\right)(1+i\beta)1!} + \frac{\left(\frac{j+\alpha+i\beta}{2}\right)\left(\frac{j+\alpha+i\beta}{2}+1\right)\left(\frac{(\lambda R)^2}{4}\right)^2}{\left(\frac{j+\alpha+i\beta+2}{2}\right)\left(\frac{j+\alpha+i\beta+2}{2}+1\right)(1+i\beta)(1+i\beta+1)\cdot 2!}\\ &\quad + \frac{\left(\frac{j+\alpha+i\beta}{2}\right)\left(\frac{j+\alpha+i\beta}{2}+1\right)\left(\frac{j+\alpha+i\beta}{2}+2\right)\left(\frac{(\lambda R)^2}{4}\right)^3}{\left(\frac{j+\alpha+i\beta+2}{2}\right)\left(\frac{j+\alpha+i\beta+2}{2}+1\right)\left(\frac{j+\alpha+i\beta+2}{2}+2\right)(1+i\beta)(1+i\beta+1)(1+i\beta+2)\cdot 3!} + \dots .\end{aligned}$$

Comparing the expressions for Φ_2 and Φ_3, we see that $\Phi_2 = \Phi_3$. Consider the expressions of the first three coefficients of each of these generalized hypergeometric functions. Let us determine the real and imaginary parts in them. Consider the function

$$\Phi_1 = {}_1F_2\left(\tfrac{j+\alpha-i\beta}{2}; 1-i\beta; \tfrac{j+\alpha-i\beta+2}{2}, \tfrac{(\lambda R)^2}{4}\right) = {}_1F_2\left(a_1 + a_1^i i;\, b_1 + b_1^i i; b_2 + b_2^i i\right) = 1 + A_1 + A_2 + A_3 + \dots .$$

Introduce the denotations:

$$(a_1)_k = \tfrac{j+\alpha-i\beta}{2}; \quad (b_1)_k = 1-i\beta, \quad (b_2)_k = \tfrac{j+\alpha-i\beta+2}{2}; \quad z = \tfrac{(\lambda R)^2}{4};$$

$$a_1 = \tfrac{j+\alpha}{2}, \quad a_1^i = -\tfrac{\beta}{2}; \quad b_1 = 1, \quad b_1^i = -\beta, \quad b_2 = \tfrac{j+\alpha+2}{2}, \quad b_2^i = -\tfrac{\beta}{2}.$$

Here

$$A_1 = \frac{\frac{j+\alpha-i\beta}{2}\frac{(\lambda R)^2}{4}}{(1-i\beta)\frac{j+\alpha-i\beta+2}{2}} = A_{11} + A_{11}^i i,$$

where

$$A_{11} = \frac{\left\{a_1(b_1b_2 - b_1^ib_2^i) + a_1^i(b_1b_2^i + b_1^ib_2)\right\}}{\left[(b_1^2 + b_1^{i2})(b_2^2 + b_2^{i2})\right]}\frac{(\lambda R)^2}{4}, \quad A_{11}^i = \frac{\left\{a_1^i(b_1b_2 - b_1^ib_2^i) - a_1(b_1b_2^i + b_1^ib_2)\right\}}{\left[(b_1^2 + b_1^{i2})(b_2^2 + b_2^{i2})\right]}\frac{(\lambda R)^2}{4};$$

$$\begin{aligned}A_2 &= \frac{\left(\frac{j+\alpha-i\beta}{2}\right)\left(\frac{j+\alpha-i\beta}{2}+1\right)\left(\frac{(\lambda R)^2}{4}\right)^2}{(1-i\beta)(1-i\beta+1)\left(\frac{j+\alpha-i\beta+2}{2}\right)\left(\frac{j+\alpha-i\beta+2}{2}+1\right)2!} = A_{12} + A_{12}^i i\\ &= \left(A_{11} + A_{11}^i i\right)\left\langle \left\{\left[(b_1+1)(b_2+1) - b_1^ib_2^i\right]a_1 + a_1^i\left[(b_1+1)b_2^i + b_1^i(b_2+1)\right]\right\}\right.\\ &\quad \left. + \left\{\left[(b_1+1)(b_2+1) - b_1^ib_2^i\right]a_1^i - \left[(b_1+1)b_2^i + b_1^i(b_2+1)\right]a_1\right\} i\right\rangle \frac{\frac{(\lambda R)^2}{4}}{\left[\left((b_1+1)^2 + b_1^{i2}\right)\left((b_2+1)^2 + b_2^{i2}\right)\right]2};\end{aligned}$$

$$A_{12} = \frac{\left\{\left[(b_1+1)(b_2+1) - b_1^ib_2^i\right]a_1 + a_1^i\left[(b_1+1)b_2^i + b_1^i(b_2+1)\right]\right\}}{\left[\left((b_1+1)^2 + b_1^{i2}\right)\left((b_2+1)^2 + b_2^{i2}\right)\right]2}\frac{(\lambda R)^2}{4},$$

$$A_{12}^i = \frac{\left\{\left[(b_1+1)(b_2+1) - b_1^ib_2^i\right]a_1^i - \left[(b_1+1)b_2^i + b_1^i(b_2+1)\right]a_1\right\}}{\left[\left((b_1+1)^2 + b_1^{i2}\right)\left((b_2+1)^2 + b_2^{i2}\right)\right]2}\frac{(\lambda R)^2}{4},$$

$$(A_2 + A_2^i i) = (A_{11} + A_{11}^i i)(A_{12} + A_{12}^i i) = (A_{11}A_{12} - A_{11}^iA_{12}^i) + (A_{11}A_{12}^i + A_{12}A_{11}^i)i;$$

$$A_3 = \frac{\left(\frac{j+\alpha-i\beta}{2}\right)\left(\frac{j+\alpha-i\beta}{2}+1\right)\left(\frac{j+\alpha-i\beta}{2}+2\right)\left(\frac{(\lambda R)^2}{4}\right)^3}{(1-i\beta)(1-i\beta+1)(1-i\beta+2)\left(\frac{j+\alpha-i\beta+2}{2}\right)\left(\frac{j+\alpha-i\beta+2}{2}+1\right)\left(\frac{j+\alpha-i\beta+2}{2}+2\right)3!} + \dots$$
$$= A_1A_2(A_{13} + A^i_{13}i) = A_1A_2\Big\langle \left\{\left[(b_1+2)(b_2+2) - b^i_1 b^i_2\right] a_1 + a^i_1 \left[(b_1+2)b^i_2 + b^i_1(b_2+2)\right]\right\}$$
$$+ \left\{\left[(b_1+2)(b_2+2) - b^i_1 b^i_2\right] a^i_1 - \left[(b_1+2)b^i_2 + b^i_1(b_2+2)\right] a_1\right\} i\Big\rangle \frac{\frac{(\lambda R)^2}{4}}{\left[\left((b_1+2)^2+b^{i2}_2\right)\left((b_2+2)^2+b^{i2}_2\right)\right]3!};$$

$$(A_3 + A^i_3 i) = (A_{11} + A^i_{11}i)(A_{12} + A^i_{12}i)(A_{13} + A^i_{13}i)$$
$$= \left[(A_{11}A_{12} - A^i_{11}A^i_{12})A_{13} - (A_{11}A^i_{12} + A_{12}A^i_{11})A^i_{13}\right]$$
$$+ \left[(A_{11}A_{12} - A^i_{11}A^i_{12})A^i_{13} - (A_{11}A^i_{12} + A_{12}A^i_{11})A_{13}\right] i.$$

For the functions Φ_2, Φ_3, the representation of the coefficients A_1, A_2, A_3 ... remain the same:

$$\Phi_2 = {}_1F_2\left(\tfrac{j+\alpha+i\beta}{2}; 1+i\beta; \tfrac{j+\alpha+i\beta+2}{2}, \tfrac{(\lambda R)^2}{4}\right) = {}_1F_2\left(a_1 + a^i_1 i; b_1 + b^i_1 i; b_2 + b^i_2 i; \tfrac{(\lambda R)^2}{4}\right)$$
$$= 1 + (A_1 + A^i_1 i) + (A_2 + A^i_2 i) + (A_3 + A^i_3 i) + \dots,$$
$$a_1 = \tfrac{j+\alpha}{2}, \quad a^i_1 = +\tfrac{\beta}{2}; \quad b_1 = 1, \quad b^i_1 = +\beta, \quad b_2 = \tfrac{j+\alpha+2}{2}, \quad b^i_2 = +\tfrac{\beta}{2};$$
$$\Phi_3 = {}_1F_2\left(\tfrac{j+\alpha+i\beta}{2}; \tfrac{j+\alpha+i\beta+2}{2}, 1+i\beta; \tfrac{(\lambda R)^2}{4}\right) = {}_1F_2\left(a_1 + a^i_1 i; b_1 + b^i_1 i; b_2 + b^i_2 i\right),$$
$$a_1 = \tfrac{j+\alpha}{2}, \quad a^i_1 = \tfrac{\beta}{2}; \quad b_1 = \tfrac{j+\alpha+2}{2}, \quad b^i_1 = \tfrac{\beta}{2}; \quad b_2 = 1, \quad b^i_2 = \beta.$$

Thus, recurrent relations are obtained for real and imaginary parts of generalized hypergeometric functions of complex arguments, what allows us to determine the temperature distribution depending on the parameters of the structure of wood and other porous materials.

Determine the solution $\tilde{T}_{wa}(\beta)$:

$$\tilde{T}_{wa}(\tau) = \int_0^\tau e^{-(\beta^2+\gamma^2)(\tau-t)} \left[\frac{\sinh \pi b}{\pi\lambda^{2\alpha}}\Big(\alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos\nu^2_n t\Big)\right] dt$$
$$= e^{-(\beta^2+\gamma^2)\tau}\frac{\sinh \pi b}{\pi\lambda^{2\alpha}} \int_0^\tau e^{(\beta^2+\gamma^2)t}\Big(\alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos\nu^2_n t\Big) dt$$
$$= \frac{\sinh \pi b}{\pi\lambda^{2\alpha}}\left[\tfrac{1-e^{-(\beta^2+\gamma^2)\tau}}{(\beta^2+\gamma^2)}\alpha_0 + \sum_{n=1}^{\infty}\alpha_n \tfrac{\left[(\beta^2+\gamma^2)\cos\nu^2_n\tau + \nu^2_n \sin\nu^2_n\tau\right] - e^{-(\beta^2+\gamma^2)\tau}(\beta^2+\gamma^2)}{(\beta^2+\gamma^2)^2 + (\nu^2_n)^2}\right]. \tag{29}$$

The solution of the Cauchy problem is the function

$$\tilde{T}(\tau,\beta) = e^{-(\beta^2+\gamma^2)\tau}\tilde{g}(\beta) + \int_0^\tau e^{-(\beta^2+\gamma^2)(\tau-t)}\left[\frac{\sinh\pi b}{\pi\lambda^{2\alpha}}\Big(\alpha_0 + \sum_{n=1}^{\infty}\alpha_n\cos\nu^2_n t\Big)\right] dt$$
$$= e^{-(\beta^2+\gamma^2)\tau}\tilde{g}(\beta) + \tilde{T}_{wa}(\tau)$$
$$= e^{-(\beta^2+\gamma^2)\tau}\Big\langle \tfrac{1}{a^2}U^{11}_{\alpha,11}(\lambda R)\pi^{-1}\sinh\pi\beta\lambda^{-(2\alpha+j)}\sum_{j=0}^{2} g_{j0}$$
$$\times \left\{\tfrac{2^{i\beta-1}\Gamma(i\beta)}{j+\alpha-i\beta}(\lambda R)^{j+\alpha-i\beta}{}_1F_2\left(\tfrac{j+\alpha-i\beta}{2}; 1-i\beta, \tfrac{j+\alpha-i\beta+2}{2}; \tfrac{(\lambda R)^2}{4}\right)\right.$$
$$\left. + \tfrac{2^{-i\beta-1}\Gamma(-i\beta)}{j+\alpha+i\beta}(\lambda R)^{j+\alpha+i\beta}{}_1F_2\left(\tfrac{j+\alpha+i\beta}{2}; 1+i\beta, \tfrac{j+\alpha+i\beta+2}{2}; \tfrac{(\lambda R)^2}{4}\right)\right\}$$
$$- X^{12}_{\alpha;11}(\lambda R,\beta)\tfrac{1}{a^2}\lambda^{(j+2\alpha)}\sum_{j=0}^{2} g_{j0}\left\{\tfrac{(\lambda R)^{j+\alpha+i\beta}}{2^{i\beta}(j+\alpha+i\beta)\Gamma(1+i\beta)}{}_1F_2\left(\tfrac{j+\alpha+i\beta}{2}; \tfrac{j+\alpha+i\beta+2}{2}, 1+i\beta; \tfrac{(\lambda R)^2}{4}\right)\right\}\Big\rangle$$
$$+ \frac{\sinh\pi b}{\pi\lambda^{2\alpha}}\left[\tfrac{1-e^{-(\beta^2+\gamma^2)\tau}}{(\beta^2+\gamma^2)}\alpha_0 + \sum_{n=1}^{\infty}\alpha_n\tfrac{\left[(\beta^2+\gamma^2)\cos\nu^2_n t + \nu^2_n\sin\nu^2_n t\right] - e^{-(\beta^2+\gamma^2)\tau}(\beta^2+\gamma^2)}{(\beta^2+\gamma^2)^2 + (\nu^2_n)^2}\right]. \tag{30}$$

Apply to the function $\tilde{T}(\tau,\beta)$ the integral operator $H_\alpha^{-1}(\tau,\beta)$. We obtain a solution of the problem. For non-stationary case we have

$$\int_0^\infty e^{-(\beta^2+\gamma^2)t}\frac{\sinh \pi\beta V_\alpha(r,\beta)\,\Omega(r,\beta)\,d\beta}{\pi\lambda^{2\alpha}}g_R = \frac{I_{q,\alpha}(\lambda r)}{U^{11}_{q,\alpha}(\lambda R)}g_R(t),$$

$$\int_0^\infty e^{-(\beta^2+\gamma^2)t}V_\alpha(r,\beta)\,V_\alpha(\rho,\beta)\,\Omega(r,\beta)\,d\beta = \varepsilon(r,\rho,q) = \frac{\lambda^{2\alpha}}{U^{11}_{q,\alpha;11}(\lambda R)}\begin{cases} I_{q,\alpha}(\lambda r)\psi^{1*}_{q,\alpha;11}(\lambda R,\lambda\rho), & 0<r<\rho<R,\\ I_{q,\alpha}(\lambda\rho)\psi^{1*}_{q,\alpha;11}(\lambda R,\lambda r), & 0<\rho<r<R.\end{cases}$$

4. Numerical analysis

Based on the formulas obtained in this and our previous work [13] for determining the temperature, moisture content at any point of the radius of wooden cylindrical beam taking into account the moving boundaries of the moisture evaporation zone at any time of drying depending on the effect of thermal diffusion, initial values of temperature and moisture, thermophysical characteristics of the material and parameters of the drying agent on the temperature of phase transitions, a software program is designed, the work of which is demonstrated for solving a specific application problem of wood drying.

To implement the numerical experiment, the characteristics of the thermophysical properties of wood were used. The dependence of the hydro conductivity of wood on temperature and moisture was derived on the basis of experimental data [19].

Numerical simulation of drying of a sample of a cylindrical pine timber beam of a circular cross-section of the temperature T_0 with a 50% moisture content was carried out. The following basic parameters of the problem were accepted: ambient temperature T_0, which is determined by the temperature of the steam-air mixture measured by a dry bulb thermometer. The drying process lasted until the temperature of the beam reached the ambient temperature $T_1 = 289\,\mathrm{K}$. Drying agent velocity $\upsilon = 2\,\mathrm{m/s}$; saturated vapor density $\rho_\nu = 0.013188\ \mathrm{kg/m^3}$; air density $\rho_{a0} = 1.29\ \mathrm{kg/m^3}$. Physical parameters of wood: the radius of cross-section of a beam $R = 0.25$ m; density 500 kg/m; moisture 0.7 kg/kg; porosity $\Pi = 0.672$. Thermal parameters of wood: initial temperature $T_0 = 290$ K, thermal conductivity coefficient $\lambda = 0.14$ W/(mK).

Computer simulation of the drying of a cylindrical beam was carried out for soft ($\approx 300\,\mathrm{K}$) and hard regimes ($\approx 370\,\mathrm{K}$), which were determined by the control functions of temperature and moisture of the steam-air mixture, which is fed into the drying chamber.

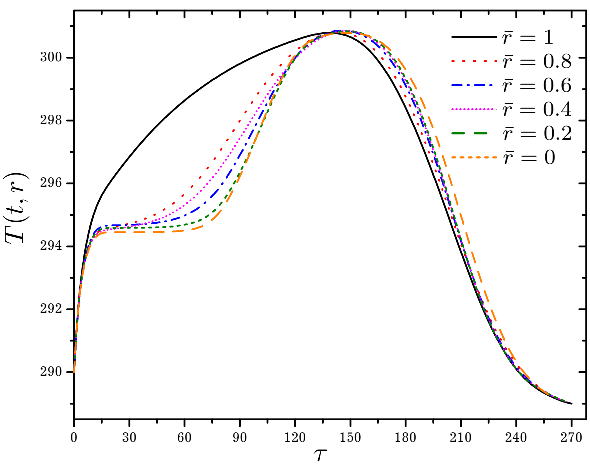

Fig. 3. Temperature distributions on the surface and inside the cylindrical beam at a drying agent temperature of 302 K (soft regime).

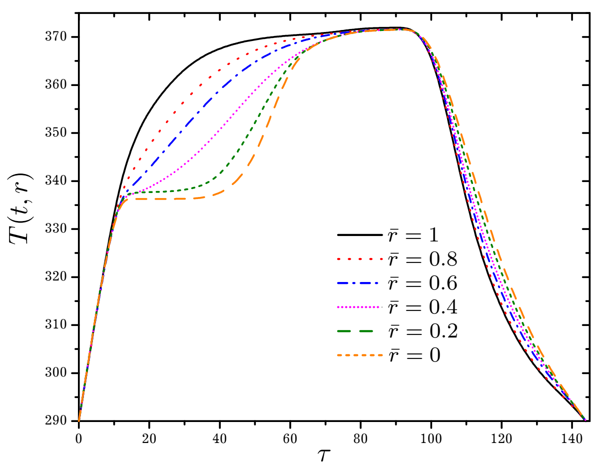

Fig. 4. Temperature distributions on the surface and inside the cylindrical beam at a drying agent temperature of 370 K (hard regime).

In Figs. 3–5, the temperature distributions in the structural elements of the cylindrical beam are presented. Figure 3 characterizes the change in temperature in the wooden beam during drying at 300K; and so does Fig 4 at 370 K, respectively. Here, the curve 1 corresponds to a unit value of dimensionless radius $\bar{r} = 1$, i.e., it shows the temperature on the surface of the cylinder; curve 2: $\bar{r} = 0.8$; curve 3: $\bar{r} = 0.6$; curve 4: $\bar{r} = 0.4$; curve 5: $\bar{r} = 0.2$; curve 6 corresponds to zero value of dimensionless radius: $\bar{r} = 0$. In Fig. 5, the temperature distributions on the surface and inside the cylindrical beam in the period of stabilization of the drying rate τ_2 in the hard regime are shown. Figure 6 shows the moisture distributions of the material along the radius of the cylinder for different drying times in the same mode. Figure 7 demonstrates the dependence of the thickness of the dried zone on the drying time.

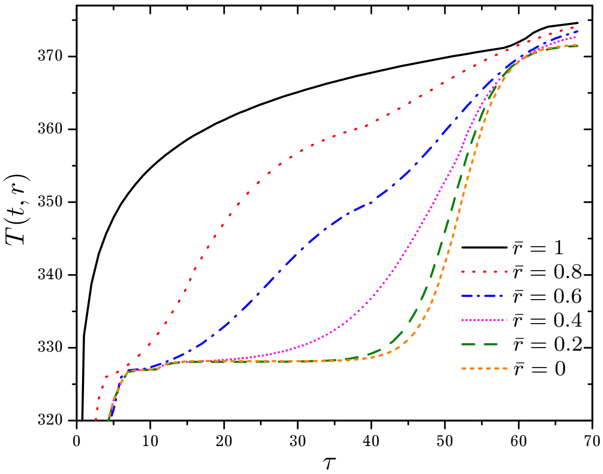

Fig. 5. Temperature distributions on the surface and inside the cylindrical beam during the period of stabilization of the drying rate τ_2 in the hard regime.

Analyzing the graphical dependences (Figs. 3, 4), we can see that in the process of drying cylindrical wood with the specified initial parameters, three characteristic stages are observed: heating, stabilization, and cooling. During the first period τ_1 (heating period) we observe intense heating of wood, and this has little effect on the change in the amount of moisture in the material (Fig. 6). Then (τ_2) we observe the stabilization of the drying rate (Fig. 5) with a noticeable loss of moisture content in depth in beam layers (Fig. 6), when the external heat exchange controls the drying rate. The temperature of the inner layers of the wood remains almost unchanged, and moisture is removed at maximum rate evenly and in proportion to the drying time. This is due to the absorption by wood layers of a large amount of heat during internal evaporation. This stage ends when the critical moisture content point is attained (Fig. 6) and the drying process proceeds to the stage of removal the bound moisture and this stage continues until equilibrium moisture is reached when the wood temperature reaches its maximum value $T_{\max}$. The duration of this period takes the majority of the total drying time (70%).

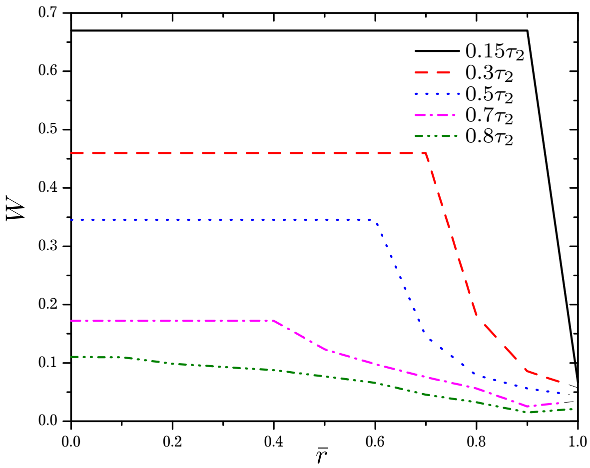

Fig. 6. Distributions of moisture content along the radius of the cylinder for different drying times τ_2 in hard drying regime.

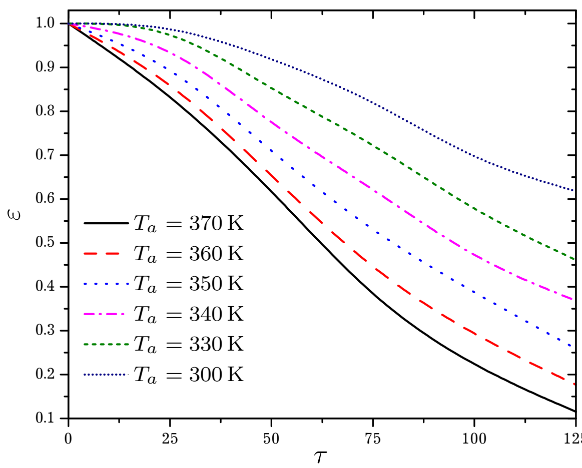

Fig. 7. Dependence of thickness of the dried area zone on time of drying.

During the third stage τ_3, the heat supply is stopped. But the wood remains in the drying medium to remove the bound moisture and to reach the required final level of moisture in the internal structure of the wood (≈ 0.05) (Fig. 6). This is a period of reduced drying rate, when drying is controlled only by internal mass transfer. At this time it is necessary to provide maximum ventilation.

For a given type of wood, we observe a high rate of moisture transfer for all drying regimes (Fig. 5). The intensity of moisture removal is proportional to the temperature of the drying agent. However, we should note that for the first period of drying, we observe a slowdown in the release of moisture for all values of the temperature of the drying agent, and in the soft mode of its removal is completely absent (curves $T_a = 300,\ 330$).

It should be noted that the temperature distributions in the cross-sectional layers of wood for the two considered drying regimes differ qualitatively and quantitatively. The temperature of the outer layer of the cylindrical beam during the entire drying period is much higher than the temperature of the middle layers, and, here, the maximum residual pressure is maintained until the end of τ_1. A temperature gradient appears, which causes the flow of moisture to move towards low temperatures, and its place is filled by steam.

At hot drying modes during the second period of stabilization we observe a significant difference in the values of the temperature of wood layers in depth, sometimes up to 10 K (Fig. 5, measurement time $0.5\tau_2$, layers $\bar{r} = 0.2,\ 0.4,\ 0.6$). Just at this time we observe the maximum values of internal residual pressures in these layers. In mild regime, an increase in the temperature of the central part of the beam is observed at the time $2/3\,\tau_2$ and a corresponding decrease in moisture content in its core layers (Fig. 6, time curve $0.7\tau_2$). From the third period τ_3, the rate of moisture removal decreases until the state of equilibrium moisture content.

5. Conclusions

The mathematical model for drying the long porous timber beam of a circular cross-section under the action of a convective-heat nonstationary flow of the drying agent in the 3-stage temperature regime is formulated. The solution of temperature distribution in the cross-section of the beam during drying at an arbitrary time of drying at any coordinate point of the radius, thermomechanical characteristics of the material, and the parameters of the drying agent has been constructed. The solution is constructed in modified Bessel functions. The relationships between the drying time and the average parameters of porous cylindrical wood and the drying agent parameters are determined. Numerical analysis is carried out. The proposed model is verified by theoretical and numerical results.

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Дослідження процесу сушіння пористої деревини циліндричної форми

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У представленому дослідженні побудовано математичну модель сушіння пористого бруса круглого перерізу під дією конвективно-теплового нестаціонарного потоку сушильного агента. При розв'язуванні задачі капілярно-пористу структуру бруса описано у термінах квазіоднорідного середовища з ефективними коефіцієнтами, які вибрані так, щоб розв'язок в однорідному середовищі збігався з розв'язком у пористому середовищі. Вплив пористої структури враховано шляхом введення в рівняння Стефана-Максвелла ефективних бінарних коефіцієнтів взаємодії. Проблема взаємного розподілу фаз вирішена з використанням принципу локальної фазової рівноваги. Приведені властивості матеріалу (теплоємність, густина, теплопровідність) вважаються функціями пористості матеріалу, а також густини та теплоємності компонентів тіла. Отримано розв'язки для визначення температури, вологості, густини пари і тиску пари в брусі в довільний момент часу сушіння в будь-якій координатній точці радіуса, термомеханічних характеристик матеріалу і параметрів сушильного агента.

Ключові слова: *плоска задача теплопровідності, циліндричний брус, сушильний агент, пористе середовище, квазіоднорідне наближення, інтегральне перетворення, функції Бесселя першого та другого роду, перетворення Конторовича–Лебедєва, теорема Стеклова, функція Гріна, рухома межа.*