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# Hybridization of Divide-and-Conquer technique and Neural Network algorithm for better contrast enhancement in medical images

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The aim of this work is to propose a new method for optimal contrast enhancement of a medical image. The main idea is to improve the Divide-and-Conquer method to enhance the contrast, and highlight the information and details of the image, based on a new conception of the Neural Network algorithm. The Divide-and-Conquer technique is a suitable method for contrast enhancement with an efficiency that directly depends on the choice of weights in the decomposition subspaces. A new hybrid algorithm was used for the optimal selection of weights, considering the optimization of the enhancement measure (EME). To evaluate the proposed model's effectiveness, experimental results were presented showing that the proposed hybrid technique is robustly effective and produces clear and high contrast images.

**Keywords:** *medical images, contrast enhancement, Divide-and-Conquer technique, Neural Network Algorithm, optimization algorithm, Gray–Scott mathematical model.*

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## 1. Introduction

Image processing is an interesting branch of applied mathematics and computer science in wide range of real applications. Image processing refers to the process of modifying or interpreting existing images. In other words, it is the process of enhancing and transforming images through a set of techniques to improve their content, quality and render them more suitable for quantitative analysis [1–3]. Image processing requires the use of digital and medical images instead of classical optical images (as perceived by eyes) [4].

Images processing software works on encrypted data contained in the image, modifies these data which are then used to build a second "transformed" image that can be visualized. The digital representation of images [5] allows to combine images obtained under different modes of macro or microscopic, representing a spatial organization (3D) from a set of successive shots (2D), archiving images and transfer them through computer networks.

The present work gives an overview of the state of the art, and describes the main areas where image processing techniques are used in the medical field. Medical image processing is a multidisciplinary and highly complex field that involves many scientific disciplines ranging from mathematics, and computer science to physics and medicine [6]. It focuses on the development of problem-specific approaches to the enhancement of raw medical image data, through selective visualization and subsequent analysis.

Numerous concepts and approaches are used to structure the field of medical image processing, focusing on different aspects of its core areas. Among them, one can cite the following three approaches:

- Image formation:
  The first integral step is the acquisition of raw image data, it contains the original information on the captured physical quantities, describing the body's internal aspects. Such information becomes the main subject of all subsequent image processing steps [7–10].
  Various types of imaging modalities may use different physical principles and may therefore involve the detection of different physical quantities. These include: the energy of incident photons in computed tomography (CT) or digital radiography (DR), the energy of photons and their detection time in positron emission tomography (PET), parameters of a X-ray signal emitted by excited atoms in magnetic resonance imaging (MRI), and parameters of acoustic echoes in Ultrasonography. Whichever the type of imaging modality, the data acquisition process can be further subdivided into detection of a physical quantity, conversion, preconditioning, and digitization of acquired raw data.
  The second step is image reconstruction [11]. It is a mathematical process used to form an image from the acquired raw data. In multi-dimensional imaging, the image reconstruction process involves combining multiple data sets captured from different angles or at different time steps. This process deals with inverse problems, which are a fundamental topic in this field, by applying analytical and iterative algorithms [12, 13]. The latter are efficient and elegant both in terms of required processing power and computation time.
- Image Computing is an interdisciplinary field that lies at the intersection of computer science, as well as information engineering, electrical engineering, physics, mathematics and medicine [14]. It develops computational and mathematical methods to solve problems arising from medical images, and their use in biomedical research and clinical care. The main objective of image computing is to extract clinically relevant information or knowledge from medical images. Their methods can be classified into several major categories: image enhancement, analysis and visualization of imaging results.
  Image enhancement is used to improve the image's transform representation to improve the interpretability of information contained in an image. Its methods can be subdivided into spatial domain technique [15,16], which operates directly on the pixels of the image, it is especially convenient for contrast optimization. The second method is the frequency domain technique [17], which uses the frequency transform, and is more suitable for refining and smoothening images through the application of different types of filters.
  Image analysis is the main core process of image computing. It consists of processing an image into fundamental components to extract meaningful information. Image analysis typically includes tasks such as pattern finding, edge detection, noise removal, object counting, and statistical analysis of texture recognition or image quality [18].
  The visualization process consists of converting (rendering) the pixels/voxels of an image into a 2D/3D graphic representation. The accurate visualization of 3D images is essential as a first step in image analysis and processing, especially when trying to understand complex structures in medical or industrial applications [19, 20]. It aims at rendering image data to visualize anatomical and physiological image information in a specific form over specified dimensions.
- The last approach in medical image processing, is the management of the acquired information. In recent years, the management of medical images has become increasingly complex as the data are produced using equipment with digital techniques. Image management covers all the techniques used to store, communicate, transmit, archive and access (retrieve) image data efficiently [21, 22]. For instance, a single grayscale radiograph in its original state may necessitate a storage capacity of several megabytes, for which compression techniques are applied.

The major benefit of medical image processing is its ability to provide in-depth, but non-invasive, visualization and exploration of the inner anatomy. It enables creating and studying 3D models of the anatomies of interest for improving patient treatment outcomes, developing improved drug delivery systems and medical devices, as well as providing a more informed diagnosis. It has become one of the main tools used for medical advances in recent years.

As mentioned above, noise reduction and contrast enhancement are two most common and significant techniques used to improve the quality of medical images.

In recent years, image enhancement have been the subject of numerous studies. Many successful image enhancement techniques have been developed which can be classified mainly in the following main categories:

- Retinex-based algorithms have received an increasing interest of scientists. The retinex theory was proposed firstly by [23, 24] to model the image process of the human visual system. This theory assumes that an image can be decomposed into illumination and reflectance. In other words, the scene in the human eye is the product of reflectance and illumination [25]. Retinex-based algorithms aim to compute both image components simultaneously, with different illumination regularization constraints [26, 27]. Most retinex-based enhancement algorithms use a variety of methods to estimate the illumination, and then remove it to obtain the reflectance of the enhanced image. The details and textures of the image can be improved by removing the illumination. However, the resulting enhanced images look overloaded and unnatural, as they do not match the human vision system.
  The Retinex multi-scale algorithm with color restoration utilizes Gaussian filtering for estimating and removing illumination, and improving image contrast as well as color consistency. However, methods based on Retinex theory tend to suffer from graying in uniform scenes and lack of effectiveness in unnatural images.
- Unsharp masking technique is a classical tool used to enhance the sharpness of an image [28, 29]. The classical linear unsharp masking (UM) technique is often used to significantly improve the visual appearance of an image, focusing on its high frequency components to enhance its edges and detail information. Although the unsharp masking method is easy to implement and functions well in many applications. It suffers from two major drawbacks. The first one is the presence of the linear high-pass filter, rendering the system extremely sensitive to noise. This results in noticeable and undesirable distortions, especially in uniform image areas which are even slightly noisy. Secondly, it enhances areas of high contrast, considerably more than areas that do not have high image dynamics. Consequently, some unpleasant overshooting artifacts can appear in the resulting image.
- Histogram equalization is an image processing technique used to adjust the contrast of an image through its histogram [30]. This contrast enhancement technique is widely used due to its high efficiency and simplicity. It is one of the most sophisticated methods used to modify the dynamic range, as well as an image's contrast by altering the image, in such a way that its intensity histogram has the desired shape. This technique can be classified into two main categories according to the transformation function used.
  - Global histogram equalization (GHE) [31], is an extremely simple and fast technique, though its contrast enhancement power is low. Such technique has been extensively applied when image enhancement is needed, as in medical image processing, radar image processing, texture synthesis, and speech recognition. In GHE, the histogram of the entire input image is used to compute the histogram transformation function. Thereby, an image's histogram dynamic range is flattened and stretched. Thus, the global contrast is improved.
  - Local histogram equalization (LHE) [32] can be more effective at improving overall contrast. It is suitable for processing local features, but suffers from halo or blocking artifacts. However, the LHE technique requires high computational power.
- The Divide-and-Conquer strategy is an enhancement method, it tends to neglect the difference between the low frequency and high-frequency components of the image. These components can not be effectively used for image enhancement. A divide-and-conquer algorithm decomposes the

original problem recursively into two or more sub-problems of either the same or related types, making them easy enough to be solved directly. Alternatively, the solutions to the sub-problems are subsequently combined to form one solution to the original problem. In particular, Divide-and-Conquer algorithm was applied to achieve cellular automata [33], parallel computers [34] and image enhancement [35]. A new method of image enhancement has therefore been developed, which uses the Divide-and-Conquer strategy.

Firstly, a series of effective linear filters which decompose the observed image into four subspaces are designed. After that, the image contrast enhancement operation on each of these subspace images is applied using the weighted gamma correction method. Then, the result of enhancement of each subspace image is obtained through the gradient distribution specification, which is the best for image naturalization. Thereafter, the entire image is then reconstructed through the fusion of these enhanced subspace images with different balancing weights. Experimental simulations show that the high-frequency information of the image can be estimated with more accuracy under the proposed framework, leading to more finely structured enhancement for better visual image quality. In both naturalization and detail promotion, the proposed method outperforms other state-of-the-art methods.

The main problem of this technique is the choice of weights. The objective of this paper is to propose an original algorithm to compute the optimal weights, through the introduction of a meta-heuristic optimization method of a well-defined cost function, based on the computation of image enhancement at each iteration [36, 37].

- Partial differential equation methods applied to digital and medical image processing, have represented a significant direction in applied mathematics over the last decades. New challenges persist in the study of digital and medical image processing, based on partial differential equation methods. In the last few years, the partial differential equations approach has received a tremendous interest from many researchers. This is due mainly to the mathematical formulation which frames any approach based on partial differential equations (PDE), as well as the good justification and interpretation of the obtained results, applying these traditional and heuristic methods in image processing [38].

  Such method exhibits better performance compared to common image processing methods, and new ideas were never considered in traditional image processing, including affine invariant feature extraction, image structure and texture decomposition, etc. Partial differential equation method is aimed at establishing the mathematical model of a partial differential equation, and then evolving the image according to the partial differential equation, and finally achieving the required effect. This method of image enhancement is beyond the reach of traditional methods.

  The Gray-Scott model is one of the most well-known and widely used mathematical models in image processing [39]. The Gray-Scott model provides a simple system inspired by the study of chemical processes. This model is a reaction-diffusion model which uses two generic chemical species $U$ and $V$. The concentration of these species at a given point in space is denoted by $u(t)$ and $v(t)$, which represent respectively the activator and the inhibitor concentration. The Gray-Scott model is described by a pair of ordinary differential equations evolving in time, given as follows

$$\begin{cases} \dfrac{du}{dt} = -uv^2 + A(1-u), \\ \dfrac{dv}{dt} = uv^2 - (A+k)v, \\ u(0) = u_0, \quad v(0) = v_0, \end{cases} \tag{1}$$

  where $A$ is the feed rate, $k$ denotes a parameter that controls the kill rate and $u_0$ refers to the initial image. This system was applied by [40] for patterns formation. More recently, this model has been used by [41] to improve the contrast of an image.

The following paragraph shows an improvement of the divide-and-conquer technique by using the Gray–Scott model on each subspace instead of the approach proposed by [35]. Despite this, the choice of balancing weights remains a major drawback. In order to determine the optimal balancing weights, the use of the proposed hybridization method of Divide-and-Conquer technique, and Neural Network algorithm represents one of the most effective methods to improve the resulting image contrast, whence the originality of the present work.

To improve the efficiency of image enhancement, a cost function defined with respect to the balancing weights, is introduced. This cost function is optimized using the improved Neural Network Algorithm (NNA). In Section 2, a modified version of the Divide-and-Conquer method together with the meta-heuristic optimization method Neural Network algorithm, are presented. The third section is devoted to the general presentation of the hybrid model which is used to improve the medical image contrast, based on the modified Divide-and-Conquer strategy with an updated version of the Neural Network Algorithm (NNA). In Section 4, the experimental results and performance analysis are presented, and the conclusion is given in Section 5.

## 2. Background and materials

This study presents a new algorithm based on the hybridization of the Divide-and-Conquer strategy and Neural Network algorithm, called the Divide-and-Conquer with Neural Network algorithm (DCNNA). The remainder of this section is concerned with a full description of Divide-and-Conquer strategy and NNA.

### 2.1. Divide-and-Conquer technique

An alternative way to increase the contrast in a digital image is the Divide-and-Conquer strategy [42, 43]. Previous literatures on the decomposition models [44, 45] had shown the difference between low-frequency (cartoon) and high-frequency (edges and texture) components of an image; the smoothing areas represent low-frequency portions, while edges and details are contained in high-frequency parts. Afterword, each component will be implemented separately. To this end, an observed image $U$ is decomposed into different low and high-frequency, using the convolution of $U$ with a predefined linear filter $h_i$:

$$U_i = U \otimes h_i \quad \text{for} \quad i = 1, 2, 3, 4, \tag{2}$$

where $h_3 = [-1, 1]$ and $h_4 = [-1, 1]^T$ are two high-frequency filters and the symbol $\otimes$ is the two-dimensional convolution operator. The sub-images $U_3$ and $U_4$ contain the high-frequency component of the entire image $U$ whereas, $U_1$ and $U_2$ form regions with low-frequency. Then, we define frequency responses $H_1$ and $H_2$ of low-frequency filters $h_1$ and $h_2$ as follows:

$$H_1 = I - H_3, \quad H_2 = I - H_4. \tag{3}$$

Where $H_3$ and $H_4$ are respectively the Fourier transform of $h_3$ and $h_4$. $I$ is a matrix where each of its elements is equal to one.

Once the decomposition process of the image $U$ is obtained, the Gray-Scott model (1) is applied to naturalize each observed subspace image $U_i$ separately to get the image $U^i_{GS}$, then the enhanced subspace image $U_i$ is reconstructed from a linear remapping approximation:

$$U_{dc} = \sum_{i=1}^{4} w_i U^i_{GS}, \tag{4}$$

where $\{w_i\}_{i=1}^4$ are the weights of balancing different sub-images. The algorithm that allows us to generate the new image is given below:

**Algorithm 1** Outline of Modified Divide-and-Conquer method **(MDC)**

Input: Observed image $U$, complete filters $\{h_i\}_{i=1}^4$, balancing weights $\{w_i\}_{i=1}^4$.
Generate observed subspaces $\{H_i\}_{i=1}^4$ via (3).
Obtain enhanced subspaces $\left\{U_{GS}^i\right\}_{i=1}^4$ via (1).
Output: Final image $U_{dc} = \sum_{i=1}^4 w_i U_{GS}^i$.

### 2.2. Description of Neural Network Algorithm (NNA)

In this work, a novel meta-heuristic optimization method, called Neural Network Algorithm (NNA), is used. This algorithm had been proposed by A. Sadollah et al. [46], and inspired by biological nervous system and Artificial Neural Networks (ANNs).

### 2.3. Artificial Neural Networks

An Artificial Neural Network is a hardware and\or software computing system with functioning based on the human Brain's neurons.

Artificial Neural Networks are a variety of deep learning technology, being a part of the machine learning subcategory of artificial intelligence.

**Definition 1.** *An artificial neuron is a biological element like a biological neuron, which receives input signal from environment as an input vector $x$ and sends it as output in environment.*

*Neural network is a mesh of numerous neurons arranged in layers. The neurons "K" of a single layer are all connected to the inputs "R". In such case, the layer is considered as entirely connected. Figure 1 is an example of a network of R-input and K-neurons.*

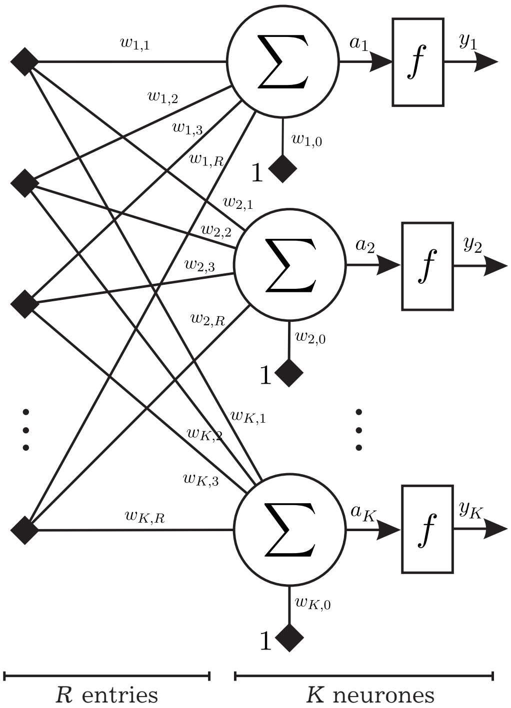



**Fig. 1.** Layer with K neurons.

Where

- The output $a_l$, $l = 1, \ldots, K$ of the integrator is obtained by applying the propagation rule

$$a_l = \sum_{j=1}^{R} w_{lj} x_j(t) + \theta_l. \tag{5}$$

- $w_{lj}$, $(l, j) \in 1, \ldots, K \times 1, \ldots, R$ is the weight between the output neuron $l$ and an input $j$. Its vector $w = [w_{1,1}, w_{1,2}, w_{1,3}, \ldots, w_{1,R}]^T$ represents the vector of synaptic weight of the neuron [47, 48].
- $\theta_l$, $l = 1, \ldots, K$ is the activation bias or threshold of the neuron which is denoted by $w_{l,0}$ for $l = 1, \ldots, K$ (see Figure 1).
- $x_j(t)$, $j = 1, \ldots, R$ the input data at time $t$.

The activation level $a = [a_1, a_2, \ldots, a_K]^T$ is transformed by a transfer function $f$ also called activation function that produces the output "$y$" of the neuron.

- $f$ is the activation function. Activation functions are a critical part of the design of a neural network. Many types of such functions exist, in particular, the most commonly used
  - — linear,
  - — sigmoid,
  - — hyperbolic tangents.

- $y_l$, $l = 1, \dots, K$, the output of neuron $i$ is given by

$$y_l = f(a_l) = f\left(\sum_{j=1}^{R} w_{lj} x_j(t) + \theta_l\right). \tag{6}$$

Finally, the Artificial Neural Network output is defined by the following sense

$$y = \sum_{l=1}^{K} w_l f\left(\sum_{j=1}^{R} w_{lj} x_j(t) + \theta_l\right) + \theta. \tag{7}$$

We define the function $h$ as

$$h(x) = \sum_{l=1}^{K} w_l f\left(\sum_{j=1}^{R} w_{lj} x_j(t) + \theta_l\right) + \theta. \tag{8}$$

Then the problem to optimize becomes

$$\min_{x \in \mathcal{D}} h(x). \tag{9}$$

The Artificial Neurons Network is formed by generating data in the field of pattern solutions denoted by $\mathcal{D}$. We reformulate the problem using the equation (9). Once the problem is rewritten, another solution can be used to solve it.

### 2.4. Neural Network Algorithm (NNA)

The concept of Artificial Neural Networks and the biological nervous system, have been used to develop a modern meta-heuristic optimization algorithm to optimally solve problems, known as the Neural Network Algorithm (NNA) [46]. One of the key features of the proposed NNA is its ability to incorporate the framework, and the definition of ANNs to build new candidate solutions, and use the other ANN operators for its search strategy. The NNA uses the dynamic nature of ANNs and their mechanism for generating new solutions in the search domain.

Similarly to the other meta-heuristic algorithms, the NNA starts with an initial population called the population of pattern solutions. Based on Artificial Neural Networks (ANNs), the target data is considered as the best obtained solution (i.e., the time-optimal solution) at each iteration in the NNA. The main objective of the NNA consists in reducing the error between the target data, and the other predicted pattern solutions (i.e., moving the other predicted patterns to the target solution).

Indeed, the NNA is designed to minimize the problem (i.e., reducing the error between the target and the predicted solutions). It is worth noting that this target solution was revised at each iteration. The different steps of the algorithm (NNA) are given below.

#### 2.4.1. Generating initial population

The NNA is a population-based algorithm. Basically, it starts with an initial population of randomly generated solutions in the search space. Each individual or search agent in the population is called a pattern solution (i.e., in the GA, it is called a chromosome). Every pattern solution is a vector of $1 \times D$ representing the input data of the NNA. $\text{Pattern Solution}_l = [x_1^l, x_2^l, \dots, x_D^l]$, where $l$ varies from $1, \dots, N$ and

$$x_j^l = LB_j + \text{rand}\left(UB_j - LB_j\right), \quad l = 1, 2, \dots, N, \quad j = 1, 2, \dots, D, \tag{10}$$

in which $LB$ and $UB$ are the minimum and maximum limits of variables, respectively.

Initially, the solution pattern matrix $X$ with size $N \times D$ is randomly generated; between the lower and upper bounds of the search space, where $N$ is the generation number and $D$ is the number of variables. The pattern solutions can be mathematically symbolized as follows:

$$X = \begin{pmatrix} x_1^1 & x_2^1 & x_3^1 & x_4^1 & x_5^1 & x_6^1 & x_7^1 & x_8^1 & \dots & x_D^1 \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 & x_5^2 & x_6^2 & x_7^2 & x_8^2 & \dots & x_D^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^N & x_2^N & x_3^N & x_4^N & x_5^N & x_6^N & x_7^N & x_8^N & \dots & x_D^N \end{pmatrix} \tag{11}$$

### 2.4.2. Weight matrix

Once the target solution is defined from the other pattern solutions, the target weight ($W^{Target}$, the weight corresponding to the target solution) needs to be selected from the weight population. In ANNs, the initial weights are random numbers to be updated as the number of iterations increase according to the network's calculated error. For the NNA, the initial weights are a square matrix of size $N \times N$, generating uniform distributed random numbers between 0 and 1 during iterations. The weight population array is given as

$$W(t) = [W_1, W_2, \cdots, W_N] = \begin{pmatrix} w_{11} & \cdots & w_{l1} & \cdots & w_{N1} \\ w_{12} & \cdots & w_{l2} & \cdots & w_{N2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{1N} & \cdots & w_{lN} & \cdots & w_{NN} \end{pmatrix}. \tag{12}$$

An additional constraint is imposed on the weight values. However, the sum of the weights for a pattern solution must not exceed one, which is defined mathematically as follows: $W$ must respect the following constraints

$$\sum_{j=1}^{N} w_{lj}(t) = 1, \quad l = 1, \dots, N, \tag{13}$$

$$w_{lj}(t) \in \mathcal{U}(0,1), \quad l, j = 1, \dots, N. \tag{14}$$

### 2.4.3. Pattern solution and weights update

Let $X(t)$ and $W(t)$ represent the population of pattern solutions and the weight of the matrix, respectively at iteration index $t$. Their values at iteration $t+1$ are given for all $l = 1, \dots, N$ by

$$X_l(t+1) = X_l(t) + \sum_{j=1}^{N} w_{jl}(t) \times X_l(t). \tag{15}$$

Once the new pattern solutions are created from the previous pattern population, based on the best weight value called the "target weight", the weight matrix needs to be updated as well. The following equations propose an updated weight matrix equation:

$$W_l(t+1) = W_l(t) + 2 \times \text{rand} \times \left(W^{Target}(t) - W_l(t)\right). \tag{16}$$

$W$ should always satisfy the constraints Eq. (13) and Eq. (14) during the optimization process.

### 2.4.4. Bias operator

The bias operator is applied to prevent the algorithm from premature convergence (specially in the early iterations). In the NNA, the bias operator is used for good exploration of the search space (exploration process). This operator is similar to the mutation operator in the Genetic Algorithm (GA). As a result, a certain percentage of the pattern solutions in the new population of pattern solutions, and the updated weight matrix is modified using the bias operator.

The modification factor $\beta$ is used to determine the percentage of the pattern solutions that should be modified using the bias operator. The initial value of $\beta$ is set to 1 that means there is a 100% chance for modifying all the individuals in the population. The modification factor value is reduced adaptively at each iteration using the following recurrence formula

$$\begin{cases} \beta(1) = 1, \\ \beta(t+1) = 0.999 \times \beta(t), \quad t = 1, \ldots, \text{MaxIteration}. \end{cases} \tag{17}$$

In the NNA, the bias operator is made up of two parts: Population Bias and Weight Matrix Bias. For population bias, a random number is produced firstly according to the following equation:

$$N_P = \text{Round}(D \times \beta). \tag{18}$$

Then the bias operator can be described as follows:

$$X_j = LB + \text{rand}(UB - LB), \quad j = 1, 2, \ldots, N_P. \tag{19}$$

Also, a random number is generated to detect the number of weights that must be updated as follows,

$$N_w = \text{Round}(N \times \beta). \tag{20}$$

Thus the weight matrix bias is defined as

$$W_j = d, \quad j = 1, 2, \ldots, N_w, \tag{21}$$

where $d$ is a random variable between 0 and 1.

### 2.4.5. Transfer function operator

In the NNA, we apply the transfer function operator to show the exploitation manner of the algorithm. This operator transfers the new pattern solutions from their original positions in the search space to new positions, in order to decrease the gap between them and the best solution (target solution). The transfer function operator (TF) is represented by the following equation

$$\begin{cases} X_l^*(t+1) = TF(X_l(t+1)) \\ \qquad\qquad\;\; = X_l(t+1) + 2 \times \text{rand} \times \left(X^{Target}(t) - X_l(t+1)\right), \quad l = 1, \ldots, N. \end{cases} \tag{22}$$

To summarize, the entire procedure of the NNA is illustrated in the figure below (Figure 2) and Algorithm 1.

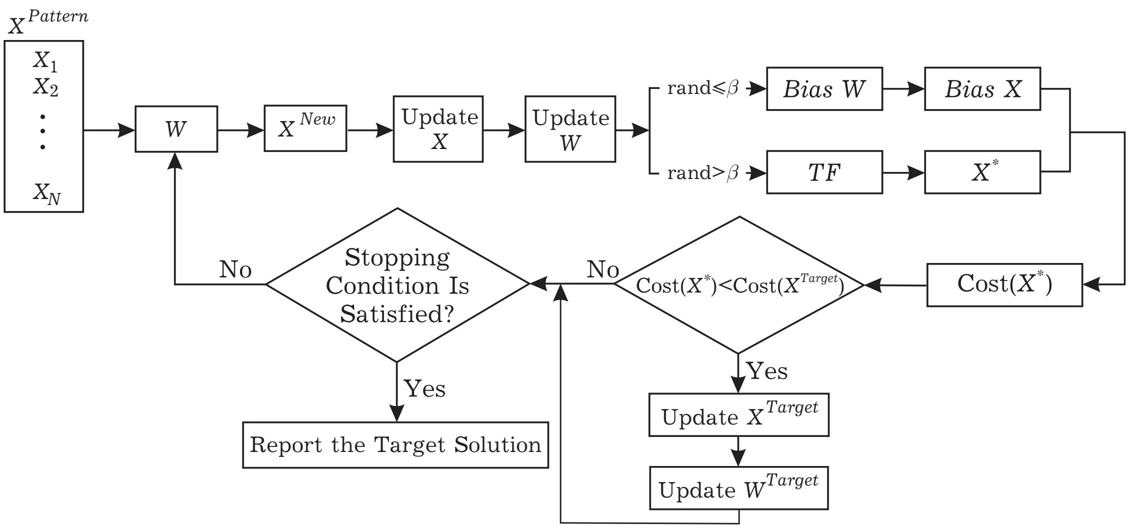



**Fig. 2.** Processes of the NNA [46].

**Algorithm 2** Pseudo code of Neural Network Algorithm **(NNA)**

1: Initialize the weight matrix and the population including $N$ solutions.
2: Calculate the fitness value of each solution and then set the target solution $X^{Target}$ and the target weight $W^{Target}$.
3: Repeat.
4: Generate the new population $X(t+1)$ via Eq. (15) and update the new weight matrix $W(t+1)$ via Eq. (16).
5: for each individual $l \in \{1, \ldots, N\}$
6: if rand $\leqslant \beta(t)$
7: Perform bias operator for updating the new pattern solution $X_l(t+1)$ by Eqs. (18)–(19), and the weight $W_l(t+1)$ by Eqs. (20)–(21)
8: else.
9: Perform the transfer function operator for updating the solution $X_l(t+1)$ via Eq. (22)
10: end if
11: end for
12: Update the new value of $\beta$ using any reduction formulation (e.g. Eq. (17)).
13: Calculate the fitness value of each solution and find the target solution $X^{Target}(t+1)$ and the target weight $W^{Target}(t+1)$.
14: Until (stop condition=false)
15: Post process results and visualization.

## 3. The proposed contrast enhancement mathematical model

In order to improve the image quality, the effective measure of enhancement $EME$ is used. It is obtained by splitting the image $U$ $k_1 k_2$ blocks $w_{k,l}(i,j)$ of sizes $l_1 l_2$, using the following equation

$$\mathrm{EME}(U) = \frac{1}{k_1 k_2} \sum_{l=1}^{k_1} \sum_{k=1}^{k_2} 20 \log \frac{u^w_{\max;k,l}}{u^w_{\min;k,l}}. \tag{23}$$

Where $u^w_{\max;k,l}$ and $u^w_{\min;k,l}$ are respectively maximum and minimum values of the image $U$ inside the block $w_{k,l}$. A higher value of EME indicates that the image is enhanced very well.

At this stage of our study, we are going to proceed to the hybridization between both methods, Modified Divide-and-Conquer and NNA algorithms.

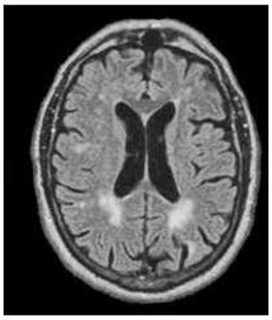
**Algorithm 3** Pseudo code of Neural Network Algorithm **(NNA)**

1: **Initialization:**
a) Input: Observed image $U$, complete filters $\{h_i\}_{i=1}^4$.
b) Generate observed subspaces $\{H_i\}_{i=1}^4$ via (3).
c) Obtain enhanced subspaces $\left\{U^i_{GS}\right\}_{i=1}^4$ via (1).
d) Generate randomly by using the uniform low, $N$ individuals: $[W^1, W^2, \ldots, W^N]$, where $W^j = (w^j_1, w^j_2, w^j_3, w^j_4)$ represent the balancing weights in Modified Divide-and-Conquer Algorithm (MDCA).
2: **Application of MDC Algorithm:**
a) For each individual $W^j = (w^j_1, w^j_2, w^j_3, w^j_4)$, obtain a new image $U^j_{dc} = \sum_{i=1}^4 w^j_i U^i_{GS}$ using Algorithm 1.
b) Calculate the cost function generated by the EME (23): $J(W^j) = \frac{1}{1+(\mathrm{EME}(U^j_{dc}))^2}$.
3: **Application of Neural-Network Algorithm:**
a) Optimize the cost function $J$ using Neural-Network Algorithm (NNA) (see Algorithm 2) to obtain $w^*_i$.
b) Output the final image: $U^* = \sum_{i=1}^4 w^*_i U^i_{GS}$.

**Remark 1.** The choice of the meta-heuristic method Neural-Network is justified since the function to minimize denoted by $J$ is not differentiable.

## 4. Results and discussion

In this section, all experimental simulations are performed in Matlab R2018 on a Dell Precision Tower 7910 Workstation with a processor: Intel® Xeon® Processor E5 v4 Family with number E5-2603V4. This machine is characterized by its sturdiness in the professional field and trimmed for best performance. The software application was written by the authors using the Matlab programming language. Here, different test images that are poorly contrasted (see Figures 3$a$, 4$a$ and 5$a$) are considered. The contrast enhancement results of the different methods on these images, are shown. In Figures 3$b$, 4$b$ and 5$b$, the resulting images obtained by Gray–Scott model, are presented. Figures 3$c$, 4$c$ and 5$c$ represent the different images enhanced using the classical Divide-and-Conquer technique. Finally, the enhanced images using the proposed hybridization of Divide-and-Conquer together with Neural Network algorithm (DCNNA), are given in Figures 3$d$, 4$d$ and 5$d$. In order to evaluate the result of the enhancement, the effective measure of enhancement EME of the enhanced images, is determined.



***a*** (Initial image with $\mathrm{EME}(4,4) = 9.3785$)

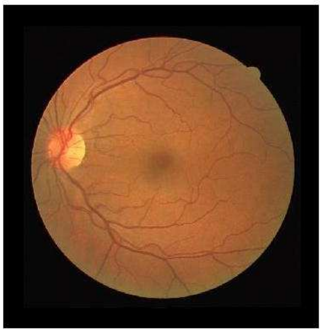
***b*** (Image obtained by GS model with $\mathrm{EME}(4,4) = 25.0329$)

***c*** (Image obtained by DC with $\mathrm{EME}(4,4) = 29.9845$ and $W = [0.7, 0.2, 0.3, 0.5]$)

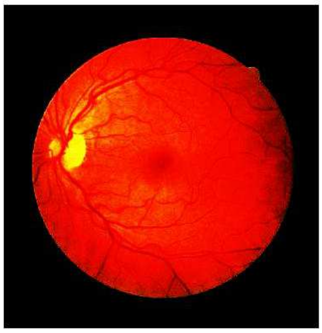
***d*** (Image obtained by DC-NNA hybridization with $\mathrm{EME}(4,4) = 38.9887$ and $W_{\text{optimal}} = [1.5, 0.2, 1.5, 0.2]$)

**Fig. 3.** Numerical results for the brain image.

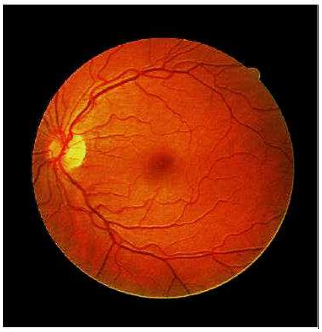
For the brain, the retina and tumor images, we calculate the EME of the initial images and compared it to the one obtained by Gray–Scott model (GS), the classical Divide-and-Conquer technique (DC) and the proposed hybridization method using Divide-and-Conquer and Neural Network algorithm (DCNNA). By applying the proposed method on the different images, one can remark that the obtained results are very satisfactory more than those obtained by Gray–Scott model and classical Divide-and-Conquer Strategy. The proposed method has always a high value of EME than GS and classical DC, which explains the good quality of the image enhanced using the DCNNA.



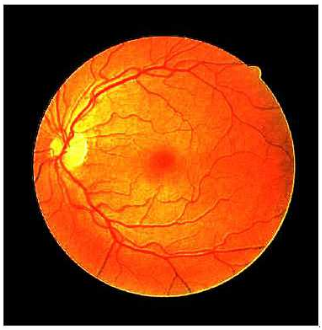
***a*** (Initial image with $\mathrm{EME}(4,4) = 8.00$)



***b*** (Image obtained by GS model with $\mathrm{EME}(4,4) = 34.0995$)

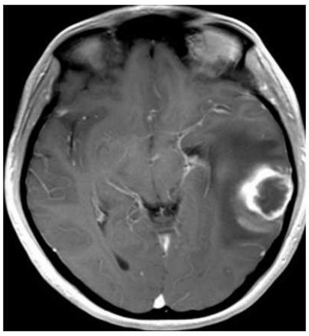


***c*** (Image obtained by DC with $\mathrm{EME}(4,4) = 33.2930$ and $W = [0.7, 0.2, 0.3, 0.5]$)

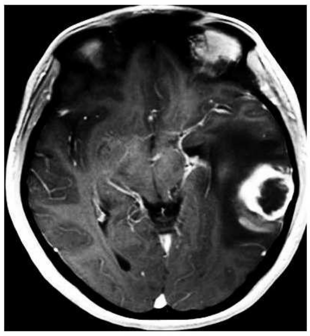


***d*** (Image obtained by DC-NNA hybridization with $\mathrm{EME}(4,4) = 41.2$ and $W_{\text{optimal}} = [0.8, 0.85, 0.82, 0.86]$)

**Fig. 4.** Numerical results for the retina image.



***a*** (Initial image with $\mathrm{EME}(4,4) = 4.4371$)



***b*** (Image obtained by GS model with $\mathrm{EME}(4,4) = 10.9224$)

***c*** (Image obtained by DC with $\mathrm{EME}(4,4) = 16.7671$ and $W = [0.7, 0.2, 0.3, 0.5]$)

***d*** (Image obtained by DC-NNA hybridization with $\mathrm{EME}(4,4) = 28.7466$ and $W_{\text{optimal}} = [1.5, 0.2, 1.5, 0.3]$)

**Fig. 5.** Numerical results for the Tumor image.

## 5. Conclusion

The numerical obtained results showed that the proposed method has consistently a high value of effective measure of enhancement EME, compared to the classical Gray-Scott system, and the classical Divide-and-Conquer technique, which explains the good quality of the enhanced image obtained by the hybrid method.

The primary focus of future work is to use other meta-heuristic algorithms rather than Neural Network Algorithm, and evaluate its performance against other algorithms, such as ABC (Artificial Bee Colony), HS (Harmony Search), CSA (Crow Search Algorithm), BOA (Butterfly Optimization Algorithm), SSA (Salp Swarm Optimization), and finally FPA (Flower Pollination Algorithm).

The second primary objective is blurred image processing through the introduction of a partial differential equation, using image restoration and contrast enhancement.

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[1] Weickert J. Anisotropic Diffusion in Image Processing. Teubner–Verlag, Stuttgart (1998).

[2] Gonzalez R. C., Woods R. E. Digital Image Processing. Pearson Prenctice Hall (2007).

[3] Alaa N. E., Zirhem M. Bio-inspired reaction diffusion system applied to image restoration. International Journal of Bio-Inspired Computation. **12** (2), 128–137 (2018).

[4] Zhang Y., Gong S., Luo M. Image quality guided biology application for genetic analysis. Journal of Visual Communication and Image Representation. **64**, 102606 (2019).

[5] Algazi V. R., Ford G. E., Hildum E. Digital representation and storage of high quality color images by anisotropic enhancement and subsampling. International Conference on Acoustics, Speech, and Signal Processing (1989).

[6] Goel N., Yadav A., Singh B. M. Medical image processing: A review. 2016 Second International Innovative Applications of Computational Intelligence on Power, Energy and Controls with their Impact on Humanity (CIPECH). 57–62 (2016).

[7] Deserno T. M. Fundamentals of biomedical image processing. In: Deserno T. (eds) Biomedical Image Processing. Biological and Medical Physics, Biomedical Engineering. Springer, Berlin, Heidelberg (2010).

[8] Morris P. G. Nuclear magnetic resonance imaging in medicine and biology (1986).

[9] Stark D. D., Bradley W. G. Magnetic resonance imaging. St. Louis, MO: Mosby (1992).

[10] Honeyman-Buck J. Deserno T. M. (ed) Biomedical image processing. Journal of Digital Imaging. **25**, 689–691 (2012).

[11] Defrise M., Gullberg G. T. Image reconstruction. Physics in Medicine & Biology. **51** (13), R139 (2006).

[12] Reader A. J., Zaidi H. Advances in PET image reconstruction. PET Clinics. **2** (2), 173–190 (2007).

[13] Vasilenko G. I., Taratorin A. M. Image reconstruction. Moscow, Izdatel Radio Sviaz (1986).

[14] Lu L., Zheng Y., Carneiro G., Yang L. Deep learning and convolutional neural networks for medical image computing. Advances in Computer Vision and Pattern Recognition (2017).

[15] Analoui M. Radiographic image enhancement. Part I: spatial domain techniques. Dentomaxillofacial Radiology. **30** (1), 1–9 (2001).

[16] Rahman S., Rahman M. M., Hussain K., Khaled S. M., Shoyaib M. Image enhancement in spatial domain: A comprehensive study. 2014 17th International Conference on Computer and Information Technology (ICCIT). 368–373 (2014).

[17] Agaian S. S., Panetta K., Grigoryan A. M. A new measure of image enhancement. In: IASTED International Conference on Signal Processing & Communication (pp. 19–22). (2000).

[18] Dhawan A. P. Medical image analysis. John Wiley & Sons. Vol. 31 (2011).

[19] Koehring A., Foo J. L., Miyano G., Lobe T., Winer E. A framework for interactive visualization of digital medical images. Journal of Laparoendoscopic & Advanced Surgical Techniques. **18** (5), 697–706 (2008).

[20] Silva S., Santos B. S., Madeira J., Silva A. Processing, visualization and analysis of medical images of the heart: an example of fast prototyping using MeVisLab. 2009 Second International Conference in Visualisation. 165–170 (2009).

[21] Deserno T. Medical image processing. Optipedia, SPIE Press, Bellingham, WA (2009).

[22] Marion J. L. Analysis of justification for modality integration. Proc. SPIE 0418, Picture Archiving and Communication Systems. **418**, 17–23 (1983).

[23] Land E. H. The retinex. American Scientist. **52** (2), 247–264 (1964).

[24] Land E. H., McCann J. J. Lightness and retinex theory. Journal of the Optical Society of America. **61** (1), 1–11 (1971).

[25] Fu X., Sun Y., LiWang M., Huang Y., Zhang X.-P., Ding X. A novel retinex based approach for image enhancement with illumination adjustment. 2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). 1190–1194 (2014).

[26] Ng M. K., Wang W. A total variation model for Retinex. SIAM Journal on Imaging Sciences. **4** (1), 345–365 (2011).

[27] Wang L., Xiao L., Liu H., Wei Z. Variational Bayesian method for retinex. IEEE Transactions on Image Processing. **23** (8), 3381–3396 (2014).

[28] Polesel A., Ramponi G., Mathews V. J. Image enhancement via adaptive unsharp masking. IEEE transactions on image processing. **9** (3), 505–510 (2000).

[29] Deng G. A generalized unsharp masking algorithm. IEEE transactions on Image Processing. **20** (5), 1249–1261 (2010).

[30] Wang Y., Chen Q., Zhang B. Image enhancement based on equal area dualistic sub-image histogram equalization method. IEEE transactions on Consumer Electronics. **45** (1), 68–75 (1999).

[31] Patel S., Goswami M. Comparative analysis of Histogram Equalization techniques. 2014 International Conference on Contemporary Computing and Informatics (IC3I). 167–168 (2014).

[32] Kim J. Y., Kim L. S., Hwang S. H. An advanced contrast enhancement using partially overlapped sub-block histogram equalization. IEEE transactions on circuits and systems for video technology. **11** (4), 475–484 (2001).

[33] Bacquey N. The packing problem: A divide and conquer algorithm on cellular automata. Automata & JAC. 1–10 (2012).

[34] Stout Q. F. Supporting divide-and-conquer algorithms for image processing. Journal of Parallel and Distributed Computing. **4** (1), 95–115 (1987).

[35] Zhuang P., Fu X., Huang Y., Ding X. Image enhancement using divide-and-conquer strategy. Journal of Visual Communication and Image Representation. **45**, 137–146 (2017).

[36] Alaa K., Atounti M., Zirhem M. Image restoration and contrast enhancement based on a nonlinear reaction-diffusion mathematical model and divide & conquer technique. Mathematical Modeling and Computing. **8** (3), 549–559 (2021).

[37] Zhuang P., Ding X. Divide-and-conquer framework for image restoration and enhancement. Engineering Applications of Artificial Intelligence. **85**, 830–844 (2019).

[38] Liu D. N., Hou R., Wu W. Z., Hua J. W., Wang X. Y., Pang B. Research on infrared image enhancement and segmentation of power equipment based on partial differential equation. Journal of Visual Communication and Image Representation. **64**, 102610 (2019).

[39] Gray P., Scott S. K. Autocatalytic reactions in the isothermal, continuous stirred tank reactor: Isolas and other forms of multistability. Chemical Engineering Science. **38** (1), 29–43 (1983).

[40] McGough J. S., Riley K. Pattern formation in the Gray–Scott model. Nonlinear Analysis: Real World Applications. **5** (1), 105–121 (2004).

[41] Alaa H., Alaa N. E., Aqel F., Lefraich H. A new Lattice Boltzmann method for a Gray–Scott based model applied to image restoration and contrast enhancement. Mathematical Modeling and Computing. **9** (2), 187–202 (2022).

[42] Li W., Shi M., Ogunbona P. A new divide and conquer algorithm for graph-based image and video segmentation. 2005 IEEE 7th Workshop on Multimedia Signal Processing. 1–4 (2005).

[43] Körting T. S., Castejon E. F., Fonseca L. M. G. The divide and segment method for parallel image segmentation. International Conference on Advanced Concepts for Intelligent Vision Systems. 504–515 (2013).

[44] Zirhem M., Alaa N. E. Texture synthesis by reaction diffusion process. Annals of the University of Craiova-Mathematics and Computer Science Series. **42** (1), 56–69 (2015).

[45] Alaa N., Zirhem M. Entropy solution for a fourth-order nonlinear degenerate problem for image decomposition. J. Adv. Math. Stud. **11**, 412–427 (2018).

[46] Sadollah A., Sayyaadi H., Yadav A. A dynamic metaheuristic optimization model inspired by biological nervous systems: Neural network algorithm. Applied Soft Computing. **71**, 747–782 (2018).

[47] Parizeau M. Réseaux de neurones. GIF-21140 et GIF-64326, 124 (2004).

[48] Dreyfus G. Réseaux de neurones: Méthodologie et applications. Eyrolles (2004).

## Гібридизація техніки "розділяй і володарюй" і алгоритму нейронної мережі для покращення контрастності медичних зображень

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Метою даної роботи є запропонувати новий метод оптимального контрастного посилення медичного зображення. Основна ідея полягає в удосконаленні методу "розділяй і володарюй" для посилення контрасту та виділення інформації та деталей зображення на основі нової концепції алгоритму нейронної мережі. Техніка "розділяй і володарюй" є відповідним методом для посилення контрасту з ефективністю, яка безпосередньо залежить від вибору вагових коефіцієнтів у підпросторах розкладання. Для оптимального вибору вагових коефіцієнтів було використано новий гібридний алгоритм з урахуванням оптимізації міри покращення (EME). Щоб оцінити ефективність запропонованої моделі, були представлені експериментальні результати, які показують, що запропонована гібридна техніка є надійно ефективною та створює чіткі та висококонтрастні зображення.

**Ключові слова:** *медичні зображення, підвищення контрастності, техніка "розділяй і володарюй", алгоритм нейронної мережі, алгоритм оптимізації, математична модель Грея–Скотта.*