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Investigation of ant colony optimization with Levy flight technique for a class of stochastic combinatorial optimization problem

El Asri F., Tajani C., Fakhouri H.

SMAD Team, Polydisciplinary Faculty of Larache,
Abdelmalek Essaadi University, Tetouan, Morocco

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The demand for efficient solutions to optimization problems with uncertain and stochastic data is increasing. Probabilistic traveling salesman problem (PTSP) is a class of Stochastic Combinatorial Optimization Problems (SCOPs) involving partially unknown information about problem data with a known probability distribution. It consists to minimize the expected length of the tour where each customer requires a visit only with a given probability, at which customers who do not need a tour are just ignored without further optimization. Since the PTSP is NP-hard, the usage of metaheuristic methods is necessary to solve the problem. In this paper, we present the Ant Colony Optimization (ACO) algorithm combined with the Levy Flight mechanism (LFACO), which is based on Levy distribution to balance searching space and speed global optimization. Experimental results on a large number of instances show that the proposed Levy ACO algorithm on the probabilistic traveling salesman problem allows to obtain better results compared with the classical ACO algorithm.

Keywords: *stochastic combinatorial optimization; probabilistic traveling salesman problem; metaheuristics; ant colony optimization; Levy flight.*

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1. Introduction

Stochastic combinatorial optimization problems (SCOPs) are complex problems that involve identifying the best possible solution in situations where there is uncertainty or variability. Advanced mathematical and algorithmic techniques are used to solve them, but these techniques can be very costly in terms of time and resources. Nevertheless, solving these problems can have a significant impact on the world by helping businesses be more efficient and profitable, as well as helping people move more quickly and easily. SCOPs have an extensive applications in the real world, and stochastic techniques are particularly appropriate for solving them.

This paper investigates a class of static stochastic combinatorial optimization problems, called the probabilistic traveling salesman problem (PTSP) introduced by Jaillet [1], which is a generalization of the traveling salesman problem (TSP). In the TSP, the goal is to find the shortest Hamiltonian cycle through a set of customers given the distances between all pairs of clients. However, in the PTSP, each customer has a probability of needing a visit, and the objective is to find a tour of the minimum expected length while respecting the given probabilities at which customers who do not need a tour are ignored without further optimization.

The PTSP is a challenging problem with various applications, and it has been extensively studied in the literature [1–3]. In this paper, a new stochastic method for solving the PTSP is proposed and its effectiveness is demonstrated through numerical experiments.

Several approximate methods have been proposed to solve the PTSP. M. Abdellahi Amar et al. have proposed an application and parallel tabu search algorithm for solving the PTSP [4], a parallel branch and bound algorithm for the probabilistic TSP [5], Balaprakash et al. have presented a hybrid optimization approach using ant colonies [6], Bianchi et al. have presented various ant colony optimization approaches [7], while Branke and Guntsch have proposed a hybrid optimization approach using ant colonies [8]. Gutjahr has proposed an SACO ant based approach [9], while, Marinakis and Marinaki have proposed a hybrid multi swarm optimization algorithm [10] and a hybrid bee mating optimization algorithm [11].

Ant Colony Optimization (ACO) is an optimization algorithm that mimics the behavior of ants searching for the shortest path between their nest and a food source. The algorithm works by maintaining a population of artificial ants, which gradually constructs a solution to the problem by incrementally building a path. The probability of selecting an edge is determined by the quantity of pheromone deposited by the ants. In the context of the PTSP, the ACO algorithm constructs a tour by probabilistically choosing the next customer to visit, taking into account the probability of needing a visit and the amount of pheromone deposited on the edge connecting the current customer to the next.

The Levy flight technique is a type of travel trajectory that intersects frequent short distances and occasional long distances. It was first introduced by French mathematician Paul Levy (1886–1979) in 1937. This technique has been applied to a variety of optimization problems, including Anomalous diffusion by Greenenko, Chechkin, and Shul'ga [12], two-species competition model by Hanert [13], artificial bee colony algorithm with Levy flight [14], and novel ant colony optimization with Levy flight [15], non-local search and simulated annealing presented by Pavlyukevich [16]; in addition to another application in image processing, where it has been used for multi-threshold segmentation [17], and the Levy flight based particle swarm optimization [18].

Experimental studies [15, 17, 19] have demonstrated that Levy's flight model is highly effective in searching for food in uncertain environments. This is because the model's frequent short-distance jumps enable individuals to conduct detailed local searches within a limited area, while the occasional long-range jumps allow them to escape from local optima and reach global ones.

In the context of using ACO to solve PTSP, ACO can sometimes get trapped in local optima, where ants repeatedly visit a suboptimal subset of cities and fail to find better solutions. To address this issue, the Levy flight technique is employed to increase exploration of the search space by allowing the ants to make occasional long-distance moves, this feature helps to prevent being trapped in local optima and to find better solutions. By incorporating Levy flights into the ACO algorithm, the ants are able to explore more of the search space to find the global optimum.

The rest of this paper is structured as follows: in section 2, we present the mathematical model of the PTSP and some cases study, some properties of the PTSP. The ant colony optimization for the PTSP is presented in section 3. Section 4 is devoted to the adaptation of the ACO with Levy flight for PTSP. In section 5, numerical results are presented for standard instances showing the efficiency of the ACO algorithm with the introduction of Levy flight technique to better approach the considered problem by also reducing the time necessary to reach it.

2. Mathematical model

2.1. Problem setting

The traveling salesman problem (TSP) is a widely studied problem in operations research that involves finding the shortest route for visiting a given set of clients. In a probabilistic context, the problem consists of determining the expected length of the optimal tour, accounting for the uncertainty in the presence of clients. To formulate the problem, a graph $G(N, B, D)$ is defined, where N is the set of nodes representing clients, B is the set of arcs connecting the nodes of N, and D is a distance matrix where $d(i, j)$ is the distance between node i and j. The goal is to find a Hamiltonian circuit of graph G, represented by $T = (i_1, \ldots, i_{|N|}, i_1)$, which is a sequence of nodes that visits each client only once and has the minimum length L_T given by the mathematical formula: $L_T = \sum_{j=1}^{|N|} d(i_j, i_{j+1})$, where $i_{|N|+1} = i_1$.

The nodes of the graph G are classified into two sets: N_1 consists of nodes that must be visited at the beginning of each tour T, and N_2 contains nodes that do not need to be visited in every tour. Each node in N_2 is present with a fixed probability p (independently of each other). It is assumed that $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \varnothing$.

2.2. Special cases of PTSP

Bellalouna [20] presented several important results regarding the relationship between PTSP and TSP.

- Firstly, if D is a non-negative over-triangular matrix and the shortest path between nodes 1 and n is $(1, n)$, then PTSP is equivalent to TSP.
- Secondly, if D is a non-negative over-triangular matrix and for $i < j$ we have $d_{i,j} \leqslant d_{k,j}$ $\forall i + 1 \leqslant k \leqslant j - 1$, then PTSP is also equivalent to TSP. Furthermore, it was demonstrated by Henchiri and Toulouse [21] that constant matrices are the only ones to have the same expected cost for any tour T, with
$$E(L_{T_{ptsp}}) = p(1-p)^{n-1} L_{T^0_{tsp}}. \tag{1}$$
- Finally, Bellalouna also proved that for a distribution matrix D, the expected cost of the PTSP tour $L_{T_{ptsp}}$ is lower-bounded by
$$p(1-p)^{n-1} \sum_{i=1}^{n} d_{ii}. \tag{2}$$

2.3. Combinatorial properties of PTSP

In this subsection, we will explore the combinatorial properties of probabilistic traveling salesman problem (PTSPs)and their relationship to the TSP.

Proposition 2 (Jaillet [1]). Let $G = (N, B, D)$ be a given graph with m cities always present, and n cities present with probability p, the TSP solves the PTSP for any graph G (TSP≡PTSP) if and only if:

1) D is symmetric and $n + m \leqslant 4$; if $m = 0$ this is true for $n = 5$;
2) D is not symmetric and $m + n \leqslant 3$.

Proposition 3 (Jaillet [1])). When the n nodes are placed on their convex envelope, TSP≡PTSP.

Proposition 4 (Bellalouna [20]). Let T_{ptsp} be the optimal tour of the PTSP through n vertices, if n odd ($n = 2k + 1$), then
$$E(L_{T_{ptsp}}) \geqslant p^2 L^{(0)}_{T_{ptsp}} \frac{1 - (1-p)^{n-1}}{1 - (1-p)^k}. \tag{3}$$

2.4. Objective function for the PTSP

We consider an instance of the probabilistic traveling salesman problem (PTSP) where there is a complete graph with nodes representing a set N of customers, each with a probability p_i of requiring a visit. The goal is to find a tour T that visits all nodes in N, and the objective is to minimize the expected tour length $E(L_T)$. This objective function is defined as the sum of the product of the distance required to visit a subset S of customers, denoted by $L_T(S)$, and the probability $P(S)$ for the subset of customers S to require a visit [7]:
$$E[L_T] = \sum_{S \subseteq N} P(S) L_T(S), \tag{4}$$
here S is a subset of the node set N, and the probability $P(S)$ can be computed as the product of the probabilities p_i for customers in S to require a visit, and the probabilities $(1 - p_i)$ for customers not

in S to not require a visit [7]:

$$P(S)=\prod_{i\in S}p_i\prod_{i\in N-S}(1-p_i). \tag{5}$$

The problem of the probabilistic traveling salesman can be approached through the use of mathematical models. One such model is the Campbell equation, which calculates the probability and expected cost resulting from any arc that may occur in the tour. The expected cost of an arc (i,j) is influenced by the realization of customers i and j, with the condition that no intermediate customers k between them $(k=i+1,\dots,j-1)$ are realized. In the Campbell equation the probability of a customer requiring a visit and the distances between them are taken into account, the equation is a summation of three terms, each representing the expected cost resulting from different scenarios in the tour [22]:

$$\sum_{j=1}^{n}p_jd_{0j}\prod_{k=1}^{j-1}(1-p_k)+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}p_ip_jd_{ij}\prod_{k=i+1}^{j-1}(1-p_k)+\sum_{i=1}^{n}p_id_{0i}\prod_{k=i+1}^{n}(1-p_k). \tag{6}$$

— The first term of the Campbell equation represents the expected cost of travel from the starting point (node 0) to the first customer node (node j). It takes into account the distance between node 0 and node j (represented by p_j), and the probabilities that no nodes between 1 and $j-1$ need to be visited (represented by the product of $(1-p_k)$ for $k=1$ to $j-1$.
— The second term in the Campbell equation represents the expected cost of traveling from customer node i to customer node j, where i and j are not the starting node. This is calculated by taking into account the distance between i and j, the probabilities that node i and j need to be visited, and the probabilities that no nodes between $i+1$ and $j-1$ need to be visited.
— The third term in the Campbell equation represents the expected cost of traveling from the last customer node n back to the starting node 0. This is calculated by taking into account the distance between node i and 0, the probability that node i needs to be visited, and the probability that no nodes between $i+1$ and n need to be visited.

By utilizing the Campbell equation, it is possible to calculate the probability and expected cost of each possible arc in the tour. This information can then be used to construct a more efficient tour that minimizes the total expected cost. This mathematical approach to solving the PTSP can be a valuable tool for businesses and organizations that need to optimize their delivery or service routes.

3. Ant colony optimization for PTSP

The PTSP is known to be an NP-hard combinatorial optimization problem which makes its resolution very difficult. Therefore, we can exploit metaheuristic methods. In particular, the standard ACO algorithm has been shown to perform well in a variety of challenging combinatorial optimization problems, including many routing problems like PTSP. One of the key inspirations behind ACO can be traced back to the work of Deneubourg et al., however, ACO was first introduced by Dorigo, Maniezzo, and Colorni [23]. Here, we adapt Dorigo and Gambardella's ACO algorithm [24] for TSP to PTSP, as described in their work.

3.1. ACO principle

The ACO algorithm is comprised of three main procedures: initialization, solution construction, and pheromone update [25]. To construct a solution, we first group the nodes already visited by the ants, which defines the possible movements at each step when ant k is on city i: N_i^k, next, we consider the visibility between nodes, which can be calculated as the reciprocal of the distance between city i and city j: $\eta_{i,j}=\frac{1}{d_{i,j}}$.

To determine the quantity of pheromone deposited on a path connecting two customers, we use the proportional transaction random parameter defined by Kube and Bonabeau (1998). According to

Dreo et al. [25], the selection probability of an edge (i,j) by ant k at time t can be expressed as follows:

$$p_{i,j}^{k}(t) = \begin{cases} \frac{[\tau_{i,j}(t)]^{\alpha}[\eta_{i,j}]^{\beta}}{\sum_{l \notin N_i^k}[\tau_{i,l}(t)]^{\alpha}\eta_{i,l}^{\beta}}, & \text{if} \quad j \notin N_i^k, \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$

where the relative importance of trail intensity and visibility is controlled by two parameters, namely α and β. After completing a full tour, each ant leaves a certain amount of pheromone $\Delta\tau_{i,j}(t)$ on the edges it traversed, which depends on the quality of the solution found. The pheromone is updated using the expected values of the circuit, as in TSP with ACO, but with a different quantity of reinforced pheromone, specifically, we have [25]:

$$\Delta\tau_{i,j}^{k}(t) = \begin{cases} \frac{Q}{E(L_k(t))}, & \text{if} \quad (i,j) \in T^k(t), \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

where Q is a fixed parameter that represents the weight to the amount of pheromone deposited by each ant, $E(L_k(t))$ is the expected path length of ant k at time t, and $T^k(t)$ is the path chosen by ant k.

To avoid being stuck in suboptimal solutions, the ACO algorithm accepts worse solutions to escape from possible local optima. We achieve this by introducing an evaporation factor γ, which represents the rate at which the pheromone disappears. The pheromone are updated using the formula [25]:

$$\tau_{i,j}(t+1) = (1-\gamma)\tau_{i,j}(t) + \sum_{k=1}^{m} \Delta_{i,j}^{k}(t), \tag{9}$$

where m is the total number of ants used.

3.2. ACO Algorithm for PTSP

The presented algorithm is ant colony optimization for the PTSP. It initializes the pheromone intensity of each edge to a starting value τ_0. For each ant, the algorithm starts by placing the ant at the starting node, which is the same for all ants, and stores this information in $Tabu_k$ to ensure that the tour starts and ends at the same node and that each city is visited only once. The tour for each ant is constructed by choosing the next node that is not in $Tabu_k$, based on the probability equation presented in the algorithm. After choosing the next node, the ant updates the local pheromone for the chosen edge. The algorithm updates the global pheromone by calculating the expected value of the tour for each ant, applying a local improvement method to the routes of all ants, and recalculating $E(L_{T_k})$, as well as storing the shortest tour found so far and its minimal expected value. Finally, the algorithm checks if the stopping condition is met and if not, the algorithm clears all $Tabu_k$ and restarts.

4. Ant colony optimization with Levy flight for PTSP

According to the PTSP function of Campbell, it is decided to position all ants at the same starting point, which corresponds to the depot, during the initialization phase to ensure that the tour forms a closed loop that starts and ends at the depot. However, as mentioned before, this strategy can lead to artificial ants getting trapped in local minima of the search space, despite pheromone updates, which can prevent the algorithm from finding an optimal solution.

To address this issue, the ant colony optimization uses Levy flight (LFACO). This strategy allows artificial ants to make random jumps of varying lengths to regions of the search space with a higher density of solutions. Thus, Levy flight is used to improve the exploration of the search space without compromising the closed loop constraint. Therefore, the starting point remains the depot for each ant.

4.1. ACO with Levy's flight

Several algorithms have been developed in the literature to generate random numbers that follow the Levy distribution. Among these methods, Mantegna's algorithm (Mantegna, 1994) stands out

Algorithm 1 ACO for the PTSP.

Step 1: initialization: $t \leftarrow 0$, $\tau_{i,j}(0) \leftarrow \tau_0$;
Step 2: starting node
for each ant k
  Place the ant on the starting node and store this information in $Tabu_k$;
Step 3: build a tour for each ant
for i from 1 to n
  for all k from 1 to m
    choose the next node j, $j \notin Tabu_k$ where j is chosen according to the probability:

$$p_{i,j}^{k}(t) = \frac{[\tau_{i,j}(t)]^{\alpha}[\eta_{i,j}]^{\beta}}{\sum_{l \notin Tabu_k}[\tau_{i,l}(t)]^{\alpha}[\eta_{i,l}]^{\beta}}$$

    Local update of the trail for chosen edge (i,j):

$$\tau_{i,j}(t) = (1-\psi)\tau_{i,j}(t) + \Delta\tau_{i,j}(t) \text{ with } \Delta\tau_{i,j} = \tau_0;$$

Step 4: global update of the trail
for each edge $(i,j) \in Cycle^*$
  Update the trail according to:

$$\tau_{i,j}(t+1) = (1-\gamma)\tau_{i,j}(t) + \sum_{k=1}^{m}\Delta\tau_{i,j}^{k}(t)$$

  with $\Delta\tau_{i,j}^{k}(t) = \frac{Q}{E(Cycle^*)}$ if $k \in (i,j)$, and 0 otherwise;
$t \leftarrow t+1$
Step 5: condition of termination
if not (end-test) **then**
  Empty all $Tabu_k$ and go to Step 2;
else
  stop.

as one of the most efficient and straight forward approaches for generating symmetric values of the Levy distribution. In this algorithm, the step size, denoted by s, can be calculated as follows (Saji et al. 2021) [26]:

$$s = \frac{u}{|v|^{\frac{1}{\lambda}}}. \tag{10}$$

Here, $\lambda \in [1,2]$, and u, v are Gaussian-centric distributions.

Unlike other commonly used distributions, such as the Gaussian or Cauchy distributions, Levy distributions are characterized by heavy tails. However, the calculation of the original Levy flight using formula (10) is complex and cannot be directly used in the ant colony optimization (ACO) algorithm. To address this issue, a Levy flight conversion formula is proposed for the candidate selection mechanism using formula (13), while formula (11) is an improved version of formula (10) (Liu et al., 2021) [27]:

$$S_{new} = \begin{cases} \frac{1}{A} * \frac{1-P_{threshold}}{1-P_{Levy}}, & \text{if } S_{new} \geqslant 1; \\ 1, & \text{else;} \end{cases} \tag{11}$$

$$1 - P_{new} = \frac{1}{S_{new}} * (1 - P_{now}); \tag{12}$$

$$P_{new} = \begin{cases} 1 - A * \frac{1-P_{Levy}}{1-P_{threshold}} * (1-P_{now}), & \text{if } P_{Levy} \geqslant P_{threshold}; \\ P_{now}, & \text{else.} \end{cases} \tag{13}$$

Here, S_{new} represents the new step length for Levy's flight, with $S_{new} \geqslant 1$, and A is a fixed parameter for the Levy flight altering ratio ($A \geqslant 0$). $P_{threshold}$ is a fixed parameter for the Levy flight threshold ($0 < P_{threshold} < 1$). P_{Levy} is a uniformly distributed variable ($0 < P_{Levy} < 1$) that represents the probability of turning on/off the Levy flight altering mechanism. Similarly, P_{now} is also a uniformly distributed variable ($0 < P_{now} < 1$) that denotes the original selection probability before the Levy flight-altering mechanism. Finally, P_{new} is a uniformly distributed variable ($0 < P_{new} < 1$) that represents the final selection probability after the Levy flight altering mechanism.

4.2. LFACO for PTSP

The Levy ACO algorithm is an improved version of ACO that incorporates the Levy flight mechanism to enhance candidate selection. This is done by using the step length of the Levy flight to alter the original random number used to select the next site. Formulas (10) and (11) are then used to implement the Levy flight mechanism in the candidate selection process, resulting in more diversified solutions. Predefined parameters such as $P_{threshold}$ and A are used to tune the algorithm for efficiency.

To illustrate the application of the LFACO algorithm to solve PTSP, we propose the following diagram that shows the different steps of the resolution process.

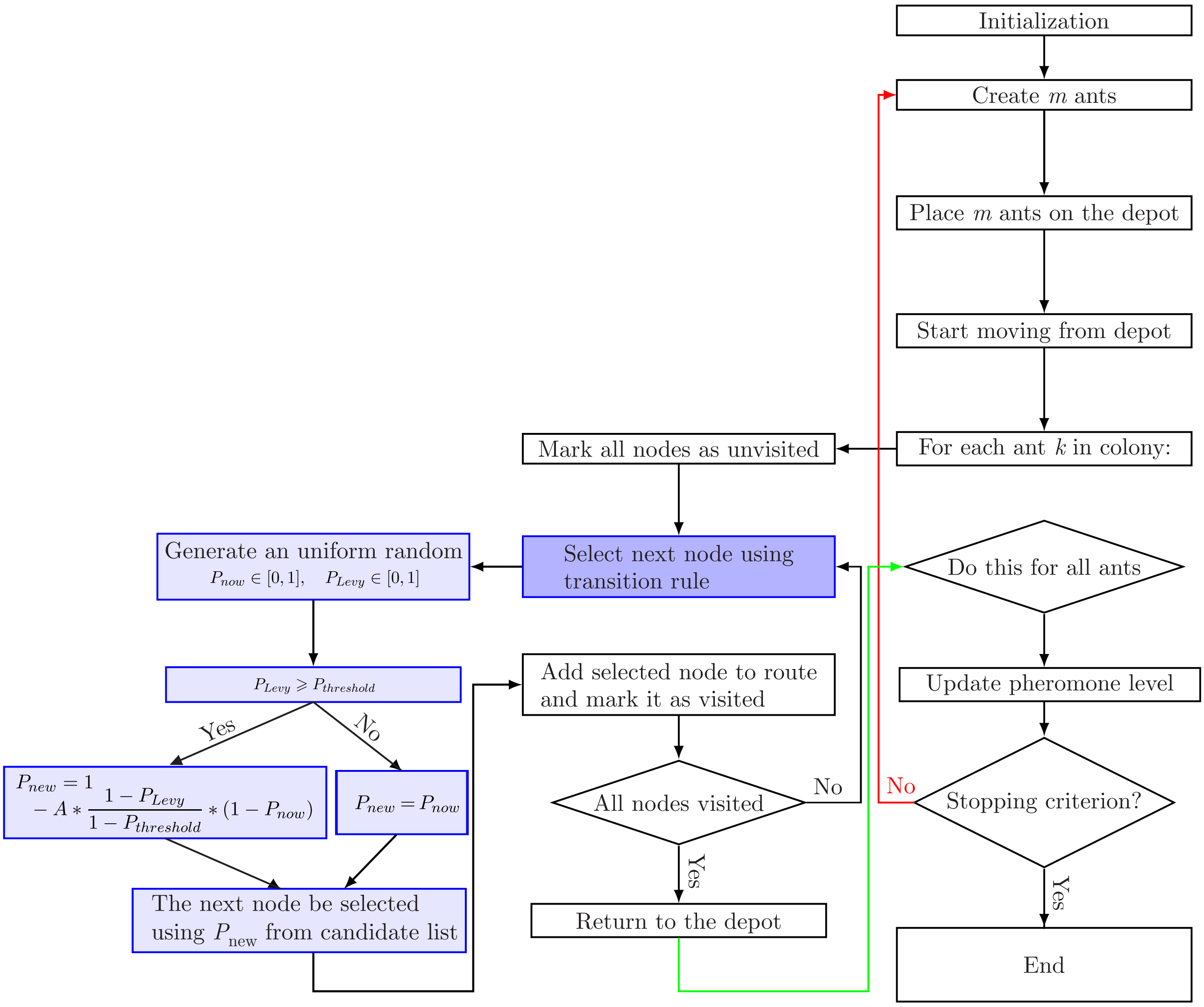

Fig. 1. Diagram illustrating the different steps of applying the LFACO algorithm to solve PTSP.

5. Computation experiment

In this section, we present the results of our experiments comparing the ACO and LFACO algorithms to solve the PTSP problem. To evaluate the performance of our proposed algorithms, we conducted experiments on reference instances from Campbell and Thomas (2008). We considered two different probability settings. The first one involved generating each customer's probability from a random number between 0 and 1, providing insight into the impact of having a larger range of customer probabilities. The second setting involved assigning probabilities of either 0.1 or 1 randomly. We referred to these two datasets as Range and Mixed, respectively, in Tables 2 and 3.

We measured the computation time required to run each algorithm and evaluated the quality of the obtained solutions. Our approach is developed entirely in Python and executed on a machine equipped with an Intel Core i5-7200U processor at 2.50 GHZ, 8GB of RAM, and a 64-bit operating system with an x64 processor. The parameter settings for the different approaches used in this paper are presented in Table 1.

Table 1. The parameters for different approaches.

Parameters	Values
τ_0	1
m	7
α	1
β	2
γ	0.5
ψ	0
$P_{threshold}$	0.8
A	1
Q	1

Table 1 presents the parameter settings for the different approaches used in our experiments to solve the PTSP problem. These parameter values were chosen to balance computation time and solution quality. The PTSP problem involves finding the shortest possible route to visit a set of cities with a minimum expected value.

We selected the value of ψ=0 based on Dennis' paper, which found that the parameter ψ, which controls diversification, did not significantly affect the quality of the algorithms' solutions. The best value for $\psi = 0$, suggests that diversification does not play a crucial role in this setting. This information can be useful in guiding parameter selection for solving the probabilistic traveling salesman problem.

Table 2. Comparison of ACO and LFACO based on expected values for the PTSP problem.

Probability	Range		Mixed	
Data set	LFACO	ACO	LFACO	ACO
22	148.16	164.20	66.87	71.44
42	175.6	177.38	163.46	195.99
62	300.98	328.82	199,91	201.25
102	412.57	420.60	382.35	400.86
152	719.37	767.56	626.50	691.98

Table 3. Comparison of ACO and LFACO based on CPU values for the PTSP problem.

Probability	Range		Mixed	
Data set	LFACO	ACO	LFACO	ACO
22	1	1	1	1
42	11	14	8	13
62	60	67	69	75
102	162	239	203	228
152	513	529	154	284

Tables 2 and 3 offer a comprehensive comparison between the performance of ACO and LFACO for the PTSP in terms of expected values and CPU time (in seconds). Table 2 indicates that LFACO consistently outperforms ACO across all data sets and problem types, with lower expected values obtained by LFACO. For example, LFACO achieved an expected value of 382.35 for the mixed problem type with a 102-city data set, which is 5.12% lower than ACO's expected value of 400.86. Furthermore, LFACO exhibited a significant improvement in the mixed problem type for the 102-city data set, with a 14.27% improvement over ACO. Table 3 indicates that LFACO requires less CPU time than ACO to obtain the same quality of solution, with a substantial difference observed in data set 152, where LFACO significantly outperforms ACO in terms of CPU time. These results suggest that LFACO is a promising algorithm for solving the PTSP problem, particularly for Range and Mixed probability types, and could potentially be useful for other stochastic combinatorial optimization problems.

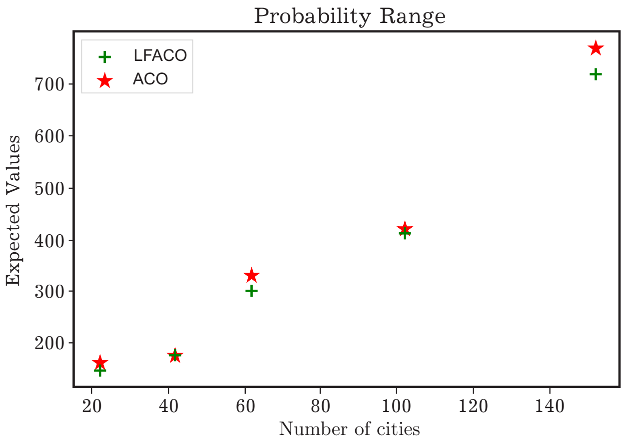

Fig. 2. Comparing ACO and LFACO for PTSP Using Expected Values and Probability Range Analysis.

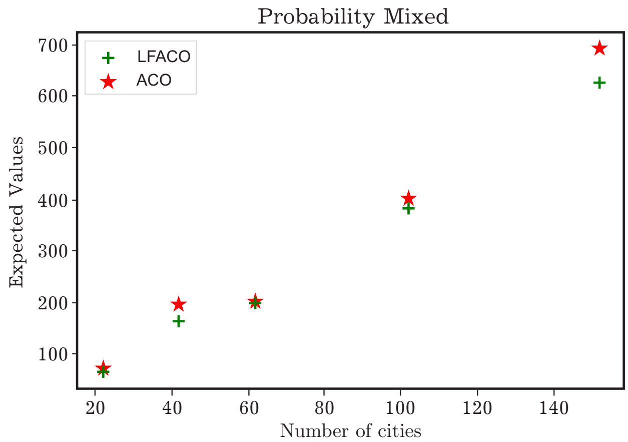

Fig. 3. Comparing ACO and LFACO for PTSP Using Expected Values and Probability Mixed Analysis.

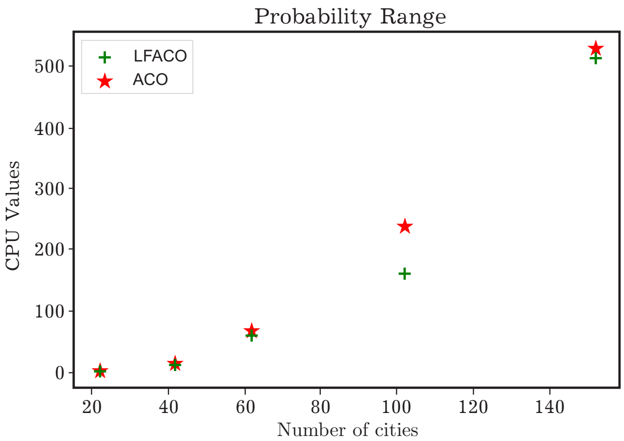

Fig. 4. Comparing ACO and LFACO for PTSP Using CPU Values and Probability Range Analysis.

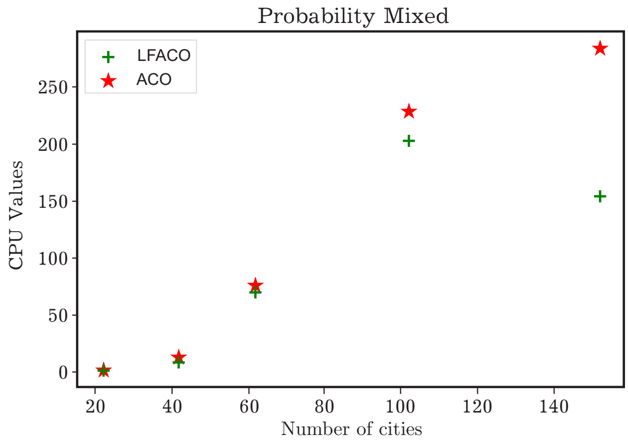

Fig. 5. Comparing ACO and LFACO for PTSP Using CPU Values and Probability Mixed Analysis.

6. Conclusion

This paper focuses on solving the probabilistic traveling salesman problem (PTSP) using mathematical modeling and the ant colony optimization (ACO) technique with the Levy-flight strategy. The presented results demonstrate the effectiveness and promise of the proposed method. Specifically, the LFACO algorithm outperforms the classical ACO algorithm in terms of solution quality and computation time, as shown in the results obtained for different standard instances of PTSP.

The proposed approach is applicable to various fields such as industry, transportation, or logistics for optimizing large datasets. These promising results can assist in solving real-world practical problems at a large scale with high solution quality. In conclusion, the ACO technique with the Levy-flight strategy is an efficient method for solving the PTSP and offers improved performance compared to existing methods for large datasets.

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Дослідження оптимізації мурашиної колонії за допомогою техніки польоту Леві для класу задач стохастичної комбінаторної оптимізації

Ель Асрі Ф., Таяні К., Фахурі Х.

Команда SMAD, Полідисциплінарний факультет Лараш, Університет Абдельмалека Ессааді, Тетуан, Марокко

Запит на ефективні розв'язки задач оптимізації з невизначеними та стохастичними даними зростає. Імовірнісна задача комівояжера (PTSP) — це клас стохастичних комбінаторних задач оптимізації (SCOP), які містять частково невідому інформацію про дані задачі з відомим розподілом ймовірностей. Вона полягає в мінімізації очікуваної тривалості туру, коли кожен клієнт вимагає відвідування лише з певною ймовірністю, за якої клієнти, яким тур не потрібен, просто ігноруються без подальшої оптимізації. Оскільки PTSP є NP-складною, для розв'язування цієї задачі необхідно використовувати метаевристичні методи. У цій статті подано алгоритм оптимізації мурашиної колонії (ACO) у поєднанні з механізмом польоту Леві (LFACO), який базується на розподілі Леві, щоб збалансувати простір пошуку та прискорити глобальну оптимізацію. Експериментальні результати на великій кількості прикладів показують, що запропонований алгоритм Леві ACO для імовірнісної задачі комівояжера дає кращі результати порівняно з класичним алгоритмом ACO.

Ключові слова: *стохастична комбінаторна оптимізація; імовірнісна задача комівояжера; метаевристика; оптимізація мурашиної колонії; політ Леві.*