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Hybrid firefly genetic algorithm and integral fuzzy quadratic programming to an optimal Moroccan diet

El Moutaouakil K.[1], Ahourag A.[1], Chakir S.[2], Kabbaj Z.[3], Chellack S.[2], Cheggour M.[2], Baizri H.[4]

[1]*Engineering science laboratory, FPT of Taza, USMBA of Fez, Morocco*
[2]*Morphoscience laboratory, FMP, CAU of Marrakech, Morocco*
[3]*Health science laboratory, FMP, CAU of Marrakech, Morocco*
[4]*Biosciences and Health Research Laboratory, Diabetes and Metabolic Diseases Endocrinology Service, Avicenne Military Hospital, FMP, UCA of Marrakech, Morocco*

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In this paper, we solve the Moroccan daily diet problem based on 6 optimization programming (P) taking into account dietary guidelines of US department of health, human services, and department of agriculture. The objective function controls the fuzzy glycemic load, the favorable nutrients gap, and unfavorable nutrient excess. To transform the proposed program into a line equation, we use the integral fuzzy ranking function. To solve the obtained model, we use the Hybrid Firefly Genetic Algorithm (HFGA) that combines some advantages of the Firefly Algorithm (FA) and the Genetic Algorithm (GA). The proposed model produces the best and generic diets with reasonable glycemic loads and acceptable core nutrient deficiencies. In addition, the proposed model showed remarkable consistency with the uniform distribution of glycemic load of different foods.

Keywords: *optimal Moroccan diet; fuzzy quadratic programming; triangular fuzzy numbers; integral ranking most function; genetic algorithm; firefly algorithm.*

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1. Introduction

Health conditions such as diabetes, cardiovascular disease, obesity, and cancer are greatly influenced by unbalanced diets [1–5]. To maintain a high life quality, healthy diets help control the development of chronic diseases and attenuate the risks of these diseases. It is all about answering the demands of the human body in an optimal manner, at all age levels. In this contribution, we propose several optimal daily regimes based on fuzzy quadratic programming, a suitable fuzzy integral ranking function, and a Hybrid Firefly Genetic Algorithm (HFGA) [6]. Through time, the dietary optimization problem has called the attention of many scientists whose suggestions diverge only with respect to the objective functions being considered. Stigler and Danzig still the first group to implement the first optimization framework in which the target function is the cost of the solution whilst the constraints reflect the demands for the correct balance of the solution [7]. In [8], the objective function of the proposed model mediates between the different meals utilizing the penalty mechanism. In this context, authors take into account the regular meals: a snack and a portion of fruit. The suggestion of G. Masset [9] was to monitor the gap between the real intake and the recommended intake that matched the nutritional requirements. Additional research has proposed supplemental diets and nutritional menus at minimal cost for children [10]. To control multiple goals simultaneously, other researchers have used multi goal mathematical models [10,11]. V. Mierlo considered nearly the similar scenario by substituting the cost of the diet with the minimization of fossil fuel depletion [12]. Cholesterol intake and glycemic load are supposed to be main factors contributing to childhood obesity and were the subject of two objective functions of the proposed multi-objective model in [13, 14].

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Because of the stochastic nature of food knowledge, including glycemic loads that vary with cooking method and maturity, it is necessary to transform the models proposed in the literature so that the best-known optimization methods can be used to estimate optimal diets. Robust programming considers nominal central values and maximum deviation when transforming the models [15–17]. This increases the complexity of these models: other terms are added to the objective function, the number of variables doubles, and the number of constraints becomes very large. The decomposition of fuzzy or interval programming problems increases the number of constraints and leads to multi-objective optimization problems that are very difficult to solve [18, 19]. The simplest transformation consists in considering the average values or the maximum values of the wave data [20]. But, the optimization methods will lose all information on the marginalized values because there is an infinite number of intervals having the same maximum value or the same center. In this work, we propose several optimal daily diets that can be part of secure regimes. To do so, we model the dietary problem in terms of an original fuzzy quadratic optimization programming that make compromise between the glycemic diet, favorable, and unfavorable daily requirements. To pass from fuzzy representation to real one, we use integral fuzzy ranking function that makes compromise between the critical values of the fuzzy glycemic load function [21]. To solve the obtained model, we use the hybrid firefly genetic algorithm that combines some advantages of the firefly algorithm and the genetic algorithm [6].

The remainder of the document is structured the following: In the second section, we introduce the triangular fuzzy number theory and the integral fuzzy ranking function. In the third section, we propose an original diet model using fuzzy quadratic programming. In the fourth section, we give the principle of the hybrid firefly genetic algorithm. In the fifth section, several experimental results are presented.

2. Integral fuzzy ranking function to triangular fuzzy numbers

Given a universe of discourse X and E a subset of X, we denote by $\mu_A(x)$ the membership grade of the element x in the set A which is a real number form the interval $[0,1]$ (see [22, 23]).

A fuzzy number $A = (a, b, c)$ is called triangular fuzzy number if $a \leqslant b \leqslant c$; in this case, the membership function of A is given by Eq. (1):

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leqslant x \leqslant b, \\ \frac{x-c}{b-c}, & b \leqslant x \leqslant c, \\ 0, & \text{otherwise}. \end{cases} \tag{1}$$

A triangular fuzzy number (a, b, c) is said to be non-negative if $0 \leqslant a$. Two triangular fuzzy numbers $A = (a, b, c)$ and $B = (a', b', c')$ are said to be equal if and only if $a = a'$, $b = b'$, $c = c'$. From now on, we consider these two triangular fuzzy numbers in the rest of this section.

Let α be a real number. In the following, we give the operations that we need to reach our goals in this paper [22]. The sum of two triangular fuzzy numbers is given by $A \oplus B = (a + a', b + b', c + c')$. The multiplication of the triangular fuzzy number A by the constant α is given by:

$$\alpha A = \begin{cases} (\alpha a, \alpha b, \alpha c), & \alpha \geqslant 0, \\ (\alpha c, \alpha b, \alpha a), & \alpha < 0. \end{cases}$$

Consider the set of convex fuzzy numbers $C = \{C_1, \ldots, C_i, \ldots, C_m\}$ and the application R mapping from C to the set of real numbers $\mathbf{R}$.

Definition 1. *The application R is said to be a ranking function if it verifies the following clauses:*
(P1): If $C_i < C_j$, then $R(C_i) < R(C_j)$,
(P2): If $C_i = C_j$, then $R(C_i) = R(C_j)$,
(P3): If $C_i > C_j$, then $R(C_i) > R(C_j)$.

Definition 2. *Consider the triangular fuzzy number $A = (a, b, c)$. The functions $f_{L,A}(x) = \frac{x-a}{b-a}$ and $f_{R,A}(x) = \frac{x-c}{b-c}$ are called the left and the right membership function of the triangular fuzzy number A, respectively.*

The functions $g_{L,A}(y) = a + (b-a)y$ and $g_{R,A}(y) = c + (b-c)y$, $y \in [0,1]$ are the inverse of the left and the right membership function of the fuzzy number A, respectively. In [21], the authors define the ranking function by: $R_\alpha(A) = \alpha I_R(A) + (1-\alpha) I_L(A)$, where $\alpha \in [0,1]$, $I_L(A) = \int_0^1 g_{L,A}(y)dy = \frac{1}{2}(a+b)$, and $I_R(A) = \int_0^1 g_{R,A}(y)dy = \frac{1}{2}(b+c)$. Thus $R_\alpha(A) = \frac{\alpha}{2}(b+c) + \frac{1-\alpha}{2}(a+b) = \frac{1}{2}(\alpha c + b + (1-\alpha)a)$.

The non-negative number α represents the degree of optimism of a decision maker. A larger α indicates a higher degree of optimism. In our case, we consider the moderate decision and we set $\alpha = 0.5$; the ranking function becomes:

$$R_\alpha(A) = \frac{a + 2b + c}{4}. \tag{2}$$

Example 1. Let us consider the following optimization problem:

$$\begin{cases} \min\ (2,3,4)x + (2.5,1,1.5)y. \\ \quad \text{Subject to:} \\ \qquad (1,1.5,1.6)x + (0,0.5,1)y \geqslant (8.5,9.5,10.5), \\ \qquad (3,4,5)x + (5.8,6,6.2)y \geqslant (8.5,9,10.5), \\ \qquad x, y \geqslant 0. \end{cases}$$

Then the fuzzy model converted to the crisp model by using the ranking function is the following:

$$\begin{cases} \min\ 6x + 2y. \\ \quad \text{Subject to:} \\ \qquad 2.8x + y \geqslant 19, \\ \qquad 8x + 12y \geqslant 18.5, \\ \qquad x, y \geqslant 0. \end{cases}$$

To solve the diet problem using a heuristic method, first, we present the glycemic load of different foods using fuzzy triangular numbers; then, we use the integral fuzzy ranking function to transform the obtained problem into line optimization program.

3. Fuzzy quadratic optimization programming to optimal diet

The fuzzy quadratic optimization programming (P) that minimizes the total glycemic load, minimizes the favorable, and unfavorable nutrients gaps is given by the equation:

$$(P)\colon \begin{cases} \min\ g^T x + \mu(Ax-b)^T(Ax-b) + \beta(Ex-f)^T(Ex-f). \\ \quad \text{Subject to:} \\ \qquad c_{car}^T x \geqslant 0.55(C^t x), \\ \qquad c_p^T x \geqslant 0.18(C^t x), \\ \qquad c_{tf}^T x \leqslant 0.29(C^t x), \\ \qquad c_{sf}^T x \leqslant 0.078(C^t x), \\ \qquad 0 \leqslant x \leqslant 6u, x \in R^{NF}. \end{cases} \tag{3}$$

Where:
- NF is the number of foods,
- $x = (x_j)_{j=1:NF}$ is the vector of the foods serving sizes,
- $g = (gl, gn, gu)$ is the matrix formed by the triangular fuzzy numbers of the foods' glycemic load. Here, the triangular fuzzy number of the food i is given by the nominal glycemic load value gn_i, the minimal glycemic load value gl_i, and the maximal glycemic load value gu_i,
- A is the matrix of the favorable nutrients, b is the vector of the favorable nutrients requirements, E is the vector of unfavorable nutrients, and f is the maximum of positive nutrients that the diet must contain,

— C is the column of different foods calories, $ccar$ is the vector of calories from carbohydrates, cp is the vector of calories from potassium, ctf is the vector of calories from total fat, and csf is the vector of calories from saturated fat,
— μ and β are penalty parameters that make balance between the objective function components. u is the vector of ones from R^{NF}.

The constraints represent the Dietary Guidelines for Americans recommendations [24]: the maximum percentage of daily calories should be set for these nutrients with respect to the total daily calorie intake for a healthy and balanced daily diet.

If we transform the proposed model using robust programming [15, 16], we obtain the following mathematical model:

$$(PR)\colon \begin{cases} \min\ gn^T x + \sum_{j\in J_0} p_{0j} + \phi_0\omega_0 + \mu(Ax-b)^T(Ax-b) + \beta(Ex-f)^T(Ex-f). \\ \quad \text{Subject to:} \\ \quad\quad \omega_0 + p_0 \geqslant (gu_j - gn_j)y_j \quad \forall j \in J_0, \\ \quad\quad -y_j \leqslant x_j \leqslant y_j \quad \forall j, \\ \quad\quad \omega_0 \geqslant 0, \\ \quad\quad p_{0j} \geqslant 0 \quad \forall j \in J_0, \\ \quad\quad y_j \geqslant 0 \quad \forall j, \\ \quad\quad c_{car}^T x \geqslant 0.55(C^t x), \\ \quad\quad c_p^T x \geqslant 0.18(C^t x), \\ \quad\quad c_{tf}^T x \leqslant 0.29(C^t x), \\ \quad\quad c_{sf}^T x \leqslant 0.078(C^t x), \\ \quad\quad 0 \leqslant x \leqslant 6u,\ x \in R^{NF}. \end{cases} \tag{4}$$

Where J_0 is the set of index in $\{1,\ldots,176\}$, $g_j \in [gn_j, gn_j + d_j]$, and ϕ_0 is a constant from $[0, |J_0|]$. Robust programming remarkably increases the complexity of the studied problem:

(1) Two linear terms are added to the objective function which increases the number of local minima;
(2) The number of variables passed from 176 to $353 + |J_0|$;
(3) The number of constraints passed from 180 to $538 + |J_0|$.

It is possible to consider x as a fuzzy triangular number (xL, xM, xU) [18]. Then, we decompose the objective function f of the problem (P) into three objective functions fl, fm, and fu where the stochastic glycemic vectors are substituted by gl, gm, and gu, respectively; the constraints are also decomposed. In this case, four major problems arise:

(1) This transformation leads to a multi-objective problem which is very difficult to solve compared to a mono-objective problem;
(2) The number of the variables becomes $3NF$ instead of NF;
(3) The number of counterparts will be multiplied by 3;
(4) A solution of the form (xl, xm, xu) is hard to exploit by nutrient experts.

One can consider gn_j instead of g_j in the problem (P). But, we lose several important knowledge about the initial mathematical model. For instance, there are an infinite number of intervals that have gn_j as center. It is possible to replace g_j with gu_j in (P) [20]. But, the optimization method loses knowledge about the bounds because there are an infinite number of intervals with the maximum gu_j.

In this work, we use, for the first time in the literature, the integral constraint ranking function in the diet problem to transform the glycemic load to real numbers considering two steps: initially, we associate a membership function to each glycemic interval (gl_j, gn_j, gu_j), then we use the ranking function to transform the obtained triangular fuzzy numbers. In our case, we consider the moderate decision and we set $\alpha = 0.5$; the mathematical model (P) becomes:

$$(P'):\ \begin{cases} \min\ \frac{(gl+2gn+gu)}{4}^T x+\alpha(Ax-b)^T(Ax-b)+\beta(Ex-f)^T(Ex-f). \\ \quad \text{Subject to:} \\ \qquad c_{car}^T x \geqslant 0.55(C^t x), \\ \qquad c_p^T x \geqslant 0.18(C^t x) \text{ pieces of} \\ \qquad c_{tf}^T x \leqslant 0.29(C^t x), \\ \qquad c_{sf}^T x \leqslant 0.078(C^t x), \\ \qquad 0 \leqslant x \leqslant 6u,\ x \in R^{NF}. \end{cases} \tag{5}$$

This transformation results in a deterministic model, taking into account all the glycemic limits, without adding any variables or constraints with almost the same objective function. In the experimental section, we use HFGA to solve the quadratic diet mathematical model (P').

4. Hybrid firefly-genetic algorithm

In this part, we present an hybrid firefly genetic algorithm that combines some advantages of the firefly algorithm and the genetic algorithm [6]. To point out the principals of this version, we work on the kernel versions of FA and GA.

4.1. Firefly algorithm

The Firefly Algorithm (FA) is inspired by firefly behavior based on light intensity (I) and attractiveness (β) [25]. Essentially, FA employs 3 rules:

(a) Fireflies are single-gender and a firefly might be attracting another firefly whatever its gender.
(b) Attraction is directly correlated to brightness. If two fireflies are blinking, the darker one will move closer to the lighter one. If there is no firefly with more light, then a random firefly will change its place.
(c) The luminosity of a firefly is decided based on the cost function of the problem to be solved.

Given the current position of the ith x_i^t and jth x_j^t fireflies, the position of the ith firefly is updated by:

$$x_i^{t+1} = x_i^t + \delta_{ij}\left(x_j^t - x_i^t\right) + \alpha_t \varepsilon_i^t, \tag{6}$$

α_t is a global random series of parameters and ε_i^t is personalized local random series of parameters linked to the ith firefly. Algorithm 1 presents the different steps of the FA algorithm.

Algorithm 1 Firefly algorithm.

```
x = [x^1, x^2, ..., x^N] ;
2 : f(x) = [f(x^1), f(x^2), ..., f(x^N)] ;
I = f(x) is the light intensity.
γ is the absorption coefficient.
while (t < FES) do
  for i ← 1 : N (all N fireflies) do
    for j ← 1 : N (all i fireflies) do
      if I_i > I_j  then
        x_j ← x_j move firefly i towards j ;
      end if
      Attractiveness varies with distance r via exp(−βr),
      where β ← 0.
      Evaluate new solutions and update light intensity I
    end for j
  end for i
  Rank the fireflies and find the current best
  t = t + 1;
end while
```

4.2. Genetic algorithm (GA)

The genetic algorithm is a global search optimization process that imitates the mechanisms of natural evolution based on the reproduction and survival of the most successful individual [26]. In GA, individual solutions progress iteratively through genetic transactions like selection, crossing over and mutation. Solutions are scored using the fitness function. The new top solutions substitute for the former bad ones in the succeeding generations:

Initialization: the first generation is performed in a random way, allowing to cover the wide spectrum of all possible solutions. Occasionally, solutions may be "segregated" in regions in which it is likely that best solutions can be reached.

Selection: In each following generation, a subset of the surviving population is screened to breed a newer generation [27]: roulette Wheel selection, rank Selection, Steady State Selection, Tournament Selection, Elitism Selection, and Boltzmann Selection.

Crossover: is a genetic process that aims to merge the DNA data of two individuals in order to breed a new child.

Mutation: Heuristics.

Algorithm 2 gives the kernel version of the genetic algorithm.

Algorithm 2 Genetic algorithm.

Set of parameters
Choose encode method
Generate the initial population
while $i < MaxIter$ and $BestFitn < MaxFitn$ **do**
  Fitness calculation
  Selection
  Crossover
  Mutation
end while
Decode the individual with maximum fitness
return the best solution

Some proposed hybrid versions are not justified and are based on algebraic tests without any relation with the principal of FA and GA [6]. The proposed hybridization version recommends genetic and firefly operations by studding the well-known phenomena in the heuristic field.

If the difference in brightness between two fireflies is large enough, it is beneficial to move one firefly towards another and improvement is most likely;

If the difference between the brightness of two fireflies is average, we can assume that both solutions are good but that they do not come from the same regions and that crossing will improve the quality of both solutions;

If the difference between the brightness of two fireflies is very small, the solutions are very similar and there is no benefit in moving one of two fireflies to the other, here the mutation will move away from the current region if it is very bad.

In HFGA [6], we create N fireflies at random and sort them depending on their light intensity I. At every iteration, given two fireflies $indf_1$ and $indf_2$, we perform the crossover operator on both $indf_1$ and $indf_2$ if $I(indf_1) > I(indf_2)$ else the mutation operator is performed on $indf_1$ and $indf_2$. Two new solutions substitute the former ones in the iteration Algorithm and their light intensity is equal to the average light intensity of the used parents. Thus, HFGA offers various ways to select individuals for crossover and mutation in a genetic algorithm. The steps of HFGA are described in Algorithm 3.

Certainly, other versions of HGFA have been proposed in the literature [28, 29]. In our case, we use the version suggested in [6] because it is powerful and simple to implement. In this context, the authors of HGFA have shown, experimentally, the superiority of this hybridization over GA and FA on academic problems. In this work, we test and we compare FA, GA, and HFGA for different values of different parameters.

Algorithm 3 Hybrid genetic firefly algorithm.

Objective function $f(x), x = (x_1, x_2, \ldots, x_d)^T$
Initialize the firefly population x_i $(i = 1, 2, \ldots, N)$
Evaluate solutions, update light intensity I and sort of fireflies
γ is the absorption coefficient.
Tr_1 and Tr_2 decision threshold.
while Stopping criteria are not met
  Keep the old solutions
  for $j \leftarrow 2 : N$
    $j \leftarrow 1 : i - 1$
    if $|I_i - I_j| > Tr_1$ **then**
      if $I_i < I_j$ **then**
        Move firefly i towards j
      else
        Move firefly j towards i
      end if
    else
      if $Tr_2 < |I_i - I_j| < Tr_1$ **then**
        $y_{ij}, z_{ij} \leftarrow \text{crossover}(x_i, x_j)$
        $a, b \leftarrow \arg\max\{I_i, I_j; I(y_{ij}), I(z_{ij})\}$
        $x_i \leftarrow a; x_j \leftarrow b$
        $update(I_i), update(I_j)$
      else
        $y_i \leftarrow mutate(x_i)$
        $z_j \leftarrow mutate(x_j)$
        $a, b \leftarrow \arg\max\{I_i, I_j; I(y_i), I(z_j)\}$
        $x_i \leftarrow a; x_j \leftarrow b$
        $update(I_i), update(I_j)$
      end if
    end if
  end for
end while
Postprocessing results and visualization.

5. Numerical experiments

We used HFGA to solve the diet problem based on our mathematical model (P') where $\mu = 0.3$ and $\beta = 0.5$. According to this choice, we inform the HGFA algorithm that the total glycemic load weights the most for us when we prepare meals for diabetic patients. Moreover, the number of favorable nutrients is 15 while the number of unfavorable nutrients is 4, thus $\|Ax - b\| \gg \|Ex - f\|$. For this, HGAFA is informed, through $\mu < \beta$, to make balance between these two terms.

The favorable and unfavorable nutrients daily requirements [30, 31] are given by: Calories (2000 kcal), Protein (91 g), Carbohydrate (271 g), Potassium (4044 mg), Magnesium (380 mg), Calcium (1316 mg), Iron (18 mg), Phosphorus (1740 mg), Zinc (14 mg), Vb6 (2.4 mg), Vb12 (8.3 μg), VC (155 mg), VA (1052 μg), VE (9.5 mg), Saturated fat (17 g), Sodium (1779 mg), Total fat (65 g), and Cholesterol (230 mg). The optimal diets are estimated based on 176 foods available in the Moroccan market.

We tested the HFGA for different population sizes ($40 \leqslant popSize \leqslant 100$) and for different attraction coefficient token from the interval ($1 \leqslant \beta_0 \leqslant 2.5$). Concerning the other parameters, we adopt the following configurations:

Genetic algorithm: initialization (random), crossover operator (multiple), crossover rate (0.8), number of iteration (5000), population size (300), mutation (gaussian), and selection function (stochastic-uniform).

Firefly algorithm: maximum iterations (500), number of fireflies (40), light absorption coefficient (1), mutation coefficient (0.2), mutation coefficient damping ratio (0.98), and mutation range (uniform: $0.05 * (VarMax - VarMin)$).

Hybrid genetic firefly algorithm: light absorption coefficient (1), mutation coefficient (0.2), mutation coefficient damping ratio (0.98), number of iteration (100), and mutation rate (0.03).

Concerning the transformation of the fuzzy vector g of different goods to deterministic vector, we consider the means nominal values, the worst case, and the integral fuzzy ranking transformation. Table 1 gives the total glycemic load, total favorable nutrients gaps, and total unfavorable nutrients excess of the diets produced by FA algorithm for fuzzy ranking transformation, mean nominal values, and worst case for different values of FA attraction coefficient β_0; the number of iterations is fixed to 500. The diets associated to β_0 less or equal to 1.5 are rejected because their total glycemic load (almost 562.08–2.9e+3), the total favorable nutrients gap (almost 3.3e+4–1.2e+5 mg), and total unfavorable nutrients excess are very large (almost 2.4e+3–3.6e+4 mg). Diets corresponding to a β_0 between 1.75 and 2.5 are acceptable and can be consumed by people with diabetes.

Table 1. Total glycemic load, total favorable nutrients gap, and Total unfavorable nutrients excess of daily diets produced by FA for different attraction coefficient base Values (β_0 between 1 and 2.5) for 500 iterations. Three transformations are considered: fuzzy ranking, the total favorable nutrients gap, and total unfavorable nutrients excess.

FA Attr. Coeff.	Tot. Glyc. Load			Tot. Fav. Nutr. Gap(mg)			Tot. Unfav. Nutr. Exc. (mg)		
	Fuz.	Mean.	Wor.	Fuz.	Mean.	Wor.	Fuz.	Mean.	Wor.
1	2.7e+3	2.9e+3	3.0e+3	1.2e+5	1.2e+5	1.1e+5	3.9e+4	3.6e+4	3.0e+4
1.25	2.2e+3	2.0e+3	2.2e+3	9.5e+4	9.7e+4	9.4e+4	2.0e+4	2.8e+04	2.6e+4
1.5	612.40	818.37	562.08	3.5e+4	3.5e+4	3.3e+4	2.4e+3	2.9e+03	2.7e+3
1.75	23.97	13.92	11.65	19.33	18.68	16.46	41.42	53.91	65.80
2	12.09	8.82	13.32	13.15	65.05	7.25	50.17	18.85	51.43
2.25	18.44	11.42	12.19	3.28	4.48	7.87	42.04	56.84	71.66
2.5	11.34	13.88	8.62	10.27	15.14	23.37	60.47	68.25	53.92

Table 2 gives total glycemic load, total favorable nutrients gap, and total unfavorable nutrients excess of the diets produced by GA algorithm for fuzzy ranking transformation, mean nominal values, and worst case for different population size of GA (between 200 and 800); the number of generations is fixed to 5000. All the produced diets by GA are rejected because their total glycemic load (almost 61–4e+3), the total favorable nutrients gap (almost 9.8–10.0e+4 mg), and total unfavorable nutrients excess are massive (almost 1.6e+4–4e+4 mg a part from the first diet). These diets are dangerous even for healthy people.

Table 2. Total glycemic load, total favorable nutrients gaps, and total unfavorable nutrients excess of the diets produced by GA algorithm for fuzzy ranking transformation, mean nominal values, and worst case for a different population size of GA; the number of generations is fixed to 5000.

GA Pop. size	Tot. Glyc. Load			Tot. Fav. Nutr. Gap(mg)			Tot. Unfav. Nutr. Exc. (mg)		
	Fuz.	Mean.	Wor.	Fuz.	Mean.	wor.	Fuz.	Mean.	Wor.
200	61.36	85.33	87.74	9.0e+3	1.1e+4	1.2e+4	0	0	0
300	2.6e+3	1.7e+3	1.9e+3	8.6e+4	8.6e+4	9.0e+4	2.8e+4	2.0e+4	1.6e+4
400	3.1e+3	2.1e+3	2.3e+3	9.8e+04	8.1e+4	9.4e+4	2.1e+4	1.5e+4	2.1e+4
500	3.0e+3	3.0e+3	2.4e+3	10.0e+4	10.0e+4	9.3e+4	3.4e+4	1.9 e+4	3.1e+4
600	2.6e+3	3.1e+3	2.8e+3	8.9e+4	1.2e+5	9.3e+04	2.2e+4	2.5e+4	2.0e+4
700	2.5e+3	2.5e+3	2.3e+3	1.1e+5	9.3e+4	9.0e+4	2.3e+4	1.8e+4	2.9e+4
800	2.3e+3	3.0e+3	2.7e+3	1.1e+5	1.1e+5	1.1e+05	3.2e+4	2.2e+4	3.4e+4

Tables 3 give the total glycemic load, total favorable nutrients gap, and total unfavorable nutrients excess of the diets produced by HFGA for fuzzy ranking transformation, mean nominal values, and worst case for different population sizes (between 40 and 100). The attraction coefficient is randomly chosen. The number of generations is fixed at 100. All the produced diets by HFGA are acceptable because their low total glycemic load (between 16 and 39), their low total favorable nutrients gap (between 5 mg and 38 mg), and their acceptable total unfavorable nutrients excess (between 1 mg and 26 mg). All these diets can be consumed by diabetic patients without any risk. Compared to fairly algorithm, HFGA requires very small number of iterations to produce good diets.

Table 3. Total glycemic load, total favorable nutrients gap, and total unfavorable nutrients excess of the regimes produced by HFGA for fuzzy ranking transformation, mean nominal values, and worst case for different population sizes (between 40 and 100).

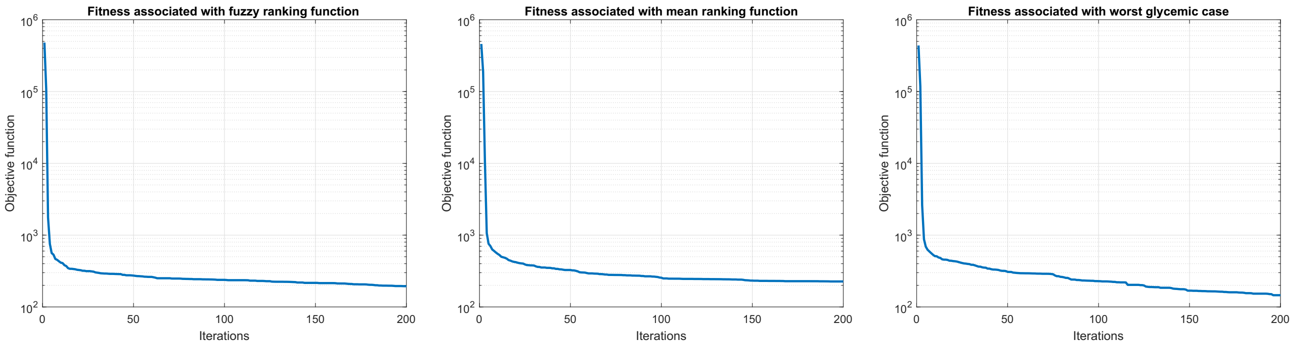
HFGA Pop. size	Tot. Glyc. Load			Tot. Fav. Nutr. Gap(mg)			Tot. Unfav. Nutr. Exc. (mg)		
	Fuz.	Mean.	Wor.	Fuz.	Mean.	Wor.	Fuz.	Mean.	Wor.
40	38.09	27.23	21.93	21.62	16.95	7.68	5.34	13.8843	25.62
50	21.07	23.56	30.12	20.23	12.40	10.43	8.42	18.89	21.01
60	19.34	24.37	25.23	13.97	28.65	12.50	10.36	1.30	17.11
70	18.54	21.25	27.60	1.22	10.25	16.84	16.09	7.62	22.49
80	28.67	16.26	20.81	6.19	17.77	5.72	9.03	12.96	18.52
90	20.21	26.50	37.47	10.17	32.31	8.18	15.56	2.57	5.07
100	18.72	29.71	21.90	10.79	6.00	19.70	2.50	2.86	2.96

The best diets produced by the system [(P)-(fuzzy ranking ‖ mean nominal ‖ worst case)-HFGA] correspond to the population of size 80. To improve the performance of the system [(P)-(fuzzy ranking ‖ mean nominal ‖ worst case)-HFGA] for this size, we solve several instances for different values of the attraction coefficient β_0. Table 4 gives total glycemic load, total favorable nutrients gap, and total unfavorable nutrients excess of the diets produced by HFGA algorithm for fuzzy ranking transformation, mean nominal values, and worst case for different values of the attraction coefficient β_0 for population of size 80; the number of iterations is only to 100. Considering three performance criterions, the worst diets are the ones corresponding to attracted coefficient of value 1: Tot. Glyc. Load (14.2–40.3), Tot. Fav. Nutr. Gap (28–40.3mg), and Tot. Unfav. Nutr. Exc. (19.48–77.84); will the best ones are the diets produced by the system [(P)-(fuzzy ranking ‖ mean nominal ‖ worst case)-HFGA] for $\beta_0 = 1.75$: Tot. Glyc. Load (7–26), Tot. Fav. Nutr. Gap (2–18 mg), and Tot. Unfav. Nutr. Exc. (9–19 mg). But all the diets produced by HFGA are good and can be used to save the diabetic people with complications and can avoid that the diabetic people to become pre-diabetic and can help them to be normal people [17].

Table 4. Total glycemic load, total favorable nutrients gap, and total unfavorable nutrients excess of the regimes produced by HFGA algorithm for fuzzy ranking transformation, mean nominal values, and worst case for different values of the attraction coefficient β_0 for population of size 80.

HFGA Attr. Coeff.	Tot. Glyc. Load			Tot. Fav. Nutr. Gap(mg)			Tot. Unfav. Nutr. Exc. (mg)		
	Fuz.	Mean.	Wor.	Fuz.	Mean.	Wor.	Fuz.	Mean.	Wor.
1	34.83	14.20	16.92	40.30	28.62	28.49	19.48	35.40	77.84
1.25	18.99	21.46	23.77	9.06	15.22	5.35	8.92	19.16	17.37
1.5	24.95	23.79	33.46	10.38	13.70	8.63	16.49	1.79	7.20
1.75	28.67	16.26	20.81	6.19	17.77	5.72	9.03	12.96	18.52
2	21.51	40.18	21.08	16.33	9.15	11.57	17.12	12.33	5.31
2.25	21.64	21.07	25.11	14.19	15.39	8.26	7.59	9.03	15.69
2.5	28.14	21.65	31.16	25.48	2.52	9.74	9.83	3.44	30.22

Figure 1 gives fitness evolution with iterations, of HFGA with $\beta_0 = 1.75$ and 80 as population size for fuzzy ranking function, means nominal value, and worst case value. HFGA needs a small number of iterations to produce acceptable diets (considering the three performance criterions). Unfortunately, Figure 1 points out a well-known phenomenon in the optimization field: HFGA stops very early in valleys that represent local minima. To overcome this problem, one can increase the mutation probability to escape from bad local minima, but the search risks becoming arbitrary.



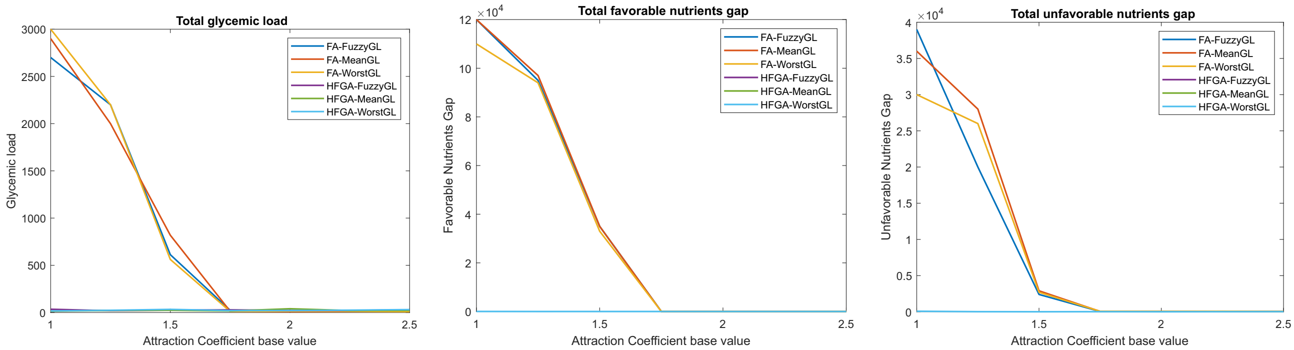
a Fitness function vs iterations of HFGA for fuzzy ranking function.

b Fitness function vs iterations of HFGA for mean nominal value.

c Fitness function vs iterations of HFGA for worst nominal value.

Fig. 1. Fitness evolution, with iterations, of HFGA with $\beta_0 = 1.75$ and 80 as population size for fuzzy ranking function, means nominal value, and worst case value.

Let us come back to the system [(P)-(fuzzy ranking $\|$ mean nominal $\|$ worst case)-FA]. Figure 2 shows that the total glycemic load (a) and the total favorable nutrients gap (b) of the diets produced by [(P)-(fuzzy ranking $\|$ mean nominal $\|$ worst case)-HFGA] are inferior than the ones produced by [(P)-(fuzzy ranking $\|$ mean nominal $\|$ worst case)-FA] for β_0 inferior than 1.75, and they become almost the same for β_0 superior than 1.75. But, the total unfavorable nutrients excess (c) of the diet produced by [(P)-(fuzzy ranking $\|$ mean nominal $\|$ worst case)-HFGA] are always smaller than the ones produced by [(P)-(fuzzy ranking $\|$ mean nominal $\|$ worst case)-FA-HFGA]. That is why the authors recommend to use [(P)-(fuzzy ranking $\|$ mean nominal $\|$ worst case)-HFGA] to solve the diet problem.



a Total glycemic load.

b Total favorable nutrients gap.

c Total unfavorable nutrients excess.

Fig. 2. Total glycemic load (a), Total favorable nutrients gap (b), and Total unfavorable nutrients excess (c) associated with daily diets produced by HFGA and FA for different values of β_0 and for different nominal values.

To compare different glycemic representations (fuzzy ranking, means nominal glycemic, and worst glycemic), we generate 50 instances of the problem (P) whose the glycemic values of different foods are generated via uniform densities from the intervals $[gl, gu]$. Then, we solve these instance using HFGA(population size =80, $\beta_0 = 1.75$), the glycimic load of the obtained diets form a vector of size 50 that we denote TGL. Then, we construct three vectors:

(a) DTGLF=TGL-TGLF, where TGLF is the total glycemic load of the diet obtained by the system [(P)-Fuzzy ranking-HFGA(population size =80, $\beta_0 = 1.75$)];
(b) DTGLN=TGL-TGLN, where TGLN is the total glycemic load of the diet obtained by the system [(P)-means glycemic-HFGA(population size =80, $\beta_0 = 1.75$)];
(c) DTGLW=TGL-TGLW; where TGLW is the total glycemic load of the diet obtained by the system [(P)-worst glycemic-HFGA(population size =80, $\beta_0 = 1.75$)].

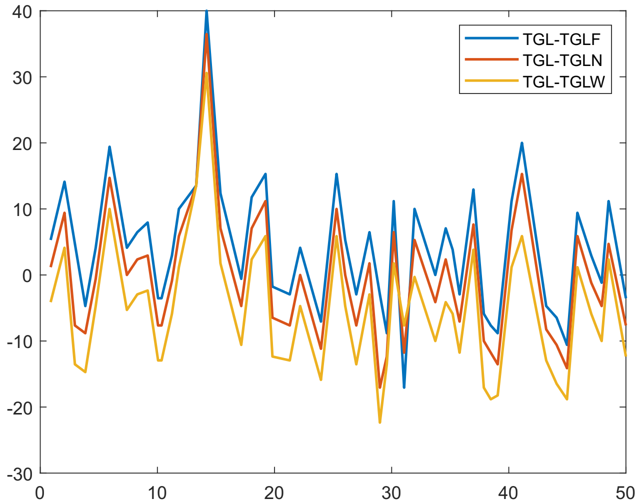


Fig. 3. Total glycemic load gap of the diet obtained by solving (P) using HFGA for Fuzzy ranking, means, and worst glycemic.

Figure 3 represents the series TGL-TGLF, TGL-TGLN, and TGL-TGLW of the 50 instances. Contrary to what was expected, it is the representation fuzzy ranking that is in the middle and not the mean glycemic representation. This means that the fuzzy ranking representation suggests a good compromise. For more precision, we calculate the mean and standard deviation of different series TGL-TGLF(mean$=-0.4175$), TGL-TGLN (mean$=3.8574$), and TGL-TGLW (mean$=-5.6001$). Exactly as expected, compared to the other series, the average of TGL-TGLF is very close to 0. This means that the total glycemic load of the instance obtained by fuzzy ranking transformation is closer to the total glycemic load of each of the 50 instances. We conclude that fuzzy ranking transformation produces a representative nominal value that summary all possible values in the intervals $[gl, gu]$. In addition, the three series have almost the same standard deviation (10.1345) which proves the consistency of the proposed model with the studied phenomena.

6. Conclusion

We solved the Moroccan daily diet problem based on original fuzzy quadratic optimization programming (P) taking into account dietary guidelines of US department of health, human services, and department of agriculture. The objective function makes a compromise between the fuzzy glycemic load, the favorable nutria Figure and unfavorable nutrients excess. To transform the proposed program into line equation, we used the integral fuzzy ranking function (R) that controls the critical values of the fuzzy glycemic load membership functions. Several experimentations were realized using a hybrid firefly genetic algorithm to estimate the optimal daily diets based on 176 foods available on the Moroccan market.

Compared to the systems [(P)-(mean glycemic load)-HFGA] and [(P)-(worst glycemic load)-HFGA], the system [(P)- (R glycemic load)-HFGA] produced the best and generic diets with reasonable glycemic loads and acceptable core nutrient deficiencies. In addition, the proposed model showed remarkable consistency with the uniform distribution of the glycemic load of different foods.

Finally, even though [(P)- (R glycemic load)-HFGA] offered the best diets, we are not going to reject all the diets offered by [(P)-(mean glycemic load)-HFGA] and [(P)-(worst glycemic load)-HFGA] because not all of them are bad. This will provide patients with a variety of diets that we can use to build long-term diets. Indeed, these diets do not have the same advantages or disadvantages, the effective strategy is to alternate these diets over the course of the diet.

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Гібридний генетичний алгоритм світлячка та інтегральне нечітке квадратичне програмування для оптимальної марокканської дієти

Ель Мутауакіл К.[1], Ахураг А.[1], Чакір С.[2], Каббадж З.[3], Челлак С.[2], Чеггур М.[2], Байзрі Х.[4]

[1]*Лабораторія інженерних наук, FPT Тази, USMBA Феса, Марокко*
[2]*Лабораторія Morphoscience, FMP, CAU Марракеша, Марокко*
[3]*Лабораторія медицинських наук, FMP, CAU Марракеша, Марокко*
[4]*Лабораторія біологічних наук і медицинських досліджень,*
Ендокринологічна служба діабета і метаболічних захворювань,
Військовий госпіталь Авіцени, FMP, UCA, Марракеш, Марокко

У цій статті розв'язується марокканська проблема денного раціону на основі 6 оптимізаційних програм (P) з урахуванням дієтичних рекомендацій Міністерства охорони здоров'я, соціальних служб і Міністерства сільського господарства США. Цільова функція контролює нечітке глікемічне навантаження, сприятливий дефіцит поживних речовин і несприятливий надлишок поживних речовин. Для перетворення запропонованої програми в лінійне рівняння використовується інтегральна функція нечіткого ранжування. Для вирішення отриманої моделі використовуємо гібридний генетичний алгоритм світлячка (HFGA), який поєднує деякі переваги алгоритму світлячка (FA) і генетичного алгоритму (GA). Запропонована модель створює найкращі та загальні дієти з прийнятним глікемічним навантаженням і прийнятним дефіцитом основних поживних речовин. Крім того, запропонована модель показала дивовижну узгодженість з рівномірним розподілом глікемічного навантаження різних харчових продуктів.

Ключові слова: *оптимальна марокканська дієта; нечітке квадратичне програмування; трикутні нечіткі числа; інтегральне ранжування більшості функцій; генетичний алгоритм; алгоритм світлячка.*