MHD Nanofluid boundary layer flow over a stretching sheet with viscous, ohmic dissipation

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(Received 8 June 2022; Revised 25 September 2022; Accepted 12 October 2022)

The objective of this research is to examine the steady incompressible two-dimensional hydromagnetic boundary layer flow of nanofluid passing through a stretched sheet in the influence of viscous and ohmic dissipations. The present problem is obtained with the help of an analytical technique called DTM-Pade Approximation. The mathematical modeling of the flow is considered in the form of the partial differential equation and is transformed into a differential equation through suitable similarity transformation. The force of fixed parameters like thermophoresis number \( N_t \), Brownian motion number \( N_b \), Prandtl number \( Pr \), Lewis number \( Le \), Magnetic field \( M \), suction/injection \( S \) and Eckart number \( Ec \) are displayed with the aid of Figures. Our outcomes showed a greater trend in the velocity profile for the parameters of magnetics \( M \), suction \( S \), and nonlinear stretching parameter \( n \). While the reverse trend is found against the temperature profile when the Prandtl number increases. Lewis number and other parameters have shown increasing behavior in the concentration profile.

Keywords: magnetic field; Eckart number; thermophoresis number; Brownian motion; heat transfer; mass transfer.

2010 MSC: 76D05, 76D10, 76A02, 80A20 DOI: 10.23939/mmc2023.01.195

1. Introduction

Heat transfer enhancement is a fascinating subject to explore in the present day because of its numerous applications in engineering fields. Conventional fluids such as water, engine oil, and ethylene glycol are being used for heat transfer, but they have limited conduction of heat. To complete the heat energy for industrial and engineering places, an alternate way to enhance the heat transfer is by adding nanoparticles to an ordinary fluid called nanofluid. Choi [1] introduced a new fluid in 1995 known as nanofluid. It is a colloidal suspension of nanoparticles, whose size is between \((1 - 100) \) nm. Nano-liquids are used in a variety of human endeavors, electrical and chemical engineering such as power systems, heat pipes, heat sinks, and thermal management of power electronics: semiconductors, air conditioning, refrigeration, fermentation, protein/cell separation, drug delivery, catalysts, lubricants, rotary seals, cancer therapy, antibacterial agents, NMR imaging, magnetic cell sorting, energy saving cooling medium, automobiles, solar energy, solar air collectors, fuel cells, refrigeration, electronic devices cooling, micro-electro-mechanical systems. The biochemical applications include gas sensing, protein and pathogen detection, DNA translocation and sequencing. In addition, utilizing magnetized nanofluid systems is a significant step toward achieving targeted medication administration and differential diagnostics.

Mustafa et al. [2] studied the flow through a stretching sheet is a significant issue in many engineering processes with industry sectors such as melt spinning, plastic and rubber sheets, production of paper, chemical engineering plant, food processing. Khan and Pop [3] deal with the boundary layer flow of nanofluid over a stretching sheet. Hassni et al. [4] depicted an analytical solution of the boundary layer flow of nanofluid by the Homotopy analysis method. Recently, Jabeel et al. [5] analyzed


The DTM has been explored and applied to a variety of linear and nonlinear issues in recent years. Rashidi et al. [19] explored a DTM method for solving both linear and nonlinear differential equations. Mujammad et al. [20] illustrated the DTM technique for unsteady nanoliquid flow and heat transfer. Dibyendu and Sanjib [21] approached the DTM and Pade approximation to derive the problems of magnetic field and mass transfer nanofluid flow through a stretching sheet with the influence of Soret and Dufour phenomena. Sheob Rashid et al. [22] solved a problem of free convection and MHD slip flow and heat transfer over a radially stretching sheet with thermal radiation using the differential transformation method.

A study of the previously mentioned literature motivated us to present this work with the effects of magnetic, ohmic and viscous dissipation in nanofluid boundary layer flow over a stretching sheet. Khashi et al. [23] discussed the effect of suction on the MHD flow in a doubly stratified micropolar fluid over a shrinking sheet. Majeed et al. [24] predicted the steady thermal boundary layer nanofluid flow was solved by the DTM-Pade approximation technique. Based on his motivation, we have expanded the work of nanoliquid motion with the effects of magnetic, ohmic and viscous dissipation in this problem.

### 2. Mathematical formulation

![Fig. 1. Physical model.](image)

The two-dimensional, laminar, steady nonlinear hydromagnetic flow of an incompressible, viscous, electrically conductive nanofluid has been addressed as Fig. 1. The flow takes place across a nonlinear stretching sheet. A variable magnetic field of strength \( B(x) = B_0 x^{\frac{n+1}{2}} \) is the applied normal force towards the sheet and parallel to the \( y \)-axis. The sheet corresponds to the plane \( y = 0 \), and the flow is confined \( y > 0 \). The \( x \)-axis is positioned in the same direction as the stretching sheet, the \( y \)-axis is angled in the opposite direction. Let \( u = U_w(x) = ax^n \) sym-
bolized as a stretching sheet velocity, where \( \alpha \) is a positive constant and \( n \) is a nonlinear stretching parameter. An energy equation takes into account both viscous and ohmic dissipation. The ambient temperature and concentration are indicated as \( T_\infty \) and \( C_\infty \), respectively. We made an assumption that the constant temperature was \( T_W > T_\infty \) and that the constant temperature was \( C_W > C_\infty \).

The governing equations for the boundary layer, which include momentum, energy, and concentration equations with dissipation effects are as follows [19, 24]

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} &= \frac{\nu^2 u}{\partial y^2} - \frac{\sigma B_0^2 (x) u}{\nu}, \\
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ \frac{\partial C}{\partial T} + \left( \frac{D_T}{D_\infty} \right) \left( \frac{\partial T}{\partial y} \right)^2 \right] + \nu \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2 u^2}{(\nu c_p)_{nf}}, \\
\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} &= D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty^2} \left( \frac{\partial^2 T}{\partial y^2} \right),
\end{align*}
\]

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) axis, \( \nu \) is the nanofluid kinematic viscosity, \( \alpha \) is the nanofluid thermal diffusivity, \( \tau = \frac{\theta \phi}{\rho C_p} \) is the ratio between the effective heat capacity of the nanoparticles and the heat capacity of the base fluid, \( \rho_{nf} \) is the density of the nanoparticle. \( D_B \) is the Brownian diffusion coefficient, \( D_T \) is the thermophoretic diffusion coefficient, and \( C \) is the volumetric expansion coefficient.

The boundary conditions are as follows:

\[
\begin{align*}
u &= V_w(x), \quad u = ax^n, \quad T = T_w, \quad C = C_w \text{ at } y = 0, \\
 u &\to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ at } y \to \infty.
\end{align*}
\]

Let us introduce a similarity transformations

\[
\begin{align*}
\eta &= y \left( a \left( \frac{n + 1}{2} \right) \right)^{\frac{1}{2}} x^{\frac{n}{2}}, \\
 u &= ax^n f'(\eta), \quad v = -av \left( \frac{n + 1}{2} \right) x^{\frac{n-1}{2}} \left( f + \left( \frac{n-1}{n+1} \right) \eta \phi' \right),
\end{align*}
\]

\[
\begin{align*}
T_\infty + \theta(\eta)(T_w - T_\infty) &= T, \\
C_\infty + \phi(\eta)(C_w - C_\infty) &= C.
\end{align*}
\]

Incorporating the above mentioned transformation into the governing equations (1) to (4) reduces to,

\[
\begin{align*}
f'''' + f f''' - \frac{2n}{n+1} f'^2 - M f' &= 0, \\
\frac{1}{Pr} \theta'' + f \theta' + Nb \phi' \theta' + Nt \theta'^2 Ec f'^2 + M f'^2 &= 0, \\
\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' &= 0,
\end{align*}
\]

and the corresponding boundary conditions are

\[
\begin{align*}
f(0) &= S, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \text{ at } \eta = 0, \\
 f'(\eta) &\to 0, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \text{ at } \eta \to \infty,
\end{align*}
\]

primes stands for differentiating with to \( \eta \). \( M \) is the magnetic parameter, \( Pr \) is the Prandtl number, \( Nb \) is the Brownian motion parameter, \( Nt \) is the thermophoresis parameter, \( Le \) is the Lewis number, \( Ec \) is the viscous dissipation parameter (Eckart number), \( S > 0 \) is the suction parameter and \( S < 0 \) is the injection parameter

\[
\begin{align*}
Pr &= \frac{\nu}{\alpha_f}, \quad M = \frac{\sigma B_0^2}{a \phi_f}, \quad Ec = \frac{U_w^2}{(CP)_{nf} (T_w - T_\infty)}, \\
Nt &= \frac{(\rho C)_{nf} D_T (T_w - T_\infty)}{(\rho C)_{nf} T_\infty^2}, \quad Nb = \frac{(\rho C)_{nf} DB (\phi_w - \phi_\infty)}{(\rho C)_{nf} \nu}.
\end{align*}
\]

The Nusselt number $Nu_x$ and Sherwood number $Sh_x$, which are defined as the parameters of practical significance

$$Nu_x = \frac{\chi q_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{\chi q_m}{D_B(C_w - C_\infty)},$$

where $q_w$ and $q_m$ are the wall heat flux and mass flux, respectively; the reduced Nusselt number $Nu_r$ and Sherwood $Sh_r$ can be written as

$$Nu_r = Re_x^{-0.5}Nu_x = -\theta'(0), \quad Sh_r = Re_x^{-0.5}Sh_x = -\phi'(0).$$

3. Differential transformation method with Pade approximation employment

The DTM is one of the famous semi analytical tools to solve the nonlinear differential equation. The boundary condition can be considered from equations (8)–(9) as,

$$F[0] = f(0) = S, \quad F[1] = f'(0) = 1, \quad F[2] = f''(0) = a,$$

$$\theta[0] = \theta(0) = 1, \quad \theta[1] = \theta'(0) = b,$$

$$\phi[0] = \phi(0) = 1, \quad \phi[1] = \phi'(0) = c.$$ Proceeding $F[3]$ for series of momentum profile, we would either predict the value of $F[2]$ or find it. For that purpose, the Pade approximation can be utilised to calculate the unknowns. We have to calculate $\theta[1]$ and $\phi[1]$ for thermal and concentration profiles.

**DTM.** The following are the basic definitions and operations of differential transformation.

Differential transformation of the function $f(\eta)$ is defined as follows

$$F(j) = \frac{1}{j!} \left[ \frac{d^j f(\eta)}{d\eta^j} \right]_{\eta=\eta_0},$$

where $F(j)$, $\theta(j)$ and $\varphi(j)$ are the transformed functions of $f(j)$, $\theta(j)$ and $\varphi(j)$, respectively; they are given by

$$f(\eta) = \sum_{j=0}^{\infty} F(j)\eta^j, \quad \theta(\eta) = \sum_{j=0}^{\infty} \theta(j)\eta^j, \quad \varphi(\eta) = \sum_{j=0}^{\infty} \varphi(j)\eta^j.$$

**Pade approximation.** If a power series is used to represent a function $f(\eta)$, then

$$f(\eta) = \sum_{i=0}^{\infty} C_i \eta^i,$$

where $C_i$, $i = 0, 1, 2, \ldots$ is reserved for the given set of coefficients. The Pade approximant is a rational function and it is given by

$$P[L, M] = \frac{a_0 + a_1\eta + a_2\eta^2 + \ldots + a_L\eta^L}{b_0 + b_1\eta + b_2\eta^2 + \ldots + b_M\eta^M}.$$

**Momentum**

$$\frac{(j+1)(j+2)(j+3)}{j!}F[j+3] + \sum_{t=0}^{j} (j+1)(j+2)F[j-t+2]F[t]$$

$$- \left( \frac{2n}{n+1} \right)^2 \sum_{t=0}^{j} (j-t+1)F[j-t+1](t+1)F[t+1] - M^2(j+1)F[j+1] = 0,$$

$$f(\eta) = S + a\eta^2 + \frac{1}{6} \left( \frac{2n}{n+1} + M^2 - 2Sa \right) \eta^3$$

$$+ \frac{1}{24} \left( \frac{8an}{n+1} + 2M^2a - 2a - S \left( \frac{2n}{n+1} + M^2 - 2Sa \right) \right) \eta^4 + \ldots.$$
Figure 4 illustrates that the velocity gradient increases when the stretching parameter increases and shows that it accelerates the momentum flow. Therefore, the boundary layer thickness increases.

Parameter $M$ the effects of the governing parameters that are nondimensional, namely, the suction parameter $S$, Lewis number $Le$, thermophoresis parameter $Nt$, Brownian motion $Nb$, Eckart number $Ec$, magnetic parameter $M$ on the velocity profile for different values of magnetic parameter $M$ in Figure 3. It is making a thinner momentum boundary layer. The increasing trend of velocity profile for different values of magnetic parameter $M$ is studied in Figure 3 and shows that it accelerates the momentum flow. Therefore, the boundary layer thickness increases.

Concentration

$$
\frac{(j+1)(j+2)}{j!}\phi[j+2] + \frac{1}{2}Le \sum_{t=0}^{j} F[j-t](t+1)\phi[t+1] + \frac{Nt}{Nb}(j+1)(j+2)\theta[j+2] = 0,
$$

According to the theoretical works data, $Ec = 0.5$, $Nb = Nt = 0.1$, $Le = 10$, $M = 0.5$ so on.

Pade approximation of temperature and concentration

$$
Pade[5, 5](\theta(\eta)) = \frac{1 - 0.025461\eta + 1.04764\eta^2 - 0.45441\eta^3 - 0.004535\eta^4 + 0.03423\eta^5}{0.9999 - 0.01622\eta + 0.01376\eta^2 - 0.00147\eta^3 + 0.00679\eta^4},
$$

$$
Pade[5, 5](\phi(\eta)) = \frac{1 - 0.01194\eta + 0.27415\eta^2 - 0.004207\eta^3 - 0.25106\eta^4 + 0.03393\eta^5}{0.9999 - 0.009763\eta + 0.001563\eta^2 - 0.010054\eta^3 + 0.004023\eta^4}.
$$

4. Results and discussion

We have used the analytical approach to solve the system of nonlinear ordinary differential equations (5)–(7) along with the boundary conditions (8) and (9). The obtained results demonstrated the effects of the governing parameters that are nondimensional, namely, the suction parameter $S$, Lewis number $Le$, thermophoresis parameter $Nt$, Brownian motion $Nb$, Eckart number $Ec$, magnetic parameter $M$ on the velocity profile $f(\eta)$. It is making a thinner momentum boundary layer. The increasing trend of velocity profile for different values of magnetic parameter $M$ is studied in Figure 3 and shows that it accelerates the momentum flow. Therefore, the boundary layer thickness increases. Figure 4 illustrates that the velocity gradient increases when the stretching parameter increases due
to the impact of volume fraction and thermal exponent. From Figure 5, we can see the diminution of thermal gradient, while increasing the Prandtl number. It means that Prandtl number controls the relative thickening of the momentum and thermal boundary layer. Figure 6 examining the Nb and Nt demonstrates that the temperature decreases when Nb and Nt increase. From Figure 7, it is clear that the temperature profiles become lower, which implies loss in the thickness of the thermal boundary layer. From Figures 8 and 9, it is seen that decreasing Ec number and increasing magnetized number cause the $\theta(\eta)$ profile decreases. In Figure 10 we observe that if the suction parameter is increasing then $\phi(\eta)$ profile also is increasing. In Figure 11, the same situation with the Pr number: with increasing Pr number the $\phi(\eta)$ profile is increasing. Figure 12: Nb and Nt parameters do not affect the concentration distribution. Figures 13 and 14 show that the effects of Lewis and Eckart numbers: their increase will enhance the concentration profile of nanofluid. The volume proportion of nanoparticles and the thickness of the concentration layer raised exponentially as the Le number increases.
Both Figures 15 and 16 indicate that there is a drop in the rate of mass transfer while there is an increase in Prandtl number, Brownian motion, and thermophoresis. So that the reduced Sherwood number is a decreasing function of dimensionless parameters Pr, Nb and Nt. The effects of Pr and Nb parameters on heat transfer rates are shown in Figures 17–19, respectively, for a variety of Le values. It is obvious that the heat transfer rates will increase when the non-dimensional parameters are enlarged. So that the reduced Nusselt number is an increasing function of each dimensionless number.

5. Conclusion

The primary goal of this work is to investigate a two-phase modeling solution for the movement of a boundary layer’s heat and mass through a stretched sheet. The current issue is resolved using the analytical method DTM with the Pade approximation method. The Pade approximation is used to solve the issue in the $[5, 5]$ order of precision since the governing equations are nonlinear and non-homogeneous.
there are some boundary conditions. Additionally, it looked for any connections between Pr, Le, suction/injection, and other parameters. In contrast to the reduced Nusselt number, which is a rising function of each dimensionless number, the reduced Sherwood number is a decreasing function of each dimensionless number, as we have learned.


