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European option pricing under model involving slow growth volatility with jump

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In this paper, we suggest a new model for establishing a numerical study related to a European options pricing problem where assets' prices can be described by a stochastic equation with a discontinuous sample path (Slow Growth Volatility with Jump SGVJ model) which uses a non-standard volatility. A special attention is given to characteristics of the proposed model represented by its non-standard volatility defined by the parameters α and β. The mathematical modeling in the presence of jump shows that one has to resort to a degenerate partial integro-differential equation (PIDE) which the resolution of this one gives a price of the European option as a function of time, price of the underlying asset and the instantaneous volatility. However, in general, an exact or closed solution to this problem is not available. For this reason we approximate it using a finite difference method. At the end of the paper, we present some numerical and comparison results with some classical models known in the literature.

Keywords: *finite difference method; jump-diffusion processes; option pricing; partial integro-differential equation; volatility smile.*

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1. Introduction

The history of the option pricing theory dates back to 1900 when the French mathematician Louis Bachelier deduced the formula for evaluating options by assuming that the stock prices followed a zero drift Brownian motion. Since then, many studies have contributed to this theory. Indeed, in the early 1970s, Fisher Black and Myron Scholes [1] and Robert Merton [2] recorded a major advance in the valuation of options by constructing the famous model so-called Black–Scholes (BS) that gives in closed form the price of a European option derivative of security dependent on a non-dividend-paying stock. However, this model based on strong assumptions such as constant volatility and normal distribution of returns (Gaussian law) is not consistent with a number of stylized facts, such as the asymmetric leptokurtic features and the volatility smile, as suggested by some empirical studies (see, for example, [3,4]). To explain this empirical phenomena, a variety of models have been proposed in the literature such as fractional Brownian motion, diffusions model with jumps and Stochastic volatility models.

To incorporate the asymmetric leptokurtic features in asset pricing, Mandelbrot, Fisher and Clavet [5] represent the price's dynamics by a Brownian movement with a multifractal time change. Heyde [6] explains these features based on the fractal activity time. Barndorff–Nielsen and Shephard [7] show that Ornstein Uhlenbeck models offer the possibility of capturing important distributional deviations from Gaussianity and for flexible modeling of dependence structures. Based on fractional stochastic volatility (FSV) of Comte and Renault [8], Gatheral et el. [9] develop a model that is able to replicate the stylized facts of the time series and conclude that the log-volatility behaves as a fractional brownian. Ibrahim et al. [10] present recently a closed-form pricing formula for call warrants under mixed-fractional Brownian motion with jump-diffusion (MFBM-MJD). Their research indicates that this model effectively captures both the long-memory phenomenon and the discontinuous behavior observed in logarithmic returns. Yanishevskyi et al. [11] derive a solution for the Fokker–Plank equation, which describes the transition probability density of fractional Brownian motion, using a path integral approach [12].

The jump-diffusion models proposed by Merton [13] and Kou [14], allow for more realistic representation of price dynamics and a greater flexibility in modeling [15]. These models have finite jump activity, unlike the more general approach with possibly infinite jump activity proposed by Carr et al. [16]. Indeed, the presence of sudden changes in the price of the underlying asset has led Merton [13] to study jumping model. The stochastic volatility models: proposed to incorporate the "volatility smile" in option pricing like (a) stochastic volatility and Autoregressive Conditionally Heteroscedastic ARCH models performed, for example, by Hull and White [17] and Gouriéroux [18]. The Heston model [19], takes into account the leverage effect and volatility clusters, which allows the volatility itself to be random and also allows it to take the non-normally distributed stock return into account. Yang et al. [20], propose a hybrid model that can capture the stochastic property of volatility rate, interest rate as well as the short term and long-term effects on the financial market. Sawal et al. [21] develop a more general model that includes jumps, stochastic volatility, and stochastic interest rate. (b) The Constant Elasticity of Variance (CEV) model proposed by Cox and Ross [22] is an extension of the stochastic volatility diffusion model that can estimate the change in asset prices in continuous time. A. Abaoud [23], derives an analytic approximation formula for European call options by combining the (CEV) process for the asset price and stochastic volatility.

It is important to take into account that with the CEV model, the variance of the asset rate of return depends on the price of the underlying asset, and time as well. In other words, the underlying asset price S_t is assumed to follow a CEV model with a constant μ in drift, and its two characterized parameters: the local volatility σ and the skew parameter α:

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma(S_t)\, dW_t, \tag{1}$$

where $\sigma(S_t) = \sigma S_t^{\alpha-1}$ and W_t is Brownian motion. The model with $0 < \alpha < 1$ is called restricted CEV. The model with $\alpha = 1$ is the standard BS lognormal process and models with $\alpha < 0$ are called unrestricted CEV. They fit the smile rather well.

This paper proposes a new model called Slow Growth Volatility with Jump (SGVJ) for European option pricing. This model was initially presented by Benjaoud and al. [24]. It is of BS type and generalizes the CEV model. The SGVJ model is quite interesting because on the one hand it incorporates a specific volatility given by

$$\sigma_t(S_t) = \sigma S_t^{\alpha-1} \log^{\beta}(1 + S_t),$$

which depends on the asset price and slowly increases with respect to this latter, where α and β are two parameters of the model to be specified. On the other hand, the jump term generally introduced to take into account certain rare phenomena observed in the financial markets. This model is a variant of the CEV model, in the case where $\beta = 0$, and extends to cover the Slow Growth Volatility SGV model developed by Benjaoud and al. [24]. It adds a jump term to the SGV model and removes the restrictive cases from the other models. The main difficulty of our model is that there is no closed form solution for European options price in our setting, due mainly to the integral term and a specific volatility. Indeed, the mathematical modeling in the presence of jump shows that one has to resort to a degenerate partial integro-differential equation (PIDE) where resolution gives a price of the European option. For this reason, we propose a numerical method that approximates the solution of the PIDE using a finite difference scheme.

The remainder of this article is organized into four sections. In Section 2, we begin by describing the SGVJ model and the resulting PIDE for European options. We also provide a numerical resolution using finite difference method by separating the integro-differential equation into two terms, differential term and the integral term. Numerical experiments and conclusions are presented in Sections 3 and 4, respectively.

2. The slow growth volatility with jump model

The model that we propose for the price of an underlying asset (for example an interest rate) combines the SGV model and the jump models under the hypotheses of Merton [13]. It consists of two parts, a continuous part modeled by a geometric Brownian motion, and a jump part, with the logarithm of the jump sizes having normal distribution and the jump times corresponding to the event times of a Poisson process. More precisely, the below stochastic differential equation (1) is used to model the asset price $S(t)$, which follows a finite activity jump-diffusion process (see for more detail Cont and Tankov [4]).

Let $(S_\tau)_{\tau\in[0,T]}$ be the price of a financial asset modeled as a stochastic process on a filtered probability space $(\Omega,\mathcal{F}_t,P)$; $\mathcal{F}_t$ represents the history of the asset S at time t and $\tilde{S}=e^{-r\tau}S$ the discounted value of the asset, with $r>0$ being the risk-free interest rate. We consider the simplified financial market that can be represented in a probabilistic space $(\Omega,\mathcal{F}_t,P)$ by the evolution of two titles namely risk-free asset and a risky asset. Their price, denoted by $S_0(t)$ and S_t respectively are represented as follows:

$$dS_0(t)=rS_0dt,$$

$$\frac{dS_t}{S_t}=\mu\,dt+\sigma S_t^{\alpha-1}\ln^{\beta}(1+S_t)\,dW_t+(\eta_t-1)\,dq_t, \tag{2}$$

where W_t is a Brownian motion, q_t is a Poisson process and where (α,β) are the parameters of the model which can be defined as follows: α is the sensitivity of volatility to changes in the price of the underlying asset, β reflects the sensitivity of volatility due to security and stability situation in a country.

The model with the term $(\eta_t-1=0)$ represents the SGV model as described by [24]. It has been reduced to the CEV model if $\beta=0$ and $\eta_t=1$. It goes back to the model Standard Black and Sholes when $\alpha=1$, $\beta=0$ and $\eta_t=1$ and finally the classic Merton pattern is obtained by taking: $\alpha=1$, $\beta=0$.

This jump model where volatility is a power function of the price of the underlying conjugated with its logarithm: either $\sigma(t,S)=\sigma S_t^{\alpha-1}\ln^{\beta}(1+S_t)$ can be designed for the valuation of a European option that has the interest rate as its underlying asset; we denote: $dq_t=0$ with probability $1-\lambda\,dt$; $dq_t=1$ with probability $\lambda\,dt$ where λ is the intensity of the Poisson process dq_t; (η_t-1) is an impulse function producing in jumps from S to $S\eta$; $\kappa=E(\eta_t-1)$ is the expected relative jump size; σ is the volatility; μ is the drift rate.

2.1. Partial integro-differential equation

From a mathematical point of view, a European contingent claim with maturity T is an arbitrary F_t-measurable random variable H. The interpretation of this definition shall be that the contingent claim is a contract which specifies that the stochastic amount H of money has to be paid out to the holder of the contract at time T. The Markov property of the price allows us to express prices of European options in terms of solutions of partial integro-differential equations (PIDE) that involve, in addition to the second-order differential operator, a nonlocal integral term.

As Merton showed in [13], W_t and q_t are independent and by using Itô's Lemma for the continuous part and an analogous lemma for the jump part we concluded that the value of the derived product $\upsilon(S,\tau)$ satisfies the following PIDE:

$$(\mathcal{P})\quad\begin{cases}\dfrac{\partial}{\partial\tau}\upsilon(S,\tau)-\dfrac{\sigma^2S^{2\alpha}\log^{2\beta}(1+S_t)}{2}\dfrac{\partial^2}{\partial S^2}\upsilon(S,\tau)-\left(r-\dfrac{\sigma^2}{2}-\lambda\overline{k}\right)S\dfrac{\partial}{\partial S}\upsilon(S,\tau)+r\upsilon(S,\tau)\\-\lambda\left(\displaystyle\int_0^{+\infty}\upsilon(S\eta,\tau)\tilde{\Gamma}_\delta(\eta)\,d\eta-\upsilon(S,\tau)\right)=0,\qquad\tau\in]0,T],\quad S>0,\\\upsilon(S,0)=\upsilon_0(S),\quad S>0\end{cases} \tag{3}$$

where T is the maturity, r is the risk-free interest rate, $\overline{k}=E(\eta_t-1)$ is the expected relative jump size and η is supposed to be a log-normally distributed jump amplitude with probability density

$$\tilde{\Gamma}_\delta(\eta) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\log(\eta)}{\delta}\right)^2\right)}{\sqrt{2\pi}\delta\eta}. \tag{4}$$

We recall that α and β are two parameters expressing the sensitivity of volatility respectively due to changes in the price of the underlying asset S and the risk associated with a country stability event. Also, we stress that the function $v(S,\tau)$ which translates the price of European option is a solution of the problem $(\mathcal{P})$.

In the case of a Call option, the boundaries conditions are: $v(S,\tau) = S - k\exp(-r\tau)$ when $S \to +\infty$; $v(S,\tau) = 0$ when $S \to 0$; $v_0 = \max(S-K,0)$.

The problem $(\mathcal{P})$ can be rewritten by changing the following variables: using the logarithmic price $x = \log S \in \mathbb{R}$, the time to maturity $t = T - \tau$ and $\eta = e^y$. Then, we call $v(S,\tau) = u(x,t)$ and therefore we can study the numerical approximation of

$$(\mathcal{P}_{\alpha\beta}) \quad \begin{cases} \dfrac{\partial u_{\alpha\beta}}{\partial t} = \dfrac{\sigma^2 e^{2(\alpha-1)x}\log^{2\beta}(1+e^x)}{2}\dfrac{\partial^2}{\partial x^2}u_{\alpha\beta}(x,t) \\ +\left(r - \dfrac{\sigma^2 e^{2(\alpha-1)x}\log^{2\beta}(1+e^x)}{2} - \lambda\overline{k}\right)\dfrac{\partial}{\partial x}u_{\alpha\beta}(x,t) - ru_{\alpha\beta}(x,t) \\ +\lambda\displaystyle\int_{-\infty}^{+\infty}\tilde{u}(x+y;t)\Gamma_\delta(y)\,dy, \qquad x\in\mathbb{R}, \quad t\in]0,T], \\ u_{\alpha\beta}(x,0) = \psi(x), \qquad x\in\mathbb{R}, \end{cases} \tag{5}$$

where Γ_δ is the Gaussian probability density:

$$\Gamma_\delta(y) = \frac{1}{\sqrt{2\pi}\delta}\exp\left(-\frac{y^2}{2\delta^2}\right), \quad \forall\delta > 0.$$

From (5) we denote $\Gamma_\delta(y) = \tilde{\Gamma}_\delta(e^y)$, $\tilde{u}(x+y,t) = u(e^{x+y},t)$ and the initial data $\psi(x)$ is the payoff function of the European options. Now, let the exercise price K be given, for the Call and the Put option we have respectively: $\psi(x) = \max(e^x - K, 0)$ and $\psi(x) = \max(K - e^x, 0)$.

2.2. Numerical solution

To apply a numerical scheme, we must truncate the space domain by considering $\Omega \subset \mathbb{R}$, the interval, where we want to compute the numerical solution, on one hand, and the integral domain on the other. To compute the non-local term, we simplify the integral by considering a finite interval instead of the whole real line. The particularity of the shape of the density function Γ_δ allows us to choose only the points for which the density has a significant value and with acceptable error (the error is not big).

We truncate the integral domain of our problem. We choose a parameter $\varepsilon > 0$ and select the finite interval $[-Y_\varepsilon, Y_\varepsilon]$ as the set of all the points y that verify:

$$\Gamma_\delta(y) \geqslant \varepsilon \Leftrightarrow \frac{1}{\sqrt{2\pi}\delta}\exp\left(-\frac{y^2}{2\delta^2}\right) \geqslant \varepsilon. \tag{6}$$

Therefore we can easily extract $-Y_\varepsilon$ and Y_ε as is shown below:

$$\sqrt{-2\delta^2\log\left(\varepsilon\delta\sqrt{2\pi}\right)} \leqslant y \leqslant \sqrt{-2\delta^2\log\left(\varepsilon\delta\sqrt{2\pi}\right)}.$$

As Γ_δ is a symmetric function with respect to its axis that can be defined as:

$$Y_\varepsilon = \sqrt{-2\delta^2\log\left(\varepsilon\delta\sqrt{2\pi}\right)}; \quad Y_\varepsilon = -Y_\varepsilon.$$

We introduce a uniform mesh on $[0,T]\times\Omega$ and we start by dividing $\Omega = [X_{\min}, X_{\max}]$ into $\bar{z}$ equal intervals of length $h = \triangle S$, and let considering the integer part of $p = [Y_\varepsilon/h]$ then we divide the time domain into N equally spaced nodes, separated by a distance $K = \triangle t$. We solve the problem $\mathcal{P}_{\alpha\beta}$ in large numerical domain $\bar{\Omega} = [X_{\min} - ph, X_{\max} + ph]$. We call $M = \bar{z} + 1 + 2p$ the total number of grid points $\{x_m, m = 0,\ldots,M-1\}$, where $x_0 = X_{\min} - ph$, $M - 1 = X_{\max} + ph$ and $P = 2p + 1$ the total number of points used for integral approximation. We shall define by z_- and z_+ the two grid indices as: $z_- = X_{\min}$ and $z_+ = X_{\max}$ and by p the integer number $p = [Y_\varepsilon/h]$. Let us set $u_m^n = u_{\alpha\beta}(x_m, nk)$

and denote by $I_h(u_m^n)$ the integral approximation and represent it as:

$$I_h(u_m^n) = h \sum_{l=-p}^{p} \alpha_l (\Gamma)_l (u_{m+l}^n),$$

where α_l indicates the weights of a quadrature rule. After defining our numerical problem on Ω, we need a limiting form for the solution u on the external set $\Omega^C = R \setminus \Omega$.

As shown by Briani [25], the integral term can be replaced (locally) by a diffusive term. We then approximate on the external set Ω^C the problem $(\mathcal{P}_{\alpha\beta})$ by a diffusive one

$$\frac{\partial u_{\alpha\beta}}{\partial t} - \frac{a_{\alpha\beta}}{2}\frac{\partial^2 u_{\alpha\beta}}{\partial x^2} - \left(r - \frac{a_{\alpha\beta}}{2} - \lambda\overline{k}\right)\frac{\partial u_{\alpha\beta}}{\partial x} + r u_{\alpha\beta} = \frac{\lambda\delta^2}{2}\frac{\partial^2 u_{\alpha\beta}}{\partial x^2}, \tag{7}$$

where

$$a_{\alpha\beta} = \sigma^2 e^{2(\alpha-1)x} \log^{2\beta}(1+e^x).$$

Then the approximated problem can be written as

$$\begin{cases} \dfrac{u_m^{n+1}-u_m^n}{k} - a_{\alpha\beta}(x_m)\dfrac{u_{m+1}^{n+1}-2u_m^{n+1}+u_{m-1}^{n+1}}{2h^2} - \left(r - \dfrac{a_{\alpha\beta}}{2} - \lambda\overline{k}\right)\dfrac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2h} \\ \qquad + r u_m^{n+1} = \dfrac{\lambda\delta^2}{2}\dfrac{u_{m+1}^{n+1}-2u_m^{n+1}+u_{m-1}^{n+1}}{h^2}, \quad m = 0,\dots,z_- - 1; \\ \dfrac{u_m^{n+1}-u_m^n}{k} - a_{\alpha\beta}(x_m)\dfrac{u_{m+1}^{n+1}-2u_m^{n+1}+u_{m-1}^{n+1}}{2h^2} - \left(r - \dfrac{a_{\alpha\beta}}{2} - \lambda\overline{k}\right)\dfrac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2h} \\ \qquad + r u_m^{n+1} = \lambda I_h(u_m^n), \quad m = z_-,\dots,z_+; \\ \dfrac{u_m^{n+1}-u_m^n}{k} - a_{\alpha\beta}(x_m)\dfrac{u_{m+1}^{n+1}-2u_m^{n+1}+u_{m-1}^{n+1}}{2h^2} - \left(r - \dfrac{a_{\alpha\beta}}{2} - \lambda\overline{k}\right)\dfrac{u_{m+1}^{n+1}-u_{m-1}^{n+1}}{2h} \\ \qquad + r u_m^{n+1} = \dfrac{\lambda\delta^2}{2}\dfrac{u_{m+1}^{n+1}-2u_m^{n+1}+u_{m-1}^{n+1}}{h^2}, \quad m = z_+ + 1,\dots,M-1. \end{cases} \tag{8}$$

For the call option, the boundary conditions are given by: $u_{\alpha\beta}(0,\cdot) = 0$ as $x \longrightarrow -\infty$ and $u_{\alpha\beta}(x,t) = e^x - Ke^{-rt}$ as $x \longrightarrow +\infty$.

3. Numerical experiments

In this section, we show some numerical examples that illustrate the performance of a European Call option price under our model. The examples are chosen to demonstrate that for the given values of the parameters, the SGVJ model converges to models without jumps and with jumps respectively for $\lambda = 0$ and $\lambda \neq 0$. We also compare the pricing results with respect to these varying models and we present a numerical analysis that yields some insight into the behavior of the European Call option price according to the parameters α, β and the strike price K.

In our sample calculation, Tables 1 and 2 list the default parameter values of the SGVJ model and the different model types respectively. All our numerical examples will use this set of parameter values unless specified differently.

Table 1. Parameter values in the SGVJ model.

λ	S_0	K	σ	r	T
0.1	100	100	0.2	0.1	1

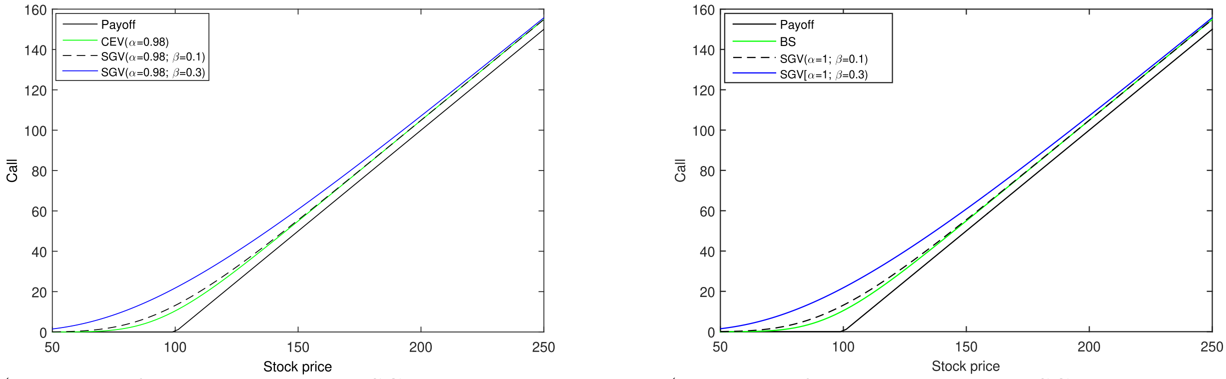
Table 2. Model types defined by the parameters: λ, α and β.

λ	α	β	Type of model
0	1	0	Black and Scholes
0	< 1	0	CEV
0	< 1	< 1	SGV
$\neq 0$	<1	0	CEV with jump
$\neq 0$	1	0	Merton's model
$\neq 0$	< 1	< 1	SGVJ

3.1. European call option price under SGVJ without jumps

As the first example, we consider the case of the European Call option price under SGVJ model when $\lambda = 0$. In this case, SGVJ is reduced to the SGV model. The results of the numerical solution of SGV model are obtained by using centered difference approximations for the first and second derivatives.

Figure 1 represents the behavior of the solution under SGV model toward the solutions under CEV and BS models when ($\alpha = 0.98$; $\beta \to 0$) and ($\alpha = 1$; $\beta \to 0$) as shown in Figures 1a and 1b respectively.



a (Behavior of the solution under SGV model toward the solution under CEV model ($\alpha = 0.98$; $\beta \to 0$))

b (Behavior of the solution under SGV model toward the solution under BS model ($\alpha = 1$; $\beta \to 0$))

Fig. 1. Behavior of the values of European Call option under SGV model when ($\alpha = 0.98$; $\beta \to 0$) (***a***) and ($\alpha = 0.98$; $\beta \to 0$) (***b***).

3.2. European call option price under SGVJ with jumps

In this second example, we evaluate the price of the European Call option under the SGVJ model in which we add the jump term by considering $\lambda \neq 0$ and using the same parameters seen in Table 1.

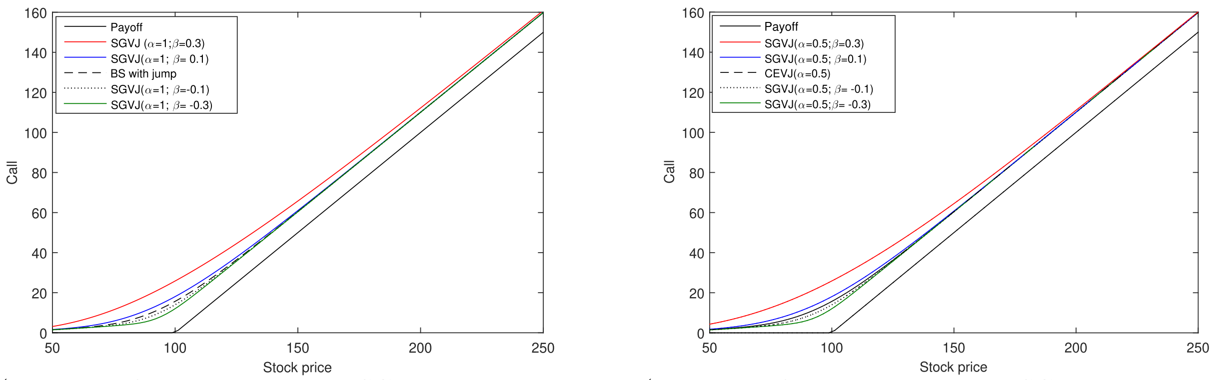
The results of the numerical solution of SGVJ model are obtained with the same method as in the previous case. Table 3 shows the values of the European Call option with one and twelve months maturity under the SGVJ model with $\lambda = 0.1$.

Table 3. Values of European Call option under the SGVJ model with $\lambda = 0.1$.

	Parameters		Time expiry	
	α	β	1 Month	1 Year
$K = 100$	0.50	1.00	46.181984	93.085476
	0.70	0.50	11.030160	38.433595
	0.90	0.3	06.252455	23.843271
	0.98	0.20	04.760329	19.180547
	1.00	0.00	02.864330	13.229330

Figures 2 and 3 illustrate the price behavior of the European Call option evaluated under the SGVJ model for different values of parameters α and β. This illustration of price behavior is compared to the results obtained by the jump models of Black-Scholes and CEV.

Figure 2 represents the behavior of the solution under SGVJ model toward the solutions under the Constant Elasticity of Variance with Jumps (CEVJ) and BS with jump models when ($\alpha = 0.5$; $\beta \to 0$) and ($\alpha = 1$; $\beta \to 0$) as shown in Figures 2a and 2b respectively.



a (Behavior of the solution under SGVJ model toward the solution under CEVJ model ($\alpha = 0.5$; $\beta \to 0$))

b (Behavior of the solution under SGVJ model toward the solution under BS model with jump ($\alpha = 1$; $\beta \to 0$))

Fig. 2. Convergence of the SGVJ model to the CEVJ and Black–Scholes with jump models when ($\alpha = 0.5$; $\beta \to 0$) and ($\alpha = 0.1$; $\beta \to 0$) respectively.

Figures 2a and 2b show that the pricing under the SGVJ model is more expensive than under the CEVJ and BS with jump models, in case where the parameter β is positive. Conversely, when the parameter β is negative the price of option under our model becomes less expensive than under the models mentioned above.

Finally, we summarize these different experiences on the behavior of the European Call option and compute the option prices for each model. Figure 3 shows the behavior of the European Call option under the SGVJ model compared to the models of pricing with and without jump. The price of the European Call option under the SGVJ model varies with the values of the parameter β. Indeed, when the parameter β is positive the price of the option under SGVJ model is higher than under the other models. Conversely, it is lower where the parameter β is negative.

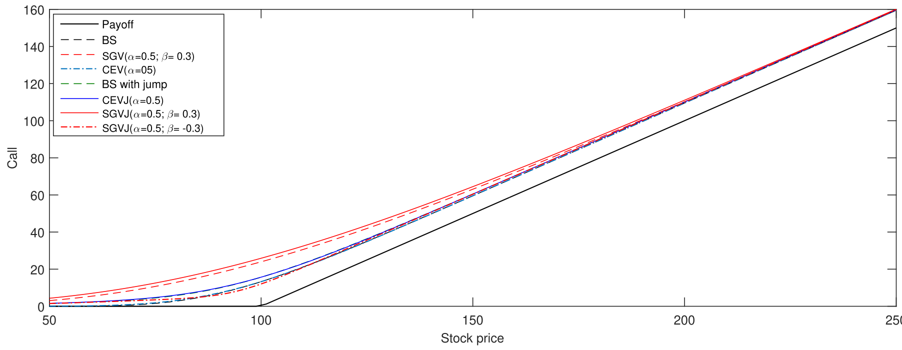


Fig. 3. Behavior of European Call option under the SGVJ model compared to the models of pricing with and without jump.

3.3. Sensitivity analysis

In this section, we present some results based on the sensitivity analysis of various key parameters. We investigate the effects of changes in the value of the parameters α, β and the Strike K, on the price of the European Call options under SGVJ model.

Effect of the parameter α. We study the effect of changes in the value of the parameter α on the price of the European call options under SGVJ model. We vary the parameter α while fixing $\beta = 0$. We use the data of Table 1.

The results are summarized in Table 4. Figure 4 illustrates the effect of the parameter α on the value of the European Call option. When the sensitivity is expressed by α tends to 1, the price of the option will have less effect and conversely it will be more affected when it tends to 0.

Table 4. The price of the European Call option corresponding to different values of α with $\beta = 0$ under SGVJ model.

	Parameter	Time expiry	
	α	1 Month	1 Year
$K = 100$, $r = 0.1$, $\sigma = 0.20$, $\lambda = 0.1$	0.50	2.864386	13.232837
	0.55	2.864372	13.232155
	0.60	2.864360	13.231548
	0.65	2.864350	13.231015
	0.70	2.864342	13.230556
	0.75	2.864336	13.230171
	0.80	2.864332	13.229854
	0.85	2.864329	13.229618
	0.90	2.864329	13.229450
	0.95	2.864330	13.229354
	1.00	2.864333	13.229330

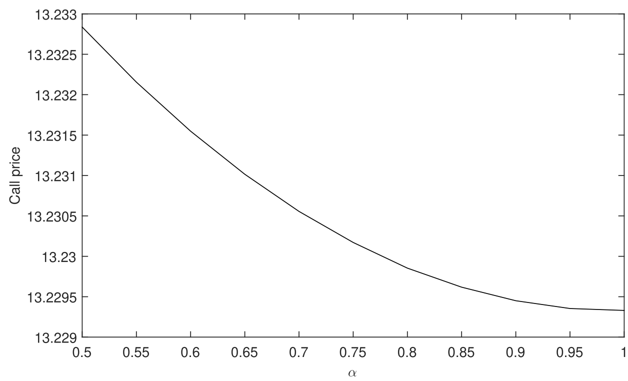


Fig. 4. Effect of the sensitivity parameter α on the price of the European Call option under SGVJ model.

Effect of the parameter β. In the second case, we present the effect of changes in the value of parameter β on the price of European Call options under SGVJ model. We choose $r = 0.05$ instead of $r = 0.1$ while the other data in Table 1 remain unchanged.

The numerical results listed in Table 5 show that when the values taken by the parameters β are negative, the value of option price remains low even if the parameter α can take the values 0 or 1. Conversely, this value will become large when the values taken by the parameters β are positive. Figure 5 shows that when the parameter β tends to 1, the price of the option will become very expensive and conversely when it tends to -1 the option will have the lowest price.

Table 5. The price of the European Call option corresponding to different values of β with ($\alpha = 0$ or $\alpha = 1$) under SGVJ model.

	Parameters		Time expiry	
	α	β	1 Month	1 Year
$K = 100$, $r = 0.05$, $\sigma = 0.2$, $\lambda = 0.1$	1.00	-1.00	1.134149	6.915810
	1.00	-0.80	1.152280	7.070847
	0.00	-0.60	1.211874	7.476273
	0.00	-0.40	1.394378	8.347444
	0.00	-0.20	1.867948	10.042045
	0.00	0.00	2.8664627	13.244050
	1.00	0.20	4.762119	19.269794
	1.00	0.40	8.290577	30.586602
	1.00	0.60	14.803199	49.552942
	1.00	0.80	26.689480	70.444556
	1.00	1.00	46.587708	85.143734

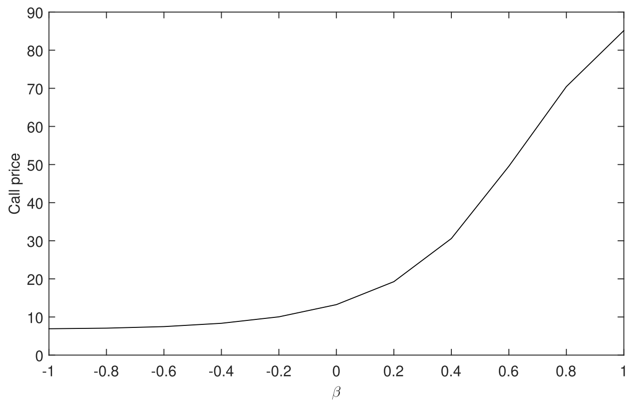


Fig. 5. Effect of security and stability expressed by the parameter β on the price of the European Call option under SGVJ model.

Effect of the strike price K. Finally, we study the effect of changes in the value of the strike price K on the European Call option price under SGVJ model. The following scenarios are suggested depending on the sign of the β: (1) $\beta > 0$, ($\alpha = 0.8$; $\beta = 0.5$); (2) $\beta < 0$, ($\alpha = 0.8$; $\beta = -0.3$) and ($\alpha = 0.8$; $\beta = -0.5$). We maintain the same parameters seen in the second case.

The numerical results which are given in Table 6 reveal the behavior of the option price under SGVJ model with respect to the strike price K, as illustrated in Figure 6. In the first scenario, we see that as the strike price increases, the call option value decreases with slower rate along the curve. In the second scenario, the call option decreases in two ways: (1) with faster rate than the first scenario when the option is In the Money (ITM) i.e., $K > S$; (2) and with slower rate when the option is Out of the Money (OTM) i.e., $K < S$.

Table 6. Behavior of the price of the European Call option according to different values of the strike price K and maturity $T = 1$.

Strike K	80	90	100	110	120
SGVJ: ($\alpha = 0.8$; $\beta = 0.5$)	45.803849	41.905259	38.424048	35.304157	32.501871
SGVJ: ($\alpha = 0.8$; $\beta = -0.3$)	25.006451	16.114749	09.058898	05.792229	04.903060
SGVJ: ($\alpha = 0.8$; $\beta = -0.5$)	24.986766	15.856488	7.836401	5.323073	04.860452

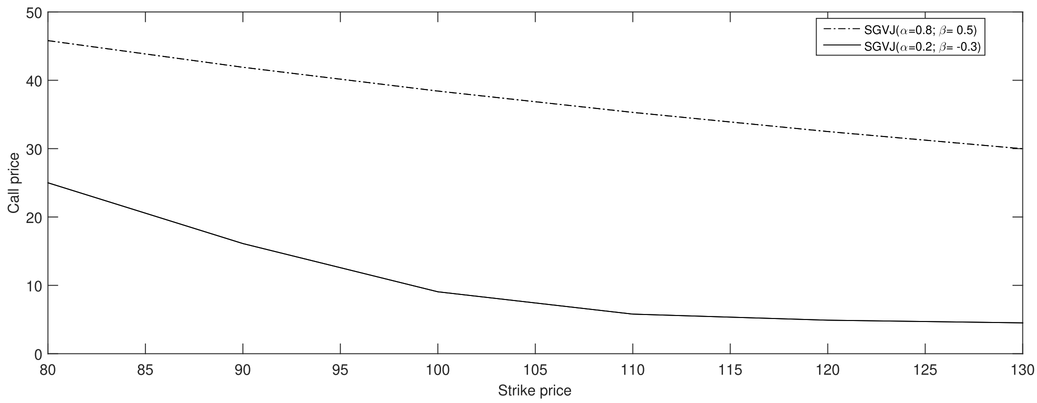


Fig. 6. A behavior of the solution under SGVJ model in respect to strike price K.

4. Conclusion

We have studied the pricing of the European Call option under Slow Growth Volatility with Jump (SGVJ) model where we have formulated the asset price evolution as a partial integro-differential equation (PIDE). Since there is not a closed solution, we solve it numerically using a finite difference method. Due to the non-locality of the integro-differential operator, we have to restrict it to a bounded domain and we have also truncated the domain of integration in the non-local part.

The experiment results have shown that the price of the European call option evaluated by our model is relatively expensive compared to the pricing under the models already seen. This is explained by the β security and stability parameter effect that characterizes our model.

Finally, we have demonstrated how the behavior of the price evolves with respect to the variations β, α and K. Future work in this area would use deep learning inside difference equation method.

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Ціноутворення європейських опціонів за моделлю, яка включає волатильність повільного зростання зі стрибком

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У статті запропоновано нову модель для створення числового дослідження, яке пов'язане з проблемою ціноутворення європейських опціонів, де ціни активів можна описати стохастичним рівнянням із розривним шляхом (модель повільного зростання волатильності зі стрибком SGVJ), яка використовує нестандартну волатильність. Особливу увагу приділено характеристикам запропонованої моделі, що полягають в її нестандартній волатильності, яка визначається параметрами α та β. Математичне моделювання за наявності стрибка показує, що потрібно вдатися до виродженого інтегро-диференціального рівняння з частинними похідними (PIDE), розв'язок якого дає ціну європейського опціону як функцію часу, ціну базового активу та миттєву волатильність. Однак, загалом, точного або замкненого розв'язку цієї задачі немає. З цієї причини апроксимовано його за допомогою методу скінченних різниць. Накінець, наведено деякі чисельні результати та результати порівняння з деякими відомими в літературі класичними моделями.

Ключові слова: *метод скінченних різниць; стрибкоподібно-дифузійні процеси; ціноутворення опціонів; інтегро-диференціальне рівняння в частинних похідних; посмішка волатильності.*