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Temperature field of metal structures of transport facilities with a thin protective coating

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A study of the temperature field in metal structures of transport facilities with corrosion-resistant coating under the conditions of changes in ambient temperature has been conducted. The results of experimentally determined temperature distribution in the surface vicinity of a galvanized metal sheet are presented. The data were obtained over the day at positive and negative surface temperatures. Given a generalized boundary condition for the heat conduction problem, with a solid heated by a localized heat flow through a thin coating, there has been obtained and analyzed a temperature field. The temperature distribution across the surface outside the heating region during heat propagation along the coating was analyzed. Experimental data and model calculations, as well as temperature calculations allowing for the coating and not, have been compared. It has been established that the effect of coating on the temperature distribution in the metal structure, when the solid is heated by a localized heat flow through a thin coating, is insignificant.

Keywords: *heating through a thin coating, generalized boundary conditions, metal structure, temperature field.*

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1. Introduction

To protect elements of road or construction structures from the harmful effects of the ambient environment, thin coatings are applied to the surface. For example, to ensure their durability, metal sheets are covered with a thin layer of 80-micron zinc. However, under the conditions of operation and external physico-mechanical influences, this surface coating tends to flake and shell (Fig. 1), which reduces the protective effect of the coating and the load-carrying capacity of the structures.

The reasons behind flaking lie partly in the difference in heat expansion of the solid material and the coating when the ambient temperature changes. When the mechanical connection between the solid and the surface coating is weakened, a change in temperature may lead to the slippage of the coating on some parts of the solid. This gradually weakens the surface coating, causing its damage [1]. In addition to the levels of temperature differences, the heterogeneity of the temperature distribution is also an important factor, with the magnitude of the temperature surface gradient being of particular interest.

2. Literature review

Temperature fields in metal structures with a thin protective coating on the surface under real weather conditions are of immense interest to researchers, which is reflected by a large number of publications on experimental measurements and analysis of temperature distributions in structural elements, as well as tackling and solving topical heat conduction problems [2–5].

Mathematical modeling and formulation of heat conduction problems for the solids with coatings consists in determining generalized boundary conditions for the solid that take into account the coating on its surface with different thermophysical characteristics from the material of the main solid [6–8].

The research into heat conduction in solids with thin coatings or internal inhomogeneity can be found in publications [9–15]. In [16, 17], it have been obtained a number of analytical solutions for boundary value problems with a single-layer coating with boundary conditions of the first and third kind, mixed conditions, and undergoing the effects of internal heat sources [18, 19]. These papers mainly deal with the heating of the solid through the coating, with the issues of head conduction over the coating and exchange of heat with the solid and the ambient environment not being elaborated on.

Papers [20–22] present temperature distributions in piecewise inhomogeneous solids and estimate their impact on the strength of structural elements. The study of the temperature field in a three-layer model (defective ferroconcrete pipe reinforced with a metal corrugated structure) [20] has established the non-uniformity of temperature distribution across the layers, which leads to a contact effect defined by non-ideal conditions of heat exchange at the junction between the layers. Similar calculation outcomes of temperature fields have been achieved in the studies on multilayered bridge structures [21, 22].

Fig. 1. Damage of zinc coating of metal sheets on the surface of structures of transport facilities.

Paper [1] introduces an analysis of the effect of non-uniform temperature distribution in the area of a galvanized sheet on the mechanical behavior of composite metal corrugated structures of transport facilities. The dependence of structure deformation on the temperature function gradient was established.

Since allowing for the layered coating on the surface of a heat-conducting solid complicates solving the problem of heat conduction, there have been improved and developed the methods for obtaining their solutions, which are presented in papers [4, 23, 24]. As regards the stationary heat conduction problems, analytical solutions from [4] can be obtained and used. In the case of transient heat conduction, it is necessary to employ numerical approaches of finite element analysis, integral transformations, etc. [23].

Doing structural analysis under the temperature effects, engineers often encounter the need to consider the inhomogeneity of the temperature distribution in the structure, which is attributed to the hardening processes of parts (heating-cooling cycles), or uneven heating during operation. As a result, this leads to the difficulties in constructing accurate analytical solutions to heat conduction problems in solids with thin coatings. In addition, there arise difficulties with initial conditions, which require a spatial temperature distribution at a certain point in time.

According to the conducted review of research into the estimation of temperature fields in structures with multilayered coatings, it has been established that there are no studies on the temperature surface distribution in metal structures of transport structures with thin coatings under the climatic temperature changes in the environment. Therefore, it is still topical to determine and examine the temperature field in layered structural elements as a result of climatic temperature changes, which in particular will enable the estimation of temperature deformations and stresses when a thin coating is in contact with metal sheets of transport facilities.

3. The purpose and tasks of the research

The purpose of the study is to determine the effect of climatic temperature changes on the heating-cooling of metal structures of transport facilities through a thin coating on their surface.

To achieve the goal, the following tasks were set:

— to conduct experimental measurements of temperature distribution on the surfaces of metal structures of transport facilities;
— to improve the heating-cooling model of metal structures through a thin coating of its surface;
— to estimate the temperature field of metal structures of transport facilities with a thin protective coating, when the solid is heated by a localized heat flux.

4. Experimental measurements of the temperature distribution on the surfaces of the construction sheets

To determine the heat fluxes acting on the metal structures of transport facilities, the temperature distribution on their surfaces was measured at positive and negative ambient temperatures (in summer and winter, respectively). The temperature was measured with a digital non-contact temperature meter NT-822. The measuring results are given in Table 1.

To determine the heat fluxes affecting the metal structures of transport facilities, the temperature distribution on their surfaces was measured at positive and negative ambient temperatures (in summer and winter, respectively). The temperature was measured by a digital non-contact temperature meter NT-822. The measurement results are presented in Table 1.

Table 1. Distribution of temperature and heat flows on the surface of metal structures.

No	Time of day	Air temperature, T_s, °C	Surface temperature of metal corrugated sheet, T_p, °C	Heat flux value, q, W/m^2
1	7.00 am	16	28.6	2996.90
2	9.45 am	22	30.4	2972.62
3	12.50 pm	27	36.8	2974.16
4	3.30 pm	28	38.7	3013.94
5	5.30 pm	27	37.8	3059.09
6	9.55 pm	26	32.9	2941.07

The maximum positive value of the surface temperature of the corrugated metal sheet was +38.7°пїS in summer and the minimum negative value was −27.5°пїS in winter. Significant temperature differences are also noted during the day. Since the structure is exposed to significant changes in temperature, with the coefficient of linear expansion differing when the temperature of the zinc coating and the base material change, the temperature deformations and stresses are affected. Constant changes in external conditions cause gradual aging of the structure surface coating.

On the basis of the experimental measurements of temperature distribution on the surfaces of metal corrugated structures conducted by the method published in [8], the values of heat fluxes affecting the structure were calculated. The calculation results of heat fluxes on the surface of the metal corrugated sheets of the structure are presented in Table 1. Using both heat radiation from the surface and from the atmosphere, and convective heat exchange, conductive heat flux formed under the influence of solar radiation, is regularly directed both from the metal sheet into the air and from the air into the sheet, cooling or heating it.

5. Heating-cooling of the solid through the thin coating of its surface

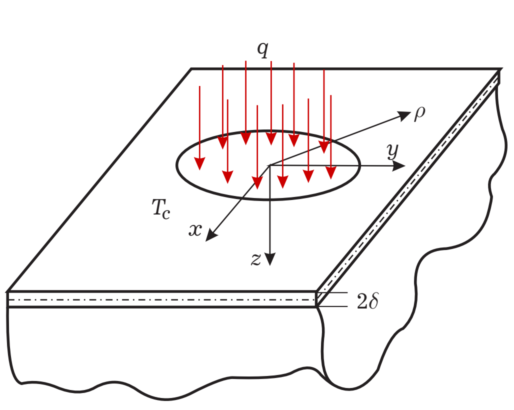

Fig. 2. Heating of the half-space through a thin coating with a thickness of 2δ.

Let us develop a mathematical model for a specific case of solid heating from the surface through the coating. Consider a half-space ($\delta < z < \infty$) with a thin coating of its thickness 2δ, which has different thermophysical characteristics compared with the main part of the solid. The temperature of the environment (air) near the surface T_c is assumed constant. Heating results from creating a uniform heat flux q in a circular area $\rho \leqslant R$ on the surface (Fig. 2).

Outside the heating region, the heat flow $q = 0$, with the heat flow having the following form on the surface

$$q_0(\rho) = \begin{cases} q & \text{when} \quad \rho \leqslant R, \\ 0 & \text{when} \quad \rho > R. \end{cases}$$

The effect of the coating on the temperature distribution in the half-space during its heating will be considered by using the generalized boundary condition (A.10), presented in Appendix A, which allows limiting this effect to the condition on the surface $z = 0$. Thus, the inhomogeneous solid with multiscale parameters is not taken into account and the heat conduction problem for a half-space with a generalized boundary condition of the type (A.10) is under consideration.

For the stationary case, given the axial symmetry of the temperature function $t(\rho, z)$, which depends on two coordinates ρ, z, the heat conduction equation for the half-space is expressed in the form

$$\frac{\partial^2 T(\rho, z)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T(\rho, z)}{\partial \rho} + \frac{\partial^2 T(\rho, z)}{\partial z^2} = 0, \quad (0 < \rho < \infty, 0 < z < \infty). \tag{1}$$

Provided there is a coating, heat exchange condition (A.14) on the surface $z = 0$ takes the following form

$$\frac{4\delta^2}{r_0}\left(\frac{\partial^2 T(\rho, z)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T(\rho, z)}{\partial \rho}\right) + (1 + r_o\mu)\,\lambda\frac{\partial T(\rho, z)}{\partial z} + \mu\,(T_c - T(\rho, z)) + q_0(\rho) = 0. \tag{2}$$

Given (1), it can also be expressed as

$$-\frac{4\delta^2}{r_0}\frac{\partial^2 T(\rho, z)}{\partial z^2} + (1 + r_o\mu)\,\lambda\frac{\partial T(\rho, z)}{\partial z} + \mu\,(T_c - T(\rho, z)) + q_0(\rho) = 0.$$

The other conditions are symmetries when $\rho = 0$ and no influence at infinity, that the temperature function is to meet are as follows:

$$\frac{\partial T(\rho, z)}{\partial \rho} = 0, \quad \rho = 0; \qquad \frac{\partial T(\rho, z)}{\partial \rho} = 0, \quad \rho \to \infty; \qquad \frac{\partial T(\rho, z)}{\partial z} = 0, \quad z \to \infty. \tag{3}$$

Let the solution (1)–(3) be expressed in the form

$$T(\rho, z) = T_c + t(\rho, z). \tag{4}$$

By applying the Hankel integral transformation

$$\theta(\eta, z) = \int_0^\infty t(\rho, z)\,\rho\,J_0(\rho\eta)\,d\rho$$

to Eq. (1) and conditions (2), (3) after substituting Eq. (4) there, the problem of function $\theta(\eta, z)$is to be considered by means of the equation

$$\frac{\partial^2 \theta(\eta, z)}{\partial z^2} - \eta^2\theta(\eta, z) = 0, \quad 0 < \eta, z < \infty, \tag{5}$$

and the conditions

$$-\frac{4\delta^2}{r_0}\frac{\partial^2 \theta(\eta, z)}{\partial z^2} + (1 + r_o\alpha_{\mathrm{T}})\,\lambda\frac{\partial \theta(\eta, z)}{\partial z} - \mu\theta(\eta, z) + Q_0(\eta) = 0, \quad z = 0; \tag{6}$$

$$\frac{\partial \theta(\eta, z)}{\partial z} = 0, \quad z \to \infty. \tag{7}$$

Here,

$$Q_0(\eta) = \int_0^\infty q_0(\rho)\,\rho\,J_0(\rho\eta)\,d\rho = q\int_0^R \rho\,J_0(\rho\eta)\,d\rho = q\frac{R}{\eta}J_1(\eta R),$$

where $J_0(x)$, $J_1(x)$ are Bessel functions of zero and first order, respectively.

The solution of auxiliary problem (5)–(7) is

$$\theta(\eta, z) = \frac{qRJ_1(\eta R)}{\left(\frac{4\delta^2}{r_0}\eta^2 + \eta\,(1 + r_o\mu)\,\lambda + \mu\right)\eta}e^{-\eta z}. \tag{8}$$

Then, the inverse transformation is applied, with the temperature function of the form:

$$t(\rho, z) = \int_0^\infty \theta(\eta, z)\,\eta\,J_0(\rho\eta)\,d\eta.$$

Using equality (8), the calculation formula for temperature distribution is derived:

$$T(\rho, z) = T_c + \int_0^{\infty} \frac{qR}{\frac{4\delta^2}{r_0}\eta^2 + \eta\,(1 + r_o\mu)\,\lambda + \mu} e^{-\eta z}\, J_1(\eta R)\, J_0(\rho\eta)\, d\eta. \tag{9}$$

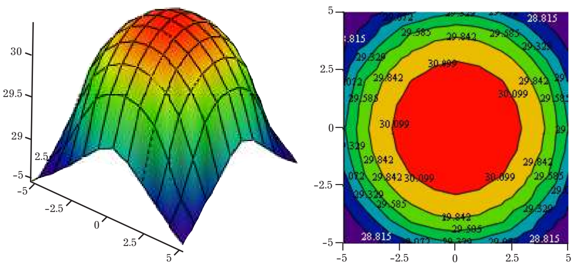

Fig. 3. Axisymmetric function $T\left(\sqrt{x^2+y^2}, 0\right)$ in surface form, left; in contour form, right.

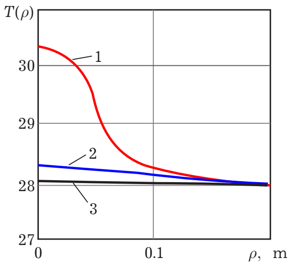

Fig. 4. Curves 1–3 correspond to the temperature distributions for $z = 0$; 0.1; 0.2 m.

To calculate the temperature distribution near the half-space surface by Eq. (9), it is assumed that the coating thickness is $\delta = 0.0001$ m, the heat conduction coefficients of the solid and coating are $\lambda = 0.6\,\frac{\text{W}}{\text{mK}}$, $\lambda_0 = 46.5\,\frac{\text{W}}{\text{mK}}$ respectively, the heat exchange coefficient on the open surface is $\mu = 163\,\frac{\text{W}}{\text{m}^2\,\text{K}}$, and the temperature of the ambient environment is $T_c = 27.8°$C. There is a heat flux $q = 3059\,\frac{\text{W}}{\text{m}^2}$ directional to the solid in the area of the circle of radius $R = 0.05$ m on the surface. The temperature distribution function $T(\rho, z)$ on the surface $z = 0$, if $\rho = \sqrt{x^2+y^2}$, is shown in Fig. 3.

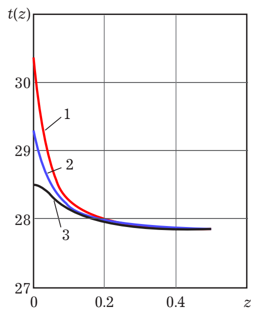

Fig. 5. Temperature profile with change of distance from the surface at different distances from the heating center. Curves 1–3 correspond to $\rho = 0$; 0.05; 0.075 m, respectively.

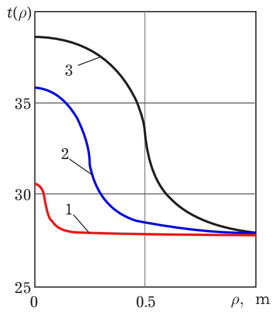

Fig. 6. Temperature distribution on the surface for different diameters of the heating region. Curves 1–3 correspond to temperature distributions for $D = 0.1$; 0.5; 1 m, respectively.

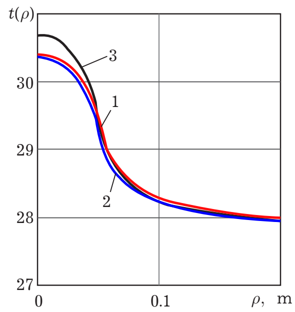

Fig. 7. Comparison of temperature distribution in steel sheet with thin coating (curve 1) and without it (curve 2). Curve 3 corresponds to ceramic coating.

Taking into account the axial symmetry $T(\rho, z)$, the temperature is calculated in terms of ρ. Figure 4 shows a comparison of temperature functions at different distances from the surface.

There is a change in temperature directly near the surface, which is explained by the heat propagation in the coating and its transfer partially to the half-space, as well as from the surface to the environment. Heat flow is not observed at greater distances from the surface, which is also evident in Fig. 5, the temperature profiles being shown according to the distance from the surface. This is especially manifested when moving away from the center of heating.

The surface temperature calculated by Eq. (9) outside the area of its heating for given thermophysical characteristics decreases rather quickly to a temperature equal to the ambient one, i.e., T_c. It should be noted that the calculated temperature value is less than the value obtained in the measurements in Table 1. The reason for this is the heating localization by the heat flow. For example, if the diameter of the heat flow area is increased, the value is close to that presented in the table. Thus, Fig. 6 provides temperature distributions on the half-space surface for different sizes of the heating region.

If a steel sheet has a zinc coating with relatively high heat conduction, this coating has little effect on the temperature of the sheet itself. However, it is different if the coating has a low value of heat conduction coefficient and a greater thickness. Then, its effect on the temperature becomes noticeable.

Figure 7 compares the temperature distributions obtained by Eq. (9) with a thin zinc coating of a steel sheet of the previously accepted parameters and characteristics and without it when their characteristics are the same, as well as the temperature distribution with a ceramic coating $\lambda_0 = 2.445\,\frac{\mathrm{W}}{\mathrm{mK}}$ with and without a greater thickness (0.001 m).

A thin coating with the characteristics used for temperature calculations has little effect on the temperature distribution in the metal structure. However, there is a temperature gradient near the surface that causes longitudinal heat flow.

6. Conclusions

Numerical calculations of temperature distributions and their comparison with measured values demonstrate the effectiveness of the proposed methodology for calculating the temperature field of metal structures, with a thin protective coating on the surface considered. This indicates the possibility of its use for determining the heat impact on the metal structures of transport facilities that are under natural heating-cooling conditions. The research is to be continued toward establishing the effects of thermoelastic deformations and stresses on the protective coating of structural elements.

Appendix A. Heat exchange conditions on the solid surface with a thin coating

Consider a homogeneous heat-conducting solid occupying the area Ω, part of whose surface is $x \in \partial\Omega_1$ covered by a thin heat-conducting layer of constant thickness 2δ. The heat (temperature) characteristics of the solid and coating are different. In the region of the solid, there is a temperature distribution $T(x, y, z, \tau)$ that satisfies the heat conduction equation in the solid region

$$\Delta T(x,y,z,\tau) - \frac{1}{a}\frac{\partial T(x,y,z,\tau)}{\partial \tau} = 0, \quad (x,y,z) \in \Omega, \quad \tau > 0, \tag{A.1}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator, $a \equiv \text{const}$ is the temperature conduction coefficient.

For the coating, an approximate description of heat conduction and heat exchange with the ambient environment is applied, which is accepted in the theory of thin heat-conducting shells [25, 26]. The temperature distribution $t(\alpha, \beta, \gamma, \tau)$ in the mixed coordinate system (α, β, γ) introduced on the middle surface of the shell satisfies the heat conduction equation

$$p^2 t(\alpha,\beta,\gamma,\tau) + \frac{\partial^2 t(\alpha,\beta,\gamma,\tau)}{\partial \gamma^2} = 0, \quad (\alpha,\beta,\gamma) \in \Omega_o, \quad \tau > 0. \tag{A.2}$$

Here $-\delta < \gamma < \delta$, the axis $\gamma$is directed to the solid under the shell (coating), $p^2 = \Delta_o - \frac{1}{a_0}\frac{\partial}{\partial \tau}$ is the operator with the derivatives of coordinates on the middle surface α, β, and time τ, $\Delta_o = \frac{1}{AB}\left[\frac{\partial}{\partial \alpha}\left(\frac{B}{A}\frac{\partial}{\partial \alpha}\right) + \frac{\partial}{\partial \beta}\left(\frac{A}{B}\frac{\partial}{\partial \beta}\right)\right]$, A, B are the coefficients of the first quadratic form of the middle surface of the shell, $a_o = \frac{\lambda_o}{c_o}$ is the heat conduction coefficient, c_o is the heat capacity, λ_o is the heat conduction of the coating. We use the index "o" to indicate the heat characteristics of a homogeneous, uniform solid coating. It should be noted that $\gamma = \delta$ corresponds to the contact solid surface and its coating. Therefore, $\frac{\partial t}{\partial \gamma}$ (when $\gamma = \delta$) corresponds to $\frac{\partial T}{\partial n}$ on the solid surface $\partial\Omega_1$.

On the open surface of the shell $\gamma = -\delta$, the boundary condition is of the form:

$$-\lambda_o \left.\frac{\partial t(\alpha,\beta,\gamma,\tau)}{\partial \gamma}\right|_{\gamma=-\delta} = \mu\big(T_c(\alpha,\beta,\tau) - t(\alpha,\beta,-\delta,\tau)\big) + q_o(\alpha,\beta,\tau). \tag{A.3}$$

For $\gamma = \delta$, there are the conditions of ideal heat contact with the solid for the shell, i.e. the equality of temperature and heat flow on both sides of the inner surface of the interface between them:

$$t(\alpha,\beta,\delta,\tau) = T(x,y,z,\tau)|_{\partial\Omega_1}, \quad \lambda_o \left.\frac{\partial t(\alpha,\beta,\gamma,\tau)}{\partial \gamma}\right|_{\gamma=\delta} = \lambda \left.\frac{\partial T(x,y,z,\tau)}{\partial n}\right|_{\partial\Omega_1}. \tag{A.4}$$

In Eqs. (A.3), (A.4), λ, λ_o are the coefficients of solid and coating heat conduction, respectively, with μ being the coefficient of heat exchange. The solid and shell are presented in their own coordinate systems. The shell surfaces of the shell $\gamma=\delta$ and solid $\partial\Omega_1$ coincide.

Depending on the relevant conditions, a temperature function, heat flux or heat transfer condition are defined for the remaining solid surface. In addition, the temperature distribution at the initial time is assumed to be known $\tau=0$.

The solution of Eq. (A.2) is formally presented in the form

$$t=p\delta\frac{\cos(p\gamma)}{\sin(p\delta)}T_1+\frac{1}{3}\frac{p^2\delta^2\sin(p\gamma)}{\sin p\delta-p\delta\cos(p\delta)}T_2. \tag{A.5}$$

Here, for the sake of simplified form, the arguments α, β, γ, τ of the temperature function are omitted, and the temperature values are averaged with regard to the shell thickness,

$$T_1=\frac{1}{2\delta}\int_{-\delta}^{\delta}t\,d\gamma,\quad T_2=\frac{3}{2\delta^2}\int_{-\delta}^{\delta}\gamma\,t\,d\gamma.$$

Further, $T_c(\alpha,\beta,\tau)$, $q_o(\alpha,\beta,\tau)$ are substituted with T_c, the ambient temperature near the coating surface $\gamma=-\delta$, and q_o, heat flow through this surface excluding convective heat transfer.

Let (A.5) be substituted into conditions (A.4) and T_1, T_2 be expressed in terms of temperature and heat flow on the contact surface $\partial\Omega_1$,

$$T_1=-\frac{1}{p^2\delta^2}\left(1+\cot^2(p\delta)\right)^{-1}\frac{\delta}{\lambda_o}\lambda\left.\frac{\partial T}{\partial n}\right|_{\partial\Omega_1}+\frac{\cot(p\delta)}{p\delta}\left(1+\cot^2(p\delta)\right)^{-1}T|_{\partial\Omega_1}; \tag{A.6}$$

$$\begin{aligned}T_2&=3\frac{1}{p^2\delta^2}\left(1-p\delta\cot(p\delta)\right)\left(1+\cot^2(p\delta)\right)^{-1}\cot(p\delta)\frac{\delta}{\lambda_0}\left.\frac{\partial T}{\partial n}\right|_{\partial\Omega_1}\\&+3\frac{1}{p^2\delta^2}\left(1-p\delta\cot(p\delta)\right)\left(1+\cot^2(p\delta)\right)^{-1}T|_{\partial\Omega_1}.\end{aligned} \tag{A.7}$$

The condition (A.3) is also transformed. By substituting the temperature function (A.5) in (A.3), excluding T_1, T_2, according to formulas (A.6), (A.7), the condition of heat exchange when $\gamma=-\delta$ is derived. It includes the temperature and the heat flow on the other surface of the coating, i.e. on the surface of the solid. Let this condition be presented as follows:

$$\begin{aligned}&\left(\cot^2(p\delta)+1\right)^{-1}\left[\left(1-\cot^2(p\delta)\right)-2\frac{\delta}{\lambda_o}\mu p^{-1}\delta^{-1}\cot(p\delta)\right]\lambda\left.\frac{\partial T}{\partial n}\right|_{\partial\Omega_1}\\&\quad+\left(\cot^2(p\delta)+1\right)^{-1}\left[-\mu(1-\cot^2(p\delta))-2\frac{\lambda_o}{\delta}p\delta\cot(p\delta)\right]T|_{\partial\Omega_1}=\left(\mu T_c+q_0\right).\end{aligned} \tag{A.8}$$

Since the coating on the surface of the $\partial\Omega_1$ solid is considered thin, its thickness 2δ is small compared to the dimensions of the solid and surface $\partial\Omega_1$. Therefore, the functions from the relations (A.5)–(A.7) are expanded into series in powers of δ in the neighborhood of zero. For the temperature (A.5), the following is obtained

$$t=T_1+\left(\frac{1}{6}-\frac{1}{2}\left(\frac{\gamma}{\delta}\right)^2\right)p^2\delta^2T_1+\frac{\gamma}{\delta}T_2+\left(\frac{1}{10}-\frac{1}{6}\left(\frac{\gamma}{\delta}\right)^2\right)\left(\frac{\gamma}{\delta}\right)p^2\delta^2T_2+O(\delta^3).$$

Moreover, the ratio $-1\leqslant\frac{\gamma}{\delta}\leqslant 1$ is considered small. The magnitude of the order δ, i.e., to linear approximations in δ, is an assumed limitation. Thus, approximations for T_1 and T_2 are of the form:

$$T_1=T|_{\partial\Omega_1}-\frac{\delta}{\lambda_o}\lambda\left.\frac{\partial T}{\partial n}\right|_{\partial\Omega_1},\quad T_2=\frac{\delta}{\lambda_o}\lambda\left.\frac{\partial T}{\partial n}\right|_{\partial\Omega_1},$$

as well as a linear approximation of the temperature

$$t=T_1+\frac{\gamma}{\delta}T_2,$$

or

$$t=T|_{x\in\partial\Omega}-\left(1-\frac{\gamma}{\delta}\right)\frac{\delta}{\lambda_o}\lambda\left.\frac{\partial T}{\partial n}\right|_{x\in\partial\Omega}.$$

The generalized boundary condition (A.8) in the same approximation has the form $\frac{4}{r_o}\delta^2 p^2 \left. T\right|_{\partial\Omega_1} + (1+r_o\mu)\,\lambda \left.\frac{\partial T}{\partial z}\right|_{\partial\Omega_1} + \mu\left(T_c - \left. T\right|_{\partial\Omega_1}\right) + q_0 = 0$.

$$\frac{4}{r_o}\delta^2 p^2 \left. T\right|_{\partial\Omega_1} + (1+r_o\mu)\,\lambda \left.\frac{\partial T}{\partial z}\right|_{\partial\Omega_1} + \mu\left(T_c - \left. T\right|_{\partial\Omega_1}\right) + q_0 = 0. \tag{A.9}$$

Here, $r_o = \frac{2\delta}{\lambda_0}$ is the heat resistance of the shell.

By substituting the operator $p^2 = \Delta_o - \frac{1}{a_0}\frac{\partial}{\partial\tau}$ in (A.9), the generalized condition of solid heat exchange in the linear approximation with the environment through a thin coating is derived:

$$\frac{4\delta^2}{r_o}\left(\Delta_o \left. T\right|_{\partial\Omega_1} - \frac{1}{a_>}\left.\frac{\partial T}{\partial\tau}\right|_{\partial\Omega_1}\right) + (1+r_o\mu)\,\lambda \left.\frac{\partial T}{\partial n}\right|_{\partial\Omega_1} + \mu\left(T_c - \left. T\right|_{x\in\partial\Omega_1}\right) + q_0 = 0. \tag{A.10}$$

The heat conduction equation (A.1) will be solved with the generalized condition of heat exchange (A.10) on the solid surface.

[1] Gera B., Kovalchuk V. A study of the effects of climatic temperature changes on the corrugated structure of a culvert of a transportation facility. Eastern-European Journal of Enterprise Technologies. **3/7** (99), 26–35 (2019).

[2] AASHTO Guide specifications: Thermal effects in concrete bridge superstructures. Washington. DC: American Association of State Highway and Transportation Officials. AASHTO (1989).

[3] Li D., Maes M., Dilger W. Thermal design criteria for deep prestressed concrete girders based on the data from confederation bridge. Canadian Journal of Civil Engineering. **31** (5), 813–825 (2004).

[4] Kulchytsky–Zhyhailo R., Matysiak S.J., Perkowski D.S. On the quasi-stationaryproblem of heat conduction for a homogeneous half-space with composite coating. Acta Mechanica. **231**, 1241–1251 (2020).

[5] Matysiak S., Perkowski D. On heat conduction problems in a composite halfspace with a nonhomogeneous coating. Heat Transfer Research. **47** (12), 1141–1155 (2016).

[6] Shevchuk V. A. Generalized boundary conditions of radiative–convective heat exchange of bodies with the environment through multi-layer non-planar coatings. Journal of Mathematical Sciences. **261**, 95–114 (2022).

[7] Shevchuk V. A. Heat conduction in plates with thin two-sided multilayer coatings under the conditions of nonstationary heating. Journal of Mathematical Sciences. **223**, 184–197 (2017).

[8] Luchko Y. Y. Algorithm for determining boundary conditions for the study of temperature strains and railway bridges beam structure deformations caused by climatic effects. Visnyk ODABA. **46**, 233–243 (2012).

[9] Perkowski D. M. On axisymmetric heat conduction problem for FGM layer inhomogeneous substrate. International Communications in Heat and Mass Transfer. **57**, 157–162 (2014).

[10] Mangerig I. Klimatische Temperaturbeanspruchung von Stahl- und Stahlverbundbrucken. Staaliche Materialpriifungsanstalt, Universitat Stuttgart, technische-wissenschaftliche Berichte. **4** (86), (1986), (in German).

[11] Shevchuk V. A., Havrys O., Shevchuk P. Nonlinear boundary–value problem of radiative–convective heat transfer of bodies with multilayer coatings. Mashynoznavstvo. **46** (6), 35–41 (2010).

[12] Gera B. Modelling of nonideal heat transfer of contacting heat-conducting layers. Physical Modeling and Information Technology. **16**, 52–60 (2012), (in Ukrainian).

[13] Gera B. Mathematical modeling of the conditions of nonideal thermal contactof layers through thin inclusion with heat sources. Physical Modeling and Information Technology. **18**, 61–72 (2013), (in Ukrainian).

[14] Gera B., Dmytruk V. Obtaining and the study of the conditions of heat transfer through inhomogeneous inclusion with presence of heat sources. Mathematical Modeling and Computing. **2** (1), 33–47 (2015).

[15] Sokolovskyy Ya., Levkovych M., Sokolovskyy I. The study of heat transfer and stress-strain state of a material, taking into account its fractal structure. Mathematical Modeling and Computing. **7** (2), 400–409 (2020).

[16] Sulym G. T., Kolodiy Y. O., Turchyn I. M. Quasi-static stresses in a coated half-space under mixed heating conditions. Visnyk Ternopilsk. National Technical University. **77** (1), 71–79 (2015).

[17] Turchyn I. M., Kolodiy Yu. O. Quasistatic plane problem of thermoelasticity for the half space with coating under mixed conditions of heating. Journal of Mathematical Sciences. **223**, 145–158 (2017).

[18] Elperin T., Rudin G. Thermal stresses in a coating–substrate assembly caused byinternal heat source. Journal of Thermal Stresses. **39** (1), 90–102 (2016).

[19] Kovalchuk V., Hnativ Yu., Luchko J., Sysyn M. Study of the temperature field and the thermos-elastic state of the multilayer soil-steel structure. Roads and Bridges (Drogi i Mosty). **19** (1), 65–78 (2020).

[20] Musii R., Zhydyk U., Mokryk O., Melnyk N. Functionally gradient isotropic cylindrical shell locally heated by heat sources. Mathematical Modeling and Computing. **6** (2), 367–373 (2019).

[21] Kovalchuk V., Onyshchenko A., Fedorenko O., Habrel M., Parneta B., Voznyak O., Markul R., Parneta M., Rybak R. A comprehensive procedure for estimating the stressed-strained state of a reinforced concrete bridge under the action of variable environmental temperatures. Eastern-European Journal of Enterprise Technologies. **2/7** (110), 23–30 (2021).

[22] Kovalchuk V., Sobolevska Y., Onyshchenko A., Fedorenko O., Tokin O., Pavliv A., Kravets I., Lesiv J. Procedure for determining the thermoelastic state of a reinforced concrete bridge beam strengthened with methyl methacrylate. Eastern-European Journal of Enterprise Technologies. **4/7** (112), 26–33 (2021).

[23] Kulchytsky–Zhyhailo R., Bajkowski A. Axisymmetrical problem of thermoelasticityfor halfspace with gradient coating. International Journal of Mechanical Sciences. **106**, 62–71 (2016).

[24] Kulchytsky–Zhyhailo R., Matysiak S., Bajkowski A. Semi-analyticalsolution of three-dimensional thermoelastic problem for half-space with gradientcoating. Journal of Thermal Stresses. **41** (9), 1169–1181 (2018).

[25] Pidstryhach Ya. S. Selected works. National Academy of Sciences of Ukraine, Pidstryhach IAPMM. Kyiv, Naukova dumka (1995).

[26] Burak Ya. Yo. Selected works. Pidstryhach IAPMM. Lviv, Akhil (2001).

Температурне поле металевих конструкцій транспортних споруд з тонким захисним покриттям

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Проведено дослідження температурного поля в металевих конструкціях транспортних споруд з антикорозійним покриттям при змінах температури навколишнього середовища. Наведено результати експериментально визначеного розподілу температури в околі поверхні металевого оцинкованого листа. Дані отримано у різний час доби при додатних та від'ємних температурах на поверхні. З використанням узагальненої граничної умови для задачі теплопровідності отримано і проаналізовано температурне поле при нагріві тіла локалізованим тепловим потоком через тонке покриття. Проаналізовано розподіл температури на поверхні за межами області нагріву при поширенні тепла вздовж покриття. Проведено співставлення експериментальних даних і модельних розрахунків, а також розрахунків температури при наявності і відсутності покриття. Встановлено, що вплив покриття на розподіл температури у металевій конструкції, при нагріві тіла локалізованим тепловим потоком через тонке покриття, є незначним.

Ключові слова: *нагрів через тонке покриття, узагальнені граничні умови, металева конструкція, температурне поле.*