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Discrete-event simulation for outpatient flow and emergency patient arrival in a haemodialysis unit

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Emergency cases among dialysis patients are uncertain and if these patients failed to obtain treatment within allocated treatment, it might risk their health conditions. In relation to that, we would like to accommodate outpatients together with the emergency patients in patient scheduling problem. Discrete-event simulation is used to estimate the outpatients flow based on the mean arrival rate, λ. A modified integer linear programming model is presented in this paper which highlighted on the patients' arrival time, patients' departure time and bed availability for emergency case. A rescheduling algorithm is also presented to accommodate existing outpatients and emergency patients. The results show that by rescheduling the existing outpatients and emergency patients in the system, there is no delaying for the outpatients' dialysis treatment. Hence, the emergency patients are able to accommodate in the system.

1. Introduction

Healthcare decision-makers frequently use discrete-event simulation (DES) as a tool to help them accomplish their research objectives and enhance workflow. To handle emergency patients with bed allocation, assess resource allocation demands, and investigate the intricate correlations among various system factors, DES can be utilized as a predictive tool [1, 2]. Patient arrival in the haemodialysis (HD) unit is further explored by using DES which utilized queuing theory. Queuing theory provides healthcare managers with insights into the cause of excessive wait times and the relationship between wait times and capacity [3]. According to Varga [4], DES is suitable when events occur at discrete points in time and take place instantly. In this study, DES is applicable if there is an emergency patient arrival or emergency situation occurs at the HD unit. For instance, when a patient is required to use the dialysis machine urgently or if a nurse is unable to attend to her duty. Therefore, this paper estimates the outpatient flow in the HD unit by using DES to accommodate emergency patients in the HD unit in case there is any. Subsection 2.1 presents queuing theory along with patient arrival rate, λ, and accommodates emergency patients. Along with that, rescheduling is then explored and examined the possibilities of accommodating emergency patients in the system without delaying outpatients' HD treatment.

2. Methodology

This section comprises methods used in this paper. Subsection 2.1 is about DES which is used to simulate the patient's arrival in the system. The parameter DOT in DES algorithm is the sum of the total duration of treatment of the patient. The value may vary according to the pre-dialysis and post-dialysis of each patient which will be discussed in Subsection 2.1. Along with that, a modified integer linear programming model is presented in Subsection 2.1. In Subsection 2.2, rescheduling algorithm will be discussed to emphasize the importance of considering emergency cases in the scheduling system.

## 2.1. Discrete-event simulation for outpatient flow in dialysis unit

Queuing theory is presented for both outpatient and emergency patient arrival at HD unit. Simulation for the outpatient arrival at the HD unit began and then followed by the emergency patient arrival. A M/M/1 queue model is proposed since the HD patient will use a dialyzer during the HD session. It is presumable that the queuing concept is founded on a circumstance of finite queuing. With a mean arrival rate, λ, as given in equation (1), patient arrival is based on the Poisson arrival distribution. $\lambda_1 = 0.1$ patient per minute is assumed because the information obtained for the pre-dialysis session from [5], the pre-dialysis session will be held within 3 minutes to 10 minutes. Therefore, $\lambda_1 = 0.1$ is generated and followed by for $\lambda_2 = 0.2$ and $\lambda_3 = 0.3$. The equation for patient arrival rate, a_i and service rate, s are as follows:

$$a_i = \frac{1}{\lambda} \tag{1}$$

$$s = \frac{1}{\mu} e^{\frac{t}{\mu}} \tag{2}$$

As a result, $a_i = 3.3$ minutes, 5 minutes, and 10 minutes is obtained, where $i = 1, 2, 3$. The patient arrival rate refers to the duration that patients need to wait to obtain their treatment. This paper examines how the patient arrival rate affects the total DOT of patients. Along with that, the mean service time, μ is selected from HD duration, $dur_{pt} = 183.15$ minutes and 246.10 minutes for both 3-hour and 4-hour sessions. This duration is selected as this study is focusing on 3-hour and 4-hour HD sessions. Meanwhile, the post-dialysis, ϕ_{pt} is exponentially distributed with probability density function computed with equation (2). $P = 20$ is selected which comprises 12 outpatients and 8 emergency patients (referred from [6]). This example has been chosen since the study was previously focused on inpatients at HD units and required bed allocation.

Scheduling process of outpatients and emergency patients are further discussed before exploring the DES algorithm. Figure 1 illustrates the flow and situation for outpatients and emergency patients' arrival at HD unit. The initial schedule indicates the outpatients that have been scheduled in the system. As shown in the flowchart, the arrival of emergency patients will be checked if there are any patients who arrived. When the emergency patients arrived, they need to be scheduled as soon as the treatment room is available by considering the bed availability in case the patients needed to be admitted to the ward and also by considering the physicians' availability. If there is no emergency patient arrived, treatments are continued as scheduled and the total duration of treatment is then computed. $Time$ refers to 720 minutes which is equal to 12 hours of HD unit operating hours. The schedule will be updated every time the treatment finishes and also when the emergency patients arrive in the system. In order to assign the emergency patients to the system, the other inpatients that have been scheduled on that day may be delayed or the usage of the treatment room on the day is extended. The scheduling process is
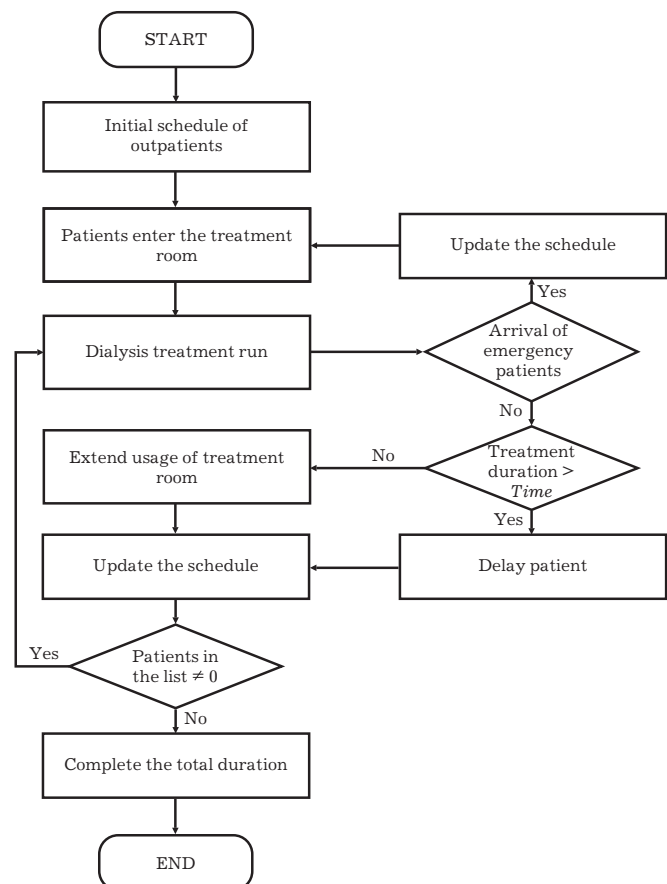
Fig. 1. The scheduling process for outpatients and emergency patients.

continued until there is no patient left on the list and hence, the total DOT is computed. However, if the treatment duration of the patient exceeds $Time$, then the patient will be considered a delayed patient. The patient will be receiving the treatment on the following day depending on the health condition and stage of CKD.

Table 1. Sets, indices, & parameter used in the modified ILP model.

Set & Indices	Description
A	Set of treatments
P	Set of patients
T	Set of periods
t	Index of periods where $t, \ldots, T$
p	Index of patients where $p, \ldots, P$
i	Index of the treatment where $i, \ldots, A$
Parameter	
dur_{pt}	HD treatment of patient p which excludes the setup and finish
j	Treatment access route
σ_{pt}	HD setup treatment of patient $p \in P$
ϕ_{pt}	Completion of HD treatment of patient $p \in P$
r_i^n	Nurse demand by treatment $i \in A$
r_i^r	Room demand by treatment $i \in A$
r_i^b	Bed demand by treatment $i \in A$
R_t^n	Nurse capacity by treatment $t \in T$
R_t^r	Room capacity by treatment $t \in T$
R_t^b	Bed capacity by treatment $t \in T$
arv_t	Arrival time of patient
dep_t	Departure time of patient
DOT	Total duration of treatment

Table 1 presents the set of notations involved in this modified model. The set of notations is modified according to our study case. The modified model is applicable for outpatient scheduling problem cases which focuses on patients' arrival and departure time. Meanwhile, for the emergency patient scheduling problem, one more constraint is added to the modified model which focuses on bed availability,

$$x_{pj} = \begin{cases} 1, & \text{if patient } p \in A \text{ assigned to vascular access route, } j, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Equation (3) is the decision variable for this optimisation model, where x_{pj} is 1 if patient $p \in A$ assigned to vascular access route, j, 0 if otherwise.

$$\min DOT = \sum_p^P \left[\sigma_{pt} + (dur_{pt} * x_{pj}) + \phi_{pt} \right]; \tag{4}$$

$$\text{subject to} \quad \sum_j x_{pj} = 1, \quad \forall p \in P; \tag{5}$$

$$DOT \leqslant dep_t, \quad \forall t \in T; \tag{6}$$

$$arv_t < dep_t, \quad \forall t \in T; \tag{7}$$

$$\sum_{j \in A} r_j^n * \sum x_{pj} \leqslant R_t^n, \quad \forall t \in T; \tag{8}$$

$$\sum_{j \in A} r_j^r * \sum x_{pj} \leqslant R_t^r, \quad \forall t \in T; \tag{9}$$

$$\sum_{j \in A} r_j^b * \sum x_{pj} \leqslant R_t^b, \quad \forall t \in T; \tag{10}$$

$$x_{pj} \in \{0, 1\}, \quad \forall p \in P, \quad j \in A. \tag{11}$$

The objective function (4) minimize the total DOT among patients during their treatment including pre-dialysis and post-dialysis sessions. Constraint (5) ensures that each treatment is scheduled exactly once. Constraint (6) is the total DOT constraint where the total DOT must be less than the departure time of the patient and constraint (7) ensures that the arrival time of the patient must be less than the departure time of the patient. Meanwhile, constraint (8) deals with nurse constraint where the demand for nurses does not exceed the nurse's capacity. The space constraint is handled by constraint (9) where the demand for the treatment rooms does not exceed the treatment room capacity. Constraint (10) deals with bed constraints where demand for the beds does not exceed the bed capacity Lastly, constraint (11) is the decision variable used in the model.

Table 2. Notations used in the DES Algorithm.

Set	Description
σ_{pt}	HD setup treatment (predialysis session) of patient $p \in P$
dur_{pt}	HD treatment of patient p which excludes the setup and finish
ϕ_{pt}	Completion of HD treatment (post-dialysis) of patient $p \in P$
arv_t	Arrival time of patient
R	Set of treatment rooms
P	Set of patients
T	Set of periods
t	Index of periods where $t, \ldots, T$
p	Index of patients where $p, \ldots, P$
r	Index of treatment rooms where $r, \ldots, R$
$DOT[p]$	Total duration of treatment for patient, p

Table 2 shows the notation and process of the DES algorithm, while Algorithm 1 presents DES of the outpatient scheduling process. Along with that, the patient list, $P = 20$ is presented in Table 3 for the DES later on. The highlighted patients are emergency patients.

Algorithm 1 Discrete-event simulation for patient arrival.

Require: σ_{pt}, dur_{pt}, ϕ_{pt};
Ensure: $DOT = \sum_p^N (\sigma_{pt} + dur_{pt} * x_{pj} + \phi_{pt})$;
 1: calculate mean values for σ_{pt}, dur_{pt} and ϕ_{pt};
 2: set arv_t along with mean values obtained at Step 1;
 3: generate patient arrival based on Poisson distribution;
 4: **while** $p \neq N$
 5:    generate for next λ;
 6: generate service rates for three stages of sessions for each patient;
 7: assign the patient according to available treatment room and assign the nurses for each patient;
 8: calculate DOT for each patient.

The simulation test will begin by calculating the mean value for pre-dialysis, dialysis, and post-dialysis for each patient. Patient arrival time will be set along with the mean value obtained in Step 1. Patient arrival rate, λ will be generated based on Poisson distribution and the λ is set up from $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, and $\lambda = 0.3$. After patient arrival is generated, service rates are generated for three stages of sessions for each patient. Following there, patients will be assigned according to the available treatment room and treatment nurses. The process will be ended after calculating DOT for each patient.

Table 3. Patient list for 20 patients, ($P = 20$).

P	$type_p$	$stage_p$	σ_{pt} (min)	dur_{pt} (min)	ϕ_{pt} (min)	DOT (min)
1	4	5	5	246.10	25	276.10
2	5	5	5	246.10	17	268.10
3	1	5	10	246.10	15	271.10
4	5	5	10	246.10	24	280.10
5	4	5	5	246.10	20	271.10
6	3	5	10	246.10	30	286.10
7	2	5	8	246.10	20	274.10
8	1	5	5	246.10	23	274.10
9	3	4	7	246.10	15	268.10
10	2	4	3	246.10	20	269.10
11	4	4	6	246.10	25	277.10
12	6	4	8	246.10	10	264.10
13	2	4	0	246.10	60	306.10
14	2	3	0	183.15	40	223.15
15	1	3	0	183.15	30	213.15
16	3	3	0	183.15	60	243.15
17	3	3	0	183.15	45	228.15
18	5	2	0	183.15	20	203.15
19	2	1	0	183.15	45	228.15
20	4	1	0	183.15	20	203.15

Table 3 consists of 20 patients where patient 1 to patient 12 is known as outpatients and 8 highlighted patients are emergency patients. These emergency patients do not have a post-dialysis session since they will be assigned to beds. This simulation test focuses on 20 patients only.

2.2. Emergency patient situation at HD unit

According to recent studies, patients missed an average of eight treatments a year for dialysis. Missed appointments typically happen when patients have higher amounts of hospitalization, ED visits, and death. Additionally, the consequences of missed treatments result in the buildup of too much fluid, electrolytes, and uremic toxins. This might affect patients' health and rescheduling is considered for those who missed their treatments.

Based on the weekly patient scheduling algorithm, there will be patients who might be leaving the system and also missing their HD treatments. Therefore, those patients who missed their treatments needed extra attention from the professionals. The algorithm is built with the aim to minimize the total DOT of the patient depending on the priority of the patient so that more patients can be treated at the beginning of the day. Higher priority will be given to patients who missed their treatments as the accumulation of excess fluids and toxins need to be taken into consideration. In that case, fewer priority patients can be rescheduled for the next time slot. After rearranging schedules, other patients in the system as per plan at the time $(T + 1)$ might be delayed. Patients that are already on the waiting list will remain in the system and there will be a calculation on how many times patients are delayed if it occurs. The process will be ended by calculating the total DOT of patients.

The rescheduling algorithm is efficient in the sense that it will schedule the patient that needs dialysis first and try to avoid moving those already on the waiting list because it will cause more delays which might affect the patient's health. Since it takes into account the urgency that is attached to patient priority, it will eliminate the possibility that urgent patients do not receive the appropriate treatment immediately.

Emergency patient arrival is categorized according to the Emergency Severity Index (ESI). According to [7], ESI is divided into five components: resuscitation (class 1), emergency (class 2), urgent

(class 3), less urgent (class 4), and non-urgent (class 5). The median time for the duration of treatment for these ESI is as shown in Table 4.

Based on the duration provided, only class 5 does not need bed allocation, meanwhile, the other 4 classes are required to stay overnight. However, this paper does not focus on overnight emergency cases, but the patients will be allocated to emergency beds. The emergency patients in this research are defined as those who needed immediate HD treatment as they have other illnesses which need to seek physicians for further treatment.

Table 4. Median time for the duration of treatment.

Class	Duration of the treatment (hours)
1	153.30 hours
2	169.00 hours
3	124.00 hours
4	34.17 hours
5	4.00 hours

Table 5. Notations used in the rescheduling algorithm.

Set	Description
d	HD treatment day
P_r	Outpatient patients
P_e	Emergency patients
P	Set of the patients
T	Set of periods
t	Index of periods where $t, \ldots, T$
p	Index of patients where $p, \ldots, P$
$system[p]$	If the value equal to 1, then patient, p is in the list, otherwise equal to 0
$dur_{pt}[p]$	Duration of HD treatment for the patient, p

Algorithm 2 Rescheduling algorithm.

Require: d, p, dur_{pt}, r;
Ensure: $d = d*$;
1: form again a list of patient that contains P_e ordered by waiting time;
2: P_r ordered according by arrival time and then from longest to shortest duration;
3: assign patient p to room r;
4: **while** $system[p] = 1$
5:   for $r = 1$, let $sum = 0$. If $system[p] = 1$, then calculate $sum+ = dur_{pt}[p]$;
6:   let $sumt[p] = sum$. If $sum > Time$, then break. Else repeat Step (ii);
7: **while** $system[p] = 0$
8:   repeat for $r + 1$;

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3. Computational result

Queuing theory experiments were performed based on discrete event simulation with inter-arrival rates set at $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, and $\lambda_3 = 0.3$. The results are shown in Tables 6–8 along with the treatment room and assigned nurses for each outpatient. Pre-dialysis events, which indicate pre-connection processes, started at predetermined intervals. Pre-dialysis events vary randomly in length from patient to patient, which usually reflects the real-world situation. Additionally, each patient's dialysis session is chosen at random from among three options that last between 183.15 and 246.10 minutes. In Table 6, for $\lambda_1 = 0.1$, 10 minutes gap is set between each patient. For the first round of HD treatment, patient 1 till patient 3 did not wait to obtain their treatment. However, starting from the 4th patient till the 12th patient, 10 minutes gap is implemented between the patients. Similarly, for Table 7, 5 minutes gap is set between the patients, and for Table 8, 3 minutes gap is set between each of them. These tables deduced from the findings that the DOT is mostly impacted by the length of both during and after dialysis

procedures but not by arrival time. Therefore, increasing the effectiveness of the during and after dialysis therapies could result in a decrease in DOT. The practices in post-dialysis like laboratory tests, radiology, and pharmacy may specifically be improved by adding personnel or equipment, and these improvements could result in a decrease in DOT but a correlating higher expenses. The administration should be made a significant impact on various preferences.

Table 6. DES result for $\lambda_1 = 0.1$.

P	σ_{pt} (min) Begin time	End time	dur_{pt} (min) Begin time	End time	ϕ_{pt} (min) Begin time	End time	Total DOT	Treatment room	Treatment nurse, T_n	Dispensing nurse, D_n
1	0.00	5.00	5.00	251.10	251.10	276.10	276.10	r_1	T_{n1}	D_{n2}
2	0.00	5.00	5.00	251.10	251.10	268.10	268.10	r_2	T_{n3}	D_{n4}
3	0.00	10.00	10.00	256.10	256.10	271.10	271.10	r_3	T_{n5}	D_{n6}
4	10.00	20.00	20.00	266.10	266.10	290.10	290.10	r_1	T_{n1}	D_{n2}
5	10.00	15.00	15.00	261.10	261.10	281.10	281.10	r_2	T_{n3}	D_{n4}
6	10.00	20.00	10.00	266.10	266.10	296.10	296.10	r_3	T_{n5}	D_{n6}
7	20.00	28.00	28.00	274.10	274.10	294.10	294.10	r_1	T_{n1}	D_{n2}
8	20.00	25.00	25.00	271.10	271.10	294.10	294.10	r_2	T_{n3}	D_{n4}
9	20.00	27.00	27.00	273.10	273.10	288.10	288.10	r_3	T_{n5}	D_{n6}
10	30.00	33.00	33.00	279.10	279.10	299.10	299.10	r_1	T_{n1}	D_{n2}
11	30.00	36.00	36.00	282.10	282.10	307.10	307.10	r_2	T_{n3}	D_{n4}
12	30.00	38.00	38.00	284.10	284.10	294.10	294.10	r_3	T_{n5}	D_{n6}

Table 7. DES result for $\lambda_2 = 0.2$.

P	σ_{pt} (min) Begin time	End time	dur_{pt} (min) Begin time	End time	ϕ_{pt} (min) Begin time	End time	Total DOT	Treatment room	Treatment nurse, T_n	Dispensing nurse, D_n
1	0.00	5.00	5.00	251.10	251.10	276.10	276.10	r_1	T_{n1}	D_{n2}
2	0.00	5.00	5.00	251.10	251.10	268.10	268.10	r_2	T_{n3}	D_{n4}
3	0.00	10.00	10.00	256.10	256.10	271.10	271.10	r_3	T_{n5}	D_{n6}
4	5.00	15.00	15.00	261.10	261.10	285.10	285.10	r_1	T_{n1}	D_{n2}
5	5.00	10.00	10.00	256.10	256.10	276.10	276.10	r_2	T_{n3}	D_{n4}
6	5.00	15.00	15.00	261.10	261.10	291.10	291.10	r_3	T_{n5}	D_{n6}
7	10.00	18.00	18.00	264.10	264.10	284.10	284.10	r_1	T_{n1}	D_{n2}
8	10.00	15.00	15.00	261.10	261.10	284.10	284.10	r_2	T_{n3}	D_{n4}
9	10.00	17.00	17.00	263.10	263.10	278.10	278.10	r_3	T_{n5}	D_{n6}
10	15.00	18.00	18.00	264.10	264.10	284.10	284.10	r_1	T_{n1}	D_{n2}
11	15.00	21.00	21.00	267.10	267.10	292.10	292.10	r_2	T_{n3}	D_{n4}
12	15.00	23.00	23.00	269.10	269.10	279.10	279.10	r_3	T_{n5}	D_{n6}

The average DOT for all average arrival rates in this study ranged from 278.46 minutes to 288.27 minutes, according to Table 9. The mean DOT is minimal at $\lambda_3 = 0.3$, and the standard deviation is 6.14 minutes. By arranging patient arrivals at close intervals to prevent system congestion, SA for an HD unit suggests that the service can be improved to consume fewer resources during the pre-dialysis period. Consequently, thorough planning of the HD unit's activities is essential since it may result in both cost savings and increased system efficiency. The findings imply that a strategic planning technique could enhance patients' DOT in HD units. The results are consistent with other studies that have shown the value of SA in the planning of HD processes.

Table 9. Comparison of DES results.

Mean arrival rate (λ_i)	Mean DOT	Standard deviation
0.1	288.27	11.33
0.2	280.77	7.04
0.3	278.46	6.14

Table 10 shows the emergency patients' arrival at HD unit. Emergency patients are able to accommodate without causing delays to the scheduled patient list. It is assumed that the emergency patients

Table 8. DES result for $\lambda_3 = 0.3$.

P	σ_{pt} (min) Begin time	σ_{pt} (min) End time	dur_{pt} (min) Begin time	dur_{pt} (min) End time	ϕ_{pt} (min) Begin time	ϕ_{pt} (min) End time	Total DOT	Treatment room	Treatment nurse, T_n	Dispensing nurse, D_n
1	0.00	5.00	5.00	251.10	251.10	276.10	276.10	r_1	T_{n1}	D_{n2}
2	0.00	5.00	5.00	251.10	251.10	268.10	268.10	r_2	T_{n3}	D_{n4}
3	0.00	10.00	10.00	256.10	256.10	271.10	271.10	r_3	T_{n5}	D_{n6}
4	3.33	13.33	13.33	259.43	259.43	283.43	283.43	r_1	T_{n1}	D_{n2}
5	3.33	8.33	8.33	254.43	254.43	274.43	274.43	r_2	T_{n3}	D_{n4}
6	3.33	13.33	13.33	259.43	259.43	289.43	289.43	r_3	T_{n5}	D_{n6}
7	7.06	15.06	15.06	261.16	261.16	281.16	281.16	r_1	T_{n1}	D_{n2}
8	7.06	12.06	12.06	258.16	258.16	281.16	281.16	r_2	T_{n3}	D_{n4}
9	7.06	14.06	14.06	260.16	260.16	275.16	275.16	r_3	T_{n5}	D_{n6}
10	10.39	13.39	13.39	259.49	259.49	279.49	279.49	r_1	T_{n1}	D_{n2}
11	10.39	16.39	16.39	262.49	262.49	287.49	287.49	r_2	T_{n3}	D_{n4}
12	10.39	18.39	18.39	264.49	264.49	274.49	274.49	r_3	T_{n5}	D_{n6}

Table 10. Emergency patients accommodation based on $\lambda_3 = 0.3$.

P	σ_{pt} (min) Begin time	σ_{pt} (min) End time	dur_{pt} (min) Begin time	dur_{pt} (min) End time	ϕ_{pt} (min) Begin time	ϕ_{pt} (min) End time	Total DOT	Treatment room	Treatment nurse, T_n	Dispensing nurse, D_n
1	0.00	5.00	5.00	251.10	251.10	276.10	276.10	r_1	T_{n1}	D_{n2}
2	0.00	5.00	5.00	251.10	251.10	268.10	268.10	r_2	T_{n3}	D_{n4}
3	0.00	10.00	10.00	256.10	256.10	271.10	271.10	r_3	T_{n5}	D_{n6}
4	3.33	13.33	13.33	259.43	259.43	283.43	283.43	r_1	T_{n1}	D_{n2}
5	3.33	8.33	8.33	254.43	254.43	274.43	274.43	r_2	T_{n3}	D_{n4}
6	3.33	13.33	13.33	259.43	259.43	289.43	289.43	r_3	T_{n5}	D_{n6}
7	7.06	15.06	15.06	261.16	261.16	281.16	281.16	r_1	T_{n1}	D_{n2}
8	7.06	12.06	12.06	258.16	258.16	281.16	281.16	r_2	T_{n3}	D_{n4}
9	7.06	14.06	14.06	260.16	260.16	275.16	275.16	r_3	T_{n5}	D_{n6}
10	10.39	13.39	13.39	259.49	259.49	279.49	279.49	r_1	T_{n1}	D_{n2}
11	10.39	16.39	16.39	262.49	262.49	287.49	287.49	r_2	T_{n3}	D_{n4}
12	10.39	18.39	18.39	264.49	264.49	274.49	274.49	r_3	T_{n5}	D_{n6}
13	–	–	–	246.10	246.10	306.10	306.10	r_1, b_1	T_{n1}	D_{n2}
14	–	–	–	183.15	183.15	223.15	223.15	r_2, b_2	T_{n3}	D_{n4}
15	–	–	–	183.15	183.15	213.15	213.15	r_3, b_3	T_{n5}	D_{n6}
16	–	–	–	183.15	183.15	243.15	243.15	r_1, b_4	T_{n1}	D_{n2}
17	–	–	–	183.15	183.15	228.15	228.15	r_2, b_5	T_{n3}	D_{n4}
18	–	–	–	183.15	183.15	203.15	203.15	r_3, b_6	T_{n5}	D_{n6}
19	–	–	–	183.15	183.15	228.15	228.15	r_1, b_7	T_{n1}	D_{n2}
20	–	–	–	183.15	183.15	203.15	203.15	r_2, b_8	T_{n3}	D_{n4}

Note: $b_1, \ldots, b_8$ are bed allocation for the emergency patients. "–" shows that there are no sessions occurring for the patients.

are from the delayed patient list, therefore the scheduled patients will carry out their treatment as usual. In case, there is an emergency patient arrives at the HD unit, then it is able to assign a treatment room with a bed and nurses for their further treatment. A modified model from the outpatient scheduling problem is used by adding a constraint for bed availability for this emergency case. The emergency patients' arrival is highlighted in Table 10.

Table 11 presents the results obtained for the scheduled patients for the rescheduling cases. This paper developed the rescheduling algorithm as mentioned in Subsection 2.2. First, the procedure starts by grouping them based on the arrival time and then sorting them according to the descending order of dialysis duration. Backtracking heuristic (BH) algorithm is used to allocate the nurse's placement to the patient after acquiring the patient's schedule. The details for BH algorithm can referred to our previous study [8].

Table 11. Schedule obtained for rescheduling.

r	P	$stage_P$	arv_t	dur_{pt} (min)	Treatment nurse, T_n	Dispensing nurse, D_n
1	1	5	8.00 am	246.10	T_{n1}	D_{n2}
	13	4	8.05 am	246.10	T_{n3}	D_{n4}
	16	3	8.15 am	183.15	T_{n1}	D_{n2}
	19	1	8.17 am	183.15	T_{n3}	D_{n4}
	4	5	8.35 am	246.10	T_{n1}	D_{n2}
	8	5	8.40 am	246.10	T_{n3}	D_{n4}
2	2	5	8.00 am	246.10	T_{n5}	D_{n6}
	14	3	8.07 am	183.15	T_{n7}	D_{n8}
	17	3	8.20 am	183.15	T_{n5}	D_{n6}
	20	1	8.25 am	183.15	T_{n7}	D_{n8}
	5	5	8.37 am	246.10	T_{n5}	D_{n6}
	9	4	8.43 am	246.10	T_{n7}	D_{n8}
	10	4	8.44 am	246.10	T_{n5}	D_{n6}
3	3	5	8.00 am	246.10	T_{n9}	D_{n10}
	15	3	8.10 am	183.15	T_{n11}	D_{n12}
	18	2	8.25 am	183.15	T_{n9}	D_{n10}
	6	5	8.45 am	246.10	T_{n11}	D_{n12}
	7	5	9.00 am	246.10	T_{n9}	D_{n10}
	11	4	9.15 am	246.10	T_{n11}	D_{n12}
	12	4	9.16 am	246.10	T_{n9}	D_{n10}

4. Conclusion

This chapter studied the patient arrival based on DES. As hospitals and clinics face ongoing competition for their service, they must afford to provide fast and efficient service to attract new patients and retain their existing patients. Therefore, patient arrival at HD unit is simulated based on DES by utilizing queuing concept. The rescheduling process is implemented since there are missed treatments/appointments among dialysis patients. Through rescheduling, those who missed their treatments will obtain their treatments immediately with the guidance of professionals.

Based on the computational results, it can be concluded that post-dialysis influenced the total duration of treatment among dialysis patients. On the other hand, the queuing theory of patient flow for a typical HD unit is modeled to reflect the real case of patient arrival. Finally, the findings imply that a small number of patients requiring emergency HD care can be accommodated during dialysis sessions lasting roughly three to four hours without substantially disrupting the system. Delayed emergency cases can be addressed by making slight alterations to the schedule of outpatients anticipated to arrive within a short period of time.

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Дискретно-подієве моделювання амбулаторного потоку та екстреного надходження пацієнтів у відділення гемодіалізу

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Екстрені випадки серед пацієнтів, які знаходяться на діалізі, невідомі, і якщо ці пацієнти не зможуть отримати лікування в межах призначеного лікування, то це може загрожувати їх здоров'ю. У зв'язку з цим ми хотіли б розмістити амбулаторних пацієнтів разом із пацієнтами невідкладної допомоги при проблемі планування пацієнтів. Дискретно-подієве моделювання використовується для оцінки потоку амбулаторних пацієнтів на основі середньої швидкості прибуття λ. У цій статті подано модифіковану модель цілочисельного лінійного програмування, яка враховує час прибуття пацієнтів, час від'їзду пацієнтів і наявність ліжка для екстрених випадків. Також подано алгоритм перепланування для розміщення наявних амбулаторних пацієнтів та пацієнтів екстреної допомоги. Результати показують, що перепланування існуючих амбулаторних пацієнтів і пацієнтів екстреної допомоги в системі не передбачає затримки амбулаторного діалізного лікування. У такий спосіб екстрені пацієнти можуть розміститися в системі.

Ключові слова: *дискретно-подієве моделювання; аварійний; перепланування; потік пацієнтів.*