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Stagnation-point flow and heat transfer over an exponentially shrinking/stretching sheet in porous medium with heat generation

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This study seeks to examine the fluid flow at the stagnation point over an exponentially shrinking and stretching sheet in a porous medium. This study also investigates the heat transfer rate in the presence of heat generation. By using the appropriate similarity transformation, we obtained ordinary differential equations (ODEs) that are reduced from the governing system of partial differential equations (PDEs). These resulting equations are subjected to new boundary conditions and solved numerically by using BVP4C in MATLAB software. The effects of the parameters involved in this study are summarized and thoroughly discussed: the skin friction coefficient, local Nusselt number, velocity profile, and temperature profile obtained. The analysis is done by using graphical and tabular data. The observed parameters are the permeability parameter K and the heat generation parameter Q towards shrinking/stretching parameter λ. It is found that a dual solution exists for $\lambda < 0$ (shrinking case), whereas the solution is unique for $\lambda > 0$ (stretching case). The analysis reveals that with heat generation being increased, the skin friction coefficient is constant. However, it increases when permeability increases. The local Nusselt number decreases with heat generation being increased. However, it increases when the permeability increases.

Keywords: *stagnation-point flow; porous medium; stretching/shrinking sheet; heat generation.*

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1. Introduction

The flow due to a stretching sheet often occurs in engineering processes, for example in manufacturing processes such as glass fiber drawing, crystal drawing, plastic extrusion, paper production, etc. The first to give a similar solution in closed analytical form for two-dimensional flow caused by stretching of a plate was Crane [1]. Since then, many authors and researchers have shown interest in this kind of problem. The problems that had been studied in the past few years on boundary layer flow and heat transfer in the region of the stagnation point on a stretching surface are mostly related to the condition where the sheet is assumed to stretch on its own plane with a velocity proportional to the distance from the stagnation point. This type of problem has been considered in the papers by Mahapatra and Gupta [2] for magnetohydrodynamic (MHD) flow, Nazar et al. [3] for micropolar fluid, Reza and Gupta [4] and Lok et al. [5] for oblique viscous flow and Lok et al. [6] for oblique micropolar fluid. Lately, interest in the boundary layer flow due to a shrinking sheet inevitably has attracted the attention of some researchers. Contrary to the stretching sheet, the velocity on the boundary for the shrinking case is towards a fixed point (Miklavcic and Wang, [7]). There are two conditions that the flow towards the shrinking sheet is likely to exist, whether an adequate suction on the boundary is

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imposed (Miklavcic and Wang, [7]) or a stagnation flow is added (Wang, [8]) so that the vorticity of the shrinking sheet is confined in the boundary layer.

Wang [8] was the first researcher that initiates the discussion of the fluid flow towards a shrinking sheet. He came out with the idea of using a similarity transform method to reduce the Navier–Stokes equations to a set of non-linear ordinary differential equations which are then integrated numerically. In his studies, both two-dimensional and axisymmetric stagnation flows are considered. It is found that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. After that, Fan [9] try to find out how the unsteady stagnation flow and heat transfer works towards a shrinking sheet. In their findings, it is seen that for a shrinking sheet, the velocity profiles approach faster those of the steady flow than for the stretching sheet. The transition from initial unsteady flow to the final steady-state flow takes place smoothly, that is without any singularity through the flow and heat transfer. It is also shown that the transition from initial unsteady flow to the final steady flow takes place smoothly for both physical quantities involved, which is skin friction coefficient and the Nusselt number.

However different findings can be found for the fluid flow over an exponentially shrinking sheet. As been studied by Bhattacharyya [10], the analysis of the conditions for the existence of steady boundary layer flow due to exponential shrinking of the sheet showed that when the mass suction parameter exceeds a certain critical value, steady flow is possible. The dual solutions for velocity and temperature distributions are obtained. With increasing values of the mass suction parameter, the skin friction coefficient increases for the first solution and decreases for the second solution. In extension of that, Bachok et al. [11] investigated boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid. The study reveals that the range of the stretching/shrinking parameter where the similarity solution exists is larger for an exponentially stretching/shrinking sheet compared with the linear stretching/shrinking case. Similar to other findings, it is found that that the solutions for a shrinking sheet are non-unique, different from a stretching sheet.

Heat transfer is one of the factors people would look over in fluid flow cases. Heat transfer is related to the fluid flow rate, specifically the velocity of the fluid flow. In the meantime, the presence of heat generation as a parameter somehow affects the fluid flow and the heat transfer over the sheet. In the paper by Pal and Mandal [12], they studied the mixed convection-radiation on the stagnation-point flow of nanofluids over a stretching/shrinking sheet in a porous medium with heat generation and viscous dissipation. The studies showed that with decreasing the value of the effective Prandtl number, there is a decrease in the value of the temperature profiles for fixed values of η for stretching as well as shrinking sheets. This means the fluid temperature can be controlled by controlling the effective Prandtl number, in other words, the temperature can be well controlled by the thermal radiation parameter in this case. Abdollahzadeh et al. [13] studied regarding stagnation-point flow of nanofluids towards a stretching sheet through a porous medium with heat generation. It was found that increasing the nanofluid concentration increases the heat transfer rate and the skin friction coefficient at the surface when the velocity ratio parameter is larger than unity ($C > 1$).

An addition of a porous medium to the problem will affect the rate of heat transfer as the velocity of the fluid flows could be affected, depending on the type of porous medium used. Decreasing the porosity of the porous medium (increasing the value of K) will widen the range of c, the shrinking sheet in which the solution exists [14]. Vyas and Srivastava [15] explored that when the sheet is placed at the bottom of a porous medium, it offers resistance to fluid traversal inside the porous matrix and contributes in some way to fluid stability, hence one may expect a rather low suction rate as compared to the case of clear fluid situation. They also found that in the case of the porous medium one requires lesser suction values to contain the vorticity in the boundary layer depending upon the porous material chosen. Whereas Pal and Mandal [12] found that the skin friction coefficient increases with an increase in permeability of porous medium for shrinking sheet, but decreases for stretching sheet. Yasin et al. [16] analysed the boundary layer flow and heat transfer past a permeable shrinking surface embedded in a porous medium with a second-order slip. In their findings, it is found that applying the slip condition alongside the presence of a porous medium increases the range of solutions. Also, the skin friction coefficient is higher for the no-slip condition compared to that of with slip condition. Hong et al. [17] did a study regarding numerical study of nonlinear mixed convection inside stagnation-point flow over surface-reactive cylinder embedded in porous media. The study reveals that the porous system deviates from local thermal equilibrium at higher Reynolds numbers and mixed convection parameters. Khan et al. [18] found that the modified porosity parameter declines the shear stress,and microrotation gradient in the stable branch outcome and augments the unstable branch outcome, while the Nusselt number decelerates in both branches of outcomes. This result was obtained from the study of the buoyancy effect on the stagnation-point flow of a hybrid nanofluid toward a vertical plate in a saturated porous medium. A research paper by Wahid et al. [19] studied the unsteady mixed convective stagnation-point flow of hybrid nanofluid in a porous medium, they discovered that the higher value of the first and second resistant parameters due to porous media concludes that these parameters aid in improving the heat transfer and skin friction rates. This investigation has proven the hybrid nanofluid's ability to reinforce heat transfer with the embedment of a porous medium.

Looking at these studies, it can be seen that many types of cases have been going through, for example, using different effects, liquids, mediums, and methods to prove or get over the curiosities [22–25]. Hence, the aim of this paper is also to modify and enhance the previous research to widen people's knowledge and interest. This present paper will explore fluid flow at stagnation-point over an exponentially permeable shrinking/stretching sheet in a porous medium. This paper also will investigate heat transfer in the presence of heat generation.

2. Mathematical Formulation

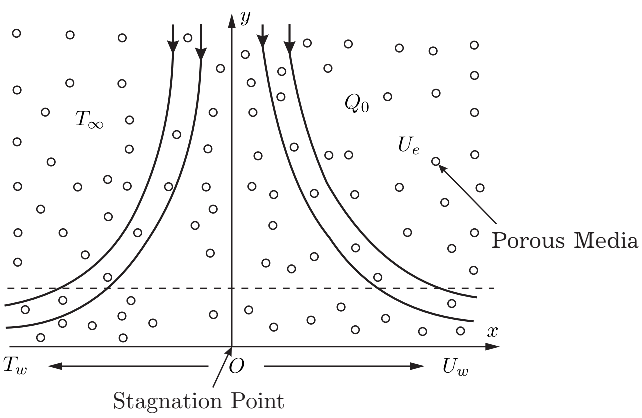

Fig. 1. A sketch of physical model and coordinate system.

We consider a steady and two-dimensional stagnation-point flow over an exponentially permeable shrinking/stretching sheet of an incompressible viscous fluid in porous medium with the presence of heat generation as shown in Figure 1.

The shrinking/stretching sheet velocity is U_w, straining velocity is U_e, T_w is the temperature at the surface and T_∞ is the temperature at surrounding where it is assumed to be constant. Under the assumptions, the governing equation for the current study can be written as:

- continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

- momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\nu}{K_1}(U_e - u), \tag{2}$$

- energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_o}{\rho c_p}(T - T_\infty) \tag{3}$$

with the boundary conditions:

$$\begin{gathered} v = v_w, \quad u = U_w(x), \quad T = T_w = T_\infty + T_o\, e^{\frac{x}{2L}} \quad \text{at} \quad y = 0, \\ u \to U_e(x), \quad T \to T_\infty, \quad \text{as} \quad y \to \infty, \end{gathered} \tag{4}$$

where the velocity components in the x direction is u and y direction is v. The shrinking/stretching velocity is $U_e(x) = ae^{\frac{x}{L}}$, ν is the kinematic viscosity, α is the thermal diffusivity, ρ is the fluid density, v_w is the mass transfer velocity, c_p is the specific heat, L is the characteristic length of the sheet, c is the shrinking/stretching velocity rate where $c < 0$ for shrinking sheet and $c > 0$ for stretching sheet, $a > 0$ is the straining velocity rate and Q_o is the dimensional heat generation coefficient.

The governing equations (1) to (3) associated with the boundary conditions (4) are transformed to ODEs by applying the similarity transformations as shown below:

$$\psi = \sqrt{2\nu \mathrm{La}} f(\eta) e^{\frac{x}{2L}}, \quad \eta = y\sqrt{\frac{a}{2\nu L}} e^{\frac{x}{2L}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{5}$$

$$u = af'(\eta)e^{\frac{x}{L}}, \quad v_w = -\sqrt{\frac{\nu a}{2L}} e^{\frac{x}{2L}} s, \quad v = -\sqrt{\frac{\nu a}{2L}} (\eta f'(\eta) + f(\eta)) e^{\frac{x}{2L}}, \tag{6}$$

where ψ and η are the stream function and similarity variable, respectively. The stream function is always be denoted as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, that comply with Eq. (1) equivalently. After fransformations, the momentum equation (2) and energy equation (3) thus reduced to the following ordinary differential equation:

$$f''' + ff'' + 2 - 2f'^2 + 2K(1 - f') = 0, \tag{7}$$

$$\frac{1}{\mathrm{Pr}} \theta'' + f\theta' - f'\theta + Q\theta = 0, \tag{8}$$

where prime symbol denotes differentiation with respect to η, and $K = \frac{\nu L}{K_1 a e^{\frac{x}{L}}}$, $\mathrm{Pr} = \frac{\nu}{\alpha}$ are permeability and Prandtl number parameters respectively. The boundary conditions are

$$\begin{gathered} f(0) = s, \quad f'(0) = \lambda, \quad \theta(0) = 1, \\ f'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad \text{as} \quad \eta \to \infty. \end{gathered} \tag{9}$$

In this study, the concerned physical quantities are skin friction coefficient C_f and the local Nusselt number Nu_x, which can respectively be written as

$$C_f = \frac{\tau_w}{\rho U_e^2}, \quad \mathrm{Nu}_x = \frac{q_w}{(T_w - T_\infty)}, \tag{10}$$

where τ_w and q_w are wall shear stress and local heat flux respectively which are declared as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{11}$$

with μ is dynamic viscosity.

Substituting (5) into Eqs. (10) and (11), we obtain

$$C_f = \frac{f''(0)}{2\,\mathrm{Re}^{1/2}}, \quad \mathrm{Nu}_x = -\frac{\mathrm{Re}^{1/2}}{2} \theta'(0), \tag{12}$$

where $\mathrm{Re} = \frac{(U_e L)}{\nu}$ is the Reynolds number.

3. Results and discussion

The system of ordinary differential equations (7) and (8) subject to the boundary conditions (9) are solved numerically by using BVP4C function in MATLAB. The numerical computations were carried out by entering distinct values of parameters into the problem and the results will be depicted in the form of graph and table. The velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$, skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$ for different parameters K, S, Pr, Q and λ are obtained by applying BVP4C method. Parameters K, S, Pr, Q and λ are permeability parameter, suction parameter, Prandtl number, heat generation parameter and shrinking/stretching parameter respectively. The results obtained are discussed further in detail by referring to the graphs and the tabulated data. In addition, comparison of the current study to Bachok et al. [11] for the case of stretching/shrinking sheet is shown in Table 1. All the comparisons have a number of similarities and therefore, this result has confirmed our confidence in our numerical approach.

Table 1 presents the comparison of present study to Bachok et al. [11] for skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$.

Table 1. Comparison values of $f''(0)$ and $-\theta'(0)$ for $s = Q = 0$ with $\mathrm{Pr} = 6.2$.

λ	Bachok et al. [11]		Present Results	
-0.5	2.1882	0.6870	2.11817	0.68700
0	1.6872	1.7148	1.68722	1.71477
0.5	0.9604	2.4874	0.96042	2.487422

The variation of the skin friction coefficient $f''(0)$ against λ for various values of parameter K is depicted in Figure 2. From the figure, we observe that up to critical value of λ_c, the solution exists. The dual solution exists for shrinking surface, where $\lambda_c < \lambda < 0$, whereas the solution is unique for stretching parameter $\lambda > 0$, λ_c is the critical value of λ. It can be seen that the skin friction coefficient $f''(0)$ increases as the value of K increases. From our computation, the critical points are $\lambda_{c1} = (-2.07065, 1.49525)$, $\lambda_{c2} = (-2.18130, 1.62321)$ and $\lambda_{c3} = (-2.23650, 1.70095)$. We can interpret that the increase of K, meaning that with the lower porosity of the porous medium, will result in widening the range of existence of solution to the governing system of differential equation considered. Also, there is no solution for $\lambda < \lambda_c$ can be observed. This is due to boundary layer separation.

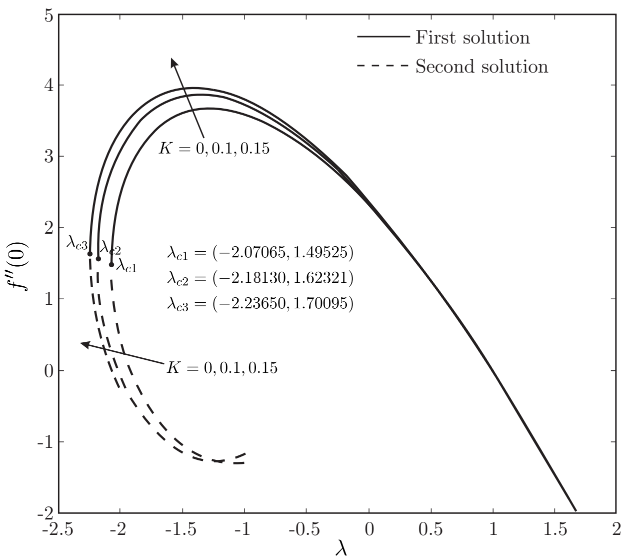

Fig. 2. Variation of the skin friction coefficient $f''(0)$ of $K = 0, 0.1, 0.15$ when $\mathrm{Pr} = 2$, $Q = 1$ and $S = 1$.

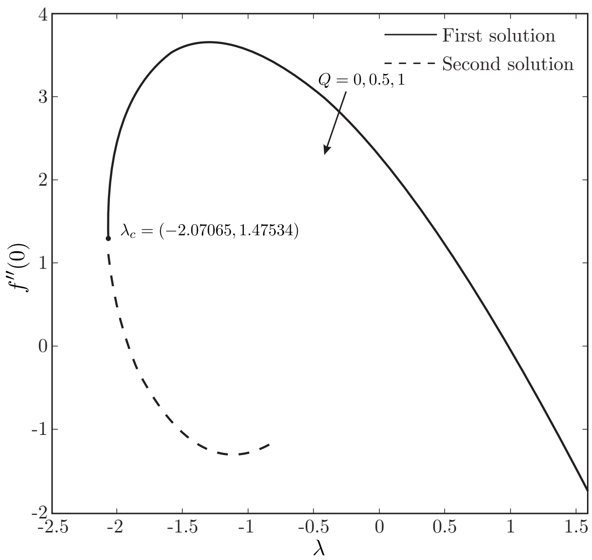

Fig. 3. Skin friction coefficient $f''(0)$ of $Q = 0, 0.5, 1$ at $\mathrm{Pr} = 1$, $K = 0$ and $S = 1$.

The graphical result of skin friction coefficient $f''(0)$ for different values of heat generation parameter Q where $\mathrm{Pr} = 1$, $K = 0$ and $S = 1$ are shown in Figure 3. From the figure, it can be observed that the skin friction lies on the same graph for different values of heat generation Q. This shows that the heat generation parameter does not affect the skin friction parameter. It comes to no surprise since the heat generation parameter is absent in the equation (14). Thus, the presence or absence of the heat generation parameter does not give any effect to the skin friction coefficient.

Figure 4 presents the influence of parameter K towards the rate of heat transfer at the surface. We can observe that the local Nusselt number $-\theta'(0)$ increases when K increases. When permeability increases, it allows easy flow of the fluid. Hence, the chances for transfer of heat to happen is high. The figure shows that dual solution exists for the shrinking surface ($\lambda < 0$), but unique for stretching surface ($\lambda > 0$).

Figure 5 presents the local Nusselt number of $Q = 0, 0.5, 1$ at $\Pr = 1$, $K = 0$ and $S = 1$. It is apparent from Figure 5 that, as the heat generation parameter Q increases with the presence of suction, the rate of heat transfer is decreases. According to Ismail et al. [20] in their paper concerning the stagnation-point flow and heat transfer over an exponentially shrinking sheet with heat generation, they found that even with the presence or without suction parameter, as the heat generation increases, the rate of heat transfer is decreases. Increasing the heat generation parameter means the rise of temperature to the fluid as supported by Fatunmbi et al. [21]. Hence, the heat transfer would be decreasing. Thus, the local Nusselt number decreases as it also depends on the wall heat transfer boundary condition.

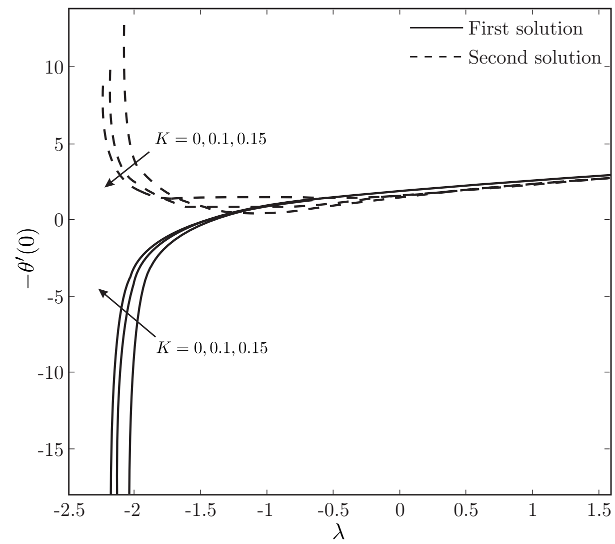

Fig. 4. Local Nusselt number $-\theta'(0)$ of $K = 0, 0.1, 0.15$ at $\Pr = 2$, $Q = 1$ and $S = 1$.

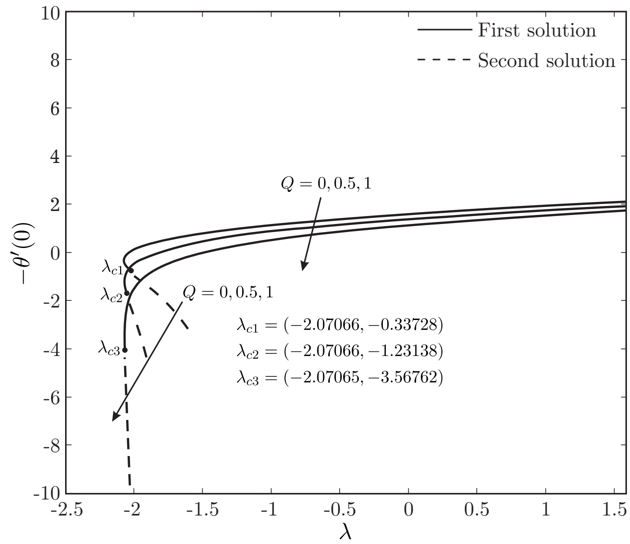

Fig. 5. Local Nusselt number $-\theta'(0)$ of $Q = 0, 0.5, 1$ at $\Pr = 1$, $K = 0$ and $S = 1$.

Figures 6–9 present the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ for various value of parameter K and parameter Q, where the dual solution existed. In these figures, we consider the value of shrinking parameter λ to be -1.6 for different K and -1 for different Q.

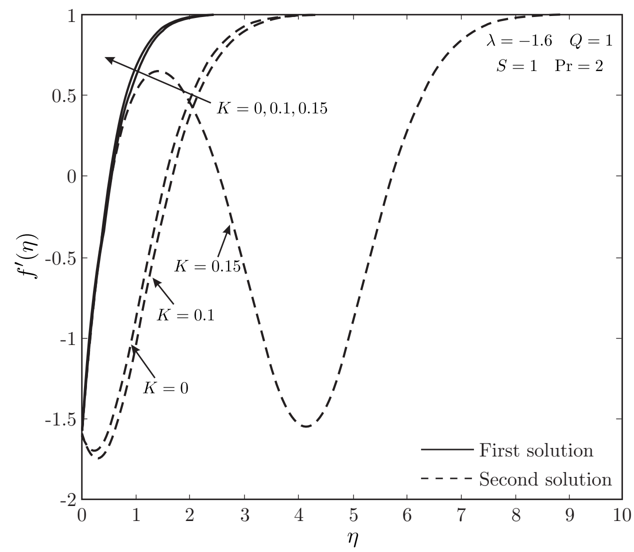

Fig. 6. Velocity profiles of $K = 0, 0.1, 0.15$ for $\lambda = -1.6$.

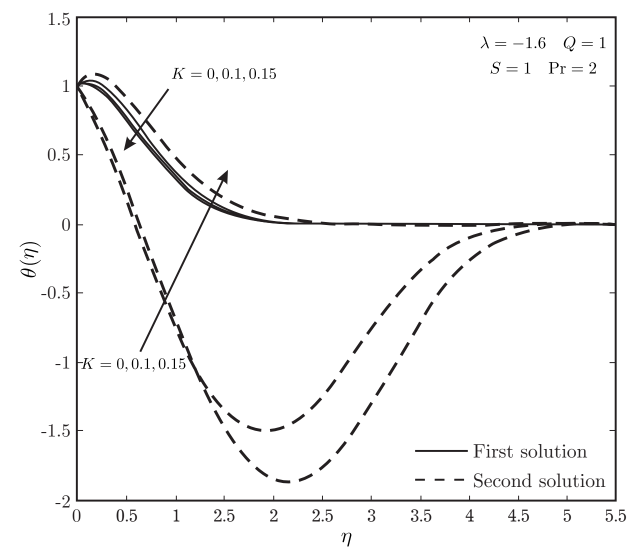

Fig. 7. Temperature profiles of $K = 0, 0.1, 0.15$ for $\lambda = -1.6$.

Figure 6 shows the velocity profile for different values of K for shrinking sheet. The velocity profile increases when parameter K increases for both the first and second solutions. Next, Figure 7 presents

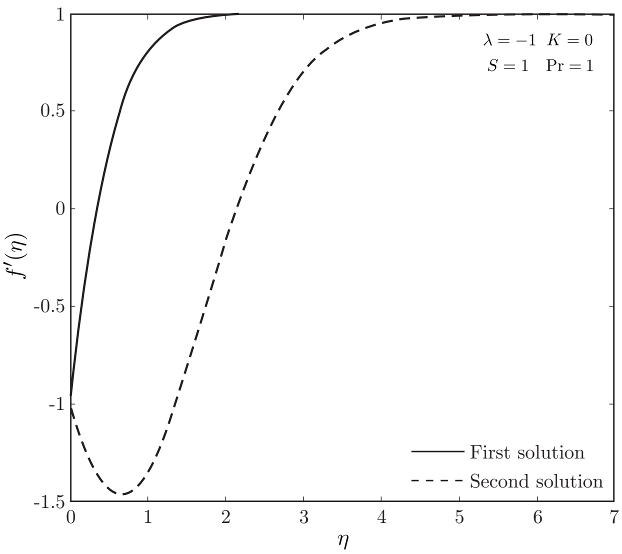

Fig. 8. Velocity profiles of $Q = 0, 0.2, 1$ for $\lambda = -1$.

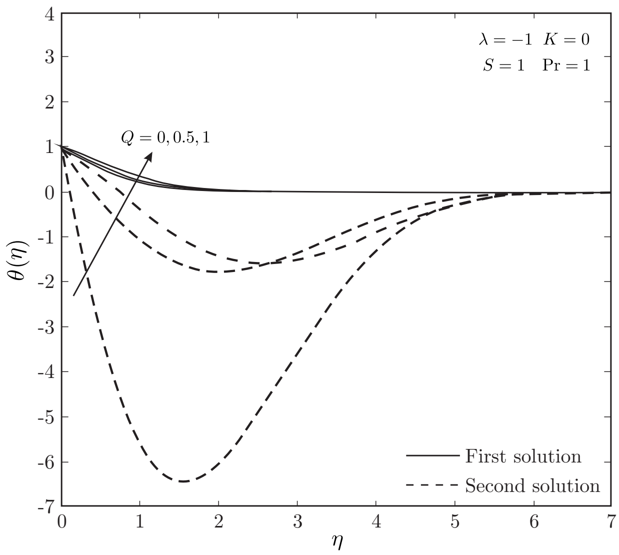

Fig. 9. Temperature profiles of $Q = 0, 0.2, 1$ for $\lambda = -1$.

the temperature profile of different value of K for $\lambda = -1.6$. It can be seen that the temperature profile is increasing when the parameter K increasing for the first and second solution. Figure 8 shows that there is no changes to the pattern towards the velocity profile when the parameter Q increases or decreases, for both first and second solutions. However, the profile postulate that there exists dual solution to the problem. In Figure 9, the temperature profile shows a pattern of increment alongside the increment of parameter Q for both the first and second solution. In these figures (Figures 6–9) the solid lines are for the first solution and the dashed lines are for the second solution. It is proved that both first and second solution profiles satisfy the far field boundary conditions asymptotically, which support the validity of the numerical results obtained. Also, this supports the dual nature of the solutions presented in Figures 6–9.

4. Conclusion

In the present study, we have investigated the effect of an exponentially shrinking and stretching sheet in a porous medium with heat generation towards stagnation point flow and heat transfer. The results acquired are also been compared to the previous studies by Bachok et al. [11] to check the validity of the method used. The results correspond to each other for both skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$. We can conclude that there exist dual solutions for skin friction coefficient and local Nusselt number for shrinking cases whereas the solution is unique for stretching cases. It was observed that an increase in the value of K leads to an increase in the skin friction coefficient and the heat transfer rate at the surface. However, the increase in the value of Q will not change the skin friction coefficient but will decrease the heat transfer rate at the surface. The velocity profiles and temperature profiles obtained satisfy the far field boundary conditions asymptotically for both first and second solutions. Thus, this supports the validity of the numerical results obtained and the dual nature of the solutions.

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Потік у точці застою та теплопередача по листу, що експоненціально стискається/розтягується, у пористому середовищі з виділенням тепла

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Метою цього дослідження є вивчення потоку рідини в точці застою по листі, що експоненціально стискається та розтягується, у пористому середовищі. Також досліджується швидкість теплопередачі за присутності виділення тепла. Використовуючи відповідне перетворення подібності, отримано звичайні диференціальні рівняння (ODE), які виведені з основної системи рівнянь у частинних похідних (PDE). Ці отримані рівняння доповнюються крайовими умовами та розв'язуються чисельно за допомогою BVP4C у програмному забезпеченні MATLAB. Вплив параметрів, які задіяні у цьому дослідженні, узагальнено та ретельно обговорено: від коефіцієнта поверхневого тертя, локального числа Нуссельта, до профілю швидкості та профілю температури. Аналіз проводиться за допомогою графічних і табличних даних. Спостережувані параметрами — це параметр проникності K, параметр теплоутворення Q та параметр стиснення/розтягування λ. Виявлено, що подвійний розв'язок існує для $\lambda < 0$ (випадок стискання), тоді як для $\lambda > 0$ (випадок розтягування) – розв'язок єдиний. Аналіз показує, що при збільшенні тепловиділення коефіцієнт шкірного тертя постійний. Однак він збільшується, коли проникність збільшується. Місцеве число Нуссельта зменшується зі збільшенням тепловиділення. Однак воно збільшується, коли проникність збільшується.

Ключові слова: *точка застою течії; пористе середовище; лист, що розтягується/стискається; утворення тепла.*