MATHEMATICAL MODELING AND COMPUTING, Vol. 10, No. 1, pp. 245–260 (2023)

# Mathematical modeling of the gaming disorder model with media coverage: optimal control approach

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(Received 20 July 2022; Revised 11 January 2023; Accepted 15 January 2023)

In this article, we propose a PEARM mathematical model to depict the dynamic of a population that reacts in the spread of the gaming disorder with media coverage. The basic reproduction number and existence of free equilibrium point and endimec equilibrium point are obtained with same fundamental properties of the model including existence and positivity as well as boundedness of equilibria are investigated. By using Routh–Hurwitz criteria, the local stability of free equilibrium point and endimec equilibrium point are obtained. Also, we propose an optimal strategy to implement the optimal campaigns through directing children and adolescents to educational and entertaining alternative means, and creating centers to restore the rehabilitation of addicts to electronic games. The existence of the optimal control are obtained by Pontryagain's maximum principle. Finally, some numerical simulations are also performed to illustrate the theoretical analysis of our results, using Matlab software. Our results show that media coverage is an effective measure to quit electronic gaming disorder.

**Keywords:** *gaming disorder; mathematical model; media coverage; optimal control; local stability.*

**2010 MSC:** 93A30, 93B05, 93C15, 74D05, 70E50 **DOI:** 10.23939/mmc2023.01.245

## 1. Introduction

In recent years, research on computer and internet gaming has increased considerably [1,2]. However, the latest (fifth) edition of the Diagnostic and Statistical Manual of Mental Disorders (DSM-5 American Psychiatric Association, 2013) [3] has reclassified "gaming disorder" as an addictive one. As a result, people with Internet Gaming Disorder have difficulty controlling or reducing the amount of time they spend gaming and may experience negative consequences, including loss of control, deception, and conflict with family members. The problem of addiction to electronic games has been classified as a major problem that can lead to the destruction of public health. The World Health Organization has officially classified (International Classification of Diseases ICD-11; WHO, 2018) continuing to play video or electronic games as an addiction leading to mania and has announced that people with this mania have certain characteristics, such as the inability to stop gaming on winning. According to the organization, a person is classified as having this disease if their addictive behaviour persists for 12 months. However, the diagnosis can be confirmed in a shorter period of time if it is certain that all symptoms are present, the warning about the severity of electronic gaming addiction is not new, but the formal classification of this addiction as a pathological obsession with the World Health Organization can be a major impetus to raise awareness of this disease and take serious action in families and societies to counter it [4].

Since the emergence and spread of electronic games, research has been conducted to study and understand the effect of electronic games on children's behaviour and health, for example, we mention Jeanne B. Funk et al. (2002) [5], correlations were examined between the preference for violent electronic gaming and adolescents' self-perception of emotional behaviours and feelings. In 2007, L. Rowell Huesmann [6], the impact of media violence was compared to other known threats to society in order to estimate the importance of the threat to consider. Tom Baranowski et al. (2012) [7], this article explains the basic characteristics of a group of different technological methods; It shows the strengths and weaknesses of each of them in meeting the needs of children of all ages. Leon Straker et al. (2014) [8], proposes a model for factors affecting children's interaction with electronic games and discusses available guidelines and their role in the wise use of electronic games by children. These guidelines provide an accessible combination of available knowledge and practical evidence-based guidelines for electronic game and related research. Duven E. C. et al. (2015) [9], diagnostic specificity to distinguish between gaming addiction and high engagement. Tindele Sosso et al. (2020) [10], this study was conducted in 2 low-income countries (Rwanda, Gabon), 6 lower-middle income countries (Cameroon, Nigeria, Morocco, Tunisia, Senegal, Cote d'Ivoire) and 1 upper-middle income country (South Africa). The nine countries selected in this study were included because their citizens have better access to gaming devices, they were ranked among the top 20 most developed African countries in terms of technology use, internet connectivity, and the use of gaming machines.

Mathematical modelling is one of the most necessary functions that contribute to the representation and simulation of ecological, social, economic phenomena and Epidemics [11–14], and convert them into mathematical equations formulated, studied, analysed, and interpreted their results. Kada Driss et al. [15], discussed the spread of addiction to electronic games phenomenon and proposed a discrete mathematical model with control strategies to limit the spread of gaming addiction. Guo and Li [16], establish a new online game addiction model with low- and high-risk exposure and use the optimal control theory to study the optimal solution problem with three kinds of control measures (isolation, education, and treatment). Kada et al. [17], proposed a mathematical model that describes the dynamic of a population that reacts in the spread of the E-game infection and study the stability of endemic equilibrium point of gaming disorder. Zeyang Wang et al. [18], considers and describes the class of cooperative differential games with non-transferable utilities and the process of construction of the optimal Pareto strategy with continuous updating. Many studies and research in the social sciences have focused on this subject [19–24].

The media is today one of the most important means of spreading the addiction to electronic games, and on the other hand, it can be relied upon to effectively reduce this spread. According to several studies, the media play an important role in the spread of certain phenomena. Hai-Feng Huo et al. [25], proposed a new social epidemic model to depict alcoholism with media coverage. Samanta et al. [26], built a mathematical model to study the impact of awareness programs by media on the emergence of infectious diseases. Liu and Cui [27], set up a model to investigate the impact of media coverage and controlling of infectious disease in a given region.

In this work, the stability analysis of the model that they proposed to show that the system is locally asymptotically stable at free equilibrium point when $\mathcal{R}_0 < 1$ and the endemic equilibrium point exists and the system becomes locally asymptotically stable when $\mathcal{R}_0 > 1$. Also, we propose an optimal control strategy to implement the optimal campaigns through prevent the potential gamers from contacting with the engaged gamers and addicted gamers, for example, the media coverage and programs of directing children and adolescents to educational alternative means, and creating centers to restore the rehabilitation of addicts to electronic games. The aim is to reduce the number of the engaged gamers and addicted gamers. Pontryagin's maximum principle is used to characterize the optimal controls and the optimality system is solved by an iterative method.

The paper is organized as follows. In Section 2, we propose a more realistic PEARM mathematical model with media coverage. Motivated by Hai-Feng Huo et al. [25], and Wang et al. [28], we use saturation function (Holling type-II) to reflect the influence of media on the gaming disorder. In Section 3, we give some basic properties of the model. In Section 4, we analyse the local stability of equilibrium points of the model investigated by using Routh-Hurwitz criteria. In Section 5, we present the optimal control problem for the proposed model and we characterize these optimal controls using Pontryagin's maximum principle. Some numerical simulations through Matlab software are presented. Finally, we conclude the paper in Section 6.

## 2. A mathematical model

### 2.1. Description of the model

In our model the total population is divided into four compartments, Potential gamers (P) represent children and youth who are vulnerable to infection or who are more likely to become addicted to electronic games. Engaged gamers (E) represent children and youth interested in electronic games and plays more than four hours a day. Addicted gamers (A) represent children and youth who are addicted to electronic games and who suffer from gaming disorders and who have no control over their gaming habits, prioritize gaming over other interests and activities, and continue to game despite its negative consequences. And recovered gamers (R) represent children and youth recovering from their addiction to electronic games. It is supposed that the cumulative density of potential gamers and recovered gamers driven by media coverage at time $t$ is $M(t)$.

The total population of individuals, $N(t)$ at time $t$ is given as,

$$N(t) = P(t) + E(t) + A(t) + R(t).$$

### 2.2. Model equations

The population flow among those compartments is shown in Figure 1. The mathematical representation of the gaming disorder model consists of nonlinear differential equations using the rate at which patients change in each compartment during separate times. Therefore, we present the model with the following system of differential equations with five state variables.

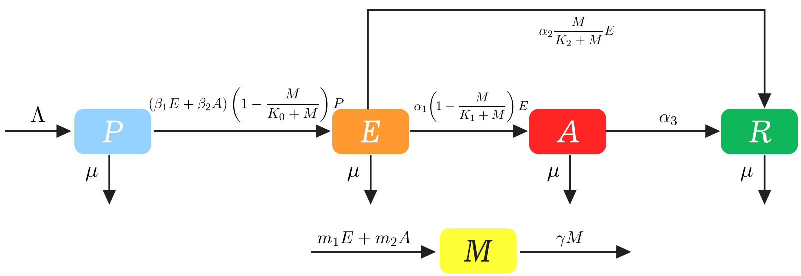


**Fig. 1.** Transfer diagram for gaming disorder model with media coverage.

$$\begin{cases} \dfrac{dP(t)}{dt} = \Lambda - \mu P(t) - (\beta_1 E(t) + \beta_2 A(t)) \left(1 - \dfrac{M(t)}{K_0 + M(t)}\right) P(t), \\ \dfrac{dE(t)}{dt} = (\beta_1 E(t) + \beta_2 A(t)) \left(1 - \dfrac{M(t)}{K_0 + M(t)}\right) P(t) - \mu E(t) \\ \qquad\qquad - \alpha_1 \left(1 - \dfrac{M(t)}{K_1 + M(t)}\right) E(t) - \alpha_2 \dfrac{M(t)}{K_2 + M(t)} E(t), \\ \dfrac{dA(t)}{dt} = \alpha_1 \left(1 - \dfrac{M(t)}{K_1 + M(t)}\right) E(t) - (\mu + \alpha_3) A(t), \\ \dfrac{dR(t)}{dt} = \alpha_3 A(t) + \alpha_2 \dfrac{M(t)}{K_2 + M(t)} E(t) - \mu R(t), \\ \dfrac{dM(t)}{dt} = m_1 E(t) + m_2 A(t) - \gamma M(t), \end{cases} \tag{1}$$

where $P(0) \geqslant 0$, $E(0) \geqslant 0$, $A(0) \geqslant 0$, $R(0) \geqslant 0$ and $M(0) \geqslant 0$ are given initial states.

$\Lambda$ is the recruitment rate of potential gamers who are more likely to become addicted to electronic games, $\mu$ is the rate of people who are older than the age limit set for people concerned with the study.

$\beta_1$ is the rate of patients who become engaged gamers because of the negative contact with the other engaged gamers, $\beta_2$ is the rate of patients who become engaged gamers because of the negative contact with the addicted gamers. $\alpha_1$ is the rate of the engaged gamers who have become addicted gamers, $\alpha_2$ is the rate of the engaged gamers who have become the recovered gamers, $\alpha_3$ is the rate of the addicted gamers who have become the recovered gamers. Large numbers of media coverage causes less interaction between potential, engaged and addicted populations to electronic games, a mathematical form of this assumption follows the Holling type-II functional form with half-saturating constants $K_0$, $K_1$ and $K_2$ (see [25, 27]). The half-saturation constants reflects the impact of media coverage on transmission. $m_1$ and $m_2$ are the implementation rates of media coverage, $\gamma$ is the depletion rate of cumulative density of media coverage due to ineffectiveness, social problems and similar factors.

## 3. The model analysis basic properties

### 3.1. Positivity of the model solutions

**Theorem 1.** *If $P(0) \geqslant 0$, $E(0) \geqslant 0$, $A(0) \geqslant 0$, $R(0) \geqslant 0$ and $M(0) \geqslant 0$ then solutions $P(t)$, $E(t)$, $A(t)$, $R(t)$ and $M(t)$ of system (1) are positive for all $t \geqslant 0$.*

**Proof.**

$$\begin{aligned}\frac{dP(t)}{dt} &= \Lambda - \mu P(t) - (\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)P(t)\\ &\geqslant -\mu P(t) - (\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)P(t).\end{aligned}$$

Then

$$\frac{dP(t)}{dt} + F(t)P(t) \geqslant 0,$$

where

$$F(t) = \mu + (\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right).$$

The both sides in the last inequality are multiplied by $\exp(\int_0^t F(s)ds)$.

We obtain

$$\exp\left(\int_0^t F(s)ds\right)\frac{dP(t)}{dt} + F(t)\exp\left(\int_0^t F(s)ds\right)P(t) \geqslant 0,$$

then

$$\frac{d}{dt}\left(\exp\left(\int_0^t F(s)ds\right)P(t)\right) \geqslant 0$$

integrating this inequality from 0 to $t$ gives:

$$P(t) \geqslant P(0)\exp\left(-\int_0^t \left(\mu + (\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)\right)ds\right).$$

So, the solution $P(t)$ is positive.

For the positivity of $E(t)$, $A(t)$, $R(t)$ and $M(t)$ we have: if the conclusion dose not satisfy, then at least one of $E(t)$, $A(t)$, $R(t)$, $M(t)$ is not positive. Thus, we have one of the following four cases.

(1) There exists a first time $t_1$ such that $E(t_1) = 0$, $\frac{dE(t_1)}{dt} < 0$, $A(t) > 0$, $R(t) > 0$, $M(t) > 0$ at $0 \leqslant t \leqslant t_1$.
(2) There exists a first time $t_2$ such that $A(t_2) = 0$, $\frac{dA(t_2)}{dt} < 0$, $E(t) > 0$, $R(t) > 0$, $M(t) > 0$ at $0 \leqslant t \leqslant t_2$.
(3) There exists a first time $t_3$ such that $R(t_3) = 0$, $\frac{dR(t_3)}{dt} < 0$, $A(t) > 0$, $E(t) > 0$, $M(t) > 0$ at $0 \leqslant t \leqslant t_3$.
(4) There exists a first time $t_4$ such that $M(t_4) = 0$, $\frac{dM(t_4)}{dt}\langle 0$, $A(t)\rangle 0$, $E(t)\rangle 0$, $R(t)\rangle 0$ at $0 \leqslant t \leqslant t_4$.

In cases (1), we have $\frac{dE(t_1)}{dt} = (\beta_2 A)(1 - \frac{M}{K_0+M})P > 0$, which is contradiction to $\frac{dE(t_1)}{dt} < 0$.
In cases (2), we have $\frac{dA(t_2)}{dt} = \alpha_1(1 - \frac{M}{K_1+M})E > 0$, which is contradiction to $\frac{dA(t_2)}{dt} < 0$.
In cases (3), we have $\frac{dR(t_3)}{dt} = \alpha_3 A + \alpha_2(\frac{M}{K_2+M})E > 0$, which is contradiction to $\frac{dR(t_3)}{dt} < 0$.
In cases (4), we have $\frac{dM(t_4)}{dt} = m_1 E + m_2 A > 0$, which is contradiction to $\frac{dM(t_4)}{dt} < 0$.
Thus, the solutions $P(t)$, $E(t)$, $A(t)$, $R(t)$, $M(t)$ of system (1) are positive for all $t > 0$. ■

### 3.2. Invariant region

It is necessary to prove that all solutions of system (1) with positive initial data will remain positive for all times $t$. we obtained the invariant region, in which the model solution is bounded. This will be established by the following lemma.

**Lemma 1.** *The set defined by*

$$\Omega = \left\{(P,E,A,R,M) \in \mathbb{R}^5_+;\ 0 \leqslant P+E+A+R \leqslant \frac{\Lambda}{\mu};\ 0 \leqslant M \leqslant \frac{\Lambda(m_1+m_2)}{\mu\gamma}\right\}$$

*with initial condition* $P(0) \geqslant 0$, $E(0) \geqslant 0$, $A(0) \geqslant 0$, $R(0) \geqslant 0$ *and* $M(0) \geqslant 0$ *are positive invariants for system* (1).

**Proof.** Using the fact $N(t) = P(t) + E(t) + A(t) + R(t)$, the system (1) reduced to following system:

$$\begin{cases} \dfrac{dN}{dt} = \Lambda - \mu N, \\ \dfrac{dM}{dt} = m_1 E + m_2 A - \gamma M \end{cases} \tag{2}$$

implies that $N(t) = N_0 e^{-\mu t} + \frac{\Lambda}{\mu}$, where $N_0 = P(0) + E(0) + A(0) + R(0)$ thus $0 \leqslant \limsup\limits_{t \longrightarrow +\infty} N(t) = \frac{\Lambda}{\mu}$. ■

Furthermore, we have $\frac{dM}{dt} = m_1 E + m_2 A - \gamma M \leqslant \frac{(m_1+m_2)\Lambda}{\mu} - \gamma M$ it follows that $0 \leqslant M(t) \leqslant M_0 e^{-\gamma t} + \frac{(m_1+m_2)\Lambda}{\gamma\mu}$ thus $0 \leqslant \limsup\limits_{t \longrightarrow +\infty} M(t) \leqslant \frac{(m_1+m_2)\Lambda}{\gamma\mu}$.

Then all possible solutions of the system (1) enter the region $\Omega$. It implies that $\Omega$ is a positively invariant set for the system (1). Hence, it is sufficient to study the dynamics of the basic model in $\Omega$.

## 4. Stability analysis of the model parameters

In this section, we will study the stability behavior of system (1) at an Disease Free Equilibrium point and an Endemic Equilibrium point. The first three equations and last equation in system (1) are independent of the variable $R$. Hence, the dynamics of equation system (1) is equivalent to the dynamics of the following equation system:

$$\begin{cases} \dfrac{dP(t)}{dt} = \Lambda - \mu P(t) - (\beta_1 E(t) + \beta_2 A(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right)P(t), \\ \dfrac{dE(t)}{dt} = (\beta_1 E(t) + \beta_2 A(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right)P(t) - \mu E(t) \\ \qquad - \alpha_1\left(1 - \dfrac{M(t)}{K_1 + M(t)}\right)E(t) - \alpha_2 \dfrac{M(t)}{K_2 + M(t)}E(t), \\ \dfrac{dA(t)}{dt} = \alpha_1\left(1 - \dfrac{M(t)}{K_1 + M(t)}\right)E(t) - (\mu + \alpha_3)A(t), \\ \dfrac{dM(t)}{dt} = m_1 E(t) + m_2 A(t) - \gamma M(t). \end{cases} \tag{3}$$

### 4.1. Equilibrium point

**The free equilibrium point.** To find the free equilibrium point $(P_0, E_0, A_0, R_0, M_0)$, we equated the right hand side of model (1) to zero, evaluating it at $E = A = 0$ and solving for the noninfected and noncarrier state variables. Therefore, the free equilibrium point $(\frac{\Lambda}{\mu}, 0, 0, 0, 0)$.

**Existence of the endemic equilibrium point.** The endemic equilibrium point $(P^*, E^*, A^*, R^*, M^*)$ it occurs when the disease persists in the community. To obtain it, we equate all the model equations (3) to zero. Then we obtain

$$\begin{cases} P^* = \dfrac{\Lambda}{\mu + (\beta_1 E^* + \beta_2 A^*)\Big(1 - \frac{M^*}{K_0 + M^*}\Big)}, \\ E^* = \dfrac{\Lambda}{\mu + \alpha_1\Big(1 - \frac{M^*}{K_1 + M^*}\Big) + \alpha_2 \frac{M^*}{K_2 + M^*}}, \\ A^* = \dfrac{\alpha_1}{(\mu + \alpha_3)}\left(1 - \dfrac{M^*}{K_1 + M^*}\right) E^*, \\ m_1 E^* + m_2 A^* - \gamma M^* = 0. \end{cases} \tag{4}$$

From the system (4), we have an equation of $M^*$ as follows:

$$a_1 (M^*)^3 + a_2 (M^*)^2 + a_3 M^* + a_4 = 0, \tag{5}$$

where

$$\begin{cases} a_1 = \gamma(\mu + \alpha_2), \\ a_2 = \gamma\mu(K_1 + K_2) + (\alpha_1 + \alpha_2)\gamma K_1 - m_1\Lambda, \\ a_3 = \gamma(\mu + \alpha_1)K_1 K_2 - m_1 \Lambda(K_1 + K_2) - \dfrac{\alpha_2 m_2 \Lambda K_1}{(\mu + \alpha_3)}, \\ a_4 = -\dfrac{(\mu m_1 + \alpha_3 m_1 + \alpha_2 m_2)\Lambda K_1 K_2}{(\mu + \alpha_3)}. \end{cases} \tag{6}$$

It is easy to see that $a_1$ is always positive and $a_4$ is always negative. Hence equation (6) has an positive root $M^*$.

### 4.2. The basic reproductive number

In our work, the basic reproduction number $\mathcal{R}_0$ is defined as the average number of secondary infections produced by an infected individual in a completely potential gamers. To obtain the basic reproduction number, we used the next-generation matrix method formulated in [29, 30].

Through the model equations system (3), then by the principle of next generation matrix, we obtained:

$$f(x) = \begin{pmatrix} (\beta_1 E(t) + \beta_2 A(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right) P(t) \\ 0 \\ 0 \\ \Lambda \end{pmatrix},$$

$$v(x) = \begin{pmatrix} \mu E(t) + \alpha_1\left(1 - \dfrac{M(t)}{K_1 + M(t)}\right) E(t) + \alpha_2 \dfrac{M(t)}{K_2 + M(t)} E(t) \\ (\mu + \alpha_3) A(t) - \alpha_1 \left(1 - \dfrac{M(t)}{K_1 + M(t)}\right) E(t) \\ \gamma M(t) - m_1 E(t) - m_2 A(t) \\ \mu P(t) + (\beta_1 E(t) + \beta_2 A(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right) P(t) \end{pmatrix}.$$

The Jacobian matrices of $f$ and $v$ at the free equilibrium point are, respectively,

$$F = \begin{pmatrix} \beta_1\frac{\Lambda}{\mu} & \beta_2\frac{\Lambda}{\mu} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} \mu+\alpha_1 & 0 & 0 & 0 \\ -\alpha_1 & \mu+\alpha_3 & 0 & 0 \\ -m_1 & -m_2 & \gamma & 0 \\ 0 & \beta_2\frac{\Lambda}{\mu} & 0 & \mu \end{pmatrix}.$$

The inverse of $V$ is given by

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\mu+\alpha_1} & 0 & 0 & 0 \\ 0 & \dfrac{1}{\mu+\alpha_3} & 0 & 0 \\ 0 & 0 & \dfrac{1}{\gamma} & 0 \\ 0 & 0 & 0 & \dfrac{1}{\mu} \end{pmatrix}$$

then

$$FV^{-1} = \begin{pmatrix} \dfrac{\beta_1\Lambda}{\mu(\mu+\alpha_1)} & \dfrac{\beta_2\Lambda}{\mu(\mu+\alpha_3)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Finally, the basic reproduction number is

$$\mathcal{R}_0 = \rho(FV^{-1}) = \frac{\beta_1\Lambda}{\mu(\mu+\alpha_1)}.$$

### 4.3. Local stability of disease free equilibrium point

**Theorem 2.** *The disease free equilibrium point is locally asymptotically stable if $R_0 < 1-\Theta$, where $0<\Theta<1$.*

**Proof.** The Jacobian matrix of system (1) at the disease free equilibrium point as follows:

$$J_0 = \begin{pmatrix} -\mu & -\beta_1\dfrac{\Lambda}{\mu} & -\beta_2\dfrac{\Lambda}{\mu} & 0 & 0 \\ 0 & -(\mu+\alpha_1-\beta_1\dfrac{\Lambda}{\mu}) & \beta_2\dfrac{\Lambda}{\mu} & 0 & 0 \\ 0 & \alpha_1 & -(\mu+\alpha_3) & 0 & 0 \\ 0 & 0 & \alpha_3 & -\mu & 0 \\ 0 & m_1 & m_2 & 0 & -\gamma \end{pmatrix}.$$

From the jacobian matrix $J_0$ we obtained a characteristic polynomial:

$$p(\lambda) = (\gamma+\lambda)(\mu+\lambda)^2(\lambda^2+a\lambda+b) = 0$$

with

$$a = \mu+\alpha_3+(\mu+\alpha_1)(1-\mathcal{R}_0), \quad b = (\mu+\alpha_1)(\mu+\alpha_3)(1-\mathcal{R}_0) - \alpha_1\beta_2\frac{\Lambda}{\mu}$$

we see that the characteristic equation $p(\lambda)$ of $J_0$ has an eigenvalues $\lambda_1 = -\mu$ and $\lambda_2 = -\gamma$ are negatives. So, in order to determine the stability of the disease free equilibrium point, we discuss the roots of the following equation:

$$\lambda^2 + a\lambda + b = 0.$$

By Routh–Hurwitz criterion, system (1) is locally asymptotically stable if $a > 0$ and $b > 0$.

Obviously we see that $a$ and $b$ to be positive, $(1-\mathcal{R}_0)$ must be positive, with the appropriate choice of transactions parameters. So, the disease free equilibrium point is locally asymptotically stable if $\mathcal{R}_0 < 1-\Theta$, with $\Theta = \frac{\alpha_1\beta_2\Lambda}{\mu(\mu+\alpha_1)(\mu+\alpha_3)}$. ■

### 4.4. Local stability of the endemic equilibrium point

**Theorem 3.** *The endemic equilibrium point is locally asymptotically stable if $\mathcal{R}_0 > 1$.*

**Proof.** The Jacobian matrix of system (1) at the endemic equilibrium point as follows:

$$J^* = \begin{pmatrix} J^*_{11} & J^*_{12} & J^*_{13} & 0 & J^*_{15} \\ J^*_{21} & J^*_{22} & J^*_{23} & 0 & J^*_{25} \\ 0 & J^*_{32} & J^*_{33} & 0 & J^*_{35} \\ 0 & J^*_{42} & J^*_{43} & J^*_{44} & J^*_{45} \\ 0 & J^*_{52} & J^*_{53} & 0 & J^*_{55} \end{pmatrix},$$

where

$$J^*_{11} = -\mu - (\beta_1 E^* + \beta_2 A^*)\left(1 - \frac{M^*}{K_0 + M^*}\right), \quad J^*_{12} = -\beta_1\left(1 - \frac{M^*}{K_0 + M^*}\right)P^*,$$

$$J^*_{13} = -\beta_2\left(1 - \frac{M^*}{K_0 + M^*}\right)P^*, \quad J^*_{14} = 0, \quad J^*_{15} = (\beta_1 E^* + \beta_2 A^*)P^*\frac{K_0}{(K_0 + M^*)^2},$$

$$J^*_{21} = (\beta_1 E^* + \beta_2 A^*)\left(1 - \frac{M^*}{K_0 + M^*}\right),$$

$$J^*_{22} = \beta_1\left(1 - \frac{M^*}{K_0 + M^*}\right)P^* - \mu - \alpha_1\left(1 - \frac{M^*}{K_1 + M^*}\right) - \alpha_2\frac{M^*}{K_2 + M^*},$$

$$J^*_{23} = \beta_2\left(1 - \frac{M^*}{K_0 + M^*}\right)P^*, \quad J^*_{24} = 0,$$

$$J^*_{25} = -(\beta_1 E^* + \beta_2 A^*)\frac{K_0}{(K_0 + M^*)^2} + \left(\frac{K_1\alpha_1}{(K_1 + M^*)^2} - \frac{K_2\alpha_2}{(K_2 + M^*)^2}\right)E^*$$

$$J^*_{31} = 0, \quad J^*_{32} = \alpha_1\left(1 - \frac{M^*}{K_1 + M^*}\right), \quad J^*_{33} = -\mu - \alpha_3,$$

$$J^*_{34} = 0, \quad J^*_{35} = -\frac{\alpha_1 K_1 E^*}{(K_1 + M^*)^2},$$

$$J^*_{41} = 0, \quad J^*_{42} = \alpha_2\frac{M^*}{K_2 + M^*}, \quad J^*_{43} = \alpha_3, \quad J^*_{44} = -\mu,$$

$$J^*_{45} = \frac{\alpha_2 K_2 E^*}{(K_2 + M^*)^2}, \quad J^*_{51} = 0, \quad J^*_{52} = m_1,$$

$$J^*_{53} = m_2, \quad J^*_{54} = 0, \quad J^*_{55} = -\gamma.$$

From the jacobian matrix $J^*$ we obtained a characteristic polynomial:

$$p(\lambda) = (\mu + \lambda)(\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4)$$

with

$$a_1 = -(J^*_{11} + J^*_{22} + J^*_{33} + J^*_{55}),$$

$$a_2 = J^*_{33}J^*_{55} + J^*_{11}J^*_{22} + (J^*_{11} + J^*_{22})(J^*_{33} + J^*_{55}) - J^*_{35}J^*_{53} - J^*_{32}J^*_{23} - J^*_{52}J^*_{25} - J^*_{21}J^*_{12},$$

$$a_3 = (J^*_{11} + J^*_{22})(J^*_{35}J^*_{53} - J^*_{33}J^*_{55}) + (J^*_{33} + J^*_{55})(J^*_{21}J^*_{12} - J^*_{11}J^*_{22}) + J^*_{32}J^*_{23}(J^*_{11} + J^*_{55})$$

$$
\begin{aligned}
&+ J_{52}^{*}J_{25}^{*}(J_{11}^{*}+J_{33}^{*}) - J_{32}^{*}J_{53}^{*}J_{25}^{*} - J_{52}^{*}J_{23}^{*}J_{35}^{*} - J_{21}^{*}J_{32}^{*}J_{13}^{*} - J_{21}^{*}J_{52}^{*}J_{15}^{*},\\
a_4 &= (J_{11}^{*}J_{22}^{*} - J_{21}^{*}J_{12}^{*})(J_{33}^{*}J_{55}^{*} - J_{35}^{*}J_{53}^{*}) + J_{21}^{*}J_{52}^{*}(J_{15}^{*}J_{33}^{*} - J_{13}^{*}J_{35}^{*}) + J_{11}^{*}J_{52}^{*}(J_{23}^{*}J_{35}^{*} - J_{25}^{*}J_{33}^{*})\\
&+ J_{32}^{*}J_{11}^{*}(J_{53}^{*}J_{25}^{*} - J_{23}^{*}J_{55}^{*}) + J_{21}^{*}J_{32}^{*}(J_{13}^{*}J_{55}^{*} - J_{53}^{*}J_{15}^{*}).
\end{aligned}
$$

We see that the characteristic equation $p(\lambda)$ of $J^*$ has an eigenvalue $\lambda_1 = -\mu$ is negative. So, in order to determine the stability of the endemic equilibrium point, we discuss the roots of the following equation:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4.$$

By Routh–Hurwitz criterion [31], system (1) is locally asymptotically stable if $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $a_4 > 0$, $a_1a_2 - a_3 > 0$ and $(a_1a_2 - a_3)a_3 - a_1^2a_4 > 0$.

If $\mathcal{R}_0 > 1$, it's obvious that $J_{ii}^{*}$ $(i = 1, 2, 3, 4)$, $J_{12}^{*}$, $J_{13}^{*}$, $J_{25}^{*}$, $J_{35}^{*}$, all are negative and $J_{15}^{*}$, $J_{21}^{*}$, $J_{23}^{*}$, $J_{32}^{*}$, $J_{42}^{*}$, $J_{43}^{*}$, $J_{45}^{*}$, all are positive. We can easily get the coefficients $a_i$ $(i = 1, 2, 3, 4)$, all are positive and we can get that the eigenvalues of the above characteristic equation have negative real parts under the conditions

$$a_1a_2 - a_3 > 0, \quad \text{and} \quad (a_1a_2 - a_3)a_3 - a_1^2a_4 > 0.$$

So, the endemic equilibrium point is locally asymptotically stable if $\mathcal{R}_0 > 1$. ■

## 5. The optimal control problem

### 5.1. Problem statement

In this section, in order to reduce the spread of the addiction of electronic games among children and adolescents, we reconsider model (1) and formulate an optimal control problem (7). Our goal is to minimize the number of engaged gamers and addicted gamers. To achieve this, we use two control variables. The control $u_1$ represents efforts intended to prevent the potential gamers from contacting with the engaged gamers and addicted gamers, for example, the media coverage and programs of directing children and adolescents to educational alternative means. The control $u_2$ represents efforts to establish rehabilitation centers of addicts to electronic games.

Thus, the controlled mathematical system is given by the following system of differential equations:

$$
\begin{cases}
\dfrac{dP(t)}{dt} = \Lambda - \mu P(t) - (1-u_1(t))(\beta_1E(t)+\beta_2A(t))\left(1-\dfrac{M(t)}{K_0+M(t)}\right)P(t),\\
\dfrac{dE(t)}{dt} = (1-u_1(t))(\beta_1E(t)+\beta_2A(t))\left(1-\dfrac{M(t)}{K_0+M(t)}\right)P(t) - \mu E(t),\\
\qquad - \alpha_1(1-u_1(t))\left(1-\dfrac{M(t)}{K_1+M(t)}\right)E(t) - \alpha_2\dfrac{M(t)}{K_2+M(t)}E(t),\\
\dfrac{dA(t)}{dt} = \alpha_1(1-u_1(t))\left(1-\dfrac{M(t)}{K_1+M(t)}\right)E(t) - (\mu+\alpha_3+u_2(t))A(t),\\
\dfrac{dR(t)}{dt} = (\alpha_3+u_2(t))A(t) + \alpha_2\dfrac{M(t)}{K_2+M(t)}E(t) - \mu R(t),\\
\dfrac{dM(t)}{dt} = m_1E(t) + m_2A(t) - \gamma M(t),
\end{cases}
\tag{7}
$$

where $P(0) \geqslant 0$, $E(0) \geqslant 0$, $A(0) \geqslant 0$, $R(0) \geqslant 0$ and $M(0) \geqslant 0$ are given initial states. Then, the problem is to minimize the objective functional:

$$J(u_1, u_2) = E(T) + A(T) + \int_0^T \left(E(t) + A(t) + \frac{F}{2}u_1^2(t) + \frac{G}{2}u_2^2(t)\right) dt, \tag{8}$$

where the parameters $F > 0$ and $G > 0$ are the cost coefficients. They are selected to weigh the relative importance of $u_1$ and $u_2$ at time $t$. $T$ is the final time. In other words, we seek the optimal controls $u_1^*$ and $u_2^*$ such that:

$$J(u_1^*, u_2^*) = \min_{(u_1,u_2)\in U_{ad}} J(u_1, u_2),$$

where $U_{ad}$ is the set of admissible control defined by $U_{ad} = \{(u,v)\colon 0 \leqslant u(t) \leqslant 1;\ 0 \leqslant v(t) \leqslant 1,\ t \in [0,T]\}$.

### 5.2. Existence of optimal control

We first show the existence of solutions of the system (7) thereafter we will prove the existence of optimal control, we use the result of Fleming and Rishel [32].

**Theorem 4.** *Consider the control problem with the system* (7). *There exists an optimal controls* $u_1^*$, $u_2^*$ *such that*

$$J(u_1^*, u_2^*) = \min_{(u_1,u_2)\in U_{ad}} J(u_1, u_2) \tag{9}$$

*subject to the controls system* (7) *with initial conditions.*

**Proof.** To prove the existence of an optimal control the following conditions must be satisfied:

(1) It follows that the set of controls and corresponding state variables is nonempty.

(2) The control set $U_{ad} = \{(u,v)\colon 0 \leqslant u(t) \leqslant 1; 0 \leqslant v(t) \leqslant 1, t \in [0,T]\}$, is convex and closed by definition. Take any controls $(u_1, v_1)$ and $(u_2, v_2) \in U_{ad}$ and $\lambda \in [0,1]$. Then $0 \leqslant \lambda u_1 + (1-\lambda)u_2$ additionally, we observe that $\lambda u_1 \leqslant \lambda$ and $(1-\lambda)u_2 \leqslant (1-\lambda)$ then $\lambda u_1 + (1-\lambda)u_2 \leqslant 1$, hence, $0 \leqslant \lambda u_1 + (1-\lambda)u_2 \leqslant 1$. Similary we prove that $0 \leqslant \lambda v_1 + (1-\lambda)v_2 \leqslant 1$ for all $(u_1, v_1)$, $(u_2, v_2) \in U_{ad}$ and $\lambda \in [0,1]$.

(3) All the right hand sides of equations of system (7) are continuous, bounded above by a sum of bounded control and state, and can be written as a linear function of $u_1$ and $u_2$ with coefficients depending on time and state.

(4) The integrand in the objective functional

$$J(u_1, u_2) = E(T) + A(T) + \int_0^T \left(E(t) + A(t) + \frac{F}{2}u_1^2(t) + \frac{G}{2}u_2^2(t)\right) dt$$

is clearly convex in $U_{ad}$.

(5) It rest to show that there exists constants $\kappa_1$, $\kappa_2$, $\kappa_3$ such that

$$E(t) + A(t) + \frac{F}{2}u_1^2(t) + \frac{G}{2}u_2^2(t) \geqslant \kappa_1 + \kappa_2|u_1|^2 + \kappa_3|u_2|^2.$$

The state variables being bounded, let $\kappa_1 = \inf_{t\in[0,T]}(E(t) + A(t))$, $\kappa_2 = \frac{F}{2}$ and $\kappa_3 = \frac{G}{2}$. Then from Fleming and Rishel [32] we conclude that there exists an optimal control. ∎

### 5.3. Characterization of optimal control

In order to derive the necessary condition for optimal control, we apply Pontryagin's maximum principle [33]. The idea is introducing the adjoint function to attach the system of differential equations to the objective functional resulting in the formation of a function called the Hamiltonian.

This principle converts into a problem of minizing Hamiltonian $H(t)$ at time $t$ defined by

$$H(t) = E(t) + A(t) + \frac{F}{2}u_1^2(t) + \frac{G}{2}u_2^2(t) + \sum_{i=1}^{5} \lambda_i(t) f_i(P, E, A, R, M), \tag{10}$$

where $f_i$ is the right side of the system of differenciel equations (7) of the $i^{\text{th}}$ state variable at time $t$.

**Theorem 5.** *Given the optimal controls $u_1^*$, $u_2^*$ and the solutions $P^*$, $E^*$, $A^*$, $R^*$ and $M^*$ of the corresponding state system (7) there exists adjoint variables $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$ and $\lambda_5$ satisfying*

$$\begin{cases} \lambda_1' = \lambda_1\mu + (\lambda_1 - \lambda_2)(1-u_1(t))(\beta_1 E(t) + \beta_2 A(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right), \\ \lambda_2' = 1 + (\lambda_1 - \lambda_2)\beta_1(1-u_1(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right)P(t) + \lambda_2\mu - \lambda_5 m_1 \\ \qquad + (\lambda_2 - \lambda_3)\alpha_1(1-u_1(t))\left(1 - \dfrac{M(t)}{K_1 + M(t)}\right) + (\lambda_2 - \lambda_4)\alpha_2\dfrac{M(t)}{K_2 + M(t)}, \\ \lambda_3' = 1 + (\lambda_1 - \lambda_2)\beta_2(1-u_1(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right)P(t) + \lambda_3\mu \\ \qquad + (\lambda_3 - \lambda_4)(\alpha_3 + u_2(t)) - \lambda_5 m_2, \\ \lambda_4' = \lambda_4\mu, \\ \lambda_5' = (\lambda_2 - \lambda_1)(1-u_1(t))(\beta_1 E(t) + \beta_2 A(t))\dfrac{K_0 P(t)}{(K_0 + M(t))^2} + \lambda_5\gamma \\ \qquad + (\lambda_3 - \lambda_2)(1-u_1(t))\alpha_1\dfrac{K_1 E(t)}{(K_1 + M(t))^2} + (\lambda_2 - \lambda_4)\alpha_2\dfrac{K_2 E(t)}{(K_2 + M(t))^2}. \end{cases} \tag{11}$$

*With the transversality conditions at time $T$: $\lambda_1(T) = 0$; $\lambda_2(T) = 1$; $\lambda_3(T) = 1$; $\lambda_4(T) = 0$ and $\lambda_5(T) = 0$.*

*Furthermore for $t \in [0, T]$, the optimal controls $u_1^*$ and $u_2^*$ are given by*

$$u_1^* = \min(u_{2\max}; \max(u_{2\min}, u_1(t))) \tag{12}$$

*with*

$$u_1(t) = \frac{1}{F}(\lambda_2 - \lambda_1)(\beta_1 E(t) + \beta_2 A(t))(1 - \frac{M(t)}{K_0 + M(t)})P(t) + (\lambda_3 - \lambda_2)\alpha_1(1 - \frac{M(t)}{K_1 + M(t)})E(t),$$

$$u_2^* = \min\left(u_{3\max}; \max\left(u_{3\min}, \tfrac{(\lambda_3 - \lambda_4)A(t)}{G}\right)\right). \tag{13}$$

**Proof.** The hamiltonian at time $t$ is given by

$$H(t) = E(t) + A(t) + \frac{F}{2}u_1^2(t) + \frac{G}{2}u_2^2(t) + \sum_{i=1}^{5}\lambda_i(t) f_i(P, E, A, R, M),$$

where

$$\begin{aligned} f_1(P,E,A,R,M) &= \Lambda - \mu P(t) - (1-u_1(t))(\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)P(t), \\ f_2(P,E,A,R,M) &= (1-u_1(t))(\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)P(t) \\ &\quad - \mu E(t) - \alpha_1(1-u_1(t))\left(1 - \frac{M(t)}{K_1 + M(t)}\right)E(t) - \alpha_2\frac{M(t)}{K_2 + M(t)}E(t), \\ f_3(P,E,A,R,M)) &= \alpha_1(1-u_1(t))(1 - \frac{M(t)}{K_1 + M(t)})E(t) - (\mu + \alpha_3 + u_2(t))A(t), \\ f_4(P,E,A,R,M) &= (\alpha_3 + u_2(t))A(t) + \alpha_2\frac{M(t)}{K_2 + M(t)}E(t) - \mu R(t), \\ f_5(P,E,A,R,M) &= m_1 E(t) + m_2 A(t) - \gamma M(t) \end{aligned}$$

for $t \in [0, T]$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum principle, such that

$$\begin{cases} \lambda_1' = -\dfrac{dH}{dP} = \lambda_1\mu + (\lambda_1 - \lambda_2)(1-u_1(t))(\beta_1 E(t) + \beta_2 A(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right), \\ \lambda_2' = -\dfrac{dH}{dE} = 1 + (\lambda_1 - \lambda_2)\beta_1(1-u_1(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right)P(t) + \lambda_2\mu - \lambda_5 m_1 \\ \qquad + (\lambda_2 - \lambda_3)\alpha_1(1-u_1(t))\left(1 - \dfrac{M(t)}{K_1 + M(t)}\right) + (\lambda_2 - \lambda_4)\alpha_2\dfrac{M(t)}{K_2 + M(t)}, \\ \lambda_3' = -\dfrac{dH}{dA} = 1 + (\lambda_1 - \lambda_2)\beta_2(1-u_1(t))\left(1 - \dfrac{M(t)}{K_0 + M(t)}\right)P(t) + \lambda_3\mu \\ \qquad + (\lambda_3 - \lambda_4)(\alpha_3 + u_2(t)) - \lambda_5 m_2, \\ \lambda_4' = -\dfrac{dH}{dR} = \lambda_4\mu, \\ \lambda_5' = -\dfrac{dH}{dM} = (\lambda_2 - \lambda_1)(1-u_1(t))(\beta_1 E(t) + \beta_2 A(t))\dfrac{K_0 P(t)}{(K_0 + M(t))^2} + \lambda_5\gamma \\ \qquad + (\lambda_3 - \lambda_2)(1-u_1(t))\alpha_1\dfrac{K_1 E(t)}{(K_1 + M(t))^2} + (\lambda_2 - \lambda_4)\alpha_2\dfrac{K_2 E(t)}{(K_2 + M(t))^2} \end{cases} \tag{14}$$

with the tranversality conditions at time $T$: $\lambda_1(T) = 0$, $\lambda_2(T) = 1$, $\lambda_3(T) = 1$, $\lambda_4(T) = 0$, and $\lambda_5(T) = 0$.

For $t \in [0, T]$, the optimal controls $u_1^*$ and $u_2^*$ can be solved from the optimality condition: $\frac{dH}{du_1} = 0$ and $\frac{dH}{du_2} = 0$ that is

$$\begin{aligned} \frac{dH}{du_1} &= F u_1(t) + (\lambda_1 - \lambda_2)(\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)P(t) \\ &\quad + (\lambda_2 - \lambda_3)\alpha_1\left(1 - \frac{M(t)}{K_1 + M(t)}\right)E(t) = 0, \\ \frac{dH}{du_2} &= G u_2(t) + (\lambda_4 - \lambda_3)A(t) = 0, \end{aligned}$$

so, we have

$$\begin{aligned} u_1(t) &= \frac{(\lambda_2 - \lambda_1)(\beta_1 E(t) + \beta_2 A(t))\left(1 - \frac{M(t)}{K_0 + M(t)}\right)P(t) + (\lambda_3 - \lambda_2)\alpha_1\left(1 - \frac{M(t)}{K_1 + M(t)}\right)E(t)}{F}, \\ u_2(t) &= \frac{(\lambda_3 - \lambda_4)A(t)}{G} \end{aligned}$$

by the bounds in $U_{ad}$ of the controls, it easy to obtain $u_1^*$ and $u_2^*$ in the form (12), (13). ∎

### 5.4. Numerical simulations and discussions

The optimality system is a two-point boundary value problem with separated boundary conditions at times step $t = 0$ and $t = T$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration we update the controls by using the characterization. We continue until convergence of successive iterates is achieved. A code is written and compiled in Matlab software using the data of Table 1.

In this section, to demonstrate the analytic results we have obtained, different simulations can be carried out using various values of parameters as shown in Table 1.

We assume that the initial condition of system (1) is $P(0) = 0.6$, $E(0) = 0.2$, $A(0) = 0.1$, $R(0) = 0.08$ and $M(0) = 0.02$.

In order to evaluate the effect of media coverage on the dynamics of gaming desorder, we choose different values of $m_1$, $m_2$ and $\gamma$ (see Figures 2, 3 and 4).

**Table 1.** The description of parameters data used for systems (1).

| Parameter | Description | Estimated value | Source |
|---|---|---|---|
| $\Lambda$ | Constant recruitment rate | 0.05 | Assumed |
| $\mu$ | natural older rate | 0.0035 | [34] |
| $\alpha_1$ | rate of engaged gamers become addicted gamers | 0.76 | [14] |
| $\alpha_2$ | rate of engaged gamers become recovered gamers | 0.3 | Assumed |
| $\alpha_3$ | rate of addicted gamers become recovered gamers | 0.1 | Assumed |
| $\beta_1$ | rate of P became E by contact with E | 0.46 | [14] |
| $\beta_2$ | rate of P became E by contact with A | 0.26 | [14] |
| $m_1$ | The implementation rate of media coverage | 0.1 | Assumed |
| $m_2$ | The implementation rate of media coverage | 0.1 | Assumed |
| $k_0$ | Half-saturating constant | 100 | Assumed |
| $k_1$ | Half-saturating constant | 80 | Assumed |
| $k_2$ | Half-saturating constant | 80 | Assumed |
| $\gamma$ | The depletion rate of media coverage | 0.02 | Assumed |

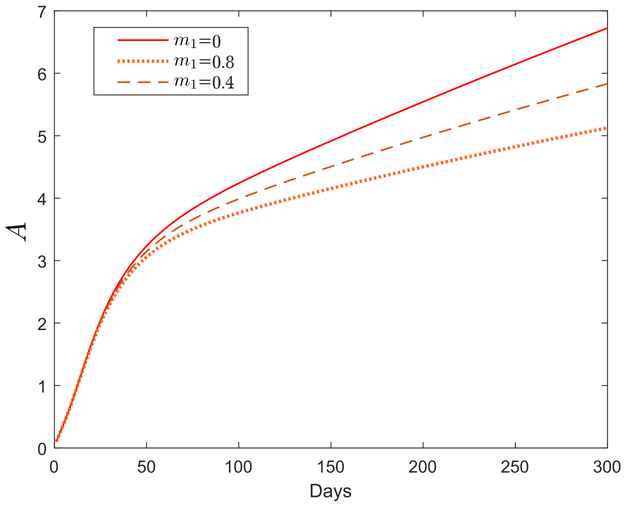


**Fig. 2.** The influence of different values of $m_1$ on the addicted gamers.

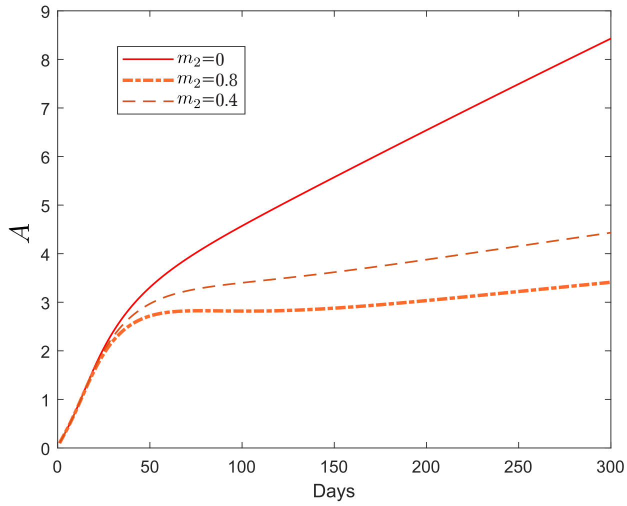


**Fig. 3.** The influence of different values of $m_2$ on the addicted gamers.

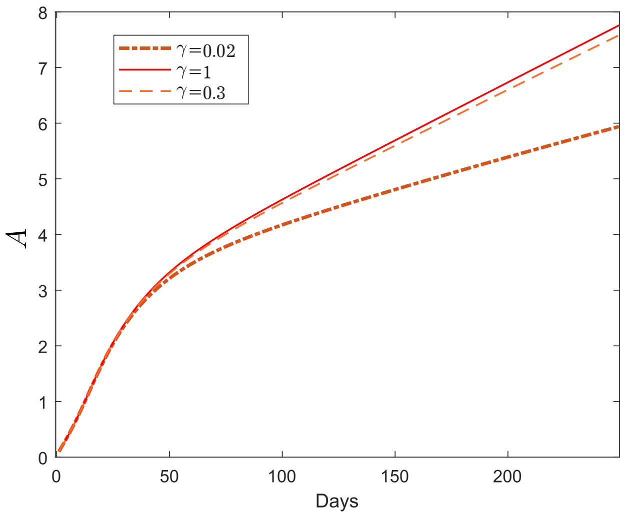


**Fig. 4.** The influence of different values of gamma ($\gamma$) on the addicted gamers.

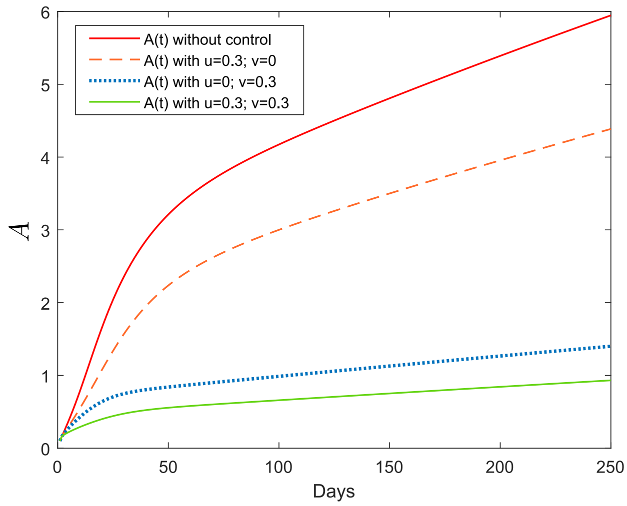


**Fig. 5.** Variations of addicted gamers density with different control $u = u_1$ and $v = u_2$.

In Figure 2, we note a decrease in the final density of addicts to electronic games, when the higher the rate of implementation of media coverage among engaged gamers.

In Figure 3, we note a decrease in the final density of addicts to electronic games, when the higher the rate of implementation of media coverage among addicts to these games.

Quite the opposite, in Figure 4, the more the depletion rate of media coverage, the higher the final addicts to electronic games density. From the above it can be said that media coverage plays a major role in reducing the spread of addiction to electronic games.

Now, we explore numerically an optimal control for the system (7).

Figure 5 indicates that the changes of addicted gamers density with the different valus of $u_1$ and $u_2$. Also shows that the best result obtained when we use $u_1 = 0.3$ and $u_2 = 0.3$.

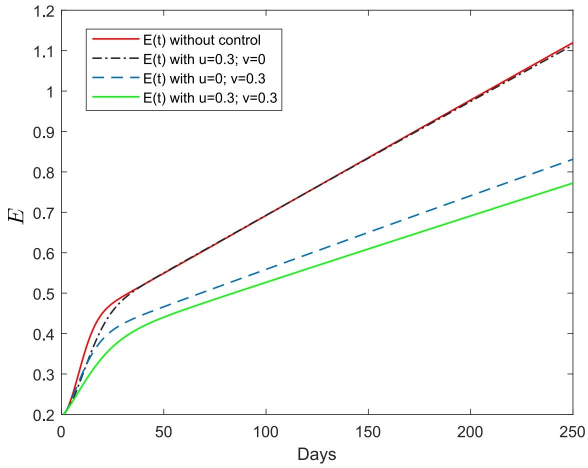


**Fig. 6.** Variations of engaged gamers density with different control $u = u_1$ and $v = u_2$.

Figure 6 indicates that the changes of engaged gamers density with the different valus of $u_1$ and $u_2$. Also shows that the best result obtained when we use $u_1 = 0.3$ and $u_2 = 0.3$.

Through an analytical study of the results obtained in Figures 5 and 6, we conclude that the combined two controls $u_1$ (efforts intended to prevent the potential gamers from contacting with the engaged gamers and addicted gamers, for example, the media coverage and programs of directing children and adolescents to educational alternative means) and $u_2$ (efforts to establish rehabilitation centers of addicts to electronic games) gives impressive results in reducing the spread of addiction to electronic games.

## 6. Conclusion

In this research, we proposed a gaming disorder epidemic mathematical model for the human population with media coverage. We form a mathematical model that describes the dynamics of gaming disorder. The qualitative analysis of the model shows that the solution of the model is bounded and positive. In addition, by using the Routh–Hurwitz criterion, the local stability of the free equilibrium point and endemic equilibrium point are obtained. Next, we establish the optimality system, including two controls: the first represents efforts intended to prevent potential gamers from contact with engaged gamers and addicted gamers (for example, the media coverage and programs directing children and adolescents to educational alternative means), and the second represents creating centers to restore the rehabilitation of addicts to electronic games. Pontryagin's maximum principle was used to characterize the optimal controls, and the optimality system was solved by an iterative method. The numerical simulation was carried out using Matlab software to simulate outcomes which we have proved. Our results show that media coverage has a substantial influence on the dynamics of gaming disorder and greatly influences the spread of addiction to electronic games.

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## Математичне моделювання моделі ігрового розладу з висвітленням у ЗМІ: підхід оптимального керування

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У цій статті запропоновано математичну модель PEARM для зображення динаміки населення, яка реагує на поширення геймінгового розладу висвітленням у ЗМІ. Базове число відтворення та існування точки вільної рівноваги та ендемічної точки рівноваги отримано за однакових фундаментальних властивостей моделі, включаючи існування та додатність, а також досліджено обмеженість рівноваги. Використовуючи критерії Рауса–Гурвіца, отримано локальну стійкість точки вільної рівноваги та ендемічної точки рівноваги. Також запропоновано оптимальну стратегію реалізації оптимальних кампаній через спрямування дітей та підлітків на освітні та розважальні альтернативні засоби та створення центрів відновлення реабілітації залежних від електронних ігор. Існування оптимального керування визначається принципом максимуму Понтрягіна. Накінець деякі чисельні моделювання виконані для ілюстрації теоретичного аналізу отриманих результатів за допомогою програмного забезпечення Matlab. Отримані результати показують, що висвітлення в засобах масової інформації є ефективним заходом для виходу з розладу електронних ігор.

**Ключові слова:** *геймінговий розлад; математична модель; висвітлення в ЗМІ; оптимальне керування; локальна стійкість.*