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**Dariia Rebot[1], Volodymyr Topilnytskyy[2]**

[1] Computer-Aided Design System Department, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: dariya.p.rebot@lpnu.ua, ORCID: 0000-0002-3583-0800
[2] Design and Operation of Machines Department, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, e-mail: topilnvol@gmail.com, ORCID 0000-0002-5191-326X

# NONLINEAR MATHEMATICAL MODEL OF THE FIVE-CONTAINER VIBRATION SYSTEM

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**Abstract.** The construction of a non-linear mathematical model of movement and interaction of the commanding and executive components of vibration systems is an important task. It implements vibration technologies of separation, grinding, mixing, compaction, transportation, surface product processing and technology for regulating the vibration effect on systems and mechanisms for their further research to increase the efficiency of vibrating machines, devices, and mechanisms and relevant technological processes. The article presents a generalized diagram of a five-container vibration system. On its basis, a mathematical model was developed, which in the future will make it possible to research the effectiveness of vibration machines, devices, and mechanisms. The calculation scheme of the system and the methods of nonlinear mechanics were used to build the mathematical model. The obtained mathematical model makes it possible to determine the horizontal and vertical components of the amplitude of any point of the containers of the vibration system. This will make it possible to investigate the influence of different modes of operation of the system on the amplitude and nature of vibrations of the containers, in particular, established regimes, influence reversing of the drive, the influence of the processing environment of the containers of the vibration system, influence processed parts.

**Keywords:** amplitude, vibration system, mathematical model, environment, generalized scheme**,** movement.

### Introduction and Problem Statement

Vibrating technologies for processing products (surface hardening, grinding, cleaning, separation, grinding) have become widespread in production as effective processes that implement devices and mechanisms with the help of vibrating machines, the range of designs of which is very diverse.

Many issues in the theory of technological processes and vibration technology machines have not been solved or even studied. They represent important tasks of vibration movement and interaction too. The developers of this equipment, when choosing the parameters of such machines, can be guided mainly only by the available data on the operation of existing types and sizes of similar machines. Such an exclusively empirical approach does not always lead to the selection of optimal solutions. The widespread use of vibration technologies requires a new approach to the calculation and design of appropriate vibration equipment (various types of drives, suspensions, container bodies, and other dynamically loaded nodes). Also, it is necessary to investigate the processes that occur with the loading of the vibration system in the container during its movement and the influence of its parameters on the processing process. The development of methods for computer design of vibration technology equipment in general, and accordingly a thorough mathematical description of the dynamic processes that occur in technological machines during the processing of products, is relevant. Due to the complexity of machine designs, vibrational processes, and the variety of assumptions and simplifications in the mathematical apparatus used in this, it remains not completely resolved even today.

Due to the great popularity of vibration technologies for the processing of products, there is a wide range of scientific works by various authors regarding the theoretical description of the vibration process of product processing [1, 2]. Although various assumptions, simplifying hypotheses, and linear apparatus are used in mathematical models construction of the corresponding vibration processes is distanced from obtaining theoretical results from practice.

The construction of a non-linear mathematical model of the movement and interaction of the controlling and executive components of vibration systems is important for increasing the efficiency of vibration machines, devices and mechanisms and the corresponding technological processes. These systems implement vibration technologies for separation, grinding, mixing, compaction, transportation, and surface treatment of products.

## Main Material Presentation

Let's choose for research a five-container vibrating system for the implementation of technologies of separation, mixing, grinding, compaction of products, and technological environments. Its generalized scheme in Fig. 1. is shown. It represents an abstracted and unified five-container vibration system. It sufficiently reproduces the picture of the movement of actual vibration systems, can be used for building mathematical models of their motion, and also as a basis for creating movement models of another, “simpler” type of vibration system, for example, a vibration system with fewer containers, or with a different container shape.

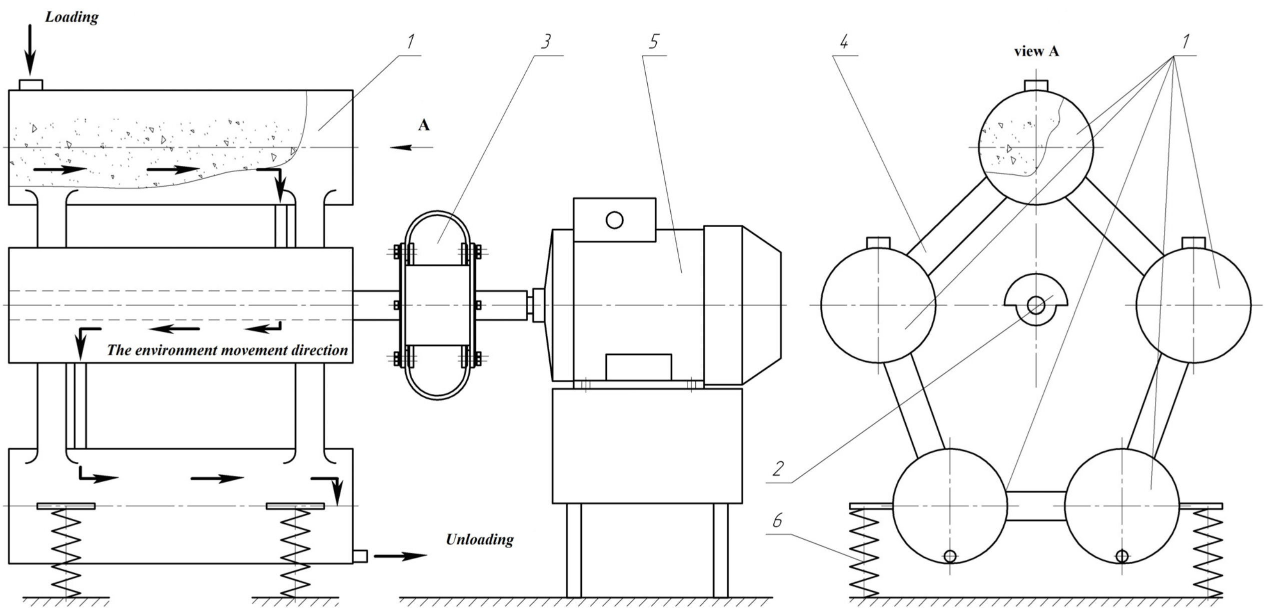


**Fig. 1.** Generalized diagram of the investigated five-container vibration system:
1 – working containers of the vibration system; 2 – unbalanced vibrator; 3 – belt clutch;
4 – frame; 5 – drive motor; 6 – elastic suspension

Analysis of the vibration system movement shows that the amplitude of vibrations of the vibration system containers in the vertical plane is much greater than the amplitude in the horizontal plane. The latter due to sufficient accuracy allows us to assume that the containers of the vibration system move only in the vertical plane (in the plane of rotation of the imbalances). That is, it is in planar motion. Therefore, the calculation scheme of the five-container vibrating system can be represented as a planar mechanical system that has six degrees of freedom (five degrees have containers and one – imbalance, which rotate around horizontal axes, in one plane). In this case, the calculation scheme will look like this (see Fig. 2).

We will introduce the following notations and assumptions: working containers have masses Mк1, Mк2, Mк3, Mк4, Mк5; $XO_6Y$ – stationary coordinate system; $X_1O_6^{'}Y_1$ – the moving coordinate system associated with the center of rotation of the imbalance and the vertical axis of symmetry of the containers and moves with it. Its coordinate origin is a point $O_6$, that coincides with the center of rotation of the imbalance; $\varphi$ – the angle of rotation of the containers relative to the initial position during their movement (the angle of rotation of the moving coordinate system relative to the stationary one). At the initial moment of time, the centers of the moving and stationary coordinate systems coincide, the common geometric center of the containers is at the point $O_6$, and its center of mass is a point S lies on the axis Y. Let $O_1$, $O_2$, $O_3$, $O_4$, $O_5$ – geometric centers of the containers of the vibration system.

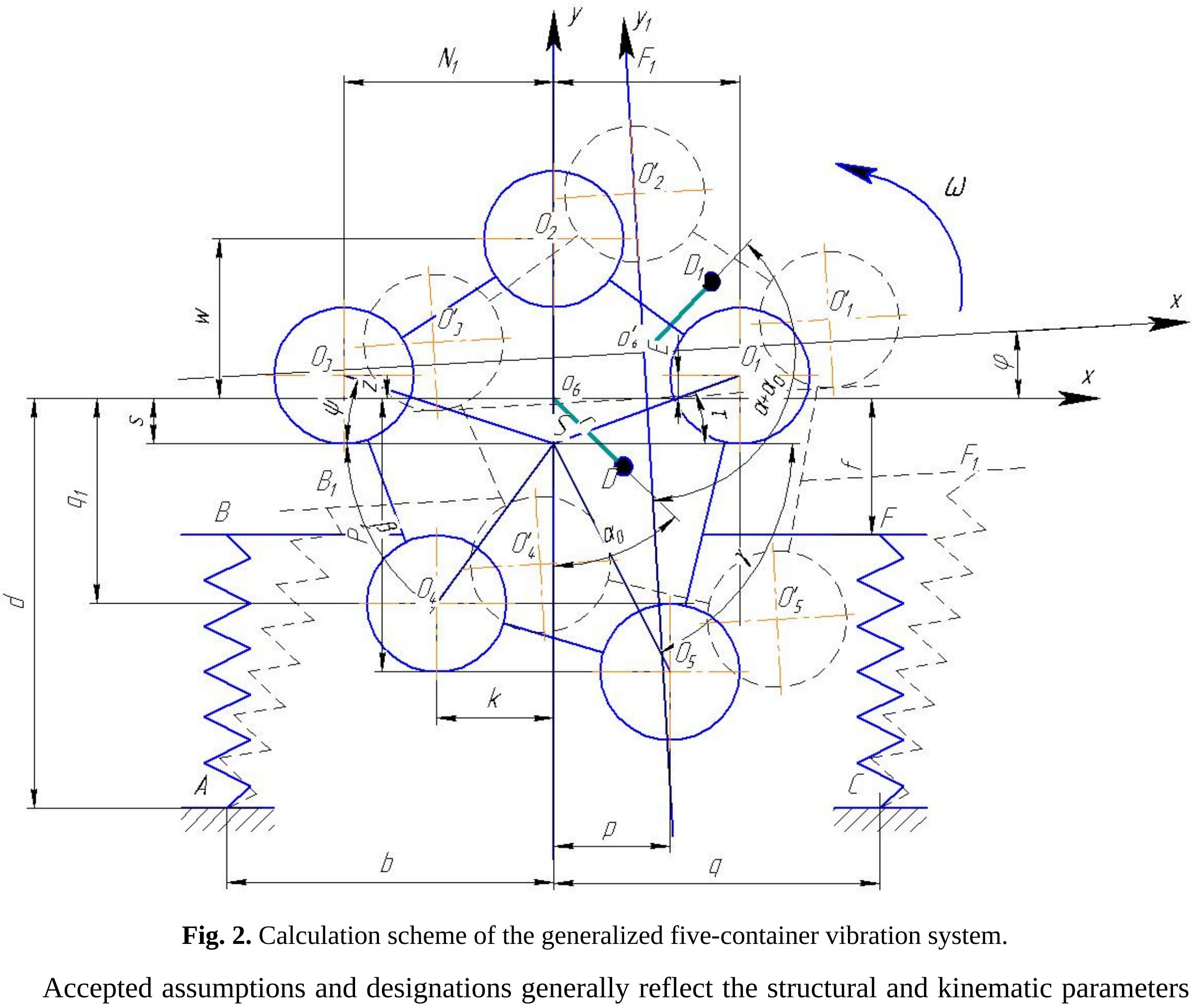


**Fig. 2.** Calculation scheme of the generalized five-container vibration system.

Accepted assumptions and designations generally reflect the structural and kinematic parameters of the five-container vibration system. It will make it possible to build an adequate parameterized unified mathematical model of movement, to investigate the influence of structural parameters and technological modes on the product's processing intensity in the system. The basis of the study of the movement of the vibration system (the motion of any point of the working containers during an arbitrary time interval of product processing) is a system of analytical expressions, which includes all its necessary parameters. These expressions are the solution of the system of differential equations that describe the motion of the vibrating system (mathematical model of its). By substituting here the coordinates of the points of the container, the movement of which must be studied, and the necessary parameters of the vibration system, we will obtain expressions for constructing the trajectories of the motion of the selected point of the containers and determining its amplitude, amplitude-frequency characteristics. The expression that will form the basis of the mathematical model of the container's movement (working body) of the vibration system is the law of motion of their certain “base” point. For example, it can be their geometric center (point O6, see Fig. 2) or their mass center and the angle of rotation containers around the center of the system mass. Time changes of the imbalance vertical and horizontal coordinates and the mass center of the containers consist of a combination of geometric and kinematic parameters of the system. It is as well as variable coordinates of the geometric center of the container – point $O_6$, that is, they are functions of the latter (accordingly, similar dependencies can be written for any point of the container).

Therefore, for the generalized coordinates of the movement of the system, we will take linear movements of the geometric center (the center of rotation of the imbalance) of the researched processing five-container vibration system $x_{0_6}$ , $y_{0_6}$ and rotation angle $\varphi$ of a moving coordinate system relative to a stationary one (angle of rotation of the vibration system around its center of mass).

The differential equations of the oscillatory motion of this mechanical system (the description of the system's motion) are derived from Lagrange's equations of the II kind.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j}-\frac{\partial L}{\partial q_j}=0, \tag{1}$$

where $L=T-(\Pi+\Pi_p)$ – the Lagrangian function, ($T=T_k+T_Д$ -the kinetic energy of the system, consisting of the sum of the kinetic energies of the containers and the energy of one imbalance, $\Pi$ – potential energy of the system (containers and imbalance), $\Pi_p$ – potential energy of the elastic suspension of containers, $q_j$ – generalized coordinates, ie $q_1=x_{0_6}$ , $q_2=y_{0_6}$ , $q_3=\varphi$, and $\dot{q}_1=\dot{x}_{0_6}$ , $\dot{q}_2=\dot{y}_{0_6}$ , $\dot{q}_3=\dot{\phi}$, respectively, their generalized velocities.

In modeling, the imbalance, as indicated above, is considered a material point in which the mass of the imbalance is concentrated, and therefore the expression for finding the kinetic energy of the material point was used to record the kinetic energy of the imbalance.

The Lagrangian of this mechanical system, taking into account the above, will have the form:

$$\begin{aligned}
L=&\frac{M_Д}{2}((\dot{x}_{0_6}+r\cos(\omega t+\phi+\alpha_0)\times\\
&(\omega+\dot{\phi}))^2+(\dot{y}_{0_6}+r\sin(\omega t+\phi+\alpha_0)(\omega+\dot{\phi}))^2)+\\
&+\frac{M_{K_1}}{2}((\dot{x}_{0_6}-\dot{\phi}F_1\sin\phi+\dot{\phi}(F_1 tg\tau+S)\cos\phi)^2+\\
&+(\dot{y}_{0_6}+\varphi F_1\cos\varphi+\dot{\varphi}(F_1 tg\tau+S)\sin\varphi)^2)+\\
&+\frac{M_{K_2}}{2}((x_{0_6}-\dot{\phi}W\cos\phi)^2+(\dot{y}_{0_6}-\dot{\phi}W\sin\phi)^2+\\
&+(\dot{y}_{0_6}-\dot{\phi}N_1\cos\phi+\dot{\phi}(N_1 tg\psi+S)\sin\phi)^2)+\\
&+\frac{M_{K_4}}{2}((\dot{x}_{0_6}+\dot{\varphi}K\sin\varphi+\dot{\varphi}(Ktg\beta+S)\cos\varphi)^2+\\
&+(\dot{y}_{0_6}+\dot{\varphi}K\cos\varphi+\dot{\varphi}(Ktg\beta+S)\sin\varphi)^2)+\frac{1}{2}j\dot{\varphi}^2+\\
&+\frac{M_{K_5}}{2}((\dot{x}_{0_6}-\dot{\phi}P\sin\phi+\dot{\phi}(Ptg\gamma+S)\cos\phi)^2+\\
&+(\dot{y}_{0_6}+\dot{\varphi}P\cos\varphi+\dot{\varphi}(Ptg\gamma+S)\sin\varphi)^2)-
\end{aligned} \tag{2}$$

$$-\left[\begin{array}{l}\frac{C_1}{2}((x_{0_6}-b\cos\varphi+d\sin\varphi+b)^2+(y_{0_6}-b\sin\varphi-\\-d\cos\varphi+d)^2-(d-f)^2)+\frac{C_2}{2}((x_{0_6}+q\cos\varphi+f\sin\varphi-q)^2+\\+(y_{0_6}+q\sin\varphi-\cos\varphi+d)^2-(d-f)^2\end{array}\right]+$$

$$+\left[\begin{array}{l}M_{K_1}g(y_{0_6}+F_1\sin\varphi-(F_1tg\tau+S)\cos\varphi-(F_1tg\tau+S))+\\+M_{K_2}g(y_{0_6}+W\cos\varphi-W)+\\+M_{K_3}g(y_{0_6}-N_1\sin\varphi-\cos\varphi(N_1tg\psi+S)-(N_1tg\psi+S))+\\+M_{K_4}g(y_{0_6}-K\sin\varphi-\cos\varphi(Ktg\beta+S)-(Ktg\beta+S))+\\+M_{K_5}g(y_{0_6}+P\sin\varphi-(Ptg\gamma+S)\cos\varphi-(Ptg\gamma+S))+\\+M_{Д}g(r\cos\alpha_0+y_{0_6}-r\cos(\omega t+\varphi+\alpha_0))\end{array}\right]$$

Let's substitute the derivative expressions for each generalized coordinate into the Lagrange equation of the II kind (1). Then, we reduce them to such a form that only the sum of the second derivative of the generalized coordinate and the product of this coordinate by any coefficient remains in the left part of the system of differential equations. The other terms we transfer to the right side of it.

By this, the equation reduces to a system of perturbed nonlinear differential equations, which will be a mathematical model of the motion of the five-container vibrating system:

$$\begin{cases}\ddot{x}_c+\omega^2x_c=\varepsilon f_x(\phi,\dot{\phi},\ddot{\phi},\omega_1t+\alpha_0,\omega_2t+\psi_0);\\ \ddot{y}_c+\omega^2y_c=\varepsilon f_y(\phi,\dot{\phi},\ddot{\phi},\omega_1t+\alpha_0,\omega_2t+\psi_0);\\ \ddot{\phi}+\omega_\phi^2(t)\phi=\varepsilon' f_\phi(\phi,\dot{\phi},\ddot{x}_c,\ddot{y}_c),\end{cases} \quad (3)$$

where $\varepsilon=\frac{1}{M}$, $\varepsilon'\approx\varepsilon$, $M$ – the total mass of the loaded containers and the unbalanced assembly; $\omega=\sqrt{\frac{C}{M}}$ – natural frequency of oscillations of containers; $\omega_\phi(t)$ – “frequency” of circular oscillations of containers, taking into account unbalance.

Note: It is obvious that $\varepsilon<<1$ is for real vibrating machines. So let's call $\varepsilon$ a small parameter.

Consider the third equation of system (3). Angle of rotation of the container $\varphi$ varies in practice by an insignificant amount – from $0^{\circ}$to $4^{\circ}$. Therefore, it can be argued that $sin\varphi\approx\varphi$ and the third equation of system (3) can be written as:

$$\ddot{\phi}+k1^2\phi=0\,,$$

where $k1=\sqrt{\frac{b_1}{d_1}}$,

$$b_1=C_1(b^2+d^2)+C_2q^2+df-M_{K_1}g(Ktg\beta+S)-M_{K_2}g(Ptg\gamma+S)+M_{K_3}gW\,,$$

$$d_1=M_{Д}r^2+J+M_{K_1}K^2+M_{K_2}P^2+M_{K_3}W^2\,.$$

Its solution can be represented as: $\varphi(t)=L_1\sin kt+L_2\cos kt$ ,

where $L_1$, $L_2$ – are determined by the initial parameters of the studied nonlinear mechanical system.

In this case, taking into account the above, the final system will look like this:

$$
\begin{aligned}
&\ddot{x}_{0_6} + \frac{C}{M} = \frac{1}{\varepsilon}\Big(k^2(L_1\sin kt + L_2\cos kt)(M_Д r\cos(\omega t + L_1\sin kt + L_2\cos kt + \alpha_0) + \\
&\quad + M_{K_1}\begin{bmatrix}(F_1 tg\tau + S)\cos(L_1\sin kt + L_2\cos kt) - \\ - F_1\sin(L_1\sin kt + L_2\cos kt)\end{bmatrix} - \\
&\quad - M_{K_2} W\cos(L_1\sin kt + L_2\cos kt) + \\
&\quad + M_{K_3}\begin{bmatrix}N_1\sin(L_1\sin kt + L_2\cos kt) + \\ + (N_1 tg\psi + S)\cos(L_1\sin kt + L_2\cos kt)\end{bmatrix} + \\
&\quad + M_{K_4}\begin{bmatrix}K\sin(L_1\sin kt + L_2\cos kt) + \\ + (Ktg\beta + S)\cos(L_1\sin kt + L_2\cos kt)\end{bmatrix} + \\
&\quad + M_{K_5}\begin{bmatrix}(Ptg\gamma + S)\cos(L_1\sin kt + L_2\cos kt) - \\ - P\sin(L_1\sin kt + L_2\cos kt)\end{bmatrix} - \\
&\quad - (L_1 k\cos kt - L_2 k\sin kt)^2 \times \\
&\times\begin{bmatrix}- M_{K_1}(F_1\cos(L_1\sin kt + L_2\cos kt) + (F_1 tg\tau + S)\sin(L_1\sin kt + L_2\cos kt) + \\ + M_{K_2} W\sin(L_1\sin kt + L_2\cos kt) + M_{K_3}(N_1\cos(L_1\sin kt + L_2\cos kt) - \\ - (N_1 tg\psi + S)\sin(L_1\sin kt + L_2\cos kt)) + M_{K_4}(K\cos(L_1\sin kt + L_2\cos kt) - \\ - (Ktg\beta + S)\sin(L_1\sin kt + L_2\cos kt)) - M_{K_5}(P\cos(L_1\sin kt + L_2\cos kt) + \\ + (Ptg\gamma + S)\sin(L_1\sin kt + L_2\cos kt))\end{bmatrix} + \\
&\quad + M_Д r(\omega + L_1 k\cos kt - L_2 k\sin kt)(-C_2 q + C_1 b) - \\
&\quad - \sin(L_1\sin kt + L_2\cos kt)(C_1 d - C_2 f) + C_2 q - C_1 b\Big) + \\
&\quad + M_{K_1}(F_1\cos(L_1\sin kt + L_2\cos kt) + \\
&\quad + (F_1 tg\tau + S)\sin(L_1\sin kt + L_2\cos kt)) - \\
&\quad - M_{K_2} W\sin(L_1\sin kt + L_2\cos kt) + \\
&\quad + M_{K_3}((N_1 tg\psi + S)\sin(L_1\sin kt + L_2\cos kt) - \\
&\quad - N_1\cos(L_1\sin kt + L_2\cos kt)) + \\
&\quad + M_{K_4}((Ktg\beta + S)\sin(L_1\sin kt + L_2\cos kt) - \\
&\quad - K\cos(L_1\sin kt + L_2\cos kt)) + M_{K_5}(P\cos(L_1\sin kt + L_2\cos kt) + \\
&\quad + (Ptg\gamma + S)\sin(L_1\sin kt + L_2\cos kt))) - \\
&\quad - (L_1 k\cos kt - L_2 k\sin kt)^2(M_{K_1}((F_1 tg\tau + S)\cos(L_1\sin kt + L_2\cos kt) - \\
&\quad - F_1\sin(L_1\sin kt + L_2\cos kt)) - M_{K_2} W\cos(L_1\sin kt + L_2\cos kt) + \\
&\quad + M_{K_3}(N_1\sin(L_1\sin kt + L_2\cos kt) + \\
&\quad + (N_1 tg\psi + S)cos(L_1\sin kt + L_2\cos kt)) + \\
&\quad + M_{K_4}(K\sin(L_1\sin kt + L_2\cos kt) + \\
&\quad + (Ktg\beta + S)\cos(L_1\sin kt + L_2\cos kt)) + M_{K_5}((Ptg\gamma + S)\times \\
&\times\cos(\omega t + L_1\sin kt + L_2\cos kt + \alpha_0) + \cos(L_1\sin kt + L_2\cos kt)(C_1 d + C_2 f) + \\
&+ \sin(L_1\sin kt + L_2\cos kt)(C_1 b - C_2 q) - d(C_1 + C_2) + \\
&+ (M_{K_1} + M_{K_2} + M_{K_3} + M_{K_4} + M_{K_5} + M_Д)\, g\Big); \\
&\qquad\qquad \ddot{\phi} + k1^2\phi = 0.
\end{aligned} \tag{4}
$$

Having reduced the obtained system of differential equations of motion of the vibration system to the system of equations of the form (3), the next step is to construct its analytical solution, in order to obtain, as a final result, the model of motion of the five-container processing vibration system.

Using the Poincaré method [3], the solution of a perturbed system of nonlinear differential equations, which describes the movement (small oscillations) of a given mechanical system (the vibrating system operates in a stationary mode), can be represented in the form of series, matched by powers $\varepsilon$, which are absolutely convergent when sufficiently small $\varepsilon$:

$$x_{0_6}(t) = x_0 \sin(\sqrt{\frac{C}{M}}t + \alpha_x) + \varepsilon\chi_x\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \varepsilon^2\chi'_x\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \dots,$$

$$y_{0_6}(t) = y_0 \sin(\sqrt{\frac{C}{M}}t + \alpha_y) + \varepsilon\chi_y\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \varepsilon^2\chi'_y\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right) + \dots,$$

where $\varepsilon = \frac{1}{M} < 0$ – the smallness condition is fulfilled; $\chi_i^j\left(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0\right)$ – some functions that are determined according to the Poincaré method.

Analyzing the system according to the weight of the parameters $\varepsilon^i = \left(\frac{1}{M}\right)^i$, where $i \in (0;\infty)$, we limit ourselves in constructing the solution to the first two members of the series – the first represents the solution of an undisturbed system, the second – the effect of an external perturbation.

In this way, the solutions of the perturbed system (the system of equations for determining the generalized coordinates – the coordinates of the center of rotation of the imbalance of the vibrating system to describe its motion) in the first approximation (taking into account the first two terms of the series) in the general case took the form:

$$x_{0_6}(t) = x_0 \sin(\sqrt{\frac{C}{M}}t + \alpha_x) + \varepsilon\int_0^t f_x(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)\sin\left(\sqrt{\frac{C}{M}}(t-u)\right)du,$$

$$y_{0_6}(t) = y_0 \sin(\sqrt{\frac{C}{M}}t + \alpha_y) + \varepsilon\int_0^t f_y(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)\sin\left(\sqrt{\frac{C}{M}}(t-u)\right)du,$$

$$\varphi(t) = L_1 \sin kt + L_2 \cos kt,$$

where $f_x(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)$ and $f_y(\phi,\dot{\phi},\ddot{\phi},\omega t + \alpha_0)$ functions of the right-hand side of the equations in generalized coordinates $x_{0_6}$ and $y_{0_6}$ of the equations system given above.

Therefore, the solution of system (4) in the first approximation will have the form:

$$\begin{aligned} x_{0_6}(t) = x_0 \sin\left(\sqrt{\frac{C}{M}}t + \alpha_x\right) + \varepsilon\int_0^t \Bigg( & k^2(L_1 \sin kt + L_2 \cos kt) \times \\ & \times (M_Д r \cos(\omega t + L_1 \sin kt + L_2 \cos kt + \alpha_0) + \\ & + M_{K_1}\begin{bmatrix} (F_1 tg\,\tau + S)\cos(L_1 \sin kt + L_2 \cos kt) \\ - F_1 \sin(L_1 \sin kt + L_2 \cos kt) \end{bmatrix} - \\ & - M_{K_2} W \cos(L_1 \sin kt + L_2 \cos kt) + \\ & + M_{K_3}\begin{bmatrix} N_1 \sin(L_1 \sin kt + L_2 \cos kt) + \\ + (N_1 tg\,\psi + S)\cos(L_1 \sin kt + L_2 \cos kt) \end{bmatrix} + \end{aligned} \quad (5)$$

$$+M_{K_4}\begin{bmatrix}K\sin(L_1\sin kt+L_2\cos kt)+\\+(Ktg\beta+S)\cos(L_1\sin kt+L_2\cos kt)\end{bmatrix}+$$

$$+M_{K_5}\begin{bmatrix}(Ptg\gamma+S)\cos(L_1\sin kt+L_2\cos kt)-\\-P\sin(L_1\sin kt+L_2\cos kt)\end{bmatrix}-(L_1k\cos kt-L_2k\sin kt)^2\times$$

$$\times\begin{bmatrix}M_{K_1}(F_1\cos(L_1\sin kt+L_2\cos kt)+(F_1tg\tau+S)\sin(L_1\sin kt+L_2\cos kt)+\\+M_{K_2}W\sin(L_1\sin kt+L_2\cos kt)+M_{K_3}(N_1\cos(L_1\sin kt+L_2\cos kt)-\\-(N_1tg\psi+S)\sin(L_1\sin kt+L_2\cos kt))+M_{K_4}\times\\\times(K\cos(L_1\sin kt+L_2\cos kt)-(Ktg\beta+S)\sin(L_1\sin kt+L_2\cos kt))-\\-M_{K_5}(P\cos(L_1\sin kt+L_2\cos kt)+\\+(Ptg\gamma+S)\sin(L_1\sin kt+L_2\cos kt))\end{bmatrix}+$$

$$+M_{Д}r(\omega+L_1k\cos kt-L_2k\sin kt)(-C_2q+C_1b)-$$

$$-\sin(L_1\sin kt+L_2\cos kt)(C_1d-C_2f)+C_2q-C_1b\Big)\sin\left(\sqrt{\frac{C}{M}}(t-u)\right)du;$$

$$y_{0_6}(t)=y_0\sin\left(\sqrt{\frac{C}{M}}t+\alpha_y\right)+\varepsilon\int_0^t\begin{pmatrix}k^2(L_1\sin kt+L_2\cos kt)\times\\\times(M_{Д}(r\sin(\omega t+L_1\sin kt+\end{pmatrix}$$

$$+L_2\cos kt+\alpha_0)+M_{K_1}(F_1\cos(L_1\sin kt+L_2\cos kt)$$

$$+(F_1tg\tau+S)\sin(L_1\sin kt+L_2\cos kt)-$$

$$-M_{K_2}W\sin(L_1\sin kt+L_2\cos kt)+$$

$$+M_{K_3}((N_1tg\psi+S)\sin(L_1\sin kt+L_2\cos kt)-$$

$$-N_1\cos(L_1\sin kt+L_2\cos kt))+$$

$$+M_{K_4}((Ktg\beta+S)\sin(L_1\sin kt+L_2\cos kt)-$$

$$-K\cos(L_1\sin kt+L_2\cos kt))+M_{K_5}(P\cos(L_1\sin kt+L_2\cos kt)$$

$$+(Ptg\gamma+S)\sin(L_1\sin kt+L_2\cos kt)))-$$

$$-(L_1k\cos kt-L_2k\sin kt)^2(M_{K_1}((F_1tg\tau+S)\times$$

$$\times\cos(L_1\sin kt+L_2\cos kt)-F_1\sin(L_1\sin kt+L_2\cos kt))+$$

$$+M_{K_4}(K\sin(L_1\sin kt+L_2\cos kt)+(Ktg\beta+S)\times$$

$$\times\cos(L_1\sin kt+L_2\cos kt))+M_{K_5}((Ptg\gamma+$$

$$+S)\cos(L_1\sin kt+L_2\cos kt)-P\sin(L_1\sin kt+L_2\cos kt)))-$$

$$-M_{Д}r(\omega+L_1k\cos kt-L_2k\sin kt)^2\cos(\omega t+L_1\sin kt+L_2\cos kt+\alpha_0)+$$

$$+\cos(L_1\sin kt+L_2\cos kt)(C_1d+C_2f)+\sin(L_1\sin kt+L_2\cos kt)(C_1b-C_2q)-$$

$$-d(C_1+C_2)+(M_{K_1}+M_{K_2}+M_{K_3}+M_{K_4}+M_{K_5}+M_{Д})g\Big)\times\sin\left(\sqrt{\frac{C}{M}}(t-u)\right)du$$

$$\varphi(t)=L_1\sin kt+L_2\cos kt.$$

The system of equations (5) describes the movement of the center of rotation of the imbalance of the vibrating system during an arbitrary time interval of its operation. Using the system of equations and the relationship between the coordinates of the points of the containers relative to two reference systems – stationary and mobile (relationship with the center of rotation of the imbalance):

$$x_n=x_{0_6}+x_i\cos\varphi(t)-y_i\sin\varphi(t),$$

$$y_n=y_{0_6}+x_i\sin\varphi(t)+y_i\cos\varphi(t),$$

it is possible to determine the horizontal and vertical components of the amplitude of vibrations at any point of the containers of the vibration system in the plane of its movement. The movement takes place in the plane of unbalanced oscillations at an arbitrary moment or during the period of operation of the vibration system during the processing of products. This makes it possible to build the trajectory of the movement of a given point during the studied time interval of processing, to find the influence of all kinematic and geometric parameters of the vibration system (embedded in the model in symbolic form) on the amplitude of oscillations of any point of the working containers. It also makes it possible to investigate their importance in terms of influence on the magnitude of the amplitude and the nature of the vibrations of the containers, as well as in the case of their arbitrary variation among themselves, over the entire interval of possible changes in these parameters.

In the latter ratio $x_n$, $y_n$ – coordinates of an arbitrary point of the containers of the vibration system relative to the stationary coordinate system; $x_i$, $y_i$ – the coordinates of an arbitrary point of the containers relative to the moving coordinate system associated with them, its geometric center, which moves with it (these coordinates are found geometrically – they determine the position of the point in the plane of movement of the container, the amplitude of oscillations and the trajectory of which must be investigated).

## Conclusions

The developed mathematical model of the movement of the five-cone processing vibration system allows you to study various modes of its operation (steady modes, the effect of drive reversal, the effect of the processing medium of the containers of the vibration system, the effect of the processed parts, to model possible non-stationary – resonant modes that may occur during the operation of the vibration system). The obtained model is fully parameterized and includes both the geometric parameters of the vibration system and the kinematic parameters of the processing of products in it.

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