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Model of Gas Turbine Plant with Concentrated Parameters for Analysis of Dynamic Properties Patterns

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Abstract

This article considers a model of gas turbine plants (GTPs) with concentrated parameters for analysing the regularities of the dynamic properties of transients with power changes. The main goal is to study and analyse the dynamic properties of GTPs, in particular their transients, taking into account power changes. To achieve this goal, the task is to develop a universal model based on approximation approaches using concentrated parameters. The key tasks of the article include: development of a mathematical model with concentrated parameters, which takes into account the current rated power of the GTP, investigation of the dynamic properties of the transient processes of the GTP using approximation models, obtaining an approximation dependence that shows the relationship between changes in the dynamic properties of transients and changes in the current power of the GTP. Focusing on the development of a universal model with approximation parameters provides the basis for a detailed analysis of the dynamic properties of GTPs and their behaviour at different power levels. These studies can have practical applications in improving the regulation and optimization of power systems based on gas turbine plants.

Keywords: dynamic properties; approximation models; power of gas turbines; regulation of power systems.

1. Literature review

The study of the dynamic properties of gas turbine units (GTUs) and their analysis to optimise and improve their performance is a relevant and important topic in the energy sector. In recent years, much attention has been paid to the development of mathematical models for describing and predicting the dynamic behaviour of gas turbines. One approach to achieving this goal is to use models with concentrated parameters, which allow generalising the dynamic characteristics of the system and making their analysis more convenient and efficient.

In their paper, Stefano Braco and co-authors [1] describe a mathematical model and research on the use of microturbines in smart microgrids, in particular in 65 kW cogeneration systems, with a focus on their efficiency and flexible simulator and control system architecture. The study is based on model validation using microturbines installed in the Savona Campus smart polygeneration microgrid at the University of Genoa in Italy.

In addition, Miller and colleagues [2] consider a methodology for modelling and analysing the dynamic behaviour of gas turbine generator rotors of active magnetic bearing units used in gas turbine cogeneration systems. The study includes a mathematical model that takes into account electromagnetic processes in active magnetic bearings, the use of the contour flow method to analyse these processes, and the comparison of the results with experimental data to confirm the effectiveness of rotor dynamics modelling.

Based on the above studies, it can be concluded that the development of models with concentrated parameters is important for analyzing the regularities of the dynamic properties of gas turbine plants. These models allow for a

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more detailed analysis of the impact of various factors on the dynamic behaviour of the system, which can help improve its operation and efficiency.

2. Purpose and objectives of the article

The aim is to analyze the regularities of the dynamic properties of transient processes when changing the capacities of gas turbine plants by developing a universal model based on concentrated parameters on the basis of approximation models.

Objectives:

1) to develop a mathematical model with concentrated parameters, taking into account the current rated capacity of the gas turbine unit;
2) to investigate the dynamic properties of the transient processes of the GTP on the basis of approximation models;
3) to obtain an approximation dependence of the change in the dynamic properties of transients on the current value of the GTP power.

3. A mathematical model with concentrated parameters taking into account the current rated capacity of a gas turbine

3.1. Assumptions

3.1.1. Equation for the turbine rotor

Modern gas turbine plants are complex nonlinear dynamic systems with mutual influence of gas-dynamic and thermophysical processes occurring in their components. These processes are usually non-stationary in terms of time and operating conditions, and some design schemes have a variable structure. The operation of a gas turbine occurs under the constant influence of internal and external disturbances, sometimes in transient modes.

When considering a turbine as a control object, we mean its rotating masses - the rotor, which is subject to forces from the flow of the working fluid, and the control valves (valves, nozzles), which can be used to change these forces within certain limits. The characteristics of the distributing bodies can be very diverse, depending on the type of machine for which this unit serves as a drive (electric generator, compressor, etc.) [3].

A schematic diagram of a single-shaft gas turbine unit (GTU) operating in a regenerative cycle is shown in Fig.1.

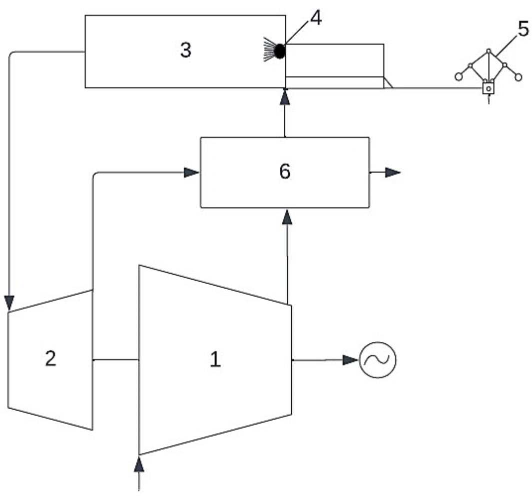

Fig.1. Schematic diagram of a single-shaft GTP with a regenerator:
1 - compressor; 2 - gas turbine; 3 - combustion chamber; 4 - fuel supply valve; 5 - speed regulator.

The gas turbine rotates an axial compressor, which is part of the gas turbine system, and an electric generator located on the same shaft. The rotor is subjected to forces from the gas turbine which generate torque. And the generator and compressor are subjected to drag moments. A parametric diagram for the rotor of a gas turbine plant can be presented as follows (Fig.2).

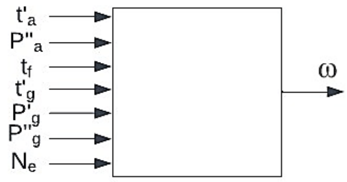

Fig.2. Parametric diagram of the GTP rotor.

The temperature of the gases upstream of the turbine is the strongest factor that determines the useful life of a gas turbine. The frequency in the circuit changes relatively insignificantly (within the range of the unevenness factor), and the main change in torque occurs under the influence of deviations in fuel consumption. Its value depends on the position of the fuel valve. The fuel consumption is determined by the static response of the fuel valve depending on the speed. Along with the fuel consumption, the gas temperature and pressure change (almost instantaneously with small tanks), which determine the flow rate of the working fluid (gases) and the torque [8].

According to the quantity-momentum theorem, if the driving and resistance torques are not equal, their difference causes a change in the rotor speed [8].

For mechanical accumulators, in accordance with the theorem of moments of quantity of motion, the equation for the steady-state mode will be as follows [4]–[8]:

$$\mathrm{M}_{df}^{0} - \mathrm{M}_{r}^{0} = 0. \tag{1}$$

For the unsteady state, equation (1) will take the form:

$$\mathrm{M}_{df} - \mathrm{M}_{r} = J\frac{d\omega}{dt},$$

where M_{df} is the moment of driving forces on the turbine blades, W·s; M_r is the moment of resistance forces, W·s; J is the moment of inertia of the turbine rotor together with the compressor and generator, kg·m ^{2}; ω angular speed of rotor rotation, s^{-1}.

Since small deviations of the GTP operating modes are considered for the study of dynamic characteristics, changes in the efficiency of turbomachines and physical properties of working bodies can be neglected and considered constant when deriving the equations of dynamics [6].

The moment of driving forces on the turbine blades M_{df} is determined by the turbine driving force torque M_T. And the moment of resistance forces M_r is determined by the compressor M_c and generator torque M_g:

$$\mathrm{M}_{df} = \mathrm{M}_T;\ \mathrm{M}_r = \mathrm{M}_c + \mathrm{M}_g.$$

The torque of a gas turbine depends on the initial parameters of the gases leaving the combustion chamber (temperature t_g' and pressure p_g'), on the backpressure p_g'' and on the rotational speed ω. The compressor torque depends on the inlet air temperature t_a', its inlet p_a' and outlet pressure p_a'' and the turbine rotor speed ω. And the torque of an electric generator depends on changes in the electrical load N_e and rotor speed.

All things considered, all of the above points can be represented in the form of functions:

$$\begin{cases} \mathrm{M}_T = f_1(p_g', p_g'', t_g', \omega); \\ \mathrm{M}_c = f_2(p_a', p_a'', t_a', \omega); \\ \mathrm{M}_g = f_3(N_e, \omega), \end{cases}$$

where p', p'' are pressures of the working fluid at the inlet and outlet of the turbine and compressor, respectively, MPa; t' is temperature of the working fluid at the inlet of the turbine and compressor inlet, °C; N_e is electric load of the generator, W.

After the known transformations, we obtain the differential equation describing the dynamics of the turbine rotor in the form [4]–[8]:

$$B\frac{d\Delta\omega}{dt} = b_1\Delta p'_g + b_2\Delta p''_g + b_3\Delta t'_g - b_4\Delta p'_a - b_5\Delta p''_a - b_6\Delta t'_a - b_7\Delta N_e, \tag{2}$$

where $\frac{\partial \mathrm{M}_r}{\partial\omega} = \frac{\partial \mathrm{M}_c}{\partial\omega} + \frac{\partial \mathrm{M}_g}{\partial\omega}$; $\frac{\partial \mathrm{M}_{df}}{\partial\omega} = \frac{\partial \mathrm{M}_T}{\partial\omega}$; $B = \frac{J}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$; $b_1 = \frac{\frac{\partial \mathrm{M}_T}{\partial p'_g}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$; $b_2 = \frac{\frac{\partial \mathrm{M}_T}{\partial p''_g}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$; $b_3 = \frac{\frac{\partial \mathrm{M}_T}{\partial t'_g}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$; $b_4 = \frac{\frac{\partial \mathrm{M}_c}{\partial p'_a}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$;
$b_5 = \frac{\frac{\partial \mathrm{M}_c}{\partial p''_a}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$; $b_6 = \frac{\frac{\partial \mathrm{M}_c}{\partial t'_a}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$; $b_6 = \frac{\frac{\partial \mathrm{M}_g}{\partial N_e}}{\frac{\partial \mathrm{M}_r}{\partial\omega} - \frac{\partial \mathrm{M}_{df}}{\partial\omega}}$.

Partial derivatives are defined by the expressions:

$$\begin{cases} \mathrm{M}_T = \dfrac{G_g \cdot H_T \cdot \eta_T}{\omega}; \\ \mathrm{M}_к = \dfrac{G_a \cdot H_c}{\omega \cdot \eta_c}; \\ \dfrac{\partial \mathrm{M}_g}{\partial\omega} = \dfrac{\mathrm{M}_g^0}{\omega^0};\ \mathrm{M}_g^0 = \dfrac{N_e^0}{\omega^0}, \end{cases}$$

where the flow rate of the working medium is determined by the Stodola formula

$$G_g = G_g^0\sqrt{\frac{p_{in}^2 - p_{out}^2}{(p_{in}^0)^2 - (p_{out}^0)^2}} \cdot \sqrt{\frac{T_{in}^0}{T_{in}}}$$

and the static characteristics of the compressor.

3.1.2. Dynamic equations for the combustion chamber

The air enters the combustion chamber from the compressor at pressure p_a and temperature t_a. The combustion of the fuel G_f in quantity heats the entire gas to a temperature t_g.

The combustion chamber requires uneven temperature and pressure fields to organise combustion. While the temperature of the gases before the turbine is strictly limited, it must be very high in the combustion chamber. Only a part of the air coming from the compressor (the primary air in the combustion chamber) is directed to the combustion hearth. The rest of the air (the secondary air) is supplied to a mixer located behind the combustion zone [8].

The ignition lag is important for the combustion process. In the first combustion zone, fuel particles are heated and evaporated. At the same time, oxidation takes place. This is followed by the slow oxidation of the fuel vapours, which precedes ignition and combustion. These processes take time to complete. The travel time of the gas particles from the fuel inlet to the turbine inlet determines the lag τ [8].

However, with integrated combustion chambers, when the volume of the combustion chamber is small and there are no connecting pipelines in the path of the combustion products, the path of gas particles from the hearth to the turbine impeller is small and amounts to a fraction of a second. Therefore, such a time τ with large rotor inertia allows us to ignore the lag effect. However, the lag can be significant if the fuel control valve is located at a considerable distance from the combustion chamber or if the capacity between the valve and the combustion chamber is large compared to the fuel flow rate [8].

The calculations show that the accumulated heat in the metal of the combustion chamber is of secondary importance and has little effect on the control dynamics [8].

Thus, taking into account the above assumptions, the gas parameters in the combustion chamber will be concentrated, and the temperature and pressure at the outlet of the combustion chamber will be set almost instantaneously depending on the valve stroke [4]–[8].

In this model, at a constant rotor speed (ω = const) the change in air flow by the compressor ΔG_a and gas flow by the turbine ΔG_T is synchronized with the fuel supply ΔG_f. The flow rates are related by the following equation:

$$\Delta G_a + \Delta G_{\mathrm{f}} = \Delta G_T.$$

The fuel component plays a significant role if the GTP operates on gaseous fuels with a low calorific value [8]. Therefore, the value of the lower calorific value of the fuel will significantly affect the amount of heat released in the combustion chamber during fuel combustion.

Based on the above, the existing mathematical model for the combustion chamber of a GTP should be improved in such a way as to neutralise the impact of the perturbation on the change in the lower heating value of fuel. Such a correction of the mathematical model will allow the development of a cogeneration power plant that allows the use of non-certified fuels.

The independent variables that determine the state of the gas in the chamber are the pressure in the chamber p_g and the temperature t_g. The temperature of the gases at the combustion chamber outlet is characterized by three heat fluxes: the amount of heat supplied by the air flow Q_a, the amount of heat supplied by the fuel Q_f and the amount of heat released in the combustion chamber during the fuel combustion process Q_{cc}.

Based on this, the heat balance equation for the steady-state regime will be as follows [4], [5]:

$$Q_a^0 + Q_{\mathrm{f}}^0 + Q_{cc}^0 - Q_g^0 = 0. \tag{3}$$

For the transient regime, equation (3) will take the form:

$$Q_a + Q_{\mathrm{f}} + Q_{cc} - Q_g = M_g \frac{\partial I_g}{\partial t},$$

where Q_a, Q_f are heat fluxes entering the combustion chamber with air and fuel, respectively, kW; Q_{cc} is amount of heat released in the combustion chamber during fuel combustion, kW; Q_g is amount of heat that goes with the gases (combustion products) to the turbine, kW; M_g is mass of gases in the combustion chamber, kg; I_g is enthalpy of gases at the combustion chamber outlet $I_g = c_g t_g$, kJ/kg.

After the known transformations, we obtain the differential equation describing the dynamics of temperature changes in the combustion chamber gases [4], [5]:

$$A\frac{\partial \Delta t_g}{\partial t} + \Delta t_g = a_1 \Delta\omega + a_2 \Delta m_{\mathrm{f}} + a_3 \Delta t_a + a_4 \Delta t_{\mathrm{f}} + a_5 \Delta LCV, \tag{4}$$

where $A = \frac{M_g c_g}{\frac{\partial Q_g}{\partial t_g}}$; $a_1 = \frac{\frac{\partial Q_a}{\partial G_a} - \frac{\partial Q_g}{\partial G_a}}{\frac{\partial Q_g}{\partial t_g}} \frac{\partial G_a}{\partial \omega}$; $a_2 = \frac{\frac{\partial Q_{\mathrm{f}}}{\partial G_{\mathrm{f}}} + \frac{\partial Q_{cc}}{\partial G_{\mathrm{f}}} - \frac{\partial Q_g}{\partial G_{\mathrm{f}}}}{\frac{\partial Q_g}{\partial t_g}} \frac{\partial G_a}{\partial m_{\mathrm{f}}}$; $a_3 = \frac{\frac{\partial Q_a}{\partial t_a}}{\frac{\partial Q_g}{\partial t_g}}$; $a_4 = \frac{\frac{\partial Q_{\mathrm{f}}}{\partial t_f}}{\frac{\partial Q_g}{\partial t_g}}$; $a_5 = \frac{\frac{\partial Q_{cc}}{\partial LCV}}{\frac{\partial Q_g}{\partial t_g}}$.

The partial derivatives are determined by the expressions for the heat fluxes:

$$\begin{cases} Q_a = G_a c_a t_a; \\ Q_f = G_f c_f t_f; \\ Q_{\mathrm{cc}} = G_f \cdot LCV \cdot \eta_{\mathrm{cc}}; \\ Q_g = G_g c_g t_g, \end{cases}$$

where G_a is air flow rate entering the combustion chamber, kg/s; G_f is fuel flow rate entering the combustion chamber, kg/s; G_g is gas flow rate at the outlet of the combustion chamber, kg/s; c_a, c_f, c_g are heat capacities of air, fuel and gases at the outlet of the combustion chamber, respectively, kJ/(kg·°C); *LCV* is lower calorific value of fuel, kJ/kg; η_{cc} is combustion chamber efficiency; t_a, t_f, t_g are temperatures of air, fuel and gases at the combustion chamber outlet, respectively, °C.

3.1.3. Dynamic equations for material energy accumulators

In gas turbine plants, there are pipes, nozzles and heat exchangers along the path of the working fluid. Such elements are often energy accumulators that influence the control process to some extent. When studying processes in gas volumes, it is necessary to distinguish between two cases: an adiabatic process and a process with heat exchange [8].

Consider the capacities of the gas volumes between the compressor (C) and the combustion chamber (CC), between the fuel supply distributor (m_f) and the combustion chamber, the gas volume of the combustion chamber itself, and the gas volume between the combustion chamber and the turbine (T), in which the process of heat exchange under pressure p takes place (Fig. 3).

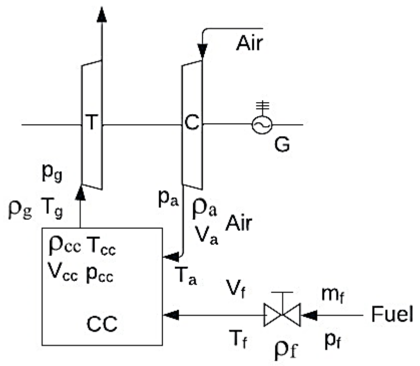

Fig.3. Design scheme of gas volumes of the GTP.

The supply or removal of heat from the outside can fundamentally change the state of the working fluid and thus affect the control process. In this case, the process is considered barotropic (when the density is a function of pressure only) [4], [8].

To establish its mode, the equation of conservation of mass in the allocated volumes will be as follows:

$$G_a^0 + G_f^0 - G_g^0 = 0.$$

For transitional mode the following equation is applied:

$$G_a + G_f - G_g = V_a \frac{\partial \rho_a}{\partial t} + V_f \frac{\partial \rho_f}{\partial t} + V_{\mathrm{cc}} \frac{\partial \rho_{\mathrm{cc}}}{\partial t} + V_g \frac{\partial \rho_g}{\partial t},$$

where V_a is the gas volume between the compressor and the combustion chamber, m^3; V_f is gas volume between the distributor and the combustion chamber, m^3; V_{cc} is gas volume of the combustion chamber, m^3; V_g is gas volume between the combustion chamber and the turbine, m^3; G_a is air flow rate through the compressor, m^3/s; G_f is fuel flow rate, m^3/s; G_g is gas flow rate through the turbine, m^3/s.

The relationship between the gas parameters in the allocated volumes is established by the Clapeyron-Mendeleev equation:

$$p\rho^{-1} = RT. \tag{5}$$

We assume that in the volumes V_a, V_f and V_g the state of the gas changes isothermally, as in an insulated pipeline. Based on this, equation (5) can be written in the form [4], [5]:

$$\frac{\partial \rho_a}{\partial t} = \frac{1}{RT_a^0}\frac{\partial \rho}{\partial t}; \qquad \frac{\partial \rho_f}{\partial t} = \frac{1}{RT_f^0}\frac{\partial \rho}{\partial t}; \qquad \frac{\partial \rho_g}{\partial t} = \frac{1}{RT_g^0}\frac{\partial \rho}{\partial t}.$$

In volume V_{cc}, the process involves heat exchange, i.e., the density cannot be considered a function of pressure alone. Therefore, the change in the density of the combustion chamber will be described by the following equation:

$$\frac{\partial \rho_{cc}}{\partial t} = \frac{1}{RT_{cc}^0}\frac{\partial p}{\partial t} - \frac{p}{R{T_{cc}^0}^2}\frac{\partial T_{cc}}{\partial t}.$$

The flow rates G_a and G_g are presented as a function of the pressure and temperature in the volume, as well as the rotor speed of the compressor and turbine. The fuel consumption G_f depends only on the position of the distributing valve. The effect of the temperature upstream of the turbine on the flow rate G_a is reflected in the pressure change. The effect of rotational speed on the turbine gas flow rate is small and can be assumed to be $\frac{\partial G_g}{\partial \omega}\Delta\omega = 0$.

The cooling air injected into the turbine was modelled using a duct passage to connect the compressor exit with the cooled turbine stage blades. Thus, the cooling air injected into the turbine affects the pressure and temperature of the main gas stream. By that it modifies the expansion line of the gas turbine component. The pressures at different locations where cooling air is injected into the turbine have determined the amount of air flow rate for respective stage cooling. The mass flow of the cooling air for each blade row can be calculated as proposed by [9] and presented in [10]:

$$m_{air} = K\frac{P_{out}}{\sqrt{T_{out}}}\sqrt{\left(1-\frac{P_{st}}{P_{out}}\right)},$$

where m_{air} is cooling air flow rate to the turbine; K is discharge coefficient, which depends on turbine design; P_{out} is air pressure at compressor output; T_{out} is air temperature at compressor output; P_{st} is turbine pressure at a specific stage.

Then, after performing the known transformations, we obtain a differential equation that describes the change in gas parameters in the selected volumes [4], [5]:

$$T_p\frac{\partial\Delta p}{\partial t} + \Delta p = T_T\frac{\partial T_{cc}}{\partial t} - k_T\Delta T_{cc} + k_m\Delta m_f + k_\omega\Delta\omega, \quad (6)$$

where $T_p = \frac{\frac{M_a+M_f+M_{cc}+M_g}{p}}{\frac{\partial G_g}{\partial p}-\frac{\partial G_a}{\partial p}}$; $T_T = \frac{\frac{V_{cc}p}{RT_{cc}^2}}{\frac{\partial G_g}{\partial p}-\frac{\partial G_a}{\partial p}}$; $k_T = \frac{\frac{\partial G_g}{\partial p}}{\frac{\partial G_g}{\partial p}-\frac{\partial G_a}{\partial p}}$; $k_m = \frac{\frac{\partial G_f}{\partial m_f}}{\frac{\partial G_g}{\partial p}-\frac{\partial G_a}{\partial p}}$; $k_\omega = \frac{\frac{\partial G_a}{\partial \omega}}{\frac{\partial G_g}{\partial p}-\frac{\partial G_a}{\partial p}}$.

3.2. Initial data

Siemens gas turbines with the following parameters (Table 1) were selected for modelling, on which the frequency change depends:

Table 1. The studied GTPs and their significant parameters.

GTP	Power, MW	Gas pressure ratio	Reduced moment of inertia, kg·m^2
SGT-300	7.9	13.7	1663.645
SGT-700	35.2	20.4	3655.078
SGT6-5000F	260	19.5	19928.17
SGT5-4000F	385	21	36342.22
SGT5-8000H	450	21	57424.56
SGT5-9000HL	593	24	102662.8

Table 2 shows the initial data used to design the model:

Table 2. Initial data.

Description	Variable	Value	Units of measurement
The temperature of the gases in front of the turbine	T_3	1200	°C
Air temperature at the compressor inlet	T_1	15	°C
The enthalpy of air at temperature T_1	h_{1a}	15.05	kJ/kg
Standard air temperature	T_{st}	25	°C
Enthalpy of air at standard temperature T_{st}	h_{sta}	25.08	kJ/kg
Enthalpy of gases at standard temperature T_{st}	h_{stg}	26.77	kJ/kg
Enthalpy of gases at temperature T_3	h_{3g}	1479.55	kJ/kg
Enthalpy of air at temperature T_3	h_{3a}	1330.08	kJ/kg
Pressure loss coefficient	λ	0.95	-
The coefficient of fuel heat consumption in the CC	η_{cc}	0.995	-

Table 2 (continued)

Description	Variable	Value	Units of measurement
Mechanical efficiency of the turbine	η_m	0.995	-
Electric generator efficiency	η_{eg}	0.982	-
Isentropic efficiency of the turbine	η_T	0.88	-
The isentropic efficiency of the compressor	η_c	0.86	-
Loss ratio	α_y	0.005	-
The minimum amount of air required for the complete combustion of 1 kg of gas	L_0	15	kg/kg
The heat of combustion	K_T	44300	kJ/kg

3.3. Set of formulae

To design the GTP model, we used the formulae presented in Table 3 – Table 6:

Table 3. Formulae used in model designing.

Explanation of the formula	Formula
Air temperature after compression	$T_2=\left((T_1+273)\cdot\left(1+\frac{\sigma^{m_a}-1}{\eta_c}\right)\right)-273$
Excess air coefficient	$\alpha=\frac{K_T\cdot\eta_{cc}+L_0\cdot h_{3a}+h_f-(1+L_0)\cdot h_{3g}}{L_0\cdot(h_{3a}-h_2)}$
Gas temperature after the turbine	$T_4=(T_1+273)\cdot\left(1-(1-(\lambda\cdot\sigma)^{m_g})\cdot\eta_T\right)-273$
Expansion work of 1 kg of gas in a turbine	$H_T=h_3-h_4$
Work consumed to compress 1 kg of air in the compressor	$H_K=h_2-h_1$
GTP operation on the unit shaft	$H_e=H_T\cdot\eta_m-\frac{\alpha\cdot L_0\cdot(1+\alpha_y)}{1+\alpha\cdot L_0}\cdot H_K$
Air flow through the turbine	$G_T=\frac{N_e}{H_e\cdot\eta_{eg}}$
Air flow rate to the compressor	$G_c=\frac{\alpha\cdot L_0\cdot(1+\alpha_y)}{1+\alpha\cdot L_0}\cdot G_T$
Fuel consumption	$B_T=\frac{G_T}{(1+\alpha\cdot L_0)}$
The power of the gas turbine	$N_T=G_T\cdot H_T$
The power consumed by the compressor	$N_c=G_c\cdot H_c$
Efficiency factor	$\varphi=\frac{N_T-N_c}{N_T}$
Electrical efficiency of the GTP	$\eta_{\mathrm{e}}=\frac{G_T\cdot H_e\cdot\eta_{eg}}{B_T\cdot K_T}$

Table 4. Formulae for calculating the coefficients of the dynamic equation for material energy accumulators.

Description of the formula	Formula
Denominator den	$den=\frac{G_T}{\sqrt{\frac{p^2-p_4^2}{p^2-p_4^2}}}\cdot\sqrt{T_3}\cdot\frac{p}{p^2-p_4^2}+0.83$
Lag coefficient T_p	$T_p=\frac{\frac{\mathrm{M}_f+\mathrm{M}_c+\mathrm{M}_{\mathrm{cc}}+\mathrm{M}_g}{p}}{den}$
T_T coefficient by channel $\frac{dT_{\mathrm{cc}}}{dt}\to p$	$T_T=\frac{\frac{V_{\mathrm{cc}}\cdot p}{R\cdot(T_3+273)^2}}{den}$
k_t coefficient by channel $\Delta T_{\mathrm{cc}}\to p$	$k_t=\frac{\frac{-G_T}{\sqrt{T_3}}\cdot\frac{T_3+273}{(T_3+273)^2}}{den}$
k_m coefficient by channel $\Delta m_f\to p$	$k_m=\frac{1}{den}$
k_w coefficient by channel $\Delta\omega\to p$	$k_w=\frac{0.38}{den}$

Table 5. Formulae for calculating the coefficients of the dynamic equation for the GTP rotor.

Description of the formula	Formula
Denominator den	$den = \frac{G_c \cdot H_T \cdot 10^3}{\omega \cdot \eta_c} \cdot \left(\frac{0.38}{G_c} + \frac{0.0024}{\eta_c} - \frac{1}{\omega}\right) + \frac{N_e 10^3}{\omega^2} + \frac{M_T \cdot 10^3}{\omega}$
Lag coefficient B	$B = \frac{I}{den}$
b_1 coefficient by channel $\Delta p'_g \rightarrow \omega$	$b_1 = \frac{\frac{G_T \cdot H_T \cdot 10^3 \cdot \eta_T}{\omega} \cdot \left(\sqrt{T_3} \cdot \frac{p_3}{p_3^2 - p_4^2} + \frac{R \cdot (T_3 + 273)}{H_T \cdot 10^3 \cdot p_3} \cdot \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}\right)}{den}$
b_2 coefficient by channel $\Delta p''_g \rightarrow \omega$	$b_2 = \frac{\frac{G_T \cdot H_T \cdot 10^3 \cdot \eta_T}{\omega} \cdot \left(\frac{k \cdot R}{(k-1) \cdot H_T 10^3} \cdot \left(1 - \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}\right) - \frac{T_3 + 273}{2 \cdot (T_3 + 273)^2 \cdot \sqrt{T_3}}\right)}{den}$
b_4 coefficient by channel $\Delta p'_a \rightarrow \omega$	$b_4 = \frac{\frac{G_c \cdot H_T \cdot 10^3}{\omega \cdot \eta_c} \cdot \left(\frac{-0.83}{G_c} + \frac{R \cdot (T_1 + 273)}{p_2 \cdot H_c \cdot 10^3} \cdot \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} + \frac{0.017}{\eta_c}\right)}{den}$
b_5 coefficient by channel $\Delta N_e \rightarrow \omega$	$b_5 = \frac{\frac{1}{\omega}}{den}$

Table 6. Formulae for calculating the coefficients of the dynamic equation for the combustion chamber.

Description of the formula	Formula
Denominator den	$den = 10^3 \cdot \left(G_a \cdot c_g + G_f \cdot c_g\right)$
Lag coefficient A	$A = \frac{M_g \cdot c_g \cdot 10^3}{den}$
a_1 coefficient by channel $\Delta\omega \rightarrow t_g$	$a_1 = \frac{10^3\left(c_a \cdot t_a + c_g \cdot t_g\right)}{den}$
a_2 coefficient by channel $\Delta m_f \rightarrow t_g$	$a_2 = \frac{10^3 \cdot \left(c_f \cdot t_f + Q_n \cdot \eta_{cc} - c_g \cdot t_g\right)}{den}$
a_3 coefficient by channel $\Delta t_a \rightarrow t_g$	$a_3 = \frac{G_a \cdot c_a \cdot 10^3}{den}$
a_5 coefficient by channel $\Delta Q_n^{\mathrm{p}} \rightarrow t_g$	$a_5 = \frac{G_{cc} \cdot \eta_{cc}}{den}$

4. Dynamic properties of transient processes

Using the differential equations (2), (4), (6), the model was implemented in the Simulink Matlab software package (Fig. 4), and using the formulae from Tables 3 – 6 and the initial data from Tables 1 – 2, the coefficients of the model were calculated (Fig.5 – 7).

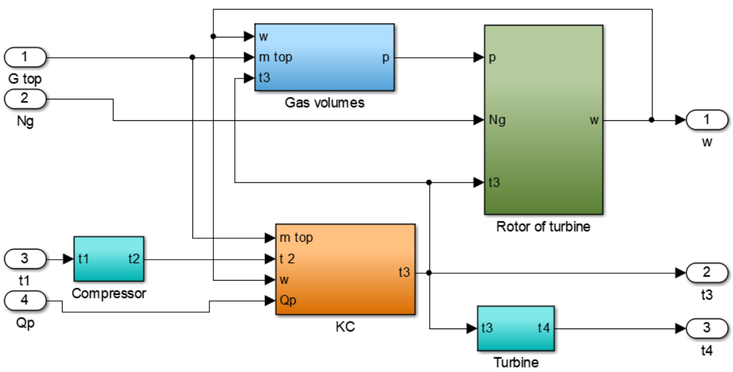

Fig.4. The GTP model.

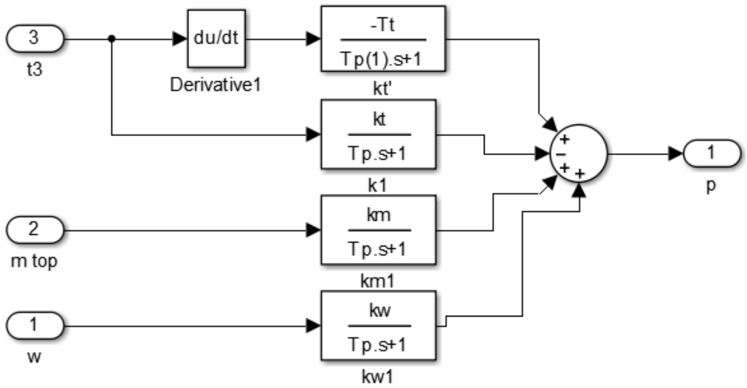

Fig.5. Gas volume model

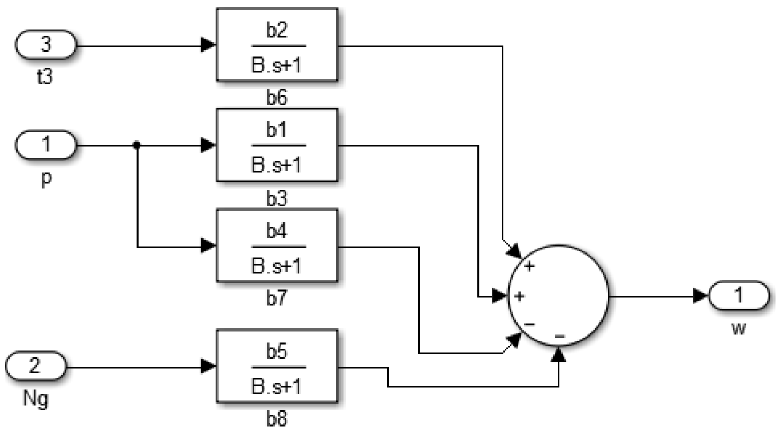

Fig.6. Turbine rotor model.

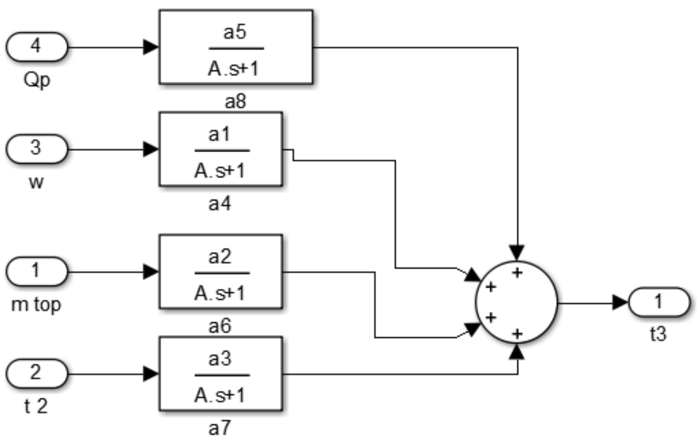

Fig.7. Model of the combustion chamber.

The following figures show the resulting transient processes of the model under the corresponding power disturbances:

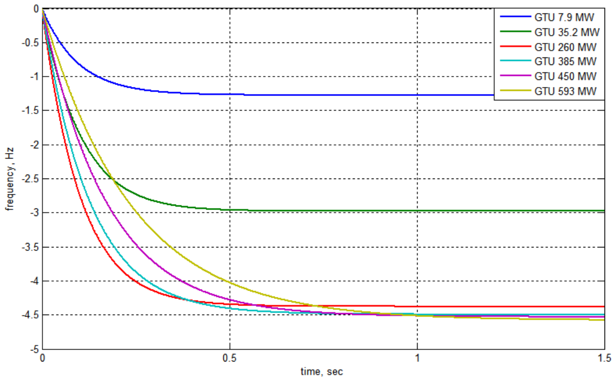

Fig.8. Transient processes during disturbance ΔN = +10%.

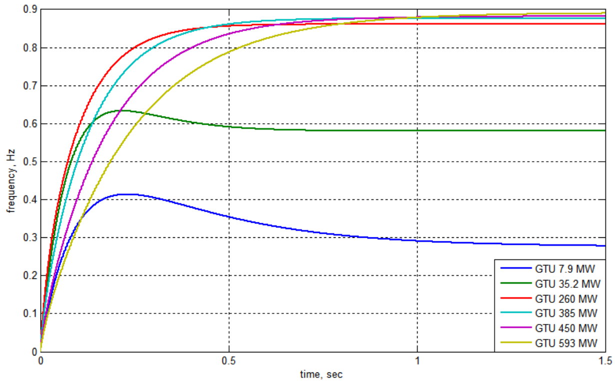

Fig.9. Transient processes during disturbance ΔN = -10%.

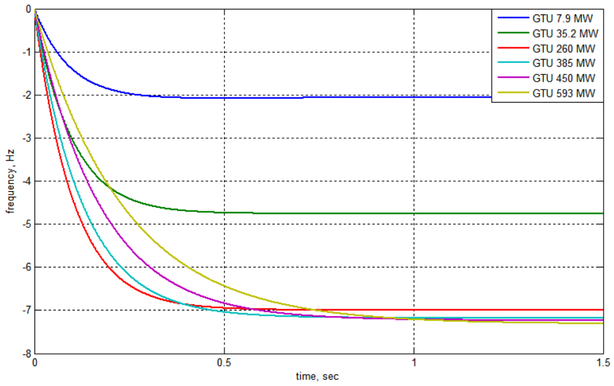

Fig.10. Transient processes during disturbance ΔN = +20%.

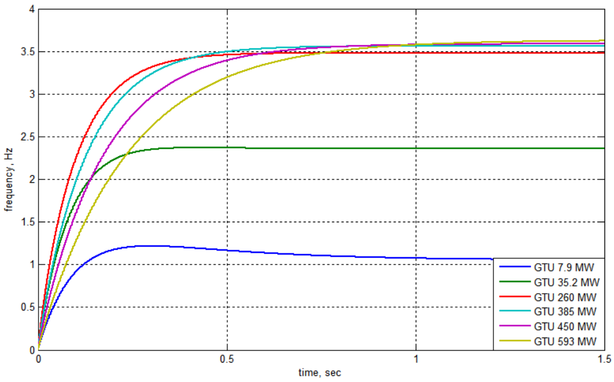

Fig.11. Transient processes during disturbance ΔN = -20%.

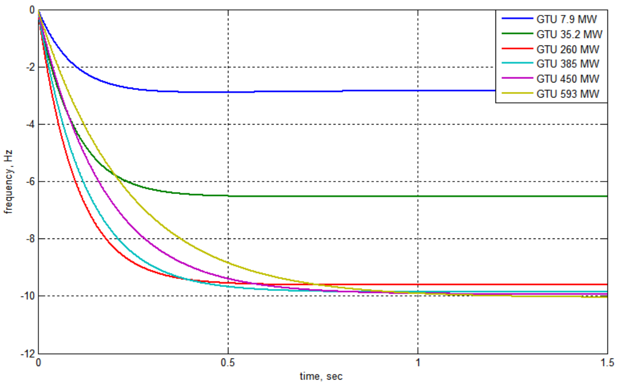

Fig.12. Transient processes during disturbance ΔN = +30%.

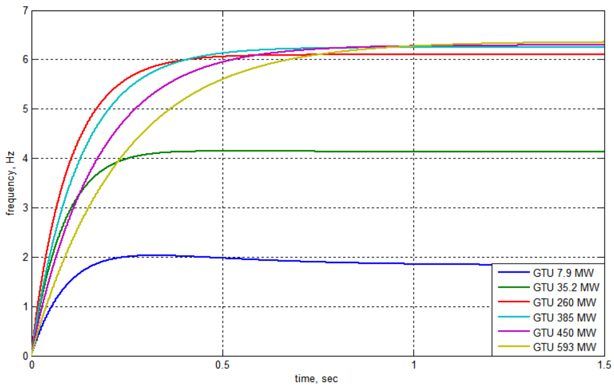

Fig.13. Transient processes during disturbance ΔN = -30%.

Since modelling of the presented transient processes is time-consuming, these transient processes were approximated by a sixth-order polynomial to save time in subsequent calculations:

$$y = a_1x^6 + a_2x^5 + a_3x^4 + a_4x^3 + a_5x^2 + a_6x + a_7,$$

where a_n are the coefficients of the polynomial.

Tables 7 – 12 show the transient processes approximation coefficients.

Table 7. Approximation coefficients of transient processes under disturbance ΔN = +10%.

GTU	Polynomial coefficients						
	a1	a2	a3	a4	a5	a6	a7
SGT-300	5.8705	-30.614	63.478	-66.662	37.31	-10.609	-0.0504
SGT-700	11.963	-63.099	132.79	-142.24	81.749	-24.062	-0.0835
SGT6-5000F	18.616	-97.698	204.29	-217	123.4	-35.876	-0.1086
SGT5-4000F	12.215	-65.734	142.38	-159.15	97.626	-31.779	-0.0487
SGT5-8000H	5.579	-31.487	72.943	-89.829	63.367	-25.086	-0.0051

Table 8. Approximation coefficients of transient processes under disturbance ΔN = -10%.

GTU	Polynomial coefficients						
	a1	a2	a3	a4	a5	a6	a7
SGT-300	-3.2159	16.668	-34.178	35.074	-18.572	4.4857	0.0303
SGT-700	-6.1564	31.12	-61.655	60.339	-30.035	6.8954	0.0733
SGT6-5000F	-4.2598	21.93	-44.743	46.033	-25.11	6.926	0.0853
SGT5-4000F	-2.6948	14.251	-30.206	32.872	-19.524	6.1316	0.0473
SGT5-8000H	-1.3174	7.216	-16.127	19.052	-12.854	4.8847	0.0258

Table 9. Approximation coefficients of transient processes under disturbance ΔN = +20%.

GTU	Polynomial coefficients						
	a1	a2	a3	a4	a5	a6	a7
SGT-300	10.416	-54.266	112.32	-117.54	65.256	-18.157	-0.0907
SGT-700	21.028	-110.23	230.05	-243.56	137.65	-39.543	-0.1619
SGT6-5000F	30.06	-157.54	328.86	-348.56	197.66	-57.279	-0.2055
SGT5-4000F	19.674	-105.74	228.7	-255.19	156.21	-50.735	-0.0967
SGT5-8000H	9.0285	-50.844	117.49	-144.28	101.48	-40.072	-0.0205

Table 10. Approximation coefficients of transient processes under disturbance ΔN = -20%.

GTU	Polynomial coefficients						
	a1	a2	a3	a4	a5	a6	a7
SGT-300	-7.761	40.318	-83.019	85.952	-46.517	12.034	0.0707
SGT-700	-15.22	78.247	-158.91	161.65	-85.936	22.376	0.1517
SGT6-5000F	-15.7	81.763	-169.29	177.58	-99.373	28.328	0.1822
SGT5-4000F	-10.15	54.254	-116.51	128.9	-78.103	25.088	0.0953
SGT5-8000H	-4.766	26.571	-60.669	73.497	-50.966	19.87	0.0412

Table 11. Approximation coefficients of transient processes under disturbance ΔN = +30%.

GTU	Polynomial coefficients						
	a1	a2	a3	a4	a5	a6	a7
SGT-300	14.96	-77.91	161.16	-168.42	93.2	-25.705	-0.131
SGT-700	30.089	-157.34	327.29	-344.86	193.55	-55.023	-0.2402
SGT6-5000F	41.498	-217.36	453.38	-480.09	271.92	-78.682	-0.3024
SGT5-4000F	27.129	-145.74	314.99	-351.2	214.79	-69.691	-0.1447
SGT5-8000H	12.477	-70.196	162.02	-198.72	139.59	-55.057	-0.0359

Table 12. Approximation coefficients of transient processes under disturbance ΔN = -30%.

GTU	Polynomial coefficients						
	a1	a2	a3	a4	a5	a6	a7
SGT-300	-12.305	63.961	-131.85	136.83	-74.461	19.581	0.111
SGT-700	-24.281	125.36	-256.14	262.96	-141.83	37.856	0.23
SGT6-5000F	-27.141	141.59	-293.82	309.11	-173.63	49.731	0.2791
SGT5-4000F	-17.608	94.251	-202.81	224.92	-136.68	44.044	0.1433
SGT5-8000H	-8.2149	45.924	-105.21	127.94	-89.078	34.856	0.0566

5. Dependence of change in dynamic properties of transient processes on current value of GTP power

To find the dependence of the change in the dynamic properties of transient processes on the current value of the GTP power, the time constant for each transient was found for different disturbances. This was done by drawing a tangent to the inflection point when the function graph resembles an oscillatory or aperiodic 2nd order link. In the case when the graph resembles a 1st order aperiodic link, the tangent was drawn to the zero point. After that, the interval between the zero point and the point of intersection of the tangent with the asymptote of the transient on the time axis was found.

Curve fitting was made by means of a second-order polynomial (7), the coefficients of which are shown in Table 13:

$$y = a_0x^2 + a_1x + a_2. \tag{7}$$

Fig.14 – 15 show the time constants for GTPs of the respective capacities at positive and negative deviations.

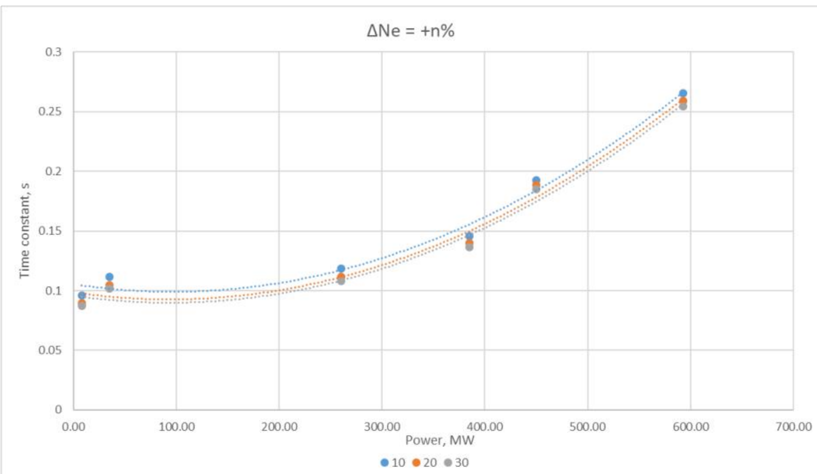

Fig.14. Dependence of time constants on power at positive deviations and their approximation.

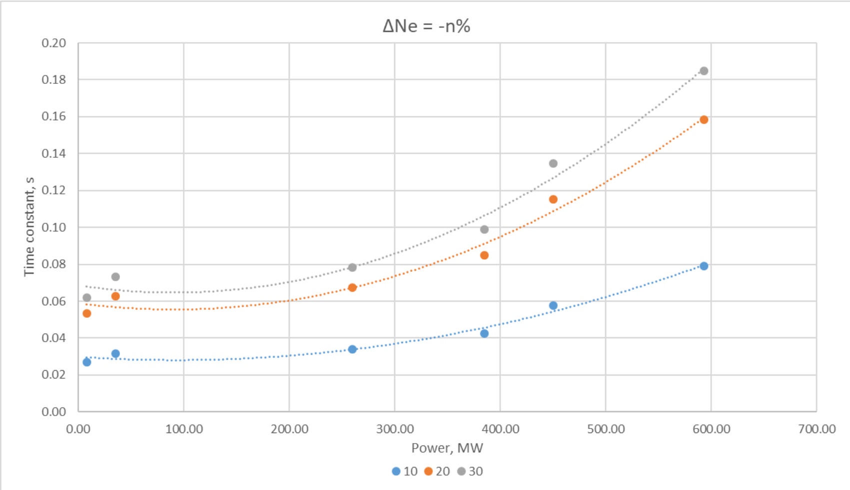

Fig.15. Dependence of time constants on power at negative deviations and their approximation.

Table 13. Coefficients of polynomials approximating the dependence of time constants on power.

Polynomial coefficients	Time constant, s					
	ΔNe = +n%			ΔNe = -n%		
	10	20	30	10	20	30
a_0	7.00E-07	7.00E-07	7.00E-07	2.00E-07	4.00E-07	5.00E-07
a_1	-0.0001	-0.0001	-0.0001	-4.00E-05	-7.00E-05	-9.00E-05
a_2	0.1054	0.0986	0.0958	0.0298	0.0589	0.0686

The obtained approximation dependencies are characterised by high approximation accuracy with a determination coefficient R^2 value of more than 0.98, which indicates high quality of the results. In addition, it should be noted that these dependencies are nonlinear and show variations depending on different levels of disturbance, in particular, with negative power changes. The obtained data can be used in the regulation of the power system of gas turbine plants with different power levels, which emphasizes their value and practical significance.

6. Conclusion

The article considers a model of gas turbine plants (GTP) with concentrated parameters for analyzing the dynamics of transients under power changes. The main purpose of the article is to study the dynamic properties of GTPs, in particular transients, taking into account power changes. The literature review indicated the relevance of studying the dynamics of GTPs in order to optimize their operation. The development of mathematical models to predict the dynamic behaviour of GTPs is an important task. Models with concentrated parameters allow simplifying the analysis of system dynamics.

The authors of this article have developed a mathematical model of a gas turbine with concentrated parameters for analyzing the dynamics of power changes. This model allows for a detailed analysis of the influence of factors on the system dynamics. To achieve this goal, the authors formulated tasks such as developing a model with concentrated parameters, studying the dynamics of transient processes and obtaining dependencies between the dynamic properties and power of a gas turbine engine. The article emphasizes the importance of modelling the dynamics of GTPs with regard to concentrated parameters. The obtained results confirm the high accuracy of the approximation dependencies, which can be used to regulate power systems of gas turbine plants of different power levels.

Thus, the article deals with the important topic of studying dynamic processes in gas turbine plants with regard to power changes. The developed model with concentrated parameters is a powerful tool for analyzing the dynamics and optimizing the operation of such systems.

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Модель газотурбінної установки зі зосередженими параметрами для аналізу закономірностей динамічних властивостей

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Анотація

У даній статті розглядається модель газо-турбінних установок (ГТУ) зі зосередженими параметрами для аналізу закономірностей динамічних властивостей перехідних процесів при зміні потужностей. Основна мета полягає в дослідженні та аналізі зміни динамічних властивостей ГТУ, зокрема їх перехідних процесів зі зміненою потужності. Для досягнення цієї мети ставиться завдання розробки універсальної моделі на основі апроксимаційних підходів з використанням зосереджених параметрів. Основні завдання статті включають: розробку математичної моделі зі зосередженими параметрами, яка враховує поточну номінальну потужність ГТУ, дослідження динамічних властивостей перехідних процесів ГТУ з використанням апроксимаційних моделей, отримання апроксимаційної залежності, яка показує взаємозв'язок зміни динамічних властивостей перехідних процесів від зміни поточної потужності ГТУ. Зосередження на розробці універсальної моделі з апроксимаційними параметрами покладає основу для детального аналізу динамічних властивостей ГТУ та їх поведінки при різних рівнях потужності. Ці дослідження можуть мати практичне застосування у покращенні регулювання та оптимізації енергосистем, що базуються на газо-турбінних установках.

Ключові слова: динамічні властивості; апроксимаційні моделі; потужність ГТУ; регулювання енергосистем.