Simulated annealing approach for outpatient scheduling in a haemodialysis unit

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National Renal Registry Malaysia has reported that the dialysis treatment demand among chronic kidney and end-stage kidney disease patients rises yearly. However, available haemodialysis (HD) units have limited facilities to meet the current and increasing demand. This leads to congestion, long waiting times, and an increase in the duration of treatment (DOT) among HD patients during their treatment sessions. Two essential factors in providing optimal treatment plans are outpatient scheduling and nurse assignment. Therefore, the objectives of this study are to minimise patients' total DOT, including the waiting time for pre-dialysis and post-dialysis sessions, which also includes determining the amount of patient flow in an HD unit. Regarding the first objective, we include simulated annealing (SA) into our simple heuristics (SH) in the patient scheduling optimisation model. Here, the initial solution obtained from the method can be improved. The backtracking heuristic (BH) is then applied to the nurse assignment problem, where at least two nurses are needed for each dialysis patient. The results show that the solutions obtained for outpatient scheduling by SA are efficient and have significantly reduced the computational time compared with the SH, even when considering more patients on the waiting list. As for total DOT, we obtain the optimum value compared to the average DOT values for both 3-hour and 4-hour sessions. Besides, a discrete-event simulation (DES) experiment of patient flow in an HD unit was performed by gradual variations in patient arrival rates, λ, to avoid congestion in the system. DES has the potential to accommodate emergency patients that seek HD treatment without causing much disruption to the system.

Keywords: *outpatient scheduling, nurse assignment, heuristic, simulated annealing, discrete-event simulation.*

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1. Introduction

Coresh et al. [1] highlighted that recent research revealed an increase in the prevalence of chronic kidney disease (CKD) and end-stage kidney disease (ESKD) worldwide. CKD consists of five stages, while ESKD is the last stage. During stage 4 or 5 of CKD, nephrologists will advise their patients to choose between dialysis or kidney transplant to ensure survival. Most patients choose dialysis treatment, which has two options: haemodialysis (HD) or peritoneal dialysis (PD), rather than a kidney transplant due to the high cost of the kidney transplant. HD is a common treatment among ESKD patients performed in a clinical setting thrice a week (Monday, Wednesday, Friday) or (Tuesday, Thursday, Saturday) for either 3-hour or 4-hour sessions [2, 3]. The patient will be connected to a machine via vascular access such as a fistula, graft, or catheter. The patient would undergo dialysis treatment and select the vascular access options as per their health requirements. They may have their preferences, including the desire for short treatment times and preferred treatment during the daytime. Moreover, dialysis facilities have to plan the treatment efficiently by optimising resources for the best service delivered to the patients [4, 5]. The management also needs to consider the nurse allocation for the HD treatment. Thus, nurse assignments need to be considered along with the case. This is because the patient scheduling problem works best when there is a staffing assignment problem, which helps the decision-makers plan the model effectively. However, these requirements make the patient scheduling process challenging due to the limited availability of the HD units, which subsequently leads to congestion, long waiting times, and an increase in the duration of treatment (DOT) among patients during their treatment sessions [6]. Thus, patient scheduling in the HD unit is considered to have great practical significance.

Outpatient scheduling problems in HD units are known as patient scheduling problems in healthcare. There are vast studies related to the patient scheduling problem in healthcare. However, as we reviewed the past studies related to patient scheduling in the HD unit, only a few studies were available for the HD patient scheduling problem. Several researchers have solved patient scheduling problems using exact optimisation methods such as an optimisation model and mixed integer linear programming. For example, Holland [7] developed an optimisation model to compare two different scheduling problems of a HD centre by considering the device utilisation, length of service and capability of the HD unit. The researcher suggested that a flexible start time for HD treatment is much more convenient for the patients as compared to a fixed start time. In another study, Pena et al. [8] explored inpatient scheduling problems for ESKD patients by developing a decision support tool based on the optimisation approach. Each patient requires a distinct number of time slots to be treated. Moreover, the dialysis devices are partitioned into blocks according to the needs of the patients. The researcher concluded that a decision support tool based on an optimisation approach could maximise the efficiency of the HD unit and thus minimise delays during the treatment. An analytical model and a decision support tool was developed by Fleming et al. [9] to meet the complex challenges of scheduling dialysis patients. The model considered clinical pathways, a limited number of nurses managing the patients and dialysis stations. The results showed a schedule could be computed efficiently using the decision support tool. However, the researchers suggested that patient-related objective functions should be considered and also suggested heuristics or metaheuristics since exact methods take a relatively longer time to solve large datasets.

Various heuristic strategies have been proposed to improve the scheduling problems in the dialysis unit. For example, Choi et al. [2] used a mathematical model of the dialysis patient for every conventional regular dialysis schedule. In contrast, the model employed a genetic algorithm (GA) to look for the optimal HD schedule. The algorithm was able to find the optimised schedule for patients and the duration of every session. GA also improved the adequacy of the conventional HD schedule and proved that frequent dialysis is more practical than conventional regular dialysis schedules. In another study, Liu et al. [10] developed a basic heuristic and a rollout algorithm (ROA) to solve multi-level treatment scheduling problems. The results showed that the ROA could schedule patients efficiently with the increasing demands of medical service and extensive use of advanced medical equipment by developing an optimisation model. According to the researchers, other approaches, such as metaheuristics or hyper-heuristics, could be used to solve this model. For example, Nwaneri et al. [11] investigated patient flow and scheduling by developing an optimisation model to minimise the length of stay (LOS) at the HD clinic. The researchers developed GA and, from their simulation results, concluded that the optimisation model effectively reduced LOS among HD patients.

We also need to consider emergency cases during patient scheduling since emergency cases are inevitable. Here, the management must be prepared for any changes that may occur in the planned scheduling system. For example, Daknou et al. [12] focused on treatment scheduling for patients at emergency department (ED). They proposed a multi-agent system in which patients and resources at the ED are able to react flexibly to changes that occur. This system also improves healthcare efficiency through monitoring and automating staff work. Meanwhile, Paulussen et al. [13] proposed a flexible agent-based approach for patient scheduling that is able to deal with emergencies in the future. The researchers aimed to minimise patients' LOS and resource idle time. They are able to improve patient scheduling practises in hospitals while providing the needed flexibility in the system.

The method used and the combination of patient and nurse assignment problems distinguish this work from previous research on patient scheduling problems. This paper focuses on outpatient scheduling and nurse assignment problems to minimise patients' DOT, including the waiting time for pre-dialysis and post-dialysis. We also want to estimate the amount of patient flow in the HD unit to reduce congestion and long waiting times in the HD unit. The scheduling problem is formulated as an optimisation model and solved using simulated annealing (SA). SA is chosen to solve the scheduling problem as it is known as a single-solution heuristic [14] and it is able to approximate the global optimum for a discrete search space. Along with that, SA is able to accept worse solutions, which allows for a more extensive search for the optimal solution in a reasonable period of time [15]. Estimation of patient flow in the HD unit for their treatment is solved using discrete event simulation. As a result, we are able to minimise patients' DOT and the optimal HD schedules can be shared with patients and HD unit management. In this paper, both HD and dialysis terms refer to haemodialysis.

The remainder of this paper is as follows. In Section 2, we formulate the optimisation model for the problem, followed by the proposed algorithms. Section 3 deals with the computational experiments, and lastly, Section 4 concludes the paper.

2. Methodology

This section discusses outpatient scheduling, nurse assignment problems and patient flow at dialysis unit. First, an optimisation model for the outpatient scheduling problem is formulated. The scheduling problem was studied by Sundar et al. [16], where they developed a simple heuristic (SH) to get the initial solution. In this study, we aim to improve the quality of the initial solution using SA and estimate the patient flow using DES. As the nurses must assist each patient during their HD session, a backtracking heuristic is used to solve the nurse assignment problem.

2.1. Outpatient scheduling problem via simulated annealing approach

Outpatient refers to the patients who receive treatment at the hospital but do not stay there for a night [17–19]. In this paper, we are focusing on outpatient scheduling problems. The scheduling process starts with the arrival of patients to the HD unit. Usually, the patients will undergo pre-dialysis, dialysis and post-dialysis sessions. During pre-dialysis, patients are required to take blood and urine tests and weigh their body before undergoing dialysis. Then, patients undergo dialysis for either 3-hour or 4-hour, depending on the health plan of the nephrologists. After the treatment, they need to weigh their bodies again and be involved in post-dialysis radiology and laboratory tests. The patients receive consultation from the nephrologists and later will collect their medicine at the pharmacy [11, 20]. According to medical requirements, the distribution of the number of patients treated in a service cycle should be as uniform as possible. A few factors that need to be considered in the scheduling process. Priority is given to those in the critical stage of chronic kidney disease. The duration for HD treatment consists of 2 sessions, which are 3-hour and 4-hour. Note that patients' preferences should be satisfied whenever it is possible. Lastly, the life span of the devices is under consideration, whereby each device must not be overused during a cycle.

The model for this problem is formulated as a single objective function. The case study for this problem is given as follows. Given N, number of patients at HD unit, each patient, p is treated per their treatment access route, j. The patient, p undergoes pre-dialysis (σ_{pt}), dialysis (dur_{pt}) and post-dialysis (ϕ_{pt}). Here, we want to determine the optimal value for the combination of both 3-hour and 4-hour sessions, which is able to minimise the DOT among patients. We selected both sessions in our study because most of the dialysis treatments in Malaysia are within these two sessions. We want to calculate the optimal value for both of the sessions separately because this may then help to plan for the patients arrival later in Section 2.3. The notations used for this model are presented in Table 1.

Table 1. Notations for the optimisation model.

Parameter	Description
A	set of treatments
dur_{pt}	dialysis treatment of patient p, excluding the setup and finish
N	set of patients
p	index of patients, where $p, \ldots, N$
j	treatment vascular access route
σ_{pt}	dialysis setup treatment of patient $p \in N$
ϕ_{pt}	completion of dialysis treatment of patient
r_i^{nurse}	nurse demand by treatment $i \in A$
r_i^{room}	room demand by treatment $i \in A$
R_t^{nurse}	nurse capacity by treatment $t \in T$
R_t^{room}	room capacity by treatment $t \in T$
T	set of periods
arv_t	arrival time of patient
dep_t	departure time of patient
DOT	total duration of treatment

The optimisation model involving the parameters in the above table is formulated as follows:

$$\text{minimize} \quad \text{DOT} = \sum_{p}^{N} [\sigma_{pt} + (dur_{pt} * x_{pj}) + \phi_{pt}], \tag{1}$$

$$\text{s.t.} \quad \sum_{j} x_{pj} = 1, \quad \forall p \in N, \tag{2}$$

$$\text{DOT} \leqslant dep_t, \quad \forall t \in T, \tag{3}$$

$$arv_t < dep_t, \quad \forall j \in N, \tag{4}$$

$$R_t^{nurse} \geqslant \sum_{j \in A} r_j^{nurse} * \sum x_{pj}, \quad \forall t \in T, \tag{5}$$

$$R_t^{room} \geqslant \sum_{j \in A} r_j^{room} * \sum x_{pj}, \quad \forall t \in T, \tag{6}$$

$$x_{pj} \in \{0, 1\}, \quad \forall p \in N, \quad j \in A. \tag{7}$$

In this model, objective function (1) minimises the total DOT for each patient. Meanwhile, constraint (2) ensures each treatment is scheduled exactly once. Moreover, constraint (3) ensures that the total DOT is less than the departure time of the patient, while constraint (4) ensures that the arrival time of the patient is less than the departure time of the patient. Furthermore, constraint (5) ensures that the demand for nurses does not exceed the nurse capacity, and constraint (6) ensures that the demand for the rooms does not exceed the room capacity. Constraint (7) is the decision variable for this model, where

$$x_{pj} = \begin{cases} 1, & \text{if patient } p \in A \text{ assigned to vascular access } j; \\ 0, & \text{otherwise.} \end{cases}$$

To minimise the total DOT, we solve the optimisation model using the SA algorithm. SA is a stochastic single point algorithm invented by Kirkpatrick et al. [21]. It was selected to solve this problem because the search space for this problem is discrete. Each patient has a specific dialysis machine that it needs to be treated on and only one treatment can be scheduled at a given time. An objective function is defined based on equation (1). The decision variables are selected, which are: σ_{pt}, dur_{pt} and ϕ_{pt}. Next, the default SA parameters are modified. The stopping criteria for SA are that when all row patients in the unscheduled list are considered, the iteration will stop, and the final schedule will be obtained. We will be using the terms "heating process" and "cooling process" to indicate the movement of the SA graph upwards and downwards, respectively. The SA's algorithm is summarised below.

Algorithm 1 Standard Simulated Annealing Algorithm

Require: σ_{pt}, dur_{pt}, ϕ_{pt};
Ensure: $\text{DOT} = \sum_{p}^{N} [\sigma_{pt} + (dur_{pt} * x_{pj}) + \phi_{pt}]$;
initialization: row = 1, add patients = unscheduled ((row;:);
choose random i and j, where $i \neq j$;
set an initial temperature $T = \text{Initial}T_0$ and $T_0' = 1$;
if $T_0 < FinalT_0$ **then**
  calculate $T_0 = \alpha T_0$;
set $T_0' = 0$;
if $(\sigma_{p(i)} + dur_{p(i)} + phi_{p(i)} \leqslant dep_t)$ and $(\sigma_{p(j)} + dur_{p(j)} + phi_{p(j)} \leqslant dep_t)$ **then**
  while $d(j) = d(i)$
    choose a random j, where $d(i) \neq d(j)$;
calculate $totalDOT(i+j)$ for $dep_{t_i} + dep_{t_j}$;
calculate $\delta = totalnewDOT(i+j) - totalcurrentDOT(i+j)$
if $\delta \leqslant 0$ **then**
  (i) $totalcurrentDOT(i+j) = totalnewDOT(i+j)$;
  (ii) dummy $= p(i)$;
  (iii) $p(i) = p(j)$;
  (iv) $p(j) =$ dummy;
else if $\delta > 0$ and $T_0' = 1$ **then**
  calculate $p = \exp \frac{-\delta}{T_0}$;
  derive random number, $r \in [0, 1]$;
if $r \leqslant p$ **then**
  loop // *infinite loop*
    under certain conditions **exit**;
else //
  choose random i and j, where $i \neq j$;
if $T_0' = 0$ **then**
  stop;
else //
  choose a random j;
if row $> N_{row}$ **then**
  stop the iteration, final schedule is obtained;

2.2. Nurse assignment problem via backtracking heuristic

Every patient scheduled in the system must be guided by the nurses assigned for that day. According to Fleming et al. [9] and Liu et al. [20], the nurse will be assigned based on their shift and availability for that day. For each scheduled treatment, a minimum of two nurses must be assigned to handle the treatment [20]. Treatment nurses are responsible for operating the dialyser and related work. Meanwhile, dispensing nurses are assigned for dispensing and related work. The second condition is the possibility of nurse rotation among them. The nurses in HD service are classified into three levels, from level 1 to level 3 and level 3 is the highest level. The nurses at higher level can replace the nurses at lower level, but not vice versa. In this case, required levels of dispensing nurse and treatment nurse are both level 1 and rotation is allowed. Thus, the treatment nurse will become the dispensing nurse for the next day of duty and vice versa. Backtracking heuristics (BH) procedures have been developed to solve the nurse assignment problem. The algorithm moves from one solution to a different solution within the search space or neighbourhood by applying the local change until an optimal solution is found. The method will continue until no better solution is found within the neighbourhood. This method ensures that every patient is assigned a treatment nurse, T_n

Table 2. Notation for the backtracking heuristic.

Notations	Descriptions
r	Index of treatment room where $r, \ldots, R$
n	Index of nurse
$TYPE1[n]$	Type of patients' background
$STATUS[n]$	Status for nurse, if 'Y' the nurse is available and if 'N' the nurse is not available

and dispensing nurse, D_n. The results obtained from the nurse assignment problem will be presented in the simulation experiment for patient arrival. Table 2 shows the notations used in the BH for nurse assignment.

Algorithm 2 Backtracking Heuristic

Require: i, n, p, r;
1: declare $STATUS[n]$ = 'Y';
2: **for** i=0
3: check for treatment room r=1 and type[p]=1;
4: **if** $TYPE1[n]$='1' and $STATUS[n]$='Y' **then**
5: assign nurse[n] to patient[p];
6: declare $STATUS[n]$='N';
7: assign nurse[n+1] to patient[p];
8: **loop** //
9: repeat Step 2–7 for $r + 1$ up to R;

2.3. Discrete event simulation for patient arrival in a haemodialysis unit

In addition to the outpatient scheduling problem, we consider patient arrival in the HD unit, which is further explored using DES. DES is applicable when an event happens at discrete instances in time and events take zero time to happen [22]. In our study, DES is applicable if an emergency occurs at the HD unit. For instances, when a patient is required to use the dialysis machine urgently or if a nurse is unable to attend to her duty. We used an M/M/15 queue model as each treatment room is allocated five dialysis machines, and there are three treatment rooms. The queue model is assumed to be based on the finite queuing situation and 'M' denotes a Markov process. Markov process usually describe a sequence of possible events by recognizing the pattern and make prediction accordingly [23]. The arrival of patients is based on a Poisson distribution with a mean arrival rate, λ_i, where i is $\{1, 2, 3\}$. For the service rates, three different service rates are adopted for the three stages of service (pre-dialysis, dialysis, and post-dialysis). The service rate for the pre-dialysis stage is exponentially distributed with a probability density function and μ is mean pre-dialysis time. Here, equation (8) refers to the patient arrival rate per minute, and equation (9) refers to the service rate:

$$a = \frac{1}{\lambda}, \tag{8}$$

$$p = \frac{1}{\mu} e^{\frac{1}{\mu}t}. \tag{9}$$

The service rate is randomly selected from the dialysis duration range, dur_{pt} [180, 240] minutes for dialysis. In this section, we combine both 3-hour and 4-hour sessions since, in reality, the dialysis management needs to tackle patient arrival at the HD unit with both duration sessions. The post-dialysis stage is exponentially distributed with a probability density function computed with equation (9). The process of the DES of HD processes is shown as follows:

Algorithm 3 Discrete-Event Simulation for Patient Arrival

Require: σ_{pt}, dur_{pt}, ϕ_{pt};
Ensure: DOT $= \sum_p^N [\sigma_{pt} + (dur_{pt} * x_{pj}) + \phi_{pt}]$;
1: calculate mean values for σ_{pt}, dur_{pt} and ϕ_{pt};
2: set arv_t along with mean values obtained at Step 1;
3: generate patient arrival based on Poisson distribution;
4: **while** $p \neq N$
5: generate for next λ;
6: generate service rates for three stages of sessions for each patient;
7: assign the patient according to available treatment room and assign the nurses for each patient;
8: calculate DOT for each patient.

3. Results and discussion

3.1. Effectiveness of patient scheduling via simulated annealing

We generate the dialysis session duration, dur_{pt} based on the uniform distribution in the range of $[180, 240]$ minutes. Based on the generated data, the mean for three sessions for the 3-hour and 4-hour dialysis treatments has been improved using SA. The stopping criteria for SA are that if row is greater than N_{row}, there is no more patients to schedule, and a final schedule is obtained. Here, the algorithms are written in Python and run on Windows 10 (CPU 2.30GHz, 4.00GB of memory) using the Spyder integrated development environment (IDE). Python was chosen because it is an open-source programming language with comprehensive standard libraries that enable model optimisation [24].

Table 3 shows the results of testing generated data on the model. The objective function, which represents the DOT values, was optimised, giving the optimum DOT values of 193.00 minutes and 255.00 minutes from the three sessions, including pre-dialysis, dialysis, and post-dialysis that were used as decision variables for the 3-hour and 4-hour dialysis sessions. Performance comparison of the mean DOT values from generated data and optimised values of DOT shown in Table 3 suggests a considerable improvement in the DOT achieved from optimisation. For the 3-hour and 4-hour dialysis procedures, the mean DOT was reduced from 235.30 minutes to 193.00 minutes and from 297.70 minutes to 255.00 minutes, respectively. Figures 1 and 2 show the optimised DOT values of 193.00 and 255.00 from the three stages of sessions. The line in the graph represents the movement of the solutions from SA, where they move upwards (heating process) and gradually move downwards (cooling process) until they reach an optimal point, which are 193.00 and 255.00, respectively.

Table 3. Optimal value of duration of treatment (DOT).

Session	σ_{pt} (min)	dur_{pt} (min)	ϕ_{pt} (min)	Mean DOT (min)	Optimal DOT (min)
3-hour	3.00	180.00	10.00	235.30	193.00
4-hour	10.00	240.00	5.00	297.70	255.00

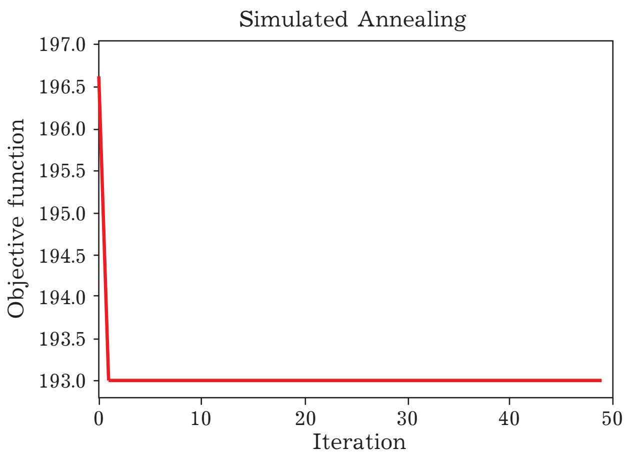

Fig. 1. DOT for 3-hour dialysis session.

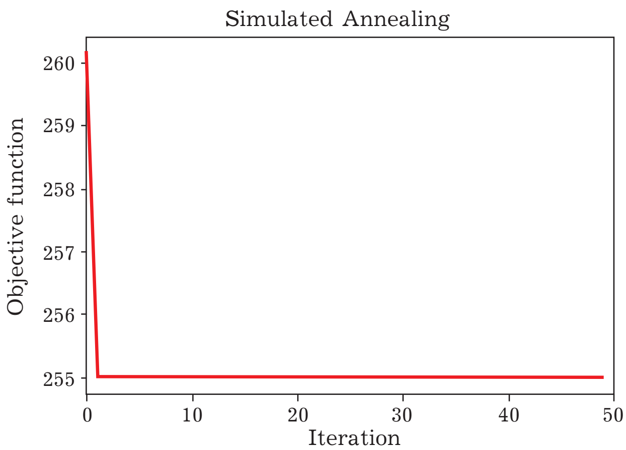

Fig. 2. DOT for 4-hour dialysis session.

3.2. Effectiveness of discrete event simulation in the patient flow

Patient arrival experiments were performed based on DES with inter-arrival rates set at $\lambda_1 = 0.1$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.3$. The results are shown in Tables 4–6 along with the treatment room and assigned nurses for each patient. The start times for pre-dialysis events, which represent pre-connection procedures, were performed at fixed intervals. The duration of pre-dialysis events varies randomly from one patient to the other, typically depicting the real case. Also, the dialysis duration of each patient is randomly selected from the two options available that last for 180 and 240 minutes.

From the result, we learned that the DOT is mostly affected by the duration of the dialysis and post-dialysis procedures but not by the arrival time. For instance, in Table 4, patient 1 spent 7.08 min for a pre-dialysis session, 180 min for dialysis treatment, and 98.02 min for post-dialysis. Thus, we can notice that post-dialysis takes more than 1 hour, which causes patients' DOT to increase. Consequently, a reduction in DOT could be achieved by enhancing the efficiencies of the dialysis and post-dialysis procedures. In particular, specific improvements by increasing manpower or equipment as well as processes in post-dialysis activities such as laboratory tests, radiology, and pharmacy could lead to a reduction in DOT but a corresponding increase in cost. Here, the management may need to make a trade-off decision depending on their preferences.

From Table 7, the mean DOT for all mean arrival rates in this study ranges from 302.96 minutes to 333.57 minutes. At $\lambda = 0.3$, the mean DOT is minimal, which is 302.96 minutes, and the standard deviation is 21.52. SA for an HD unit implies that the system can be optimised to use fewer resources at the pre-dialysis stage by scheduling patient arrivals at short intervals to avoid congestion in the system. Overall, careful planning of the operations of the HD unit is necessary as it could lead to a reduction in the cost as well as an increase in the efficiency of the system. The results suggest that patients' DOT in HD units could be improved with a well-planned scheduling strategy. Our findings are in agreement with similar studies which have demonstrated the effectiveness of SA in planning HD processes. On the other hand, the DES of patient flow for a typical HD unit is modelled to reflect the real-case scenario of patient arrival. The results suggest that a few patients seeking emergency HD service can be accommodated for dialysis treatments that last for $2-3$ hours without causing much disruption to the system. Furthermore, by having minor adjustments in the schedule of regular patients expected to arrive at a short interval, unscheduled emergency patients are able to be treated.

Table 4. DES result for $\lambda_1 = 0.1$.

N	σ_{pt} (min)	dur_{pt} (min)	ϕ_{pt} (min)	DOT (min)	r	T_n	D_n
1	7.08	180	98.02	285.10	r_1	n_2	n_1
2	12.05	180	75.01	267.06	r_2	n_3	n_4
3	22.06	240	60.01	322.07	r_3	n_5	n_6
4	32.06	240	31.01	303.07	r_1	n_2	n_1
5	42.06	240	65.01	347.07	r_2	n_3	n_4
6	52.01	240	50.01	342.02	r_3	n_5	n_6
7	62.06	240	65.01	367.07	r_1	n_2	n_1
8	72.06	240	65.01	377.07	r_2	n_3	n_4
9	82.06	240	20.01	347.07	r_3	n_5	n_6
10	93.01	240	45.01	378.02	r_1	n_2	n_1

Table 5. DES result for $\lambda_2 = 0.2$.

N	σ_{pt} (min)	dur_{pt} (min)	ϕ_{pt} (min)	DOT (min)	r	T_n	D_n
1	7.08	180	98.02	285.10	r_1	n_2	n_1
2	7.05	180	75.01	262.06	r_2	n_3	n_4
3	12.06	240	60.01	312.07	r_3	n_5	n_6
4	17.06	240	31.01	288.07	r_1	n_2	n_1
5	22.06	240	65.01	327.07	r_2	n_3	n_4
6	27.01	240	50.01	317.02	r_3	n_5	n_6
7	32.06	240	65.01	337.07	r_1	n_2	n_1
8	37.06	240	65.01	342.07	r_2	n_3	n_4
9	42.06	240	20.01	302.07	r_3	n_5	n_6
10	47.01	240	45.01	332.02	r_1	n_2	n_1

Table 6. DES result for $\lambda_3 = 0.3$.

N	σ_{pt} (min)	dur_{pt} (min)	ϕ_{pt} (min)	DOT (min)	r	T_n	D_n
1	7.08	180	98.02	285.10	r_1	n_2	n_1
2	5.38	180	75.01	260.39	r_2	n_3	n_4
3	8.73	240	60.01	308.74	r_3	n_5	n_6
4	12.06	240	31.01	283.07	r_1	n_2	n_1
5	15.39	240	65.01	320.40	r_2	n_3	n_4
6	18.68	240	50.01	308.69	r_3	n_5	n_6
7	22.06	240	65.01	327.07	r_1	n_2	n_1
8	25.39	240	65.01	330.40	r_2	n_3	n_4
9	28.73	240	20.01	288.74	r_3	n_5	n_6
10	32.01	240	45.01	317.02	r_1	n_2	n_1

Table 7. Comparison of simulation results.

Mean Arrival Rate (λ_i)	Mean DOT	Standard Deviation
0.1	333.56	36.42
0.2	310.56	24.60
0.3	302.96	21.52

4. Conclusion

In this paper, we developed an outpatient schedule for the HD unit using an optimisation model to reduce the duration of treatment (DOT). As the demand for dialysis from CKD and ESKD increases in Malaysia, there is a need to improve the processes in the HD unit to reduce congestion, long waiting times, and increased DOT. Using the SA algorithm, we improved the quality of the solution for patient scheduling. The results showed that SA performed better than simple heuristics, and the model was therefore effective in reducing patients' DOT. Apart from that, from the queuing theory concept for patient arrival at the HD unit, the inter-arrival rate revealed no significant differences in patients' DOT. DES suggests that a few patients seeking emergency dialysis services can be accommodated for dialysis treatment without causing much disruption to the system. This shows that our scheduling system is flexible and can handle emergencies. We also found that the DOT is mostly determined by the duration of the dialysis procedure rather than the arrival time when patient arrival rates were at closely spaced intervals.

Future work will include rescheduling if there is a sudden change in the existing schedule or if there is a missed treatment or appointment. We can consider rescheduling for patients as well as re-rostering for nurses in this case. We would also like to formulate the patient scheduling problem as a multi-objective model by maximising the efficiency of the dialysis unit and minimising the cost of dialysis.

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Підхід імітованого відпалу для амбулаторного планування для відділення гемодіалізу

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Згідно зі звітом Національного ниркового реєстру Малайзії попит на лікування діалізом серед пацієнтів із хронічною хворобою нирок і термінальною стадією хвороби нирок зростає з кожним роком. Однак наявні гемодіалізні (ГД) апарати мають обмежені можливості для задоволення поточного та зростаючого попиту. Це призводить до перевантаження медичних установ, тривалого часу очікування та збільшення тривалості лікування (ТЛ) серед пацієнтів з ГД під час сеансів лікування. Двома важливими факторами для створення оптимальних планів лікування є графік амбулаторних прийомів і призначення медсестри. Отже, цілі цього дослідження полягають у мінімізації загальної ТЛ пацієнтів, включаючи час очікування сеансів перед і після діалізу, що також включає визначення обсягу потоку пацієнтів у відділенні ГД. Що стосується першої мети, ми включаємо моделювання відпалу (МВ) в нашу просту евристику (ПЕ) у моделі оптимізації планування пацієнтів. Тут вихідний розв'язок, отриманий за допомогою цього методу, може бути вдосконалено. Потім евристика зворотного відстеження (ЗЕ) застосовується до проблеми призначення медсестри, де для кожного пацієнта на діалізі потрібні щонайменше дві медсестри. Результати показують, що розв'язки, отримані для амбулаторного планування за допомогою МВ, є ефективними та значно скоротили час обчислень порівняно з ПЕ, навіть якщо припускати більшу кількість пацієнтів у списку очікування. Що стосується загальної ТЛ, отримуємо оптимальне значення порівняно із середніми значеннями ТЛ як для 3-годинного, так і для 4-годинного сеансу. Крім того, було проведено експеримент моделювання дискретних подій (МДП) потоку пацієнтів у відділенні ГД шляхом поступових варіацій частоти прибуття пацієнтів, λ, щоб уникнути перевантаження в системі. МДП має потенціал для розміщення невідкладних пацієнтів, які звертаються за лікуванням ГД, без спричинювання значних збоїв у системі.

Ключові слова: *амбулаторне планування, призначення медсестри, евристика, імітація відпалу, моделювання дискретних подій.*