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On modeling a lexicographic weighted maxmin–minmax approach for fuzzy linear goal programming

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In this paper, a novel approach for solving fuzzy goal programming is proposed. This approach utilizes the weighted maxmin and weighted minmax methods simultaneously. Relative weight is assigned to each fuzzy goal according to the preference of the decision maker. A model for each of the two methods is separately stated; hence the two models are merged into one. Moreover, the lexicographic maximization technique is applied to guarantee efficient solutions. Therefore, the proposed approach allows the decision maker to compromise between the two methods. Furthermore, the proposed approach can be implemented to concave piecewise linear membership functions. This type of membership function is represented using the min-operator. The effectiveness of the proposed approach is illustrated by a numerical example.

Keywords: *fuzzy goal programming; weighted maxmin; weighted minmax; lexicographic maximization; efficiency.*

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1. Introduction

Both weighted maxmin and minmax approaches have been utilized extensively in fuzzy mathematical programming. Zimmermann [1] proposed the application of fuzzy linear programming methods to the linear vector-maximum problem in one of his early works. Hannan [2] provided one of the most well-known models for solving fuzzy goal programming problems when the fuzzy goals have linear membership functions. Yaghoobi and Tamiz [3] proposed an extension of Hannan's model that deals with unbalanced triangular linear membership functions. They applied the conventional minmax approach for solving fuzzy goal programming problems. Furthermore, their model is demonstrated to be equivalent to a model provided by Yang et al. [4]. Additionally, a weighted model is stated and compared to the model given by Kim and Whang [5]. Lin [6] presented a weighted maxmin approach for fuzzy goal programming. His approach can be adapted to complicated membership functions, such as quasiconcave membership functions. Iskander [7] utilized the approach by Lin [6] in stochastic fuzzy goal programming. In his paper, the weights are considered as either trapezoidal or triangular fuzzy numbers. It is required that the sum of the least values of all fuzzy weights should be less than one, while the sum of the highest values of all fuzzy weights should be greater than one. Also, Amid et al. [8] applied the approach by Lin [6] to the fuzzy multi-objective supplier selection problem. Their methodology is based on determining the weights of the objective functions using an analytical hierarchy process. Moreover, the weighted minmax approach is used to solve the normalized fuzzy goal programming problems [9]. Cheng et al. [10] proposed an approach for solving fuzzy multi-objective linear programming problems where all the coefficients are triangular fuzzy numbers and all the constraints are fuzzy equality or inequality. In their work, the fuzzy multi-objective linear programming problem is transformed into a crisp linear programming problem using the deviation degree measures and weighted maxmin method. Furthermore, Iskander [11] merged the maxmin approach with the lexicographic maximization approach in fuzzy goal programming according to four dominance criteria. In each fuzzy goal, the coefficients and the aspiration level are considered either trapezoidal or triangular fuzzy numbers. Umarusman [12] proposed the De novo programming and the minmax goal programming approaches. In his study, each goal constraint had both positive and negative deviation variables. Banik and Bhattacharya [13] made improvements to the study by Umarusman [12]. Moreover, Umarusman [14] utilized a minmax approach and a fuzzy goal programming approach to solve a multiple De novo programming problem.

Zangiabadi and Maleki [15] applied the minmax form of goal programming to the fuzzy model of multi-objective transportation problems. They showed how to solve the multi-objective transportation problem using fuzzy goal programming to find the best compromise solution. Besides, they assigned each objective function a special form of nonlinear (hyperbolic) membership function to define each fuzzy goal. Venkatasubbaiah et al. [16] proposed a fuzzy goal programming approach for solving multi-objective transportation problems. A fuzzy maxmin operator is utilized to demonstrate the efficiency of their proposed approach. Ikeagwuani et al. [17] incorporated a minmax fuzzy goal programming model into the Taguchi optimization method to optimize additives for the enhancement of the properties of expansive soil. On the other hand, Raskin et al. [18] utilized the fuzzy concept in the multi-criteria optimization problem when the relative weights, which reflect the importance of the criteria, are not clearly defined.

In this paper, both the weighted maxmin and weighted minmax methods are simultaneously utilized within the lexicographic maximization approach to solving fuzzy goal programming problems. The membership functions of the fuzzy goals can be presented in the form of concave piecewise linear membership functions, with multiple linear sub-functions. The remainder of this paper is organized as follows. In Section 2, the fuzzy goal programming model with its crisp equivalent weighted maxmin and weighted minmax programs are presented. In Section 3, the general lexicographic weighted maxmin-minmax program is provided. Section 4 presents a numerical example to demonstrate the implementation of the proposed approach. Finally, in Section 5, conclusions are drawn.

2. Weighted maxmin and minmax fuzzy goal programs

Let the fuzzy goal programming model [19] be presented as:
Find $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$ to satisfy the following m fuzzy goals:

$$\sum\nolimits_{j=1}^{n} a_{ij}x_j \gtrsim g_i, \quad i = 1, 2, \ldots, k, \tag{1}$$

$$\sum\nolimits_{j=1}^{n} a_{ij}x_j \lesssim g_i, \quad i = k+1, k+2, \ldots, m, \tag{2}$$

$$\text{subject to} \quad \boldsymbol{B}\boldsymbol{x} \begin{pmatrix} \leqslant \\ = \\ \geqslant \end{pmatrix} \boldsymbol{b}, \tag{3}$$

$$\boldsymbol{x} \geqslant 0, \tag{4}$$

where $\boldsymbol{x}$ is an n-vector of non-negative decision variables, a_{ij} is the coefficient of x_j in the ith fuzzy goal, $\gtrsim$ and $\lesssim$ mean approximately greater than or equal to and approximately less than or equal to, respectively, while g_i is the aspiration level of the ith fuzzy goal. Constraint set (3) represents the set of linear crisp system constraints, where $\boldsymbol{B}$ is a $p \times n$ matrix of coefficients and $\boldsymbol{b}$ is a $p \times 1$ vector of constants. Let $(\boldsymbol{ax})_i = \sum_{j=1}^{n} a_{ij}x_j$. Then, the membership function $\xi_i((\boldsymbol{ax})_i)$ of the ith fuzzy goal in (1) and (2) is, respectively, defined as [6]:

$$\xi_i((\boldsymbol{ax})_i) = \begin{cases} 1 & \text{if } (\boldsymbol{ax})_i \geqslant g_i, \\ f_i((\boldsymbol{ax})_i) & \text{if } \ell_i \leqslant (\boldsymbol{ax})_i < g_i, \\ 0 & \text{if } (\boldsymbol{ax})_i < \ell_i, \end{cases} \tag{5}$$

and

$$\xi_i((\boldsymbol{ax})_i) = \begin{cases} 1 & \text{if } (\boldsymbol{ax})_i \leqslant g_i, \\ f_i((\boldsymbol{ax})_i) & \text{if } g_i < (\boldsymbol{ax})_i \leqslant u_i, \\ 0 & \text{if } (\boldsymbol{ax})_i > u_i, \end{cases} \tag{6}$$

where ℓ_i is the lower tolerance limit of the ith fuzzy goal according to (1), and u_i is the upper tolerance limit of the ith fuzzy goal according to (2). Moreover, $f_i((\boldsymbol{ax})_i)$ takes the form of a linear or a concave piecewise linear function. Hence, by utilizing the min-operator, the common membership function μ_i of the ith fuzzy goal in (1) and (2) can be represented as follows:

$$\begin{aligned} \mu_i &= \min\{f_i((\boldsymbol{ax})_i),\, 1\}, \quad && i = 1, 2, \dots, m, \\ \mu_i &\geqslant 0, && i = 1, 2, \dots, m, \end{aligned} \tag{7}$$

or, equivalently,

$$\begin{aligned} \mu_i &\leqslant f_i((\boldsymbol{ax})_i), && i = 1, 2, \dots, m, \\ \mu_i &\leqslant 1, && i = 1, 2, \dots, m, \\ \mu_i &\geqslant f_i((\boldsymbol{ax})_i) - (1 - q_i)R, \quad && i = 1, 2, \dots, m, \\ \mu_i &\geqslant 1 - q_i R, && i = 1, 2, \dots, m, \\ q_i &\in \{0, 1\},\ \mu_i \geqslant 0, && i = 1, 2, \dots, m, \end{aligned} \tag{8}$$

where q_i is the ith binary variable and R is a large positive number. According to the General Algebraic Modeling System (GAMS) software, using the min-operator in (7) makes it in the form of a nonlinear program with discontinuous derivatives, while its equivalent transformation (8) takes the form of a mixed zero-one linear program. The number of constraints in (8) is significantly greater than that in (7). Moreover, when $f_i((\boldsymbol{ax})_i)$ is a concave piecewise linear function and in accordance with (8), the number of both the constraints and the binary variables increases as the number of sub-functions in the piecewise function increases. For instance, if $f_i((\boldsymbol{ax})_i)$ consists of two sub-functions, then q_i should be replaced by two binary variables (q_{i1} for the first sub-function and q_{i2} for the second). Also, each ith constraint of $f_i((\boldsymbol{ax})_i)$ has to be represented by two constraints. Finally, the constraint $\mu_i \geqslant 1 - q_i R$ must take the form of $\mu_i \geqslant 1 - (q_{i1} + q_{i2})R$.

In Section 4, the importance of utilizing the min-operator to represent the membership functions is illustrated.

2.1. The weighted maxmin goal program

According to the approach by Lin [6], the weighted maxmin goal program can be stated as follows:

$$\begin{aligned} &\text{Maximize} && \lambda_1 \\ &\text{subject to} && w_i \lambda_1 \leqslant \mu_i, \quad i = 1, 2, \dots, m, \\ &&& \text{and } (3), (4), \end{aligned} \tag{9}$$

where μ_i represents the value of the membership function (achievement degree) of the ith fuzzy goal, which is evaluated either according to the min-operator (7) or (8), and w_i is a positive relative weight of the ith fuzzy goal, $\sum_{i=1}^{m} w_i = 1$. Lin's proposition states that this program provides solutions in which the ratio between any two positive achievement degrees is as close as possible to the ratio of their corresponding relative weights [6].

2.2. The weighted minmax goal program

This program aims to minimize the largest weighted underachievement of the fuzzy goals. Therefore, the weighted minmax goal program can be presented as follows:

$$\begin{aligned} &\text{Minimize} && \lambda_2 \\ &\text{subject to} && \mu_i + d_i = 1, \quad && i = 1, 2, \dots, m, \\ &&& w_i d_i \leqslant \lambda_2, && i = 1, 2, \dots, m, \\ &&& \text{and } (3), (4), \end{aligned} \tag{10}$$

where d_i is the underachievement of the ith fuzzy goal, $0 \leqslant d_i \leqslant 1$. This program gives solutions in which the ratio between the positive underachievement of any two fuzzy goals is as close as possible to the reciprocal ratio of their corresponding relative weights [9]. This means that the products of these underachievements by their corresponding relative weights are as close as possible.

3. Lexicographic weighted maxmin-minmax goal program

Both the weighted maxmin and weighted minmax approaches can be merged as one approach to represent the general weighted maxmin-minmax program as follows:

$$\begin{aligned}
\text{Maximize} \quad & \alpha\lambda_1 - (1-\alpha)\lambda_2 & & \\
\text{subject to} \quad & w_i\lambda_1 \leqslant \mu_i, & & i = 1, 2, \dots, m, \\
& \mu_i + d_i = 1, & & i = 1, 2, \dots, m, \\
& w_i d_i \leqslant \lambda_2, & & i = 1, 2, \dots, m, \\
& \text{and } (3), (4), & &
\end{aligned} \tag{11}$$

in addition to either (7) or (8). The value of α, $\alpha \in [0, 1]$, is set by the decision maker, who can obtain different solutions by adjusting the values of α. Note that when $\alpha = 1$, only the weighted maxmin approach is utilized, and when $\alpha = 0$, only the weighted minmax approach is utilized. Accordingly, the other different values of α compromise between the two approaches, which provide a more flexible decision support.

However, Ogryczak [20] stated that when the minmax and a regularization term (sum of the weighted underachievements) are used within a lexicographic minimization technique, efficient solutions are guaranteed. Hence, the lexicographic maximization technique can be utilized in program (11) by incorporating the regularization term either as the sum of the weighted membership functions or the sum of the membership functions. Accordingly, the objective function in program (11) can be represented by either

$$\text{Lexicographically maximize} \quad \left\{\alpha\lambda_1 - (1-\alpha)\lambda_2,\ \sum_{i=1}^{m} w_i\mu_i\right\}, \tag{12}$$

or

$$\text{Lexicographically maximize} \quad \left\{\alpha\lambda_1 - (1-\alpha)\lambda_2,\ \sum_{i=1}^{m} \mu_i\right\}. \tag{13}$$

The sum of the membership functions according to (13) is greater than or equal to the sum of the membership functions according to (12). As a result, the decision maker may prefer to use (13). Notably, the impact of maximizing $\sum_{i=1}^{m} w_i\mu_i$ is equivalent to the impact of minimizing $\sum_{i=1}^{m} w_i d_i$, and the impact of maximizing $\sum_{i=1}^{m} \mu_i$ is equivalent to the impact of minimizing $\sum_{i=1}^{m} d_i$.

The proposed lexicographic weighted maxmin-minmax goal program is implemented in the next section.

4. Numerical example

In this section, the proposed approach is illustrated using the first numerical example by Lin [6]. This example considers the case when the membership functions of the fuzzy goals are concave piecewise linear functions. Therefore, the fuzzy goal programming problem is presented as follows:

Find $\boldsymbol{x} = (x_1, x_2, x_3)^T$

$$\begin{aligned}
\text{to satisfy} \quad & z_1 = 3x_1 + x_2 + x_3 \gtrsim 7, \\
& z_2 = x_1 - x_2 + 2x_3 \gtrsim 8, \\
& z_3 = x_1 + 2x_2 \gtrsim 5, \\
\text{subject to} \quad & 4x_1 + 2x_2 + 3x_3 \leqslant 10, \\
& x_1 + 3x_2 + 2x_3 \leqslant 8, \\
& x_3 \leqslant 5, \\
& x_1, x_2, x_3 \geqslant 0.
\end{aligned}$$

Then, the membership functions of the three fuzzy goals are, respectively, given as follows:

$$\xi_1(z_1) = \begin{cases} 1 & \text{if } z_1 \geqslant 7, \\ 0.2(z_1 - 6) + 0.8 & \text{if } 6 \leqslant z_1 < 7, \\ 0.3(z_1 - 5) + 0.5 & \text{if } 5 \leqslant z_1 < 6, \\ 0.5(z_1 - 4) & \text{if } 4 \leqslant z_1 < 5, \\ 0 & \text{if } z_1 < 4, \end{cases}$$

$$\xi_2(z_2) = \begin{cases} 1 & \text{if } z_2 \geqslant 8, \\ 0.15(z_2 - 4) + 0.4 & \text{if } 4 \leqslant z_2 < 8, \\ 0.2(z_2 - 2) & \text{if } 2 \leqslant z_2 < 4, \\ 0 & \text{if } z_2 < 2, \end{cases}$$

$$\xi_3(z_3) = \begin{cases} 1 & \text{if } z_3 \geqslant 5, \\ 0.2(z_3 - 4) + 0.8 & \text{if } 4 \leqslant z_3 < 5, \\ 0.4(z_3 - 2) & \text{if } 2 \leqslant z_3 < 4, \\ 0 & \text{if } z_3 < 2. \end{cases}$$

Hence, according to Lin [6], the crisp weighted maxmin linear goal program takes the following form:

$$\begin{aligned} \text{Maximize} \quad & \lambda \\ \text{subject to} \quad & 0.4\lambda \leqslant 0.2(z_1 - 6) + 0.8, \\ & 0.4\lambda \leqslant 0.3(z_1 - 5) + 0.5, \\ & 0.4\lambda \leqslant 0.5(z_1 - 4), \\ & 0.35\lambda \leqslant 0.15(z_2 - 4) + 0.4, \\ & 0.35\lambda \leqslant 0.2(z_2 - 2), \\ & 0.25\lambda \leqslant 0.2(z_3 - 4) + 0.8, \\ & 0.25\lambda \leqslant 0.4(z_3 - 2), \\ & 4x_1 + 2x_2 + 3x_3 \leqslant 10, \\ & x_1 + 3x_2 + 2x_3 \leqslant 8, \\ & x_3 \leqslant 5, \\ & x_1, x_2, x_3 \geqslant 0, \end{aligned} \tag{14}$$

where the relative weights of the three fuzzy goals are equal to 0.4, 0.35, and 0.25, respectively. Alternatively, by utilizing the min-operator, the weighted maxmin goal program according to (7) and (9) can be stated as follows:

$$\begin{aligned} \text{Maximize} \quad & \lambda_1 \\ \text{subject to} \quad & \mu_1 = \min\{0.2(z_1 - 6) + 0.8,\ 0.3(z_1 - 5) + 0.5,\ 0.5(z_1 - 4),\ 1\}, \\ & \mu_2 = \min\{0.15(z_2 - 4) + 0.4,\ 0.2(z_2 - 2),\ 1\}, \\ & \mu_3 = \min\{0.2(z_3 - 4) + 0.8,\ 0.4(z_3 - 2),\ 1\}, \\ & 0.4\lambda_1 \leqslant \mu_1, \\ & 0.35\lambda_1 \leqslant \mu_2, \\ & 0.25\lambda_1 \leqslant \mu_3, \\ & 4x_1 + 2x_2 + 3x_3 \leqslant 10, \\ & x_1 + 3x_2 + 2x_3 \leqslant 8, \\ & x_3 \leqslant 5, \\ & \lambda_1,\ x_1,\ x_2,\ x_3 \geqslant 0, \end{aligned} \tag{15}$$

where the non-negativity of the membership functions is represented by the non-negativity of λ_1. To illustrate the importance of using the min-operator, both programs (14) and (15) are solved when the relative weights are 0.6, 0.35, and 0.05, and the right-hand sides of the three crisp system constraints are 20, 18, and 6 instead of 10, 8, and 5, respectively. Table 1 shows the results of this case (the relaxed case) for the two programs.

Table 1. The results of programs (14) and (15) for the relaxed case.

	λ^a	x_1	x_2	x_3	z_1	z_2	z_3	μ_1	μ_2	μ_3
Program (14)	3.125	2.825	0	2.900	11.375	8.625	2.825	1	1	0.330
Program (15)	1.667	4.867	0	0.178	14.778	5.222	4.867	1	0.583	0.973

[a] It represents λ_1 in program (15).

From Table 1 and according to program (14), $0.2(z_1-6)+0.8=1.875$, while $0.15(z_2-4)+0.4=1.094$. Hence, as $z_1=11.375$ and $z_2=8.625$, then $\mu_1=1$ and $\mu_2=1$. However, in this case, the values of the three membership functions have no impact on the value of λ, as well as on the verification of Lin's proposition, since $1.875/0.6 \simeq 1.094/0.35 \simeq 3.13$. Conversely, in program (15), the value of λ_1 depends on the value of at least one membership function since $1/0.6 \simeq 0.583/0.35 \simeq 1.67$.

It should be mentioned that, according to the relaxed case, program (15) has alternative optimal solutions. Therefore, the objective function (12) when $\alpha=1$, as well as the min-operator (8), instead of (7), are utilized in program (15) to guarantee an efficient solution. This efficient solution is $x_1=0$, $x_2=2.286$, $x_3=5.143$, $\mu_1=1$, $\mu_2=1$, and $\mu_3=0.914$. Hence, the maximum sum of the three weighted membership functions is 0.996, while the sum of the three membership functions is 2.914. Then again, if the objective function (13) is used, instead of (12), the efficient solution becomes $x_1=0$, $x_2=2.5$, $x_3=5$, $\mu_1=1$, $\mu_2=0.925$, and $\mu_3=1$. Thus, the maximum total achieved value of the three membership functions is 2.925. Therefore, whether according to (12) or (13), the sum of the three membership functions of program (15) is greater than that of program (14).

On the other hand, the crisp lexicographic weighted maxmin-minmax goal program, for Lin's example, according to (7), (11), and (12) is given as follows:

$$
\begin{aligned}
&\text{Lexicographically maximize} && \{\alpha\lambda_1-(1-\alpha)\lambda_2,\ 0.4\mu_1+0.35\mu_2+0.25\mu_3\}\\
&\text{subject to} && \mu_1=\min\{0.2(z_1-6)+0.8,\ 0.3(z_1-5)+0.5,\ 0.5(z_1-4),\ 1\},\\
& && \mu_2=\min\{0.15(z_2-4)+0.4,\ 0.2(z_2-2),\ 1\},\\
& && \mu_3=\min\{0.2(z_3-4)+0.8,\ 0.4(z_3-2),\ 1\},\\
& && 0.4\lambda_1 \leqslant \mu_1,\\
& && 0.35\lambda_1 \leqslant \mu_2,\\
& && 0.25\lambda_1 \leqslant \mu_3,\\
& && \mu_1+d_1=1,\\
& && \mu_2+d_2=1,\\
& && \mu_3+d_3=1,\\
& && 0.4d_1 \leqslant \lambda_2,\\
& && 0.35d_2 \leqslant \lambda_2,\\
& && 0.25d_3 \leqslant \lambda_2,\\
& && 4x_1+2x_2+3x_3 \leqslant 10,\\
& && x_1+3x_2+2x_3 \leqslant 8,\\
& && x_3 \leqslant 5,\\
& && \lambda_1, x_1, x_2, x_3 \geqslant 0.
\end{aligned}
\tag{16}
$$

Program (16), which represents the proposed approach, is solved when $\alpha=1$ (weighted maxmin), $\alpha=0.5$ (a balance between weighted maxmin and weighted minmax), and $\alpha=0$ (weighted minmax).

Three different cases, according to the assumed values of the relative weights, are considered. In the first case, the relative weights are 0.4, 0.35, and 0.25 (the original relative weights). In the second case, the relative weights are 0.1, 0.7, and 0.2. In the third case, the relative weights are 0.1, 0.45, and 0.45. Table 2 shows the results of the three cases.

Table 2. The results of program (16) for the three cases.

	Case 1 (0.4, 0.35, 0.25)			Case 2 (0.1, 0.7, 0.2)			Case 3 (0.1, 0.45, 0.45)		
α	1	0.5	0	1	0.5	0	1	0.5	0
λ_1	0.820	0.808	0.019	0.539	0.539	0	0.604	0.604	0
λ_2	0.269	0.251	0.230	0.436	0.436	0.397	0.328	0.328	0.328
x_1	0.602	0.663	0.767	0.264	0.264	0.222	0.360	0.360	0.360
x_2	0.955	0.921	0.717	1.003	1.003	0.889	1.160	1.160	1.160
x_3	1.893	1.836	1.833	2.313	2.313	2.444	2.080	2.080	2.080
μ_1	0.328	0.372	0.425	0.054	0.054	0	0.160	0.160	0.160
μ_2	0.287	0.283	0.343	0.377	0.377	0.433	0.272	0.272	0.272
μ_3	0.205	0.202	0.080	0.108	0.108	0	0.272	0.272	0.272

In Case 1, the results change as α changes. When $\alpha = 1, 0.328/0.4 = 0.287/0.35 = 0.205/0.25 = 0.82$, whereas when $\alpha = 0, (1 - 0.425)(0.4) = (1 - 0.343)(0.35) = (1 - 0.080)(0.25) = 0.23$. Hence, the properties of the weighted maxmin and the weighted minmax are verified. Furthermore, the value of the total achievement of the three fuzzy goals ($\mu_1 + \mu_2 + \mu_3$) for $\alpha = 1$, 0.5, and 0 is 0.82, 0.857, and 0.848, respectively. This means that the weighted minmax may be preferable than the weighted maxmin; however, the balance between the two approaches may be recommended than each of the two.

In Case 2, for $\alpha = 0$, the first and the third fuzzy goals are completely unachieved ($\mu_1 = \mu_3 = 0$); hence, the trend of the achievements of the fuzzy goals is inconsistent with the relative weights. However, for $\alpha = 1$ and 0.5, the results are much better and the same. Here the decision maker has the option to select the weighted maxmin approach rather than the weighted minmax approach.

In Case 3, the results do not change as α changes, except for λ_1 when $\alpha = 0$, which represents a stable case. This case may also be preferable to the decision maker.

In general, for any optimal solution when $\alpha = 1$, λ_2 may take different values. In contrast, for any optimal solution when $\alpha = 0$, λ_1 may take different values. Notably, in all cases of program (16), the regularization term has no impact on the optimal solutions. Therefore, in these cases, the solutions are efficient irrespective of the regularization term.

Finally, the CONOPT solver is utilized for the min-operator (7), while the CPLEX solver is utilized for the min-operator (8). Both solvers are embedded in the GAMS win32 23.8.2 software.

5. Conclusions

This paper utilizes both the weighted maxmin and the weighted minmax approaches in one approach through the lexicographic maximization technique. In any given situation, the proposed method allows the decision maker to choose the more satisfactory approach among the two approaches. Moreover, the two approaches can be merged to varying degrees. This may provide the decision maker with different sets of solutions. Also, merging the two approaches may provide a total value of the membership functions (total achievement degree) better than that if only one of the two approaches is utilized. This is shown in Case 1, in the numerical example, which illustrates the efficiency of the proposed approach. Nevertheless, the proposed program utilizes the min-operator to represent the concave piecewise linear membership functions. For future research, the implementation of the proposed approach to other types of membership functions, such as quasiconcave piecewise linear membership functions, could be considered.

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Про моделювання лексикографічного зваженого maxmin-minmax підходу для нечіткого лінійного цільового програмування

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У цій статті пропонується новий підхід до вирішення нечіткого цільового програмування. У цьому підході одночасно використовуються методи зваженого maxmin і зваженого minmax. Відносна вага призначається кожній нечіткій цілі відповідно до пріоритетів особи, яка приймає рішення. Модель для кожного з двох методів вказана окремо; тому дві моделі об'єднані в одну. Крім того, для забезпечення ефективних розв'язків застосовано техніку лексикографічної максимізації. У такий спосіб запропонований підхід дозволяє особі, яка приймає рішення, знайти компроміс між двома методами. Крім того, запропонований підхід може бути реалізований для увігнутих кусково-лінійних функцій належності. Цей тип функції належності представлений за допомогою оператора min. Ефективність запропонованого підходу проілюстровано на числовому прикладі.

Ключові слова: *програмування нечітких цілей; зважений maxmin; зважений minmax; лексикографічна максимізація; ефективність.*