MATHEMATICAL MODELING AND COMPUTING, Vol. 9, No. 4, pp. 968–976 (2022)

Modeling of a bulk material stress state in a conical hopper hole under vibration action

Maherus N. I., Sholovii Yu. P., Tymoshenko N. M., Kuchma M. I.

Lviv Polytechnic National University,
12 S. Bandera Str., 79013, Lviv, Ukraine

(Received 22 January 2022; Accepted 1 November 2022)

A model of a fine-grained bulk material stress state in a conical hopper hole under vibration during material unloading is proposed. For the study, it is used a model of a discrete medium based on a balance of a small thickness elementary volume, in which the stress redistribution occurs by opening an outlet at the hopper bottom. A formula for determining the material radial stress in the discharge conical hopper hole is obtained, taking into account all force factors influencing the bulk material behavior. The influence of humidity, shape and geometric parameters of the hopper, and vibration intensity on the change of the bulk material stress state is investigated. As a result, the efficiency of vibration to improve the conditions of the fine-grained bulk material leakage from the hoppers is established.

Keywords: *fine-grained bulk material, stress state, unloading hole, hopper, vibration parameters, leakage conditions.*

2010 MSC: 70F40 **DOI:** 10.23939/mmc2022.04.968

1. Introduction

Modeling is the most effective way to study and predict complex technical systems for different purposes both at the stage of their design and during operation. The need of solving issues of the modeling of the system behavior appears not only for scientists but also for professionals who implement complex technological processes. Modern modeling technologies can significantly improve the understanding of a complex process for both the modeling specialist and the specialist who uses these results, especially when it comes to processes which are difficult to predict due to the many factors influence. An example of such processes is a leakage of bulk materials (BM) from the hoppers during unloading. Complex physical and mechanical characteristics of the BM, the ability to change these properties in a wide range is the reason for the formation of a free-fall arch over the outlets of the unloading hopper holes, which counteract the material leakage. This is especially important during working with fine-grained BMs, which are able to form strong structures of the free-fall arch, which are difficult to destroy [1]. To effectively fight with such a negative phenomenon, as well as to prevent them, it is used the dynamic effect of vibration on bulk materials.

Nowadays ensuring of the BM continuous flow from the hoppers [2], BM separation [3], as well as combating the formation of the free-fall arch over the hopper outlets [4] are the most common issues, which scientists are working on, during studying the BM behavior. Some studies are based directly on an experimental study of the vibration effects on the bulk product. In particular, the influence of the vibration frequency and the vibration amplitude of hopper hole located at the angle on the particle velocity of granular material and on the hopper productivity is established [5]. The conditions, which the BM enters the vibration fluidization state under, are established, and the influence of vibration parameters on the BM velocity and motion modes is investigated [6]. It was confirmed that an increase in the vibration amplitude in processes such as mixing, segregation and vibration treatment is more effective than an increase in the vibration frequency to ensure the BM fluidization [6]. However, these studies are based mainly on the experimental determination of the main indicators of the BM leakage process without mathematical modeling of the product stress state in critical hopper areas.

Due to the problems that appear in the process of the fine-grained BM movement, the question of the BM fluidity improvement remains open. The mathematical model was developed, that allows effectively investigating fine-grained BMs in the vibration boiling state [7]. However, to provide the vibration boiling flow of fine-grained BMs, it is required a significant vibration intensity [7]. It is additionally used BM aeration in some studies [8, 9] because of the significant adhesion forces in the material. However, due to the significant pollution that occurs in case of BM air saturation, vibration remains a more effective way to improve the material fluidity. Properly selected vibration parameters allow changing the critical diameter of the hopper hole and minimizing the probability of the free-fall arch formation over the outlet [10].

Numerous studies [11] have shown that different BM movement forms, which are directly related to the BM stresses, depend on the BM properties, the shape and parameters of the hoppers. Therefore, it is very important to study the stress state of the fine-grained bulk material, as well as its changes under the vibration action. Radial stress deserves special attention, because there is a transition from general to radial stress fields in the unloading hopper hole, and the hopper output is determined by the BM stresses near the outlet.

2. Development of the bulk material stress state model

An important structural element of the lower part of the round section hopper is a conical hole, where the problems associated with continuous BM unloading are. The BM movement in this hopper part is accompanied by the occurrence of a passive stress state, because of the fact, that the reducing the cross-sectional area leads to compaction and compression of the BM. To study the fine-grained BM leakage from the conical hopper hole, a model of a discrete medium is used, which the stress redistribution occurs due to the opening the hopper outlet in. The model developed by A. Katalimov was chosen as a basis [11]. This approach allows studying the processes occurring in the BM at the hopper outlet with a high accuracy, because of the fact, that it takes into account the linear dependence of tangential stress on the radial coordinate and rejects the assumption of the radial stress invariability in cross section. The material movement is considered as a continuous process of formation and destruction of the free-fall arch over the outlet. This model also allows estimating the pressure on the bottom and walls of the loading hopper due to changes in the physical and mechanical characteristics of the BM.

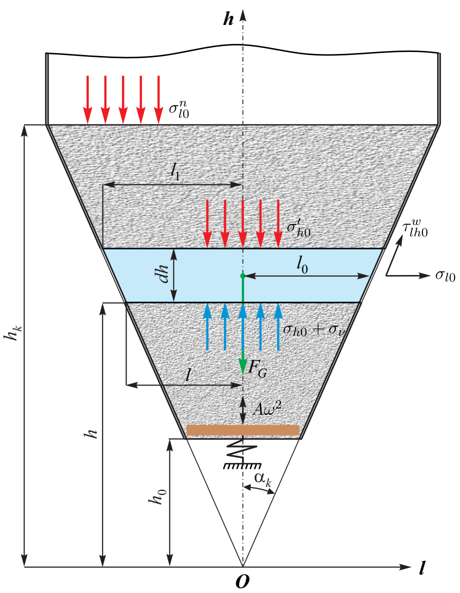

Fig. 1. Scheme for estimating the bulk material stresses in the unloading conical hopper hole.

The mathematical model based on the balance of the elementary volume $V_{e.\nu}$ with thickness dh was developed to study the BM stress state in the conical hopper hole in Fig. 1.

The following assumptions were used to develop the mathematical model:

- vertical stress σ_{h0} is a function of a coordinate h only;
- radial stress σ_{l0} is constant in a cross section;
- radial stress σ_{l0} is constant in a cross section;
- tangential stress τ_{lh0} varies from its maximum value near the hole wall to the minimum on the axis Oh;
- inertial component σ_ν of the vertical stress resulting from the vibration action is directed upwards, because it improves the BM leakage conditions and reduces the radial stress.

The BM stress state is changed in the process of loading and unloading the hopper. The BM transition from the passive state (immobile state) to active state (movement) is characterized by certain changes of the internal friction coefficient and the external friction coefficient of the BM, as well as the bulk density. The hopper loading process is accompanied by an increase in the radial stress and a decrease in the vertical stress, which leads to the passive state and the formation of the free-fall arch over the outlet, provided the following condition $\sigma_{h0} < \sigma_{l0}$. BM passes from the immobile state to movement at the time of the outlet opening due to an increase in the vertical stress. Therefore, it is important to analyze and study the BM stress state in the critical hopper area, which determines the conditions of BM continuous leakage.

Thus all the above stresses can be written as

$$\sigma_{l0} = f(h), \quad \tau_{lh0} = \tau_{lh0}^{w} \frac{l}{l_0} = \sigma_{l0} f_w \frac{\tan\alpha}{\tan\alpha_k}$$

with σ_{l0} is the radial stress in the section under consideration; τ_{lho} and τ_{lh0}^{w} are the tangential stresses in the BM layer and near the hopper hole walls respectively; f_w is the external friction coefficient of the BM to the hopper hole walls; l and h are the radial and vertical coordinates respectively; l_0 is the hopper hole radius on the vertical section middle line of the selected elementary volume; $0 \leqslant \alpha \leqslant \alpha_k$; α_k is the angle inclination of unloading hopper walls to the vertical.

Using the Mohr–Coulomb balance condition, the connection between a radial, vertical and tangential stresses can be written as

$$(\sigma_{h0} - \sigma_{l0})^2 + 4\tau_{lh0}^2 = \sin^2\psi(\sigma_{h0} + \sigma_{l0})^2$$

with ψ is the internal friction angle of the BM.

Taking into account that $\tau_{lh0}^{w} = \sigma_{l0} \cdot f_w$, an expression to determine the vertical stress can be written [11]:

$$\sigma_{h0} = \sigma_{l0} \left[1 + 2f^2 \pm \sqrt{(1+2f^2)^2 - 1 - 4f_w^2 \left(\frac{\tan\alpha}{\tan\alpha_k}\right)^2 (1+f^2)} \right] \tag{1}$$

with $f = \tan\psi$ is the internal friction coefficient of the BM; $f_w = \tan\psi'$ is the external friction coefficient of the BM to the hopper walls; ψ' is the external friction angle of the BM to the hopper hole walls.

In the equation (1) the signs "+" and "−" are used to point the active and passive stress states respectively.

To establish the force factors influencing the fine-grained BM behavior, the balance of the material elementary volume is considered and written as the force balance equation in the projection on the axis Oh:

$$F_{\sigma_{h0}} + F_{\sigma_\nu} + F_{\tau_{lh0}^{w}} + F_{\sigma_{l_0}} = F_{\sigma'_{h0}} + F_G \tag{2}$$

with $F_{\sigma_{h0}}$ is the force gain caused by the vertical stress acting on the upper part of the selected elementary volume; F_G is the weight gain of the material; F_{σ_ν} is the inertial force gain, that occurs in the BM under the vibration action; $F_{\sigma'_{h0}}$ is the force gain caused by the vertical stress acting on the lower part of the selected elementary volume; $F_{\tau_{lh0}^{w}}$ is the force gain caused by the tangential stress near the hopper hole walls; $F_{\sigma_{l_0}}$ is the force gain caused by the radial stress.

The force caused by the vertical stress acting on the upper part of the selected elementary volume of the BM can be defined in the polar coordinate system l, φ as the product of the stress and the area, which stress acts on

$$F_{\sigma_{h0}} = \iint_F \sigma_{h0}\, dF = \int_0^{2\pi} \int_0^{l_0} \sigma_{h0} l\, dl\, d\varphi = 2\pi \int_0^{l_0} \sigma_{h0} l\, dl,$$

Using the formula (1), last relation can be represented as

$$F_{\sigma_{h0}} = \pi l_0^2 \sigma_{l0} C_{1,2} \tag{3}$$

with

$$C_{1,2} = 1 + 2f^2 \pm \frac{\sqrt{[(1+2f^2)^2-1]^3} - \sqrt{[(1+2f^2)^2-1-4f_w^2(1+f^2)]^3}}{6f_w^2(1+f^2)}$$

is the coefficient of active and passive stress states respectively.

The weight of the elementary volume can be determined by a formula:

$$F_G = \rho g V_{e.\nu},$$

where ρ is the bulk density of the BM; g is the gravitational acceleration.

To find the volume $V_{e.\nu}$, a truncated cone volume is considered (Fig. 1). A formula for finding the volume $V_{e.\nu}$ can be written as a dependence:

$$V_{e.\nu} = \frac{1}{3}\pi\, dh\big(l^2 + l\, l_1 + l_1^2\big) \tag{4}$$

with l and l_1 are the unloading hole radiuses, respectively, the lower and upper base of the selected elementary volume.

It is written the radiuses of the upper and lower bases of the selected volume in the next form for the convenience of the mathematical model study:

$$l_1 = l_0 - \frac{dh}{2}\tan\alpha_k, \quad l = l_0 + \frac{dh}{2}\tan\alpha_k. \tag{5}$$

Substituting relations (5) into the equation (4), it is obtained an expression for the elementary volume $V_{e.\nu}$:

$$V_{e.\nu} = \frac{1}{3}\pi\left[3l_0^2 dh - l_0\tan\alpha_k dh^2 + \frac{1}{4}\tan^2\alpha_k dh^3\right]. \tag{6}$$

Since the height dh of the selected element is small compared to the height of the conical hopper hole h_k, so it is neglected the last two terms in the formula (6). Then the volume $V_{e.\nu}$ can be written as

$$V_{e.\nu} = \pi\, l_0^2 dh.$$

Taking into account last formula, the weight is determined as

$$F_G = \rho\, g\, \pi\, l_0^2 dh. \tag{7}$$

Similar to the formula (3), it is determined the value of the force caused by the vertical stress acting on the selected volume lower part:

$$F_{\sigma'_{h0}} = \pi\, l_0^2 C_{1,2}(\sigma_{l0} + d\sigma_{l0}) \tag{8}$$

with $d\sigma_{l0}$ is the gain of the radial stress.

The force $F_{\tau^w_{lh0}}$ caused by the tangential stress τ^w_{lh0} and the force $F_{\sigma_{l0}}$ caused by the radial stress σ_{l0} have a significant effect on the lateral surface of the BM elementary volume in the unloading conical hole. The value of the force caused by the tangential stress can be found by a formula:

$$F_{\tau^w_{lh0}} = \sigma_{l0}\, f_w\, P_{e.\nu}$$

with $P_{e.\nu}$ is the lateral surface area of the selected elementary volume.

Since the selected elementary volume is the truncated cone, the lateral surface area is determined by a dependence:

$$P_{e.\nu} = 2\pi\, l_0 \frac{dh}{\cos\alpha_k}. \tag{9}$$

Then taking into account (9) the formula determining the force arising from the tangential stress action will take a form:

$$F_{\tau^w_{lh0}} = \pi\sigma_{l0} f_w \frac{2l_0}{\cos\alpha_k} dh. \tag{10}$$

Another important force acting on the selected elementary volume is the force caused by the radial stress:

$$F_{\sigma_{l0}} = \sigma_{l0}\sin\alpha_k P_{e.\nu}.$$

Taking into account the formula (9), the value of the force has a form:

$$F_{\sigma_{l0}} = \pi\sigma_{l0}\sin\alpha_k \frac{2l_0}{\cos\alpha_k}\,dh. \tag{11}$$

To study the vibration effect on the BM stress state in the unloading conical hopper hole, the inertial force arising as a result of the dynamic vibration action on the material is considered. The inertial force can be determined by the stress σ_ν:

$$F_{\sigma_\nu} = \iint_F \sigma_\nu\,dF.$$

The stress, which occurs due to the vibration perturbation of the flow, can be determined by a formula:

$$\sigma_\nu = \frac{4F(t)}{\pi D_\nu^2} \tag{12}$$

with $F(t) = m_{e.\nu}A\omega^2\cos(\omega t + \varphi_0)$ is the generalized forcing force; $m_{e.\nu}$ is the selected elementary volume mass; A is the vibration amplitude; $\omega = 2\pi\nu$ is the angular frequency; ν is the ordinary frequency, Hz; φ_0 is the initial phase of oscillations; D_ν is the diameter of the vibrating surface.

To study the critical values of the BM stress state during the material outflow, it is important to use the forcing force amplitude $F(t) = m_{e.\nu}A\omega^2$ of the vibrating surface. Also it is considered the elementary volume near the outlet $2l_0 \approx D_\nu$, where a high probability of the free-fall arch formation is. By (12) it follows that

$$\sigma_\nu = \frac{A\omega^2 m_{e.\nu}}{\pi l_0^2}.$$

Due to the fact that $m_{e.\nu} = \rho\,\pi\,l_0^2\,dh$, the value of the force caused by the vibration action, can be represented as

$$F_{\sigma_\nu} = \pi A\,\omega^2\rho\,l_0^2 dh. \tag{13}$$

Using the dependences (3), (7), (8), (10), (11), (13), the force balance equation,(2) in the projection on the axis Oh can be written as

$$\pi l_0^2\sigma_{l0}C_{1,2} + \pi\,A\,\omega^2\rho\,l_0 dh + \pi\,\sigma_{l0} f_w\frac{2l_0}{\cos\alpha_k}dh + \pi\,\sigma_{l0}\sin\alpha_k\frac{2l_0}{\cos\alpha_k}dh = \pi\,l_0^2\,C_{1,2}\big(\sigma_{l0} + d\sigma_{l0}\big) + \rho\,g\,\pi l_0^2 dh. \tag{14}$$

After performing the transformation and taking into account the expression $l_0 = \frac{h\tan\alpha_k}{\cos^2\alpha_k}$, the equation (14) takes the form:

$$-\frac{d\sigma_{l0}}{dh} + \sigma_{l0}\frac{2(f_w + \sin\alpha_k)\cos\alpha_k}{\tan\alpha_k\,C_{1,2}h} = -\frac{A\,\omega^2\rho}{C_{1,2}} + \frac{\rho\,g}{C_{1,2}}. \tag{15}$$

We denote a_1 and a_2 by

$$a_1 = \frac{2(f_w + \sin\alpha_k)\cos\alpha_k}{\tan\alpha_k\,C_{1,2}}, \qquad a_2 = -\frac{A\,\omega^2\rho}{C_{1,2}} + \frac{\rho\,g}{C_{1,2}}. \tag{16}$$

According to notations (16), the equation (15) has the form:

$$-\frac{d\sigma_{l0}}{dh} + \frac{a_1\sigma_{l0}}{h} = a_2.$$

Using Bernoulli's method, the solution of last equation is written as

$$\sigma_{l0} = -\frac{a_2}{1-a_1}h + C\,h^{a_1}.$$

Satisfying the boundary conditions $h = h_k$, $\sigma_{l0} = \sigma_{l0}^n$, it is obtained

$$\sigma_{l0} = \frac{a_2 h}{a_1 - 1}\left[1 - \left(\frac{h}{h_k}\right)^{a_1 - 1}\right] + \sigma_{l0}^n\left(\frac{h}{h_k}\right)^{a_1}$$

with σ_{l0}^n is the radial stress at the upper boundary of the conical hopper hole.

According to notations (16), last relation for determining the BM radial stress in the unloading hopper hole has a form:

$$\sigma_{l0} = \frac{\left(\frac{\rho g}{C_{1,2}} - \frac{A\omega^2\rho}{C_{1,2}}\right) h}{\frac{2(f_w+\sin\alpha_k)\cos\alpha_k}{\tan\alpha_k C_{1,2}} - 1} \left[1 - \left(\frac{h}{h_k}\right)^{\frac{2(f_w+\sin\alpha_k)\cos\alpha_k}{\tan\alpha_k\, C_{1,2}} - 1}\right] + \sigma_{l0}^{n} \left(\frac{h}{h_k}\right)^{\frac{2(f_w+\sin\alpha_k)\cos\alpha_k}{\tan\alpha_k\, C_{1,2}}}. \quad (17)$$

3. Main results

Thus, the analytical dependence obtained on the basis of the developed mathematical model allows modeling and investigating the factors that directly affect the BM stress state. Since the fine-grained bulk product belongs to the hydrophilic materials, the moisture contained in material pores is a very important factor that determines its behavior during transportation. The presence of moisture in the BM directly affects the bulk density of the product [11]:

$$\rho = \frac{\rho_d(1 + W_{\text{abs}})}{1 + \frac{W_{\text{abs}}\rho_p}{3\rho_w}}$$

with ρ_d is the bulk density of the dry BM, ρ_p is the density of BM particles, ρ_w is the water density, W_{abs} is the absolute humidity.

Using the obtained dependence (17), the simulation of the stress states of fine-grained BM with the following input parameters $\psi = 35°$, $\psi' = 28°$, $\rho_d = 400\,\text{kg/m}^3$, $\rho_p = 500\,\text{kg/m}^3$, $\rho_w = 998\,\text{kg/m}^3$, $h_0 = 0.2\,\text{m}$, $h = 0.3\,\text{m}$, $h_k = 0.65\,\text{m}$ was performed.

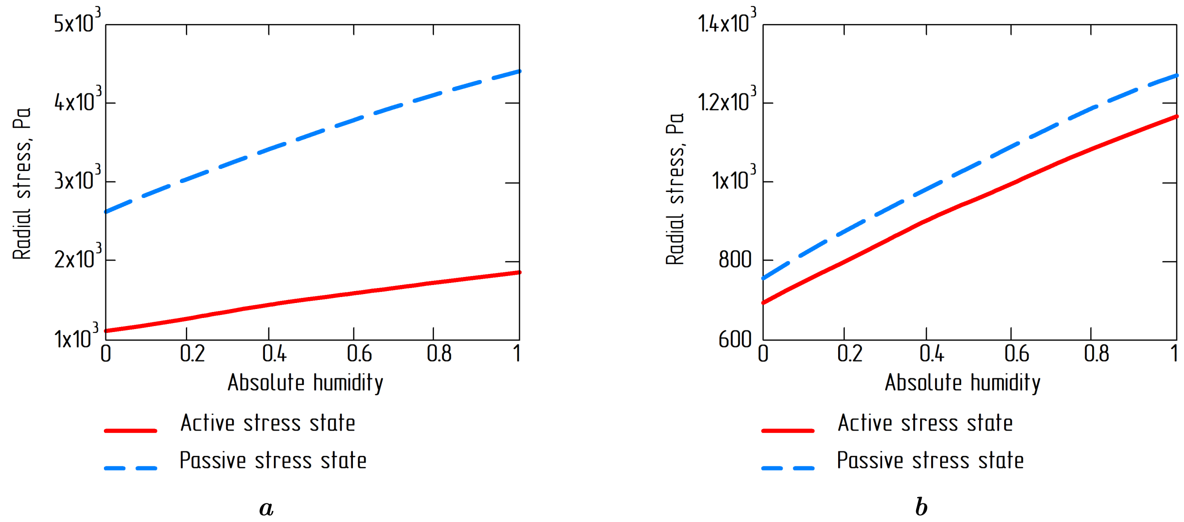

Fig. 2. Dependence of the BM radial stress on the BM absolute humidity for (***a***) $\alpha_k = 45°$, (***b***) $\alpha_k = 15°$.

As it is seen in Fig. 2, analysis of graphical dependences shows that the radial stress is slightly above 1000 Pa in the conical hopper hole with the angle inclination of the walls $\alpha_k = 45°$ (Fig. 2*a*) in case the moisture is absent in the fine-rained BM at the active stress state, while the radial stress is 2500 Pa under the same conditions at passive stress state. It was also found that an increase in the BM humidity leads to an increase in the radial stress and, thus, an increase in the probability of the free-fall arch formation. Graphical dependences (Fig. 2) confirm that the effect of moisture on the material stress is much greater at the passive stress state than at the active state. The angle inclination of the hopper hole walls also significantly affects the BM stress state: reducing the angle inclination of the hopper hole walls leads to a decrease in the radial stress at both active and passive stress states. This fact is confirmed experimental studies [11].

The following input parameters were used to study the dependence of the radial stress on the ratio $\frac{h}{h_k}$: $\rho = 460\,\text{kg/m}^3$, $h_0 = 0.2\,\text{m}$, $h_0 \leqslant h \leqslant h_k$, $h_k = 0.65\,\text{m}$, $\alpha_k = 45°$.

It is analyzed the change of BM stresses by the coordinate h in Fig. 3. The fact, that the hopper cylindrical part filled of BM is absent (Fig. 3a) or it is present (Fig. 3b), is very important for this study. It was found that the maximum stress is observed in the outlet area in case the hopper cylindrical part filled of BM ($\sigma_{l0}^{n} = 0$) is absent (Fig. 3a). However, the stress fields approach zero, when the coordinate h approaches the free edge of the BM volume in the conical hopper hole, provided it is fully loaded, regardless of the BM stress state. The maximum difference between the radial stresses at the active and passive stress states is realized at the ratio $\frac{h}{h_k} \approx 0.6$.

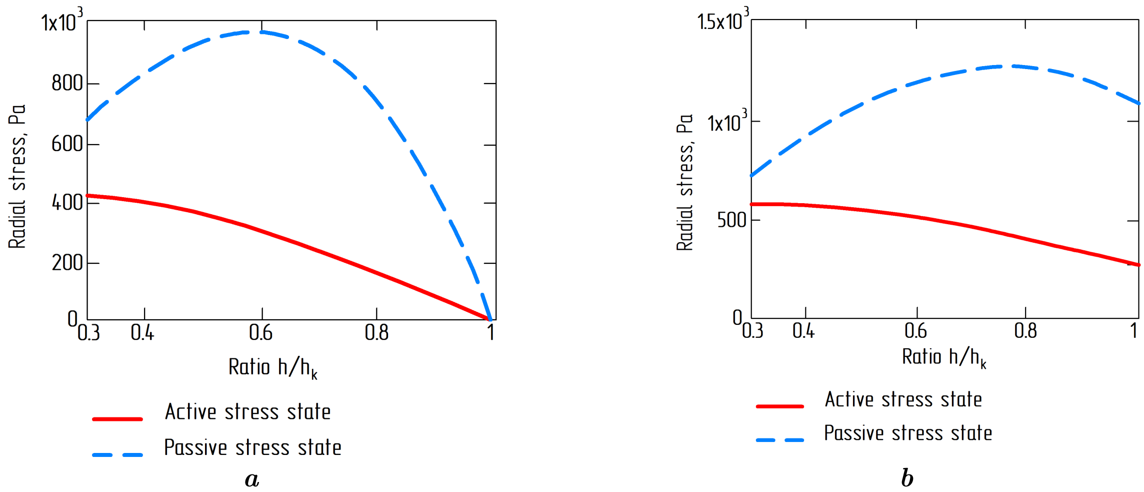

Fig. 3. Dependence of the BM radial stress on the ratio $\frac{h}{h_k}$ in case (***a***) the hopper cylindrical part is absent, (***b***) the hopper cylindrical part filled with BM is present.

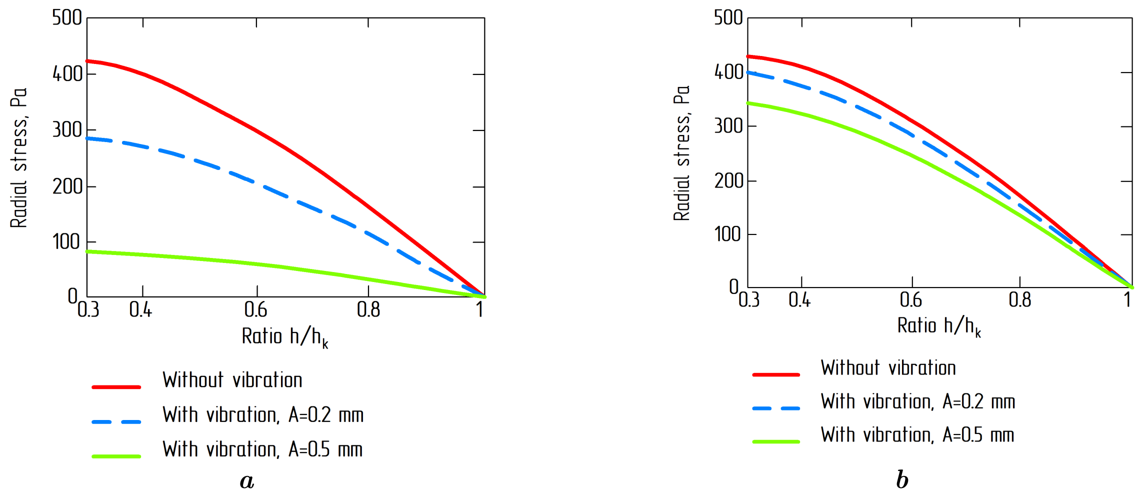

Fig. 4. Dependence of BM radial stress on vibration parameters at (***a***) $\nu = 20\,\text{Hz}$, (***b***) $\nu = 10\,\text{Hz}$.

Another situation is observed in the unloading hopper hole, which contains also a cylindrical part filled with fine-grained BM in Fig. 3b. The stress σ_{l0}^{n} was determined according to the recommendations [11] for this study. It is seen that the BM stress is different from zero in case it is researched the stresses that occur at the boundary of the transition from the conical hole to the cylindrical part of the hopper, provided the hopper is fully loaded. Thus, the radial stress is slightly higher than 1000 Pa at the passive stress state, and it is about 300 Pa at the active stress state. Thus, the presence of the filled hopper cylindrical part requires additional ways to stimulate the BM flow. The maximum difference between radial stresses at the active and passive stress states is realized at the ratio $\frac{h}{h_k} \approx 0.8$.

To study the effect of vibration on the radial stress magnitude, the active stress state is simulated, because even minor mechanical oscillations drive the BM in motion. The input parameters used to obtain the graphical dependences in Fig. 4a and Fig. 4b are similar to the previous study.

The BM transition from the immobile state to the movement causes changes in stress state, which depends on many factors. In addition to the BM properties and the geometric parameters of the hopper hole the stress state change is significantly influenced by the acceleration of the hopper surface, which is under the vibration action. It can be concluded from the graphical dependences (Fig. 4a) that the use of vibration (even small intensity vibration) significantly reduces the radial stress. Thus, at the vibration amplitude of 0.2 mm, the radial stress in the outlet area decreases from 440 Pa to 300 Pa, and the use of vibration with the amplitude of 0.5 mm reduces this parameter to less than 100 Pa. The stresses in the hopper decrease significantly when the coordinate h approaches the upper limit of the BM volume due to a decrease in the volume of the material, which is placed above. The vibration effect on the stress is also significantly reduced in this area. As it is seen in Fig. 4b, analysis of graphical dependences shows that an increase in the vibration amplitude does not lead to a significant decrease in the radial stress at the lower vibration frequency in contradistinction to the results obtained in Fig. 4a.

4. Conclusions

An increase in the BM absolute humidity leads to an increase in the BM stresses at both active and passive stress states. The results of mathematical modeling show that the radial stresses decrease with a decrease in the angle inclination of the hopper hole walls, and therefore the vibration intensity required for the BM transition from the active to the passive stress states is significantly reduced. The simulation results show that the presence of a cylindrical hopper part filled with BM directly affects an increase in the material stresses in the conical hopper hole. This fact contributes to the appearance of the BM passive stress state in the unloading hole area, which is characterized by material compaction and the formation of the free-fall arch above the outlet, due to the pressure of the upper layers of the loaded material. It was found that the use of vibration significantly reduces the BM radial stresses in the critical hopper area, helping to improve leakage conditions. Moreover, to ensure the continuous BM flow from the hopper, it is advisable to use low-frequency vibration of the vibrating bottom.

[1] Hayvas B., Dmytruk V. Torskyy A., Dmytruk A. On methods of mathematical modeling of drying dispersed materials. Mathematical Modeling and Computing. **4** (2), 139–147 (2017).

[2] Rabinovich E., Kalman H., Peterson P. F. Granular material flow regime map for planar silos and hoppers. Powder Technology. **377**, 597–606 (2020).

[3] Di Carlo D., Edd J. F., Humphry K. J., Stone H. A., Toner M. Particle segregation and dynamics in confined flows. Physical Review Letters. **102**, 094503 (2009).

[4] Park S. S., Kim E. S. Jamming probability of granular flow in 3D hopper with shallow columns: DEM simulations. Granular Matter. **22** (2), 77 (2020).

[5] Du J., Liu C., Tong L., Yin S., Wang L. Effects of vibrations on tilted silo discharge. Chemical Engineering Research and Design. **171**, 247–253 (2021).

[6] Hashemnia K., Pourandi S. Study the effect of vibration frequency and amplitude on the quality of fluidization of a vibrated granular flow using discrete element method. Powder Technology. **327**, 335–345 (2018).

[7] Sholovii Y. P., Maherus N. I., Zyska T., Sagymbekova A., Askarova N. Modelling of the finely-dispersed noncoherent material flow from the loading hopper under vibration. Proceedings Volume 11045, Optical Fibers and Their Applications 2018. 1104508 (2019) .

[8] Hesse R., Krull F., Antonyuk S. Experimentally calibrated CFD-DEM study of air impairment during powder discharge for varying hopper configurations. Powder Technology. **372**, 404–419 (2020).

[9] Zhu L., Lu H., Poletto M., Liu H., Deng Z. Hopper discharge of cohesive powders using pulsated airflow. AIChE Journal. **66** (7), 16240 (2020).

[10] Jafari A., Abolghandi A., Gharibi A., Khalili Parizi M. V., Begheri Jamebozorgi A. Effects of local vibration on silo discharge and jamming: employing an experimental approach. Journal of Particle Science & Technology. **4** (2), 91–100 (2018).

[11] Katalymov A. V., Liubartovych V. A. Dosage of viscous and bulk materials. Lviv, Khimiia (1990).

Моделювання напруженого стану сипкого матеріалу в конічній лунці бункера під дією вібрації

Магерус Н. І., Шоловій Ю. П., Тимошенко Н. М., Кучма М. І.

Національний університет "Львівська політехніка",
вул. С. Бандери, 12, 79013, Львів, Україна

Запропоновано модель напруженого стану дрібнодисперсного сипкого матеріалу в конічній розвантажувальній лунці бункера під дією вібрації в процесі його розвантаження. Для дослідження використано модель дискретного середовища, що базується на рівновазі елементарного об'єму малої товщини, в якому перерозподіл напружень відбувається за рахунок відкривання випускного отвору в днищі бункера. Враховуючи усі силові фактори, що впливають на поведінку сипкого матеріалу, отримано формулу для визначення його радіального напруження в розвантажувальній лунці бункера. Досліджено вплив вологості, форми та параметрів бункера, інтенсивності коливань на зміну напруженого стану сипкого матеріалу. Встановлено ефективність використання вібрації для покращення умов витікання дрібнодисперсних сипких матеріалів з бункерів.

Ключові слова: *дрібнодисперсний сипкий матеріал, напружений стан, розвантажувальна лунка, бункер, параметри вібрації, умови витікання.*