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Algorithm for Defining the Amount of Energy Transferred by Dry Saturated Steam

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Abstract

The algorithm for determining the amount of energy transferred by the dry saturated steam has been developed. The steam flow rate is measured by means of the differential pressure method with application of a standard long radius nozzle of a high-ratio type and a low-ratio type. The equations for determining the thermodynamic temperature of dry saturated steam when measuring its absolute pressure was applied together with the equations for determining the absolute pressure of dry saturated steam when measuring its thermodynamic temperature. A new non-iterative equation for calculating the mass flow rate of the heat energy carrier was obtained. The proposed method and equation for determining the amount of energy can be applied in digital devices for both technological and custody transfer metering of fluid energy carriers. Application of the developed algorithm in the microprocessor controllers and calculators provides the increase of computational speed at measurement of the amount of energy transferred by the fluid energy carrier with application of a long radius nozzle.

Keywords: amount of energy; differential pressure method; long radius nozzle; dry saturated steam; algorithm.

1. Introduction

The amount of energy transferred by a fluid energy carrier is usually measured by means of the differential pressure method. The main principle of this method and the requirements for its application are described in the international standards ISO 5167–1,2,3,4,6:2022 [1] – [5] and national standards of Ukraine DSTU 8.586.1,2,3.4,5 [6] – [10]. To calculate the parameters of the fluid energy carriers, including dry saturated steam, the international standards IAPWS R7-97 [11] and IAPWS R12-08 [12] are applied.

The differential pressure method is a complex indirect method for measurement of the flow rate and volume of fluid energy carriers, which involves measuring the pressure difference across the primary device, the absolute pressure and the temperature of the fluid in the pipe. The amount of energy can be defined based on the measured volume of the fluid energy carrier. Since the coefficients of the mass flow rate equation, according to the standards described above, depend on the mass flow rate of the fluid, the process of calculating the mass flow rate is iterative. In the algorithm for calculating the mass flow rate and the amount of energy, in addition to the iterative process of calculating the values of the fluid parameters and the coefficients of the mass flow rate equation, it is necessary to implement checking of the conditions and limitations for the application of the differential pressure method in a certain sequence. To do this, the following parameters should be checked:

- the range of allowable values of the energy carrier parameters;
- the characteristics of the primary device;
- the characteristics of the measuring pipe, its straight lengths and fittings;

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- eccentricity;
- steps caused by a misalignment and/or change in the diameter of adjacent pipe sections.

By implementing all the above-mentioned requirements, a complex algorithm for indirect calculation of the mass flow rate, the volume of fluid and the amount of energy transferred by the fluid energy carrier is obtained. This algorithm should be implemented in a microprocessor calculator of the amount of energy in real time within the limits of the computing cycle duration and the computing power of the calculator. Therefore, it is important to simplify the mass flow rate calculation algorithm and eliminate the iterative process of mass flow rate calculation, which occupies a significant part of the computing cycle of the calculator.

2. Analysis of publications and research

Using the differential pressure method to determine the mass flow rate, the volume of fluid energy carrier and the amount of energy transferred by the fluid, the following problems must be solved for a new flow meter:

- to perform a direct calculation of the differential pressure flow meter, during which the diameter of the orifice or throat of a standard primary device is determined according to the given value of the mass flow rate of the fluid;
- to make a standard primary device and check its characteristics for compliance with the standards;
- to perform a reverse calculation of the differential pressure flow meter, which involves the calculation of the mass flow rate of the fluid energy carrier for the given characteristics of a standard primary device and measuring pipe.

To measure the mass flow rate, the volume of the energy carrier and the amount of energy transferred by this energy carrier, the following standard primary devices are used: orifice plate [2], [7]; ISA 1932 nozzle, long radius nozzle, Venturi nozzle [3], [8]; Venturi tube of any type [4], [10]; wedge meters [6]. The use of a long radius nozzle has the following advantages when measuring the flow rate of the fluid energy carrier compared to the orifice plate:

- stable characteristics during continuous operation;
- a lower pressure loss;
- the diameter ratio of the primary device up to 0.8 is possible.

Therefore, it is often used to measure the mass flow rate of dry saturated steam.

Based on the mathematical model of the differential pressure method, the discharge coefficient of a long radius nozzle depends on the Reynolds number and, therefore, on the mass flow rate of the fluid energy carrier, and thus the mass flow rate equation is an implicit equation.

The calculation of the mass flow rate of the fluid energy carrier with application of a long radius nozzle of a high-ratio type and a low-ratio type can be made by an iterative method.

According to ISO 5167-1:2022 [1], it is recommended to perform the calculation using the Reynolds number or the mass flow rate value of the fluid energy carrier [1] or [10]. This increases the amount of memory and the duration of calculation of the mass flow rate, the volume of fluid energy carrier and the amount of energy transferred by this energy carrier, and, thus, raises the requirements for microprocessors or microcontrollers used in calculators of the mass flow rate and energy of fluid energy carrier.

It should be noted that simplified algorithms could be used to calculate the mass flow rate and the amount of energy of the fluid. These algorithms, however, would increase the relative error of the result of calculating the mass flow rate of the fluid, the maximum value of which would have to be added to the uncertainty of the result of the mass flow rate calculation. However, the problem here is that the additional error caused by the simplification of the algorithm can be systematic, and therefore can create an additional imbalance during transfer metering of energy carriers in distribution and transmission networks.

To verify the algorithms for calculating the mass flow rate of the fluid (regardless of whether the algorithms are simplified or not), it is necessary to have tables of control points (tests). They help determine the value of mass flow rate of the fluid, the amount of energy transferred by this fluid, and the relative expanded uncertainty of the result of calculating the amount of energy transferred by the fluid energy carrier in a given range of measurements of the absolute pressure or temperature of the fluid. The possibility to create such tables is implemented in Raskhod-RU CAD [13] and [14].

Based on the mathematical model of the differential pressure flow meter in Raskhod-RU CAD [13] and [14], the method for calculating the mass flow rate of fluid energy carrier has been introduced, implemented in the form of an appropriate optimized and formalized algorithm with cyclic calculation. The coefficients of the mass flow rate equation that depend on the mass flow rate of the fluid, in particular the discharge coefficient *C*, Reynolds number Re, fluid parameters, as well as a number of other coefficients of the mass flow rate equation that do not depend on the mass flow rate of the fluid are calculated in the cycle body. At the end of the cycle body, the relative deviation of the current value of the mass flow rate of the fluid energy carrier from the previous value of the mass flow rate of the fluid energy carrier is calculated. The exit from the cycle is carried out by comparing the relative deviation with the permissible error of the calculation, which, in accordance with the standard [10], is equal to 0.001%.

3. Goal of the research

The goal of this research is to develop a new algorithm for calculating the mass flow rate (q_m) and the amount of energy (Q) transferred by the dry saturated steam in order to reduce the required memory of the controller and increase the computing rate of the microprocessor calculator when calculating the mass flow rate, the volume of fluid energy carrier and the amount of energy transferred by this energy carrier. To reach this goal the analytical dependencies for calculating the following values should be developed:

- the absolute pressure of dry saturated steam when measuring its thermodynamic temperature;
- the thermodynamic temperature of dry saturated steam when measuring its absolute pressure;
- the density of dry saturated steam;
- the isentropic exponent of dry saturated steam;
- the dynamic viscosity of dry saturated steam;
- the mass flow rate of dry saturated steam;
- the enthalpy of dry saturated steam;
- the amount of energy transferred by dry saturated steam.

4. Presentation and discussion of the research results

The amount of heat Q transferred by the dry saturated steam during the time interval from the beginning of sampling $\tau_{\mathrm{b}1}$ to the end of sampling $\tau_{\mathrm{e}1}$, is calculated by the equation

$$Q = \sum_{i=1}^{n} \left[\frac{(q_{mi}H_{mi} + q_{mi+1}H_{mi+1})}{2} (\tau_{\mathrm{e}i} - \tau_{\mathrm{b}i}) \right], \tag{1}$$

where q_{mi}, q_{mi+1} are the values of mass flow rate of dry saturated steam calculated at the beginning of sampling $\tau_{\mathrm{b}i}$ and at the end of sampling $\tau_{\mathrm{e}i}$; H_{mi}, H_{mi+1} are the enthalpy values of dry saturated steam calculated at the beginning of sampling $\tau_{\mathrm{b}i}$ and at the end of sampling $\tau_{\mathrm{e}i}$, obtained according to the thermodynamic temperature values T_1 and T_2 or according to the absolute pressure values p_1 and p_2 of dry saturated steam.

Dry saturated steam is a state of water vapour in which the value of the absolute pressure p_s of dry saturated steam uniquely depends on the value of its thermodynamic temperature T_s, and, conversely, the value T_s of the thermodynamic temperature of dry saturated steam depends on the value p_s of its absolute pressure. Therefore, to measure the mass flow rate and enthalpy of dry saturated steam, either thermometers or pressure gauges are used.

According to the IAPWS R7-97 standard [11], the state of dry saturated steam is described by the saturation line equation as

$$\beta^2\vartheta^2 + n_1\beta^2\vartheta + n_2\beta^2 + n_3\beta\vartheta^2 + n_4\beta\vartheta + n_5\beta + n_6\vartheta^2 + n_7\vartheta + n_8 = 0, \tag{2}$$

where

$$\beta = \left(\frac{p}{p^*}\right)^{0{,}25} \tag{3}$$

and

$$\vartheta = \frac{T}{T^*} + \frac{n_9}{\frac{T}{T^*} - n_{10}} \tag{4}$$

with $p^* = 10^6$ Pa and $T^* = 1$ K; the values of coefficients from n_1 to n_{10} are given in Table 1.

Table 1. Values of coefficients from n_1 to n_{10} of equation (2).

i	n_i	i	n_i
1	$0.11670521452767 \times 10^4$	6	$0.14915108613530 \times 10^2$
2	$-0.72421316703206 \times 10^6$	7	$-0.48232657361591 \times 10^4$
3	$-0.17073846940092 \times 10^2$	8	$0.40511340542057 \times 10^6$
4	$0.12020824702470 \times 10^5$	9	$-0.23855557567849 \times 10^0$
5	$-0.32325550322333 \times 10^7$	10	$0.65017534844798 \times 10^3$

4.1. Calculating the absolute pressure of dry saturated steam

The value of the absolute pressure of dry saturated steam is calculated using the equation obtained from the thermodynamic formula for describing the saturation line of water vapour (2). Having written the equation (2) in the form of

$$a_1\beta^2 + b_1\beta + c_1 = 0, \tag{5}$$

where

$$a_1 = \vartheta^2 + n_1\vartheta + n_2;$$

$$b_1 = n_3\vartheta^2 + n_4\vartheta + n_5;$$

$$c_1 = n_6\vartheta^2 + n_7\vartheta + n_8$$

and taking into account the formula (3), we obtain the following relationship for determining the absolute pressure of dry saturated steam

$$p = p^* A^2\left(\sqrt{1+B} - 1\right)^4, \tag{6}$$

where the values of coefficients A and B are calculated by the equations:

$$A = \left(\frac{b_1}{2a_1}\right)^2;$$

$$B = -\frac{c_1}{a_1 A}.$$

The equation (6) is valid for calculating the absolute pressure of dry saturated steam along the entire saturation line, from the value of thermodynamic temperature of the triple point of water to the value of the critical thermodynamic temperature of water vapour, and can be simply extrapolated to 273.15 K

$$273.15\ \text{K} \le T_s \le 647.096\ \text{K}.$$

4.2. Calculating the thermodynamic temperature of dry saturated steam

The values of thermodynamic temperature of dry saturated steam are calculated by the equation, which is obtained for measuring the absolute pressure of dry saturated steam. To do this, let us convert the equation (4) to the form

$$\left(\frac{T}{T^*}\right)^2 + d_s\left(\frac{T}{T^*}\right) + e_s = 0, \tag{7}$$

and the equation (2) to the form

$$a_2\vartheta^2 + b_2\vartheta + c_2 = 0, \tag{8}$$

where the coefficients are defined by the following equations:

$$d_s = n_{10} + \vartheta;$$

$$e_s = n_{10}\vartheta + n_9;$$

$$a_2 = \beta^2 + n_3\beta + n_6;$$

$$b_2 = n_1\beta^2 + n_4\beta + n_7;$$

$$c_2 = n_2\beta^2 + n_5\beta + n_8;$$

$$\vartheta = -\frac{b_2}{2a_2}\left(1 - \sqrt{1 - \frac{4a_2c_2}{b_2^2}}\right). \tag{9}$$

Solving the system of equations (7) and (8), we obtain the equation for determining the thermodynamic temperature of dry saturated steam

$$T = T^*E_s\left(1 - \sqrt{1 - \frac{e_s}{E_s^2}}\right), \tag{10}$$

where

$$E_s = \frac{d_s}{2}.$$

The equation (10) applies within the same range as the equation (6). This means that the value of thermodynamic temperature of dry saturated steam is calculated for the value of its absolute pressure taken from the range

$$611.213 \text{ Pa} \le p \le 22.064 \cdot 10^6 \text{ Pa}.$$

The value of 611.213 Pa corresponds to the absolute pressure when the equation (10) is applied to a thermodynamic temperature value of 273.15 K.

4.3. Calculating the density of dry saturated steam

To calculate the density of dry saturated steam, the Gibbs free energy g equation is used [11]. This equation is expressed in dimensionless form, $\gamma = \frac{g(\pi,\tau)}{RT}$, and consists of two parts: the ideal gas γ_0 and the remainder γ_r

$$\frac{g(p,T)}{RT} = \gamma(\pi,\tau) = \gamma^o(\pi,\tau) + \gamma^r(\pi,\tau). \tag{11}$$

where π is the gauge pressure of dry saturated steam, the value of which is calculated using the equation $\pi = \frac{p}{p^*}$; τ is the relative thermodynamic temperature of dry saturated steam, the value of which is calculated using the equation $\tau = \frac{T^*}{T}$. $p^* = 10^6$ Pa, $T^* = 540$ K, the value of the constant is

$$R = 461.526\ \frac{\text{J}}{\text{kg·K}}. \tag{12}$$

The equation for the ideal gas $\gamma^o(\pi,\tau)$ of the dimensionless Gibbs free energy is calculated as follows

$$\gamma^o(\pi,\tau) = \ln\pi + \sum_{i=1}^{9}\left(n_i^o \tau^{J_i^o}\right). \qquad (13)$$

Table 2 contains the coefficients n_i^o and powers J_i^0 of the equation (13).

Table 2. Numerical values of coefficients n_i^o and powers J_i^0 of ideal gas γ^0 of the dimensionless Gibbs free energy (13).

i	J_i^o	n_i^o	i	J_i^o	n_i^o
1	0	-0.96927686500217 × 10^{1}	6	-2	0.14240819171444 × 10^{1}
2	1	0.10086655968018 × 10^{2}	7	-1	-0.43839511319450 × 10^{1}
3	-5	-0.56087911283020 × 10^{-2}	8	2	-0.28408632460772 × 10^{6}
4	-4	0.71452738081455 × 10^{-1}	9	3	0.21268463753307 × 10^{-1}
5	-3	-0.40710498223928 × 10^{0}			

The residual part $\gamma^r(\pi,\tau)$ of the dimensionless Gibbs free energy is calculated as follows

$$\gamma^r(\pi,\tau) = \sum_{i=1}^{43}\left[n_i \pi^{I_i}(\tau-0.5)^{J_i}\right]. \qquad (14)$$

The values of coefficients n_i and powers I_i and J_i of the equation (14) are given in Table 3.

Table 3. Numerical values of coefficients n_i and powers I_i and J_i of residual part $\gamma^r(\pi,\tau)$ of the dimensionless Gibbs free energy (14).

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1[a]	1	0	-0.17731742473213 × 10^{-2}	23	7	0	-0.59059564324270 × 10^{-17}
2[a]	1	1	-0.17834862292358 × 10^{-1}	24	7	11	-0.12621808899101 × 10^{-5}
3	1	2	-0.45996013696365 × 10^{-1}	25	7	25	-0.38946842435739 × 10^{-1}
4	1	3	-0.57581259083432 × 10^{-1}	26	8	8	0.11256211360459 × 10^{-10}
5	1	6	-0.50325278727930 × 10^{-1}	27	8	36	-0.82311340897998 × 10^{1}
6	2	1	-0.33032641670203 × 10^{-4}	28	9	13	0.19809712802088 × 10^{-7}
7	2	2	-0.18948987516315 × 10^{-3}	29	10	4	0.10406965210174 × 10^{-18}
8	2	4	-0.39392777243355 × 10^{-2}	30	10	10	-0.10234747095929 × 10^{-12}
9	2	7	-0.43797295650573 × 10^{-1}	31	10	14	-0.10018179379511 × 10^{-8}
10	2	36	-0.26674547914087 × 10^{-4}	32	16	29	-0.80882908646985 × 10^{-10}
11	3	0	0.20481737692309 × 10^{-7}	33	16	50	0.10693031879409 × 10^{0}
12	3	1	0.43870667284435 × 10^{-6}	34	18	57	-0.33662250574171 × 10^{0}
13	3	3	-0.32277677238570 × 10^{-4}	35	20	20	0.89185845355421 × 10^{-24}
14	3	6	-0.15033924542148 × 10^{-2}	36	20	35	0.30629316876232 × 10^{-12}
15	3	35	-0.40668253562649 × 10^{-1}	37	20	48	-0.42002467698208 × 10^{-5}
16	4	1	-0.78847309559367 × 10^{-9}	38	21	21	-0.59056029685639 × 10^{-25}
17	4	2	0.12790717852285 × 10^{-7}	39	22	53	0.37826947613457 × 10^{-5}
18	4	3	0.48225372718507 × 10^{-6}	40	23	39	-0.12768608934681 × 10^{-14}
19	5	7	0.22922076337661 × 10^{-5}	41	24	26	0.73087610595061 × 10^{-28}
20	6	3	-0.16714788451061 × 10^{-10}	42	24	40	0.55414715350778 × 10^{-16}
21	6	16	-0.21171472321355 × 10^{-2}	43	24	58	-0.94369707241210 × 10^{-6}
22	6	35	-0.23895741934104 × 10^{2}	-	-	-	-

Taking into account the IAPWS97 standard [13], the value of dry saturated steam density is calculated by the equation

$$\rho = \frac{p^*}{RT\gamma_\pi}, \qquad (15)$$

where the value γ_π is calculated by the formulae

$$\gamma_\pi = \frac{1}{\pi}\left\{1 + \sum_{i=1}^{43}\left[n_i I_i \pi^{I_i}(\tau-0.5)^{J_i}\right]\right\}. \qquad (16)$$

4.4. Calculating the isentropic exponent of dry saturated steam

Since the dry saturated steam is described by the Gibbs free energy saturation line (11) [11], the equation for calculating the value of the isentropic exponent κ of dry saturated steam is as follows

$$\kappa = \frac{\tau^2 \gamma_\pi \gamma_{\tau\tau}}{\pi\left[\left(\gamma_\pi - \tau\gamma_{\pi\tau}\right)^2 - \tau^2 \gamma_{\pi\pi}\gamma_{\tau\tau}\right]}. \tag{17}$$

where the values of the functions $\gamma_{\pi\pi}$, $\gamma_{\tau\tau}$, $\gamma_{\pi\tau}$ according to the international standard IAPWS97 [11] are calculated by the equations:

$$\gamma_{\pi\pi} = \frac{1}{\pi^2} \sum_{i=1}^{43} \left[n_i I_i (I_i - 1) \pi^{I_i} (\tau - 0.5)^{J_i}\right]; \tag{18}$$

$$\gamma_{\tau\tau} = \sum_{i=1}^{9} \left[n_i^o J_i^o (J_i^o - 1) \tau^{J_i^o - 2}\right] + \sum_{i=1}^{43} \left[n_i \pi^{I_i} J_i (J_i - 1)(\tau - 0.5)^{J_i - 2}\right]; \tag{19}$$

$$\gamma_{\pi\tau} = \frac{1}{\pi(\tau - 0.5)} \sum_{i=1}^{43} \left[n_i I_i \pi^{I_i} J_i (\tau - 0.5)^{J_i}\right]. \tag{20}$$

4.5. Calculating the dynamic viscosity of dry saturated steam

The dynamic viscosity of dry saturated steam is calculated by the equation [12]

$$\mu = 10^{-6}\overline{\mu}. \tag{21}$$

The dimensionless dynamic viscosity $\overline{\mu}$ of dry saturated steam is determined in accordance with [12] by the equation

$$\overline{\mu} = \overline{\mu}_0\left(\overline{T}\right)\overline{\mu}_1\left(\overline{T},\overline{\rho}\right), \tag{22}$$

where $\overline{T}$ is the dimensionless temperature of dry saturated steam; $\overline{\rho}$ is the dimensionless density of dry saturated steam; $\overline{\mu}_0\left(\overline{T}\right)$ is the first factor which represents the viscosity in the dilute-gas limit; $\overline{\mu}_1\left(\overline{T},\overline{\rho}\right)$ is the second factor which represents the contribution to viscosity due to finite density.

The values of dimensionless physical quantities are calculated by the equations:

- the dimensionless pressure of dry saturated steam

$$\overline{p} = \frac{p}{22.064 \cdot 10^6}; \tag{23}$$

- the dimensionless temperature of dry saturated steam

$$\overline{T} = \frac{T}{647.096}; \tag{24}$$

- the dimensionless density of dry saturated steam

$$\overline{\rho} = \frac{\rho}{322.0}. \tag{25}$$

The first factor $\overline{\mu}_0\left(\overline{T}\right)$ is calculated by the equation

$$\overline{\mu}_0\left(\overline{T}\right) = \frac{100\sqrt{\overline{T}}}{\sum_{i=0}^{3} \frac{H_i}{\overline{T}^i}}, \tag{26}$$

the numerical values of coefficients H_i of which are given in Table 4.

Table 4. Numerical values of coefficients H_i for the first factor $\overline{\mu}_0(\overline{T})$.

i	H_i	i	H_i
0	1.67752	2	0.6366564
1	2.20462	3	-0.241605

The second factor $\overline{\mu}_1(\overline{T},\overline{\rho})$ is calculated by the equation

$$\overline{\mu}_1(\overline{T},\overline{\rho}) = \exp\left\{\overline{\rho}\sum_{i=0}^{5}\left\{\left(\frac{1}{\overline{T}}-1\right)^i\sum_{j=0}^{6}\left[H_{ij}(\overline{\rho}-1)^j\right]\right\}\right\}, \tag{27}$$

the values of coefficients H_{ij} of which are given in Table 5.

Table 5. Numerical values of coefficients H_{ij} for the second factor $\overline{\mu}_1(\overline{T},\overline{\rho})$.

i	j	H_{ij}	i	j	H_{ij}
0	0	5.20094×10^{-1}	2	2	-7.72479×10^{-1}
1	0	8.50895×10^{-2}	3	2	-4.89837×10^{-1}
2	0	-1.08374×10^{0}	4	2	-2.57040×10^{-1}
3	0	-2.89555×10^{-1}	0	3	1.61913×10^{-1}
0	1	2.22531×10^{-1}	1	3	2.57399×10^{-1}
1	1	9.99115×10^{-1}	0	4	-3.25372×10^{-2}
2	1	1.88797×10^{0}	3	4	6.98452×10^{-2}
3	1	1.26613×10^{0}	4	5	8.72102×10^{-3}
5	1	1.20573×10^{-1}	3	6	-4.35673×10^{-3}
0	2	-2.81378×10^{-1}	5	6	-5.93264×10^{-4}
1	2	-9.06851×10^{-1}	-	-	-

4.6. Calculating the mass flow rate of dry saturated steam

The mass flow rate q_m of dry saturated steam in accordance with the standards of Ukraine [6] is determined by the equation

$$q_m = \frac{\pi}{4} d_{20}^2 K_{PD}^2 E C \varepsilon \sqrt{2\Delta p \rho}\ , \tag{28}$$

where d_{20} is the diameter of the throat of a long radius nozzle at the fluid temperature of 20°C; K_{PD} is the coefficient which takes into account the change in diameter of a long radius nozzle throat caused by deviation of the fluid temperature from 20°C; E is the velocity of approach factor; C is the discharge coefficient of the primary device; Δp is the pressure difference across the primary device; ρ is the density of the fluid under operating conditions at the thermodynamic temperature T of dry saturated steam or at the absolute pressure p of dry saturated steam; ε is the expansibility coefficient of dry saturated steam.

The velocity of approach factor depends on the diameter ratio β of the primary device and is determined by the equation [1, 6]

$$E = \frac{1}{\sqrt{1-\beta^4}}, \tag{29}$$

where the diameter ratio β is calculated by the equation

$$\beta = \frac{d}{D_{20} K_{\mathrm{T}}}, \tag{30}$$

where D_{20} is the internal diameter of the measuring pipe at the fluid temperature of 20°C; K_{T} is the coefficient that takes into account the change in the internal diameter of the measuring pipe caused by a deviation of the fluid temperature from 20°C.

The discharge coefficient *C* of a long radius nozzle of a high-ratio type and a low-ratio type in accordance with the National Standard of Ukraine DSTU GOST 8.586.3:2009 [8] is determined by the equation

$$C = 0.9965 - 0.00653\sqrt{\frac{10^6\beta}{\mathrm{Re}}}, \tag{31}$$

where Re is the Reynolds number, showing the ratio of the flow inertial force to the viscosity force, determined by the equation [10]

$$\mathrm{Re} = \frac{\overline{w}D\rho}{\mu}, \tag{32}$$

where $\overline{w}$ is the longitudinal component of the average local velocity of dry saturated steam in the measuring pipe; μ is the dynamic viscosity of dry saturated steam.

Taking into account that the value of the longitudinal component of the average local velocity of dry saturated steam in the measuring pipe in accordance with [6] is calculated by the equation

$$\overline{w} = \frac{4q_m}{\pi D^2\rho}, \tag{33}$$

and, substituting it in (33), we obtain the following equation for calculating the value of the Reynolds number

$$\mathrm{Re} = \frac{4q_m}{\pi D\mu} \tag{34}$$

The value of the coefficient of expansibility ε of dry saturated steam is calculated by the equation

$$\varepsilon = \sqrt{\frac{\kappa\tau^{\frac{2}{\kappa}}}{\kappa-1}\frac{1-\beta^4}{1-\beta^4\tau^{\frac{2}{\kappa}}}\frac{1-\tau^{1-\frac{1}{\kappa}}}{1-\tau}}, \tag{35}$$

where τ is calculated by the equation $\tau = 1 - \frac{\Delta p}{p}$.

If the ratio is $\frac{\Delta p}{p} > 0.25$, then the calculation of the mass flow rate of dry saturated steam continues, but it is noted and recorded, and an alarm message is issued.

By substituting the equation for the Reynolds number calculation (34) into the equation for calculating the value of the discharge coefficient *C* for a long radius nozzle (31) and the equation for determining the coefficient *C* into the equation (28) for calculating the mass flow rate of dry saturated steam, we obtain the following equation for calculating the mass flow rate of dry saturated steam q_m

$$q_m = \frac{\pi}{4}d_{20}^2K_{PD}^2E\varepsilon\left(0.9965 - 3.265\sqrt{\pi d_{20}K_{PD}\mu}\frac{1}{\sqrt{q_m}}\right)\sqrt{2\Delta p\rho}\,. \tag{36}$$

As we can see, the equation (36) for calculating the mass flow rate q_m of dry saturated steam is the function of the mass flow rate of dry saturated steam itself, i.e. $q_m = \mathrm{f}(q_m)$, and can be solved as a solution of a cubic equation. Let us convert the formula (36) into a form that can be solved analytically:

$$q_m\sqrt{q_m} + 3p_0\sqrt{q_m} + 2q_0 = 0\,, \tag{37}$$

where

$$p_0 = -\frac{0.9965}{3}X; \tag{38}$$

$$q_0 = 1.6325\sqrt{\pi\mu d_{20} K_{PD}} X; \tag{39}$$

$$X = \frac{\pi}{4} d_{20}^2 K_{PD}^2 E\varepsilon\sqrt{2\Delta p\rho}. \tag{40}$$

By substituting

$$y = \sqrt{q_m} \tag{41}$$

in the equation (37), we obtain the following cubic equation

$$y^3 + 3p_0 y + 2q_0 = 0\ . \tag{42}$$

Since the value of $p_0 < 0$, then the values of the roots of the equation (42) depend on the discriminant D_0 , the value of which is calculated by the equation

$$D_0 = q_0^2 + p_0^3. \tag{43}$$

If $\mathrm{D}_0 > 0$, the equation (43) will have one solution (one real root and two imaginary roots).

If $\mathrm{D}_0 < 0$, the equation (43) will have three solutions (three real different roots).

If $D_0 = 0$, the equation (43) will have one solution:

- if $p_0^3 = -q_0^2 \neq 0$, the equation (43) will have two solutions (three real roots, two of which coincide);
- if $p_0 = q_0 = 0$, the equation (43) will have one solution (three real coincident zero roots).

The research has established that $D_0 < 0$ and $p_0 < 0$, and the root of equation (43) is determined by the relationship

$$y = 2\sqrt{-p_0}\cos\frac{\pi - \arccos\left[\frac{q_0}{(-p_0)^{1.5}}\right]}{3}\ . \tag{44}$$

Applying the equations (38) – (41) and (44), we obtain the following equation with the help of which the mass flow rate of dry saturated steam is calculated by the differential pressure method using a long radius nozzle

$$q_m = -4p_0 \cos^2\left\{\frac{\pi - \arccos\left[\frac{q_0}{(-p_0)^{1.5}}\right]}{3}\right\}. \tag{45}$$

4.7. Calculating the enthalpy of dry saturated steam

The enthalpy value of dry saturated steam in accordance with the international standard IAPWS97 [11] is calculated by the equation

$$H_m = RT\tau\gamma_\tau, \tag{46}$$

where γ_τ is calculated by the formula

$$\gamma_\tau = \sum_{i=1}^{9}\left[n_i^o J_i^o \tau^{J_i^0 - 1}\right] + \sum_{i=1}^{43}\left[n_i J_i \pi^{I_i}(\tau - 0.5)^{J_i - 1}\right]. \tag{47}$$

4.8. Algorithm for calculating the amount of energy transferred by dry saturated steam with application of a long radius nozzle

Based on sections 4.1-4.7, we create the algorithm for calculating the amount of energy transferred by dry saturated steam using a long radius nozzle and applying the differential pressure method:

1) when measuring the thermodynamic temperature T of dry saturated steam, the absolute pressure p of dry saturated steam is calculated by the equation (6). When measuring the absolute pressure p of

dry saturated steam, the thermodynamic temperature T of dry saturated steam is calculated by the equation (10);

2) based on the values of thermodynamic temperature and absolute pressure of dry saturated steam, the density ρ of dry saturated steam is calculated by the equation (15);
3) based on the values of thermodynamic temperature and absolute pressure of dry saturated steam, the value of the isentropic exponent κ of dry saturated steam is calculated by the equation (17);
4) based on the values of thermodynamic temperature and absolute pressure of dry saturated steam, the dynamic viscosity μ of dry saturated steam is calculated by the equation (21);
5) using a differential pressure gauge, the pressure difference Δp_1 in a long radius nozzle is measured. Using the equation (45), the mass flow rate q_{m1} of dry saturated steam is calculated;
6) the value of enthalpy H_{m1} is calculated by the equation (46);
7) using a differential pressure gauge, the pressure difference Δp_2 in a long radius nozzle is measured. Using the equation (45), the mass flow rate q_{m2} of dry saturated steam is calculated;
8) the value of enthalpy H_{m2} is calculated by the equation (46);
9) using the equation (1), the amount of heat Q transferred by dry saturated steam during the time interval $\tau_{ei} - \tau_{bi}$ is calculated.

5. Conclusion

The research proposes the equation (1) for calculating the amount of energy transferred by the dry saturated steam, the mass flow rate of which is measured by means of the differential pressure method, applying a standard long radius nozzle. The values of thermophysical parameters are determined by measuring the thermodynamic temperature of dry saturated steam, based on which the value of its absolute pressure is determined, using the equations that were developed. The value of the absolute pressure of dry saturated steam is measured, while the thermodynamic temperature of dry saturated steam is calculated by the new equation. The equations developed by us made it possible to obtain the equation for calculating the thermophysical parameters of dry saturated steam, namely the density of dry saturated steam (equation (15)), isentropic exponent of dry saturated steam (equation (17)) and dynamic viscosity of dry saturated steam (equation (21)). The equation for calculating the mass flow rate (45) allows calculating the flow rate for both gas and liquid fluids without iteration. Based on the equation (1), the algorithm was created for calculating the amount of energy of the fluid, which is proposed to be applied in microprocessor calculators of the amount of energy transferred by the fluid energy carrier.

Based on the results of comparing the flow rates obtained using the developed non-iterative algorithm with the flow rates obtained using the standard iterative algorithm, the authors found that the developed non-iterative algorithm ensures the required accuracy of the flow rate calculation determined by the DSTU GOST 8.586.5:2009 standard. Thus, the proposed equation and algorithm for the flow rate calculation can be applied in flow meters with the use of a standard long radius nozzle for both technological and custody transfer metering of fluid energy carriers. The application of the developed algorithm in the microprocessor controllers allows increasing the computing rate when calculating the flow rate of the energy carrier.

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Алгоритм визначення кількості енергії, що переноситься сухою насиченою парою

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Анотація

Розроблено алгоритм визначення кількості енергії, що переноситься сухою насиченою парою, витрату якої визначають за методом змінного перепаду тиску із застосуванням стандартного еліпсного сопла з малим і великим відносним діаметром. Застосовано рівняння для визначення термодинамічної температури сухої насиченої пари при вимірюванні її абсолютного тиску, або абсолютного тиску при вимірюванні її термодинамічної температури. Отримано нове безітераційне рівняння для розрахунку масової витрати теплового енергоносія. Запропонований метод і рівняння визначення кількості енергії можуть бути застосовані у цифрових пристроях як для технологічного так і комерційного обліку плинних енергоносіїв. Застосування розробленого алгоритму збільшує швидкість розрахунку кількості енергії, що переноситься плинним енергоносієм, за допомогою еліпсного сопла із застосуванням мікропроцесорних контролерів та обчислювачів.

Ключові слова: кількість енергії; метод змінного перепаду тиску; еліпсне сопло; суха насичена пара; алгоритм.