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Magneto-hydrodynamic boundary layer flow and heat transfer of hybrid carbon nanotube over a moving surface

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The boundary layer flow and heat transfer of hybrid carbon nanotubes over a moving surface with magneto-hydrodynamic effect are studied numerically in this paper. Single-wall (SWCNT) and multi-wall (MWCNT) carbon nanotubes are combined with water as the base fluid to form hybrid carbon nanotubes. The governing partial differential equations were transformed into a set of nonlinear ordinary differential equations using the similarity transformation, which were then numerically solved in the Matlab software using bvp4c. The influence of the nanoparticle volume fraction, magnetic parameter and velocity ratio parameter, on velocity and temperature profiles, local skin friction and local Nusselt number are discussed and presented in graphical forms. The results show that dual solutions appear when the free stream and plate move in the opposite direction, and the rate of heat transfer for hybrid carbon nanotubes is higher than viscous fluid and carbon nanotubes.

Keywords: *hybrid carbon nanotube; moving surface; boundary layer; magnetohydrodynamics effect.*

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1. Introduction

Due to the rapid growth of nanotechnology and modern sciences, nanofluid has drawn the interest of many researchers. Kakaç and Pramuanjaroenkij [1] state that when fundamental heat transfer fluids with nanometer-sized particles are suspended in a fluid, it is referred to as a "nanofluid". In nanofluids, nanoparticles are frequently formed of metals, oxides, or carbon nanotubes (CNTs) [2]. Water, ethylene glycol, and oil are examples of typical base fluids. Along with the evolution of technologies, carbon nanotubes (CNTs) are increasingly studied due to their thermo-physical properties, benefiting the nanotechnology industry and advancing technologies. There are two types of carbon nanotubes: multi-wall and single-wall carbon nanotubes (MWCNTs and SWCNTs). There are numerous research on boundary layer flow problems concerning carbon nanotubes with different type of surfaces under a variety of effects [3–7]. Later on, the idea of using hybrid nanofluids fluorished due to the better performance of heat transfer compared to single nanoparticle nanofluids. Typically, metallic and non-metallic nanoparticles are combined with conventional heat transfer fluids to create hybrid nanofluids. Then, many researchers are interested in the study of hybrid carbon nanotubes that are formed by combining two types of nanoparticles which are single-wall and multi-wall carbon nanotubes into the base fluid. In recent works, several studies on hybrid carbon nanotubes with different type of surfaces and effects were explored [8–10].

In recent years, numerous researchers have studied the boundary layer flows caused by moving surfaces on different fluids such as nanofluids [11–14], carbon nanotubes [15–17], hybrid nanofluids [18–20] and hybrid carbon nanotubes [9]. Sakiadis [21] has studied the classical problem of the boundary layer flow resulting from a continuously solid moving surface moving at a constant velocity. This problem appeared in numerous industrial applications, including paper manufacturing, aerodynamic plastic sheet extrusion, the cooling of an endless metallic plate in a cooling bath, and material handling conveyors [22]. There are two types of moving plate which are horizontal and vertical. Gravity is taken into account by a vertical moving plate however not by a horizontal plate.

There are several industries, particularly in physics, engineering, and medicine, where the study of boundary layer flow and heat transfer in the presence of magnetic fields is useful. Therefore, the boundary layer flow problem in the presence of magnetic field has been studied by numerous researchers. Salleh et al. [23] analyzed the numerical solution of nanofluid flow over a moving thin needle with magneto-hydrodynamics effect. The study reveals that the presence of magnetic field reduces the skin friction and heat transfer coefficients while it increases the mass transfer coefficient on the needle surface. Nithya and Vennila [24] examined the hydromagnetic nanofluid flow passing through a stretching sheet in the presence of viscous and ohmic dissipations. According to the findings, the velocity profile for the parameters of magnetics M, suction S, and nonlinear stretching parameter n exhibited an increased trend. Besides, Kalpana et al. [25] conducted a study on magneto-hydrodynamic hybrid nanofluid flow with the thermophoresis and Brownian motion in an irregular channel. The findings show that the magnetic field decreases the boundary layer thickness and accelerates heat transfer. Khashi'ie et al. [26] studied the problem of magneto-hydrodynamic hybrid nanofluid flow with heat transfer on a moving plate with Joule heating. It is found that dual solutions are obtained when the plate is moved oppositely from the free stream flow and magnetic parameter enhanced the heat transfer process.

Recent experimental and numerical finding by Anuar et al. [8] found that hybrid carbon nanotube had a higher rate of heat transfer when compared to carbon nanotube and conventional fluid. However, the above mentioned study is restricted to the problem in a permeable vertical plate. Hence, this study is performed to analyze the behavior of hybrid carbon nanotube due to a moving plate with magneto-hydrodynamics effect which motivated by the previous researcher.

2. Methodology

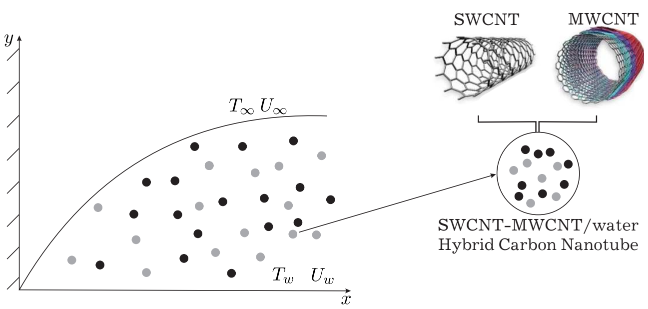

Fig. 1. Physical model of horizontal moving plate.

Consider a steady two-dimensional hybrid carbon nanotube flow in horizontal moving surface with hydromagnetic effect. The problem is shown in Figure 1. The x and y axes are the Cartesian coordinates, with x-axis measured along the plate and y-axis perpendicular to the plate, with the flow being in the region $y \geqslant 0$. The free stream of hybrid carbon nanotube has a fixed velocity U. Moreover, the plate is moving with the velocity $U\lambda$, where indicates moving parameter. Similarity solutions are possible, according to Aly et al. [27], if the magnetic field has the particular form of $B = B_0 x^{-1/2}$. Additionally, the small magnetic Reynolds number caused by the induced magnetic field is ignored. The far field has a temperature of T_∞, while the moving plate has a temperature of T_w.

The basic governing equations for a hybrid carbon nanotube's continuity, momentum, and energy can be expressed as [26]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hcnt}}{\rho_{hcnt}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hcnt}}{\rho_{hcnt}} B^2 (u - U), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hcnt}}{(\rho C_p)_{hcnt}}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

and the associate boundary conditions are

$$u = \lambda U, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0, \tag{4}$$
$$u \to U_\infty, \ T = T_\infty \quad \text{as} \quad y \to \infty,$$

where u and v are the components of velocity for x and y axes, respectively. T is the temperature of the fluid, μ is the dynamic viscosity, ρ is the density, $\sigma = k/(\rho C_p)$ is the thermal diffusivity, k is the thermal conductivity, (ρC_p) is the heat capacity and U is the composite velocity which defined as $U = U_w + U_\infty$. The subscript '$hcnt$' represents hybrid carbon nanotube. Meanwhile, the magnetic field is defined as $B = B_0 x^{-1/2}$. The following thermophysical properties of hybrid carbon nanotubes are given by Aladdin and Bachok [9] in Table 1.

Table 1. Hybrid carbon nanotube's thermophysical properties.

Properties	Hybrid carbon nanotube
Density	$\rho_{hcnt} = (1-\varphi_2)[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}] + \varphi_2\rho_{s2}$
Heat Capacity	$(\rho C_p)_{hcnt} = (1-\varphi_2)\left[(1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{s1}\right] + \varphi_2(\rho C_p)_{s2}$
Dynamic Viscosity	$\mu_{hcnt} = \frac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Thermal Conductivity	$k_{hcnt} = \left[\frac{1-\varphi_2+2\varphi_2\left(\frac{k_S}{k_S-k_{nf}}\right)\ln\left(\frac{k_S+k_{nf}}{2k_{nf}}\right)}{1-\varphi_2+2\varphi_2\left(\frac{k_{nf}}{k_S-k_{nf}}\right)\ln\left(\frac{k_S+k_{nf}}{2k_{nf}}\right)}\right] k_{nf}$
	$k_{nf} = \left[\frac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f+\varphi_1(k_f-k_{s1})}\right] k_f$

Table 1 lists the thermophysical properties of hybrid carbon nanotube, where φ_1 and φ_2 represent the solid volume fractions of nanoparticles for SWCNT and MWCNT, respectively. The subscripts 'f', 'nf', and '$hcnt$', denote three different types of fluids: fluids, nanofluids, and hybrid carbon nanotubes, respectively. Besides, $S = s_1 + s_2$ and s_1 is definitely for SWCNT while s_2 is for MWCNT nanoparticles. Table 2 displays the physical characteristics of the base fluid and carbon nanotubes.

Table 2. Base fluid and carbon nanotube properties (Anuar et al. [8]).

	Physical properties		
	k $(\mathrm{W \cdot m^{-1} \cdot K^{-1}})$	C_p $(\mathrm{J \cdot kg^{-1} \cdot K^{-1}})$	ρ $(\mathrm{kg \cdot m^{-3}})$
Base fluid:			
Water	0.613	4179	997.1
Carbon nanotube:			
SWCNT	6600	425	2600
MWCNT	3000	796	1600

Further, to obtain the similarity solutions for the governing equations, the following transformation are introduced as follows:

$$\eta = y\sqrt{\frac{U}{\nu_f x}}, \quad \psi = \sqrt{\nu_f x U} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{5}$$

where the composite velocity, U is defined as $U = U_w + U_\infty$ and kinematic viscosity, ν_f is defined as $\nu_f = \mu_f/\rho_f$. The stream function, ψ helps in the transformation of velocity components into dimensionless parameters that can be defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Thus Equation (1) is identically satisfied. By substituting Equations (5), (2) and (3) are reduced into

$$\frac{\mu_{hcnt}/\mu_f}{\rho_{hcnt}/\rho_f} f''' + \frac{1}{2} f f'' - \frac{\sigma_{hcnt}/\sigma_f}{\rho_{hcnt}/\rho_f} M(f' - 1) = 0, \tag{6}$$

$$\frac{1}{\Pr} \frac{k_{hcnt}/k_f}{(\rho C_p)_{hcnt}/(\rho C_p)_f} \theta'' + \frac{1}{2} f\theta' = 0 \tag{7}$$

with the boundary conditions (4) for the similarity transformation are as follows:

$$f(0)=0,\quad \theta(0)=1,\quad f'(0)=\lambda,$$
$$f'(\eta)\to 1-\lambda,\ \ \theta(\eta)\to 0\ \text{ as }\ \eta\to\infty, \tag{8}$$

where Prandtl number, Pr is defined as $\mathrm{Pr}=\nu_f/\alpha_f$ and λ is the moving parameter.

The physical quantities involve in this study are local skin friction C_f and Nusselt number, Nu_x which are given by:

$$C_f=\frac{\mu_{hcnt}}{\rho_f U^2}\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad \mathrm{Nu}_x=-\frac{x k_{hcnt}}{k_f(T_w-T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{9}$$

Substituting Equation (5), $\nu_f=\frac{\mu_f}{\rho_f}$ into Equation (9), we have

$$\sqrt{2}\,\mathrm{Re}_x^{\frac{1}{2}}C_f=\frac{\mu_{hcnt}}{\mu_f}f''(0),\quad \sqrt{2}\,\mathrm{Re}_x^{-\frac{1}{2}}\mathrm{Nu}_x=-\frac{k_{hcnt}}{k_f}\theta'(0), \tag{10}$$

where local Reynolds number, Re_x is defined as $\mathrm{Re}_x=U_x/\nu_f$.

3. Results and discussion

The system of Equations (6) and (7) with boundary conditions in (8) are solved using the bvp4c solver numerically in Matlab software. Table 3 shows the relative values of reduced skin friction coefficient, $f''(0)$ when $\varphi_1=\varphi_2=0$ and $M=0$ for different values of λ, which indicates a good correlation between the current results and the results obtained from the previous studies by Bachok et al. [28] and Anuar et al. [5]. Thus, it is assured that the fluid flow and heat transfer of the current problem can be solved reliably using the present numerical method.

The range of nanoparticle volume fraction (φ_1, φ_2) used in this study is taken from the previous study by Anuar et al. [8] which in the range of $0\leqslant\varphi_1,\varphi_2\leqslant 0.02$. Next, moving parameter λ are taken over the range of $-0.50<\lambda<-0.40$. The previous study done by Anuar et al. [5] shows that unique solutions exist when $\lambda>0$ where the plate and the free stream move in the same direction, while dual solutions exist when $\lambda_c<\lambda<0$ where the plate and the free stream move in the opposite direction with each other. Next, the range of magnetic parameter, M are taken from Khashi'ie et al. [29] in the range of $0\leqslant M\leqslant 0.02$. For the Prandtl number, since this study only considered water as the base fluid, it is fixed to 6.2 which was introduced by Oztop and Abu–Nada [30].

Table 3. $f''(0)$ comparison values for hybrid nanofluid when $\varphi_1=\varphi_2=0$, $M=0$ and $\mathrm{Pr}=6.2$.

λ	Bachok et al. [28]		Anuar et al. [5]		Present result	
	First	Second	First	Second	First	Second
−0.5	0.3979	0.1710	0.3978	0.1710	0.3978	0.1710
−0.4	0.4357	0.0834	0.4356	0.0834	0.4356	0.0834
−0.3	0.4339	0.0367	0.4339	0.0367	0.4339	0.0367
−0.2	0.4124	0.0114	0.4124	0.0114	0.4124	0.0114
−0.1	0.3774	0.0010	0.3774	0.0011	0.3774	
0	0.3321		0.3321		0.3321	
0.5	0		0		0	
1	−0.4438		−0.4438		−0.4438	

Figure 2 shows the result for four different types of fluid, which are viscous fluid ($\varphi_1=\varphi_2=0$), SWCNT/water ($\varphi_1=0.01$, $\varphi_2=0$), MWCNT/water ($\varphi_1=0$, $\varphi_2=0.01$) and SWCNT-MWCNT/water or hybrid carbon nanotube ($\varphi_1=0.01$, $\varphi_2=0.01$) on reduced skin friction, $f''(0)$ and reduced heat transfer, $-\theta'(0)$. Generally, the results show that the unique solution exist when $\lambda>0$, however the dual solutions exist when $\lambda_c<\lambda<0$ and no solution exist when $\lambda<\lambda_c$. Other than that, the results obtained show that hybrid carbon nanotube has the highest value of λ_c when compared to the other fluids. This indicates that hybrid carbon nanotube delays the boundary layer separation. In addition, hybrid carbon nanotube has the lowest value of skin friction, however this fluid has the highest rate of heat transmission when compared to viscous fluid, SWCNT/water and MWCNT/water.

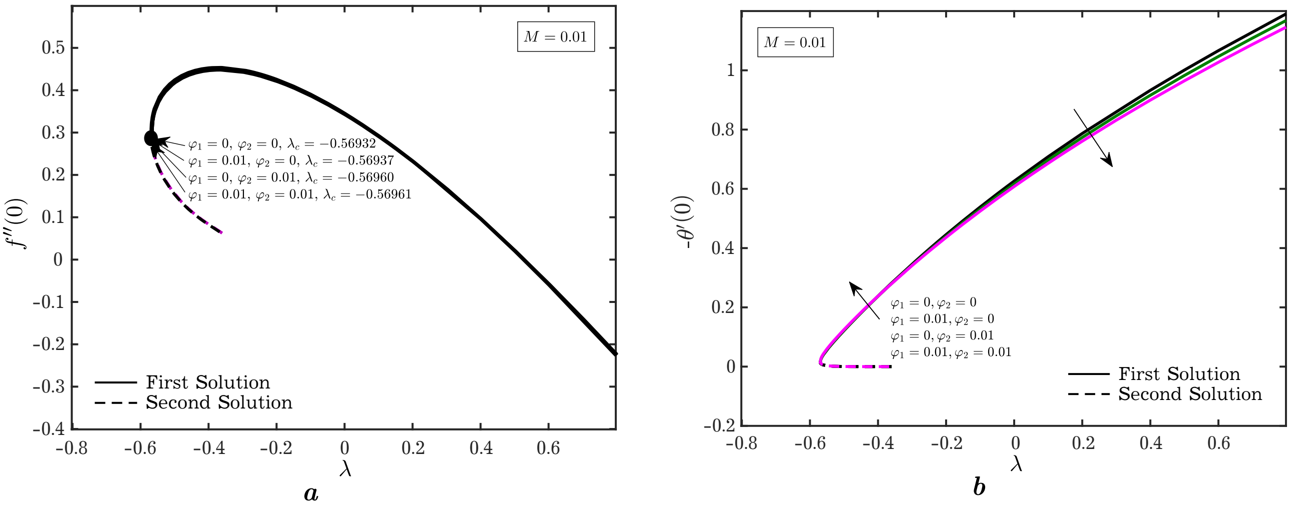

Fig. 2. (***a***) Effect of φ on reduced skin friction $f''(0)$. (***b***) Effect of φ on reduced heat transfer $-\theta'(0)$.

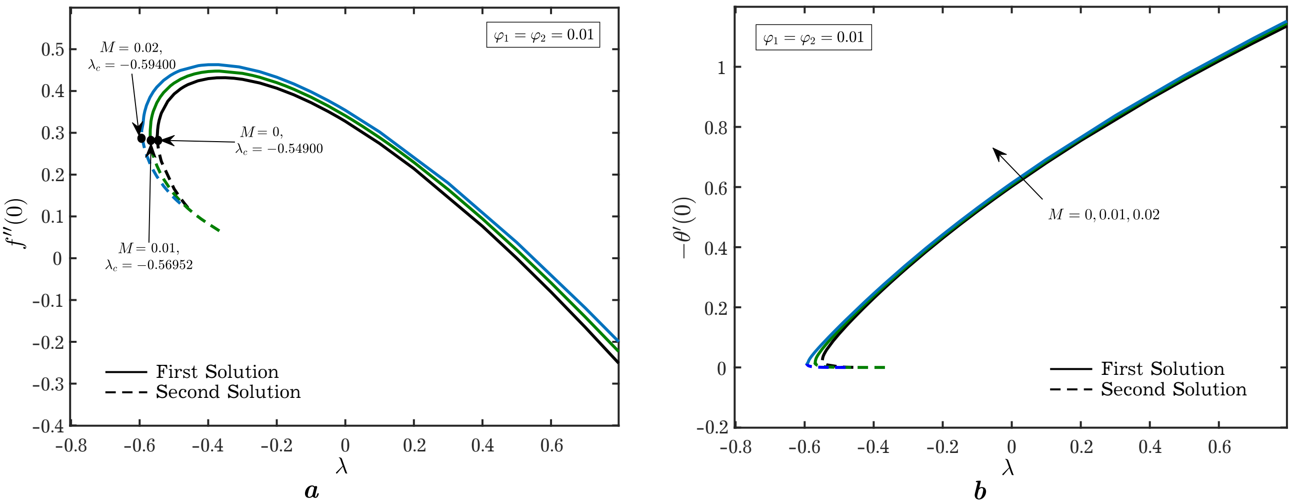

Fig. 3. (***a***) Effect of M on reduced skin friction $f''(0)$. (***b***) Effect of M on reduced heat transfer $-\theta'(0)$.

Next, Figure 3 illustrates the result of reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$ with $\varphi_1 = \varphi_2 = 0.01$ for different values of M $(0, 0.01, 0.02)$. The findings show that the unique solution exists when $\lambda > 0$; nevertheless, the dual solutions exist when $\lambda_c < \lambda < 0$; and there is no solution when $\lambda < \lambda_c$. The graph shows that the critical values, λ_c for $M = 0$, 0.01, 0.02 are $\lambda_c = -0.54900$, -0.56952 and -0.59400, respectively. It is noticed that the values of skin friction coefficient, $f''(0)$ and heat transfer rate, $-\theta'(0)$ slightly increase as M increases. Fundamentally, the magnetic field caused a drag or resistance Lorentz force that opposed the motion of the fluid and hence delayed the separation of the boundary layer. The delay in the separation of the thermal boundary layer increases with the increasing of M value.

Figure 4 depicts the influence of M on skin friction, C_f and Nusselt number, Nu_x with φ, where the value of φ_1 is the same as φ_2 at $\lambda = 0.2$. As example, $\varphi = 0.01$ means $\varphi_1 = 0.01$ and $\varphi_2 = 0.01$. Figure 4a shows that the value of C_f increases with the increase of M and decreases with the increase

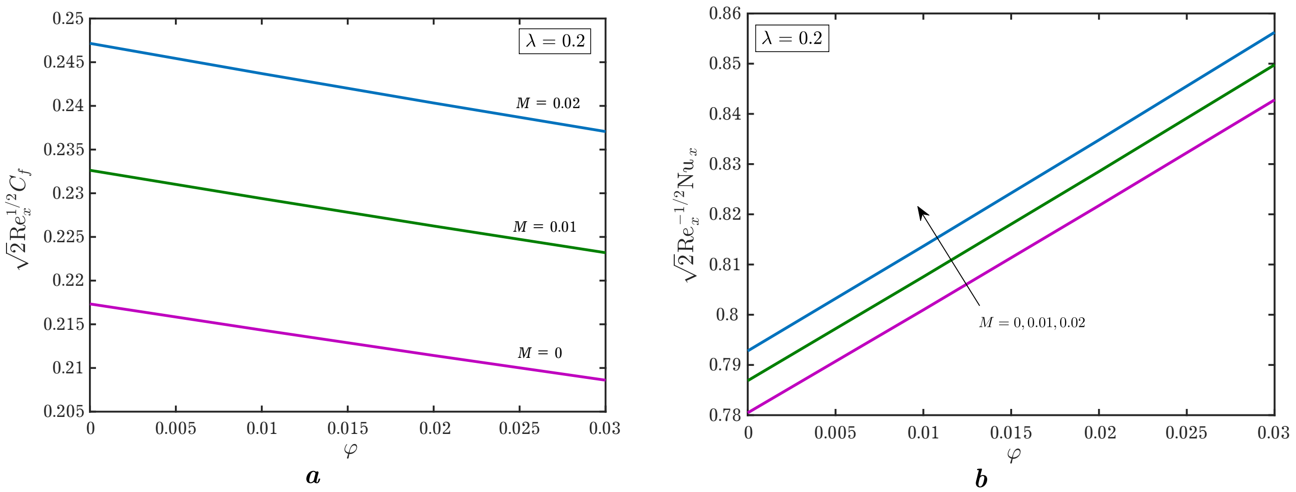

Fig. 4. (*a*) Effect of M on skin friction. (*b*) Effect of M on Nusselt number.

of φ value. On the other hand, the value of Nu_x increases with the increase of M and also increases with the increase of φ value in Figure 4*b*.

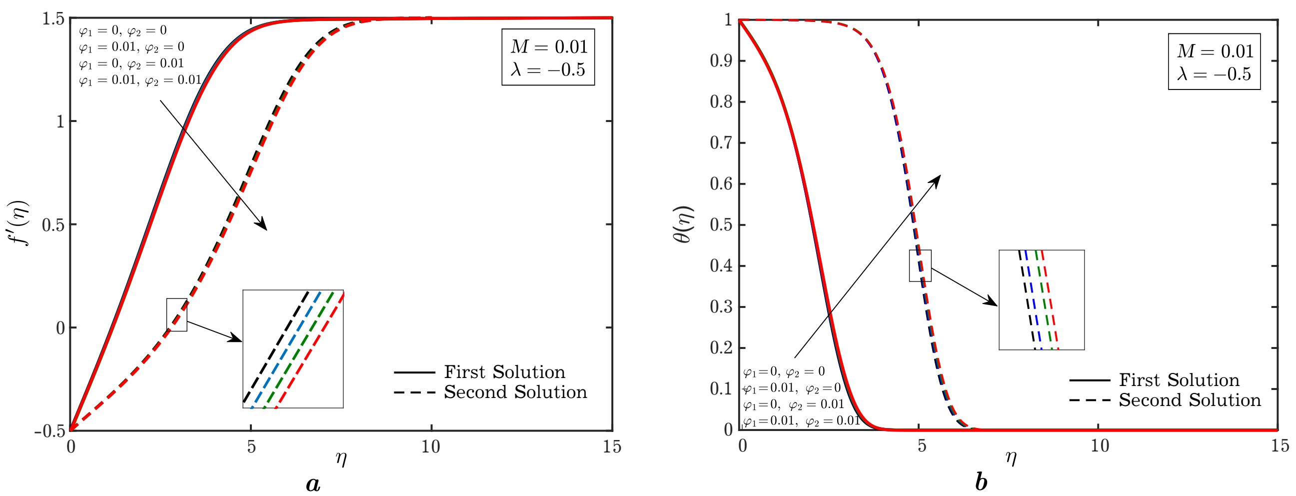

Fig. 5. (*a*) Effect of various φ on velocity profile. (*b*) Effect of various φ on temperature profile.

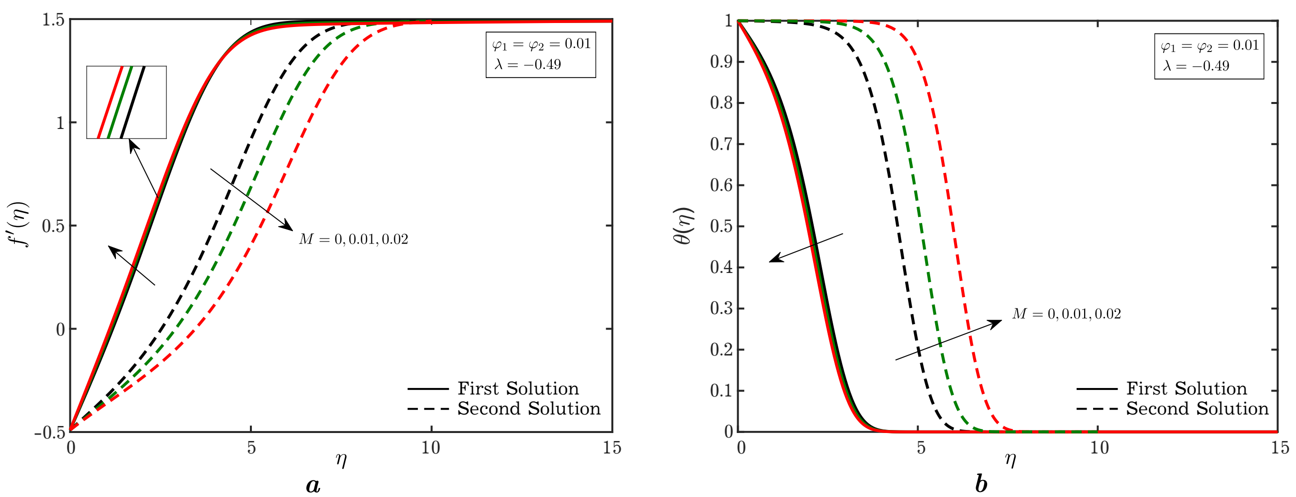

Fig. 6. (*a*) Effect of various M on velocity profile. (*b*) Effect of various M on temperature profile.

Figures 5–7 portray the variation of velocity profile, $f'(\eta)$ and temperature profile, $\theta(\eta)$ for several parameters which are φ, M and λ. It is observed that the duality solutions exist in all graphs of $f'(\eta)$ and $\theta(\eta)$. Figure 5 shows the effect of various value of φ on velocity profile, $f'(\eta)$ and temperature profile, $\theta(\eta)$. These graphs focus on the results of several different fluids which are viscous fluid ($\varphi_1 = \varphi_2 = 0$), SWCNT/water ($\varphi_1 = 0.01$, $\varphi_2 = 0$), MWCNT/water ($\varphi_1 = 0$, $\varphi_2 = 0.01$) and SWCNT-MWCNT/water or hybrid carbon nanotube ($\varphi_1 = 0.01$, $\varphi_2 = 0.01$) with the value of parameters of M and λ are fixed to $M = 0.01$ and $\lambda = -0.5$. It clearly shows that hybrid carbon nanotubes ($\varphi_1 = \varphi_2 = 0.01$) have higher velocity profile and lower temperature profile compared to the other fluids. Figure 6 illustrates the effect of various M on velocity and temperature profile with $\varphi_1 = \varphi_2 = 0.01$ and $\lambda = -0.49$. The value of magnetic parameter applied in both graphs are $M = 0, 0.01, 0.02$. It is observed that the graphs for the second solution have wider gap for each range of M parameter than the graphs for first solution. The velocity profile, $f'(\eta)$, increases for the first solution while decreases for the second solution as M is increased. The temperature profile, $\theta(\eta)$ for the first solution decreases as M is increased, while it increases for the second solution. Lastly, Figure 7 shows the influence of various value of λ on velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$. The value of velocity ratio parameter, λ used in both graphs are as follows, $\lambda = -0.50, -0.45, -0.40$ while φ_1 and φ_2 are both set to the value of 0.01 and $M = 0.01$. Furthermore, it is noticed that all the velocity and temperature profiles satisfied the boundary condition, Equation (8) asymptotically.

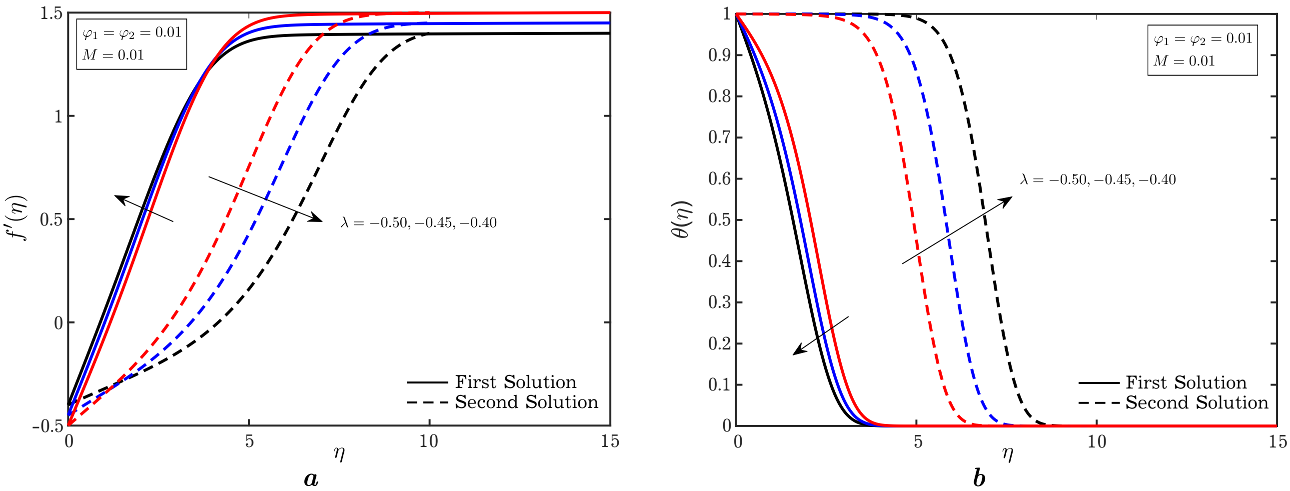

Fig. 7. (***a***) Effect of various λ on velocity profile. (***b***) Effect of various λ on temperature profile.

4. Conclusion

The boundary layer flow and heat transfer in hybrid carbon nanotube past a moving surface with magneto-hydrodynamic effect is discussed in this study. The numerical results are presented in specific values parameter graphically by using the bvp4c solver in the Matlab software. As a result, the findings of this study are as follows:

- When the plate moves in the same direction ($\lambda > 0$), unique solutions exist, while when the plate moves in the opposite direction ($\lambda_c < \lambda < 0$) dual solutions exist.
- The range of dual solutions widens as the M value increases.
- Hybrid carbon nanotube has higher rate of heat transmission, however it has lower value of skin friction compared to viscous fluid, SWCNT/water and MWCNT/water.
- Skin friction coefficient $f''(0)$ and heat transfer rate $-\theta'(0)$ values both slightly increase as M increases.
- Skin friction, C_f increases and Nusselt number, Nu_x decreases as M increases.
- The velocity and temperature profiles of hybrid carbon nanotube ($\varphi_1 = \varphi_2 = 0.01$) are higher than viscous fluid, SWCNT/water and MWCNT/water.

- With an increase in the value of M, the velocity profile of first solution increases while the second solution decreases and vice versa for temperature profile for the first and the second solution.
- Skin friction, C_f increases and Nusselt number decreases as M increases.

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Магнітогідродинамічний потік у граничному шарі та теплообмін гібридної вуглецевої нанотрубки над рухомою поверхнею

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У статті чисельно досліджено течію в граничному шарі та теплообмін гібридних вуглецевих нанотрубок над рухомою поверхнею з магнітогідродинамічним ефектом. Одностінні (SWCNT) та багатостінні (MWCNT) вуглецеві нанотрубки поєднано з водою як основною рідиною для утворення гібридних вуглецевих нанотрубок. Основні диференціальні рівняння в частинних похідних були перетворені в набір нелінійних звичайних диференціальних рівнянь за допомогою перетворення подібності, які потім були чисельно розв'язано за допомогою програмного забезпечення Matlab з використанням bvp4c. Вплив об'ємної частки наночастинок, магнітного параметра та параметра відношення швидкостей на профілі швидкості та температури, локальне поверхневе тертя та локальне число Нуссельта обговорюються та представлені у графічній формі. Результати показують, що подвійні розв'язки виникають, коли вільний потік і пластина рухаються в протилежних напрямках, а швидкість теплопередачі для гібридних вуглецевих нанотрубок вища, ніж для в'язкої рідини та вуглецевих нанотрубок.

Ключові слова: *гібридна вуглецева нанотрубка; рухома поверхня; граничний шар; магнітогідродинамічний ефект.*