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Multi-attribute group decision-making problem of medical consumption products based on extended TODIM–VIKOR approach with Fermatean fuzzy information measure

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The fundamental goal of this research is to develop a MAGDM (Multi-Attribute Group Decision Making) problem of Medical Consumption Products. We propose TODIM–VIKOR approach in this paper, which combines the TODIM (an acronym in Portuguese for Interactive and Multi-criteria Decision-Making) and VIKOR (Vlsekriterijumska Optimizacija I Kompromisno Resenje) procedures under Fermatean fuzzy information. A new Fermatean fuzzy scoring function is presented for dealing with comparison problems. In addition, we introduce a novel entropy measure for assessing the degree of fuzziness associated with an FFS. We also offer a Jensen–Shannon divergence measure for the Fermatean Fuzzy set that can be used to compare the discrimination information of two FFSs. This suggested measure meets all mathematical standards for being considered a measure. We introduced entropy and divergence measures to determine the objective weight in the TODIM–VIKOR approach. Meanwhile, to deal with multiple attribute group decision-making, a new decision procedure based on the suggested Entropy and Jensen–Shannon divergence measure was proposed in a Fermatean Fuzzy environment. In this article, TODIM has in view to find out the overall dominance degree, and VIKOR is to determine the compromise solution. Finally, we manage a supplier selection problem to verify the performance of the suggested Fermatean fuzzy TODIM–VIKOR method by comparing the ranking solution to the rankings of existing methodologies. We investigate the reliability and effectiveness of our proposed methodology.

Keywords: *entropy; divergence measure; Fermatean Fuzzy Set; MAGDM.*

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1. Introduction

We cannot continually supply correct and efficient information for the alternatives in genuine MADM (multi-attribute decision making) difficulties because of the uncertainty/indeterminacy of experts and decision problems. To overcome this problem [1], unique fuzzy set (FS) used the crisp membership degree rather than an exact or precise real number, to measure the indeterminacy inherent in estimation outcomes. Engineering, image processing, medical sciences, social sciences and clustering are just a few of the fields where fuzzy theory has been applied. Since fuzzy set theory is based solely on measurement, it is unable to demonstrate intuition. As a result, [2] has established a perspective on the intuitionistic fuzzy set (IFSs) theory by adding a new dimension to non-membership degree. Various researchers have looked into the uses of IFSs. As a result, IFS can provide a more accurate depiction in a variety of domains, but its application is limited in particular instances. In order to deal with these issues, [3] generalised the IFS theory and introduced the "pythagorean fuzzy set (PyFS)" as a new set theory. Do the squares of the membership (g) and non-membership (f) grades in PyFSs, then determine the sum that is lesser or equal to 1. Currently, the PyFS theory has proven to be a useful tool for dealing with ambiguous data in real-life scenarios. The case cannot be solved using the IFS theory if a decision maker offers a membership grade of 0.6 and a non-membership grade of 0.7 in their evaluation. Furthermore, in a fuzzy context, the PyFSs are extremely capable of dealing with new information. As evidenced by $0.6^2 + 0.7^2 \leqslant 1$. PyFS is more expansive than IFS, to explain the more decision-making problems. In the previous literature, research on PyFSs has been done very effectively. MADM difficulties were examined using some new pythagorean fuzzy information metrics by [4]. In a pythagorean fuzzy environment, [5] investigated cumulative prospect theory. Authors have begun to examine and solve decision-making problems using the PyFS environment [6–8]. With the boundary requirement that $g^2 + f^2 \leqslant 1$, PyFS has a relatively limited range. PyFS was then expanded to Fermatean fuzzy sets (FFSs) by [9] which is a new type of IFS and FFS in which the sum of the cubes of membership grade (g) and negative membership (f) grade is $\leqslant 1$. FFS is similar to IFS and PyFS in terms of functionality, but it has more flexibility for expressing confusing data. It means that FFSs are more knowledgeable and assertive in their advocacy for membership grade. For example, suppose an expert offers a membership grade of 0.8 and a non-membership grade of 0.7 in their judgement. The usefulness of IFS and PyFS is limited in the current situation since $0.8 + 0.7 > 1$ and $0.8^2 + 0.7^2 > 1$, but $0.8^3 + 0.7^3 \leqslant 1$. Therefore, experts can issue membership and non-membership grades independently using FFSs. Thus, FFSs are more skilled and adjustable to handle the realistic information which is vague or uncertain. Various researchers initiated to do the study on FFSs theory in many areas after the successful development of FFSs.

In the case of MADM and MAGDM, decision makers can provide possible language terms so that these given numbers can lead to diversity distribution opportunities. The problems of deciding the status of multiple attributes can often be considered as complete forms of ambiguity and uncertainty. Over the past few decades, the problems of MADM or MAGDM have become increasingly common in everyday life problems in many mysterious are as [10–16]. To incorporate [17], theory of TODIM was proposed, and human bias was considered in multilateral decision-making.

TODIM is a valuable strategy for identifying the dominance of one thing over another in decision-making situations. [18–21] scholars have successfully employed the TODIM approach. The VIKOR method provided by [22] has been deemed an effective and logical strategy that may be employed in decision-making challenges in recent years when considering a compromise option. The VIKOR approach has several advantages, including providing the most effective level outcomes from a kit of other feasible choices, as well as unambiguous and contradictory decision-area circumstances. This method produces the greatest results when the decision maker (DM) wants to maximise revenues while minimising risk variables that aren't important to the DM [23]. Because it improves choice quality, the VIKOR approach a decision-making tool. Despite the fact that TODIM and VIKOR models can answer challenges for effective decision making. As a result of these approaches, several specialists have developed enhanced versions of TODIM and VIKOR, which successfully address decision-making difficulties in ambiguous situations. Literature reviews on VIKOR incomprehensible and TODIM methods are shown in Table 1 in various uncertain situations.

A divergence measure is widely used in image segmentation, medical diagnosis, pattern recognition and decision-making challenges because it is better at distinguishing between two items or probability distributions. [24] presented a divergence metric for FSs based on logarithmic information functions. Various academics created divergence measures for fuzzy sets as a result [25–29]. The purpose of this exercise is to create a divergence metric using Jensen inequality and Shannon entropy. FFSs are, as previously stated, extensions of Atanassov's intuitionistic fuzzy sets. Divergence metrics for Fermatean fuzzy sets have been increasingly important in the recent past for reducing uncertainty and supporting decision-makers in circumstances with limited information. Numerous writers have published a fairly limited amount of work on divergence measurements for Fermatean fuzzy environments. We construct a broader version of Jensen–Shannon Fermatean fuzzy divergence in this research using an adjustable approach that provides the user more power over the choice. Any measure's quality can be considered to be determined by its attributes. Some mathematical properties of the new divergence measure have been established in order to demonstrate its applicability. The new FFS divergence measure not only meets distance measure's axiomatic definition, but it significantly improves the discrimination of differences across FFSs. Then, the new divergence metric can produce more accurate results.

The most important approaches of domain MAGDM are entropy and divergence measurements. These can be considered as effective mathematical techniques for determining the objective weights of qualities while measuring ambiguous information. Entropy measures are good alternatives to aggregation operators and algebraic operations because they are capable of providing equal smooth approximations. The distance metrics are based on Hamming distances, which ignore the independent effects of membership rate and also non-membership rates. As a result, in order to expand the distance estimates, we provide a new FFS divergence metric based on the Jensen–Shannon measure. However, entropy and divergence measurements on FFSs have received little attention inspired by these principles, based on the entropy rate and the degree of divergence of FFSs handling Fermatean fuzzy MAGDM within FFSs, we created a Fermatean fuzzy MAGDM technique. In addition, none of the MAGDM papers use the TODIM–VIKOR integrated strategy instead of FFS. As a result, the TODIM and VIKOR approaches are extremely useful in dealing with decision-making challenges. The primary premise of this manuscript is to evaluate a combination of TODIM and VIKOR methods in an FFs environment with entropy and divergence measurements, where everything is done by FFN and test information about individual weights is determined using entropy and divergence measures. As a result, the TODIM model and VIKOR technique under the Fermatean fuzzy set have been improved in this study in order to get a consistent and trustworthy standard of options.

The present work has a rare contribution in the situations of medical consumption products that have not been studied previously. Several models were established in a prior study on FSs and their extensions; some of them are only interested in attribute/criterion entropy information. However [8] should improve the entropy measure employed in the entropy approach because many decision outcomes are illogical. This study attempted to fill in the gaps using a new measurement approach in the literature based on measurement methods of entropy and divergence within Fermatean fuzzy information. Such integration helps to achieve real-world decision-making standards and to achieve greater stability in a wide range of responsibilities. However, the combined TODIM–VIKOR model with unknown Fermatean numbers (FFNs) has not been thoroughly investigated. As a result, the FF–TODIM–VIKOR model must be considered. The current approach is to develop an integrated model that uses TODIM and VIKOR methodologies as well as FF knowledge to efficiently set up MAGDM challenges. Some fundamental definitions, scoring function, accuracy function, entropy, and divergence measure of FFSs are all included in this article.

The suggested activity's major goal is to execute computations using a Fermatean fuzzy backdrop and integrate new formulas to obtain more realistic qualities and decision weights, which will improve the TODIM–VIKOR approach's stability. The important aspects of FFS are membership numbers and degree memberships, as well as precise statistics for non-membership degrees. As a result, paying particular attention to group decision-making challenges using Fermatean fuzzy knowledge is an encouraging topic. In summary, this current paper contains the following: **(1)** A new Fermatean fuzzy entropy was introduced on the basis of Shannon entropy and proves its validity; **(2)** A step of separation introduced by FFNs and discusses some of the refined features, which are useful for the proposed scale; **(3)** The desired mix of entropy and divergence measure to figure out the weight of the attribute; **(4)** The new MAGDM method is introduced thus according to the TODIM–VIKOR method of the Fermatean fuzzy nature. **(5)** In order to demonstrate the efficacy of the planned strategy, a case of a medical product is created. It has specified functions that deal with the realities of decision-making in the real world. Finally, the proposed FF–TODIM–VIKOR methods are compared with existing methods, intended for the confirmation of result obtained by a particular image.

The paper's superfluity is classified into the following categories: Section 2 suggests some basic ideas related to IFS, PyFS, FFS. The new Fermatean fuzzy entropy and its key structures are given in Section 3 and prove its competence on the basis of language variability. Section 4 established a new divergence rate and studied its key properties. In order to evaluate a suitable supplier, Section 5 employs the TODIM–VIKOR new approach based on Fermatean fuzzy measurements. Section 6 includes a numerical model of the items, as well as a comparison of the proposed approach to existing methods, to illustrate the new method's dependability and performance. Section 7 describes conclusion of this work as well as the supplementary comments.

Table 1. In many environments, approaches of fuzzy VIKOR and TODIM are used.

Authors	Applications	Methods
[17]	Decision problem	Classical TODIM
[16]	Project investment	Classical TODIM
[18]	Investment problem	IF–TODIM
[30]	Problem of construction minerals industry	Fuzzy TODIM–DELPHI
[31]	Site selection problem	Interval IF–TODIM
[32]	Hotels Selection	TODIM for intuitionistic linguistic numbers
[33]	The executive person selection problem	Shapley function-based IF–TODIM
[14]	Decision making	TODIM
[34]	Performance Appraisal	interval IF–TODIM
[35]	Tourism attraction	ANP–TODIM
[36]	Service quality assessment	interval-valued IF–TODIM
[12]	High-tech risk evaluation	VIKOR and prospect theory
[8]	renewable energy technoloies	Classical VIKOR
[37]	Green supplier programme	VIKOR
[38]	Evaluate the performance of Emerging eco-industrial	VIKOR and GRA
[39]	Group decision making	VIKOR
[40]	Flood control operation	Compromise ratio model
[41]	Decision making model	VIKOR
[42]	financing risk assessment	VIKOR
[43]	Selection of investment problem	Interval IF–VIKOR
[44]	Selection of a cooperative partner	Hesitant fuzzy (HF) VIKOR
[45]	Evaluate and find rank of the service quality	Hesitant fuzzy VIKOR HF–VIKOR
[46]	MADM approach	Classical VIKOR
[47]	Selection of an ERP system	HF Linguistic VIKOR

2. Basic notions

We will go through the basics of IFs, PyFSs, and FFSs in this segment.

Definition 1 (Ref. [2]). *An Atanassov intuitionistic fuzzy set S on K is explained as*

$$S = \{(b_i, g_S(b_i), f_S(b_i)) : b_i \in K\},$$

where $g_S\colon K \to [0,1]$ is membership function and $f_S\colon K \to [0,1]$ a non-membership function such that $0 \leqslant g_S(b_i) + f_S(b_i) \leqslant 1$, for all $b_i \in K$. Moreover, $\phi_S(b_i) = 1 - g_S(b_i) - f_S(b_i)$, $b_i \in K$ is named as intuitionistic degree.

Definition 2 (Ref. [3]). *A pythagorean fuzzy set S on K is explained as*

$$S = \{(b_i, g_S(b_i), f_S(b_i)) : b_i \in K\}$$

$g_S\colon K \to [0,1]$ is known as membership function and $f_S\colon K \to [0,1]$ is a non membership function such that $g_S^2(b_i) + f_S^2(b_i) \leqslant 1$ for all $b_i \in K$. Moreover, $\phi_S(b_i) = \sqrt{1 - g_S^2(b_i) - f_S^2(b_i)}$ is pythagorean index for each $b_i \in K$.

Definition 3. *A Fermatean fuzzy set A on K is explained as*

$$S = \{(b_i, g_S(b_i), f_S(b_i)) : b_i \in K\}$$

known a membership $g_S\colon K \to [0,1]$ and a non membership function $f_S\colon K \to [0,1]$ with the inequality $0 \leqslant g_S^3(b_i) + f_S^3(b_i) \leqslant 1$, for all $b_i \in K$. Moreover, for all $b_i \in K$, Fermatean fuzzy index is defined as $\Phi_S(b_i) = \sqrt[3]{1 - g_S^3(b_i) - f_S^3(b_i)}$.

Definition 4 (Ref. [48]). *For any Fermatean fuzzy number, $\mu = (g, f)$. The score value and accuracy value of μ are defined as:*

$$\mathrm{Sco}(\mu) = g^3 - f^3,$$

where $\mathrm{Sco}(\mu) \in [-1, 1]$.

$$\mathrm{Acu}(\mu) = g^3 + f^3,$$

where $\mathrm{Acu}(\mu) \in [0, 1]$.

Definition 5 (Ref. [48]). *For any $\mu_1, \mu_2 \in FFN$, we have*

- *If $\mathrm{Sco}(\mu_1) > \mathrm{Sco}(\mu_2)$, then $\mu_1 > \mu_2$;*
- *If $\mathrm{Sco}(\mu_1) = \mathrm{Sco}(\mu_2)$, then*
  - *if $\mathrm{Acu}(\mu_1) > \mathrm{Acu}(\mu_2)$, then $\mu_1 > \mu_2$;*
  - *if $\mathrm{Acu}(\mu_1) = \mathrm{Acu}(\mu_2)$, then $\mu_1 = \mu_2$.*

Definition 6. *With in universe K, any two FFSs S and T are specified by*

$$S = \{b_i, g_S(b_i), f_S(b_i)\colon b_i \in K\} \quad \textit{and} \quad T = \{b_i, g_T(b_i), f_T(b_i)\colon b_i \in K\};$$

then some mathematical operations are given as below:

1. *$S \subseteq T \Leftrightarrow \forall b_i \in K$, $g_S(b_i) \leqslant g_T(b_i)$ and $f_S(b_i) \geqslant f_T(b_i)$;*
2. *$S = T \Leftrightarrow \forall b_i \in K$, $g_S(b_i) = g_T(b_i)$ and $f_S(b_i) = f_T(b_i)$;*
3. *co $S = S^c = \{b_i, f_S(b_i), g_S(b_i)\colon b_i \in K\}$;*
4. *$S \cup_3 T = \{\langle b_i, \max(g_S(b_i), g_T(b_i)), \min(f_S(b_i), f_T(b_i))\rangle : b_i \in K\}$;*
5. *$S \cap_3 T = \{\langle b_i, \min(g_S(b_i), g_T(b_i)), \max(f_S(b_i), f_T(b_i))\rangle : b_i \in K\}$.*
6. *$S \oplus_3 T = \left\{\left(b_i, \sqrt[3]{g_S^3 + g_T^3 - g_S^3 g_T^3}, g_S^3 g_T^3\right)\colon b_i \in K\right\}$;*
7. *$S \oplus_3 T = \left\{\left(b_i, f_S^3 f_T^3, \sqrt[3]{f_S^3 + f_T^3 - f_S^3 g_T^3}\right)\colon b_i \in K\right\}$;*
8. *$\gamma *_3 S = \left\{\left(b_i, \sqrt[3]{1 - \left(1 - g_\gamma^3(b_i)\right)^3}, g_S^\gamma(b_i)\right)\right\}$;*
9. *$S \wedge_3 \gamma = \left\{\left(b_i, f_S^\gamma(b_i), \sqrt[3]{1 - \left(1 - g_\gamma^3(b_i)\right)^\gamma}\right)\right\}$.*

2.1. Property

If S in the axiom E4 is crisper than T, then we have

$$\left|g_S^3(b_i) - \frac{1}{\sqrt[3]{3}}\right| + \left|f_S^3(b_i) - \frac{1}{\sqrt[3]{3}}\right| + \left|\Phi_S^3(b_i) - \frac{1}{\sqrt[3]{3}}\right| \geqslant \left|g_T^3(b_i) - \frac{1}{\sqrt[3]{3}}\right| + \left|f_T^3(b_i) - \frac{1}{\sqrt[3]{3}}\right| + \left|\Phi_T^3(b_i) - \frac{1}{\sqrt[3]{3}}\right|$$

and

$$\left[g_S(b_i) - \frac{1}{\sqrt[3]{3}}\right]^3 + \left[f_S(b_i) - \frac{1}{\sqrt[3]{3}}\right]^3 + \left[\Phi_S(b_i) - \frac{1}{\sqrt[3]{3}}\right]^3 \geqslant \left[g_T(b_i) - \frac{1}{\sqrt[3]{3}}\right]^3 + \left[f_T(b_i) - \frac{1}{\sqrt[3]{3}}\right]^3 + \left[\Phi_T(b_i) - \frac{1}{\sqrt[3]{3}}\right]^3.$$

3. A new entropy measure for FFSs

Here, we construct a new entropy measure for fermatean fuzzy set based on the probability type. Let

$$\Sigma_n = \left\{ K = (b_1, b_2, \ldots, b_n)\colon \sum_{i=1}^{n} b_i = 1, b_i \geqslant 0 \right\}, \quad n \geqslant 2$$

be a sequence of discrete probability distributions. For $K \in \Sigma_n$, corresponding to the Shannon entropy [49] we define a new fermatean fuzzy entropy measure $M_{fd}(S)$ for a FFS S given by the following:

$$M_{fd}(S) = -\frac{1}{n} \sum_{i=1}^{n} \left[g_S^3(b_i) \log\left(g_S^3(b_i)\right) + f_S^3(b_i) \log\left(g_S^3(b_i)\right) + \phi_S^3(b_i) \log\left(\phi_S^3(b_i)\right) \right]. \tag{1}$$

Theorem 1. *Let $K = (b_1, b_2, \ldots, b_n)$ be an universe of discourse (non-empty). The proposed entropy $M_{fd}(S)$ for a FFS S satisfies the following conditions.*

E1: Let us consider a crisp set S which has membership values either $g_S^3(b_i) = 1$ and $f_S^3(b_i) = \Phi_S^3(b_i) = 0$ or $f_S^3(b_i) = 1$ and $g_S^3(b_i) = \Phi_S^3(b_i) = 0$ or $\Phi_S^3(b_i) = 1$ and $g_S^3(b_i) = f_S^3(b_i) = 0$.

Then easily we can say, $M_{fd}(S) = 0$.

Conversely, if $M_{fd}(S) = 0$ we have

$$\left[g_S^3(b_i) \log\left(g_S^3(b_i)\right) + f_S^3(b_i) \log\left(g_S^3(b_i)\right) + \phi_S^3(b_i) \log\left(\phi_S^3(b_i)\right) \right] = 0.$$

This will be included three possible cases as:

1) either $g_S^3(b_i) = 1$ and $f_S^3(b_i) = \Phi_S^3(b_i) = 0$ or
2) $f_S^3(b_i) = 1$ and $g_S^3(b_i) = \Phi_S^3(b_i) = 0$ or
3) $\Phi_S^3(b_i) = 1$ and $g_S^3(b_i) = f_S^3(b_i) = 0$.

S is a crisp set if and only if $Mfd(S) = 0$ in the three examples above.

E2: Since $g_S^3(b_i) + f_S^3(b_i) + \Phi_S^3(b_i) = 1$, in order to calculate the maximum value of fermatean fuzzy entropy $M_{fd}(S)$, we compose $H(g_S, f_S, \Phi_S) = g_S^3(b_i) + f_S^3(b_i) + \Phi_S^3(b_i) - 1$ and the following Lagrange function will be constructed as follows:

$$G^*(g_S^3, f_S^3, \Phi_S^3) = M_{fd}(g_S^3, f_S^3, \Phi_S^3) + \beta_1 H(g_S^3, f_S^3, \Phi_S^3), \tag{2}$$

where β_1 is the Lagrange's multiplier. To get the maximality of $M_{fd}(S)$, differentiating (2) partly with regard to g_S^3, f_S^3, Φ_S^3 and β_1 and set each of them to 0, we have got $g_S^3(b_i) = f_S^3(b_i) = \Phi_S^3(b_i) = \frac{1}{\sqrt[3]{3}}$. The stationary points of $M_{fd}(S)$ is $g_S^3(b_i) = f_S^3(b_i) = \Phi_S^3(b_i) = \frac{1}{\sqrt[3]{3}}$. Next, by using Hessian matrix, we prove that the function $M_{fd}(S)$ is concave at stationary points.

Definition 7. *The Hessian matrix (HM) of any three-variable function $S(b_1, b_2, b_3)$ is given by*

$$\lceil HM \rceil(S) = \begin{bmatrix} \dfrac{\partial^2 S}{\partial b_1^2} & \dfrac{\partial^2 S}{\partial b_2 \partial b_1} & \dfrac{\partial^2 S}{\partial b_3 \partial b_1} \\ \dfrac{\partial^2 S}{\partial b_1 \partial b_2} & \dfrac{\partial^2 S}{\partial b_2^2} & \dfrac{\partial^2 S}{\partial b_3 \partial b_2} \\ \dfrac{\partial^2 S}{\partial b_1 \partial b_3} & \dfrac{\partial^2 S}{\partial b_2 \partial b_3} & \dfrac{\partial^2 S}{\partial b_3^2} \end{bmatrix}$$

here S is strictly convex in the domain at a point when $\lceil HM \rceil(S)$ is defined as positive definite and its strictly concave if $\lceil HM \rceil(S)$ is defined as negative definite.

The HM of $M_{fd}(S)$ is stated as follows:

$$\lceil HM \rceil (M_{fd}(S)) = (\sqrt[3]{3}) \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

it is negative definite. As an outcome, $M_{fd}(S)$ is strictly concave function with a maximum value at $g_S^3(b_i) = f_S^3(b_i) = \Phi_S^3(b_i) = \frac{1}{\sqrt[3]{3}}$ (stationary points).

E3: Since, $M_{fd}(S)$ is a concave function of $S \in FFS(K)$ having maximum value at stationary points, then if max $\{g_S^3(b_i), f_S^3(b_i), \Phi_S^3(b_i)\} \leqslant \frac{1}{\sqrt[3]{3}}$, then $g_S^3(b_i) \leqslant g_T^3(b_i), f_S^3(b_i) \leqslant f_T^3(b_i)$ implies $\Phi_S^3(b_i) \geqslant \Phi_T^3(b_i) \geqslant \frac{1}{\sqrt[3]{3}}$. Then, by using property (2.1), we shall see that $M_{fd}(S)$ proves the condition $E3$.

Similarly, if $\min\{g_S^3(b_i), f_S^3(b_i)\} \geqslant \frac{1}{\sqrt[3]{3}}$, then $g_A^3(b_i) \geqslant g_T^3(b_i), f_S^3 b_i) \geqslant f_T^3(b_i)$. We can show that $M_{fd}(S)$ holds the condition E3 by using property (2.1).

E4: For any FFS, $M_{fd}(S) = M_{fd}(S^c)$, it is easy to check.

Theorem 2. *For any two $S, T \in FFS(K)$, such that for all $b_i \in K$ either $S \subseteq T$ or $BT \subseteq S$; then,*

$$M_{fd}(S \cup T) + M_{fd}(S \cap T) = M_{fd}(S) + M_{fd}(T).$$

Proof. Separate K into two halves, say K_1 and K_2, to establish the theorem 2

$$K_1 = \{b_i \in K \colon S \subseteq T\}, \quad \text{and} \quad K_2 = \{b_i \in K \colon T \supseteq S\},$$

$$g_S^3(b_i) \leqslant g_T^3(b_i), f_S^3(b_i) \leqslant f_T^3(b_i) \quad \forall b_i \in K_1,$$
$$g_S^3(b_i) \geqslant g_T^3(b_i), f_S^3(b_i) \geqslant f_T^3(b_i) \quad \forall b_i \in K_2,$$

$$\begin{aligned} M_{fd}(S \cup T) = &-\frac{1}{n}\sum_{i=1}^{n}\Big[g_{(S\cup T)}^3(b_i)\log\big(g_{(S\cup T)}^3(b_i)\big) \\ &+ f_{(S\cup T)}^3(b_i)\log\big(f_{(S\cup T)}^3(b_i)\big) + \phi_{(S\cup T)}^3(b_i)\log\big(\phi_{(S\cup T)}^3(b_i)\big)\Big] \\ = &-\frac{1}{n}\sum_{K_1}\Big[g_T^3(b_i)\log\big(g_T^3(b_i)\big) + f_T^3(b_i)\log\big(f_T^3(b_i)\big) + \phi_T^3(b_i)\log\big(\phi_T^3(b_i)\big)\Big] \\ &- \frac{1}{n}\sum_{K_2}\Big[g_S^3(b_i)\log\big(g_S^3(b_i)\big) + f_S^3(b_i)\log\big(f_S^3(b_i)\big) + \phi_S^3(b_i)\log\big(\phi_S^3(b_i)\big)\Big]. \end{aligned}$$

Similarly, we get

$$\begin{aligned} M_{fd}(S \cap T) = &-\frac{1}{n}\sum_{K_1}\Big[g_S^3(b_i)\log\big(g_S^3(b_i)\big) + f_S^3(b_i)\log\big(f_S^3(b_i)\big) + \phi_S^3(b_i)\log\big(\phi_S^3(b_i)\big)\Big] \\ &- \frac{1}{n}\sum_{K_2}\Big[g_T^3(b_i)\log\big(g_{(T}^3(b_i)\big) + f_T^3(b_i)\log\big(f_T^3(b_i)\big) + \phi_T^3(b_i)\log\big(\phi_T^3(b_i)\big)\Big]. \end{aligned}$$

Now, adding above equations, we have

$$M_{fd}(S \cup T) + M_{fd}(S \cap T) = M_{fd}(S) + M_{fd}(T).$$

∎

As a result, the proposed entropy measure satisfies some effective features, demonstrating its utility in certain applications.

4. Divergence measure for FFSs

History: Shannon was a pioneer in the subject of entropy and he proposed the below statement for a set S:

$$M_F(S) = -\sum_{i=1}^{n}(b_i)\log(b_i).$$

In a mixed distribution, a decomposition of total diversity $\frac{S+T}{2}$ can be obtained from the concavity behaviour of $M(A)$ as follows:

$$M_f\left(\frac{S+T}{2}\right) = \frac{1}{2}\big(M_f(S) + M_f(T)\big) + \frac{1}{2}J_n(S,T). \tag{3}$$

The first half of the equation (3) described above, i.e., $\frac{S+T}{2}$ the average diversity within distributions is described, where the second part, i.e.,

$$J_n(S,T) = \big(-M_f(S) - M_f(T)\big) - 2\left(-M_f\left(\frac{S+T}{2}\right)\right) \tag{4}$$

is called Jensen difference that raises due to the convexity of $M(S)$ which is $\geqslant 0$ (i.e., non-negative) and vanishes if and only if $S = T$. Also, between two finite probability distributions S and T it computes a unique divergence measure.

We assume that equation (4) originates as a result of the generalised technique of entropy functions, which incorporates the entropy investigated by [49] and referred to as a Jensen–Shannon divergence, and that it has the convex property. According to the well-known Shannon entropy, the property of convex for Jensen divergence equation (4) is straightforward to understand. Jensen–Shannon divergence measure defined by [50] for two finite probability distributions S and T, corresponding to the weight vectors $\sigma_1 + \sigma_2 = 1$, is simple to understand, as per the well-known Shannon entropy.

For two finite probability distributions S and T, corresponding to the coefficient weight vectors $\sigma_1 + \sigma_2 = 1$,

$$JS(S,T) = M_f(\sigma_1 S + \sigma_2 T) - \sigma_1 M_f(S) - \sigma_2 M_f(T).$$

The Jensen–Shannon divergence measure has a critical property that different weights can be allocated for each probability distribution. Such characteristic are appropriate to study the various decision problems emerge, whenever weights are prior probabilities. For two probability distributions, the majority of divergence measure has been built Jensen-Shannon divergence was the inspiration for this piece. We build a new Jensen–Shannon divergence measures for FFSs which is advanced stage of Jenson–Shannon divergence measure for IFSs and recall some particular cases, which shows its validation. In particular, several aspects of the constructed measure are discussed.

4.1. Proposed divergence measure for FFSs

Definition 8. *Let $S, T \in FFSs(K)$. Now we represent*

$$\Omega^i_{MF}(S,T) = \big(g_S^3(b_i) + g_T^3(b_i)\big)\log\frac{g_S^3(b_i) + g_T^3(b_i)}{2} - g_S^3(b_i)\log\big(g_S^3(b_i)\big) - g_T^3(b_i)\log\big(g_T^3(b_i)\big),$$

$$\Omega^i_{NMF}(S,T) = \big(f_S^3(b_i) + f_T^3(b_i)\big)\log\frac{f_S^3(b_i) + f_T^3(b_i)}{2} - f_S^3(b_i)\log\big(f_S^3(b_i)\big) - f_T^3(b_i)\log\big(f_T^3(b_i)\big),$$

$$\Omega^i_{II}(S,T) = (\phi_S^3(b_i) + \phi_T^3(b_i))\log\frac{(\phi_S^3(b_i) + \phi_T^3(b_i))}{2} - \phi_S^3(b_i)\log\big(\phi_S^3(b_i)\big) - \phi_T^3(b_i)\log\big(\phi_T^3(b_i)\big).$$

Now by using above three equations we explain the Jensen–Shannon fermatean fuzzy divergence measure as below:

$$\Omega_{ff}(S,T) = -\frac{1}{n}\sum_{i=1}^{n}\left(\Omega^i_{MF}(S,T) + \Omega^i_{NMF}(S,T) + \Omega^i_{II}(S,T)\right). \tag{5}$$

Definition 9. *Let $S,T \in FFSs(K)$. A real-valued function $\Omega_{ff}\colon FFSs(K)\times FFSs(K)\to \mathrm{Re}$ it is known as the FFS divergence measure, if it meets the following criteria:*
1) $\Omega_{ff}(S,T)\geqslant 0$, and $\Omega_{ff}(S,T)=0$ iff $S=T$;
2) $\Omega_{ff}(S,T)=\Omega_{ff}(T,S)$;
3) $\Omega_{ff}(S,T)\leqslant \Omega_{ff}(S,R)$ and $\Omega_{ff}(T,R)\leqslant \Omega_{ff}(S,R)$.

Properties of proposed divergence measure

1. $\Omega_{ff}(S,S\cup T)=\Omega_{ff}(S\cap T,T)=\Omega_{ff}(S,T)$.
2. $\Omega_{ff}(S\cup T,S\cap T)=\Omega_{ff}(S\cap T,S\cup T)=\Omega_{ff}(S,T)$.
3. $\Omega_{ff}(S,S\cup T)+\Omega_{ff}(S,S\cap T)=2\Omega_{ff}(S,T)$.
4. $\Omega_{ff}(T,S\cup T)+\Omega_{ff}(T,S\cap T)=2\Omega_{ff}(S,T)$.
5. $\Omega_{ff}(S\cup T,R)\leqslant \Omega_{ff}(S,R)+\Omega_{ff}(T,R)$.
6. $\Omega_{ff}(S\cap T,R)\leqslant \Omega_{ff}(S,R)+\Omega_{ff}(T,R)$.
7. $\Omega_{ff}(S\cup T,R)+\Omega_{ff}(S\cap T,R)=\Omega_{ff}(S,R)+\Omega_{ff}(T,R)$.
8. $\Omega_{ff}(S,T)=\Omega_{ff}(S^c,T^c)$.
9. $\Omega_{ff}(S,T^c)=\Omega_{ff}(S^c,T)$.
10. $\Omega_{ff}(S,T^c)+\Omega_{ff}(S^c,T)=\Omega_{ff}(S^c,T^c)+\Omega_{ff}(S,T^c)$.

Proof. First, we will separate set K into two subsets K_1 and K_2, such that

$$K_1=\{b_i\in K\colon S\subset T\}\quad\text{and}\quad K_2=\{b_i\in K\colon S\supset T\}\tag{6}$$

From Equation (6), for all $b_i\in K_1$,

$$g_S^3\leqslant g_T^3\quad\text{and}\quad f_S^3\geqslant f_T^3;$$

for all $b_i\in K_2$,

$$g_S^3\geqslant g_T^3\quad\text{and}\quad f_S^3\leqslant f_T^3.$$

1. Property 1 as follows. Since

$$\begin{aligned}
\Omega_{ff}(S,S\cup T)&=-\frac{1}{n}\sum_{i=1}^{n}(\Omega_{MF}^i(S,S\cup T)+\Omega_{NMF}^i(S,S\cup T)+\Omega_{II}^i(S,S\cup T))\\
&=-\frac{1}{n}\sum_{i=1}^{n}\Bigg[\big(g_S^3(b_i)+g_{S\cup T}^3(b_i)\big)\log\frac{g_S^3(b_i)+g_{S\cup T}^3(b_i)}{2}\\
&\quad+\big(f_S^3(b_i)+f_{S\cup T}^3(b_i)\big)\log\frac{f_S^3(b_i)+f_{S\cup T}^3(b_i)}{2}+\big(\phi_S^3(b_i)+\phi_{S\cup T}^3(b_i)\big)\log\frac{\phi_S^3(b_i)+\phi_{S\cup T}^3(b_i)}{2}\\
&\quad-\big(g_S^3(b_i)\log\big(g_S^3(b_i)\big)+f_S^3(b_i)\log\big(f_S^3(b_i)\big)+\phi_S^3(b_i)\log\big(\phi_S^3(b_i)\big)\big)\\
&\quad-\big(g_{S\cup T}^3(b_i)\log\big(g_{S\cup T}^3(b_i)\big)+f_{S\cup T}^3(b_i)\log\big(f_{S\cup T}^3(b_i)\big)+\phi_{S\cup T}^3(b_i)\log\big(\phi_{S\cup T}^3(b_i)\big)\big)\Bigg].
\end{aligned}$$

This implies

$$\begin{aligned}
\Omega_{ff}(S,S\cup T)=&-\frac{1}{n}\sum_{k_1}\Bigg[\big(g_S^3(b_i)+g_T^3(b_i)\big)\log\frac{g_S^3(b_i)+g_T^3(b_i)}{2}\\
&+\big(f_S^3(b_i)+f_T^3(b_i)\big)\log\frac{f_S^3(b_i)+f_T^3(b_i)}{2}+\big(\phi_S^3(b_i)+\phi_T^3(b_i)\big)\log\frac{\phi_S^3(b_i)+\phi_T^3(b_i)}{2}\\
&-\big(g_S^3(b_i)\log\big(g_S^3(b_i)\big)+f_S^3(b_i)\log\big(f_S^3(b_i)\big)+\phi_S^3(b_i)\log\big(\phi_S^3(b_i)\big)\big)\\
&-\big(g_T^3(b_i)\log\big(g_T^3(b_i)\big)+f_T^3(b_i)\log\big(f_T^3(b_i)\big)+\phi_T^3(b_i)\log\big(\phi_T^3(b_i)\big)\big)\Bigg]\\
&-\frac{1}{n}\sum_{k_2}\Bigg[\big(g_S^3(b_i)+g_S^3(b_i)\big)\log\frac{g_S^3(b_i)+g_S^3(b_i)}{2}\\
&+(f_S^3(b_i)+f_S^3(b_i))\log\frac{f_S^3(b_i)+f_S^3(b_i)}{2}+\big(\phi_S^3(b_i)+\phi_S^3(b_i)\big)\log\frac{\phi_S^3(b_i)+\phi_S^3(b_i)}{2}
\end{aligned}$$

$$
\begin{aligned}
&- \left(g_S^3(b_i)\log\left(g_S^3(b_i)\right) + f_S^3(b_i)\log\left(f_S^3(b_i)\right) + \phi_S^3(b_i)\log\left(\phi_S^3(b_i)\right)\right) \\
&- \left(g_S^3(b_i)\log\left(g_S^3(b_i)\right) + f_S^3(b_i)\log\left(f_S^3(b_i)\right) + \phi_S^3(b_i)\log\left(\phi_S^3(b_i)\right)\right)\Bigg] \\
\leqslant & -\frac{1}{n}\sum_{i=1}^{n}\Bigg[\left(g_S^3(b_i)+g_T^3(b_i)\right)\log\frac{g_S^3(b_i)+g_T^3(b_i)}{2} \\
&+ \left(f_S^3(b_i)+f_T^3(b_i)\right)\log\frac{f_S^3(b_i)+f_T^3(b_i)}{2} + \left(\phi_S^3(b_i)+\phi_T^3(b_i)\right)\log\frac{\phi_S^3(b_i)+\phi_T^3(b_i)}{2} \\
&- \left(g_S^3(b_i)\log\left(g_S^3(b_i)\right) + f_S^3(b_i)\log\left(f_S^3(b_i)\right) + \phi_S^3(b_i)\log\left(\phi_S^3(b_i)\right)\right) \\
&- \left(g_T^3(b_i)\log\left(g_T^3(b_i)\right) + f_T^3(b_i)\log\left(f_T^3(b_i)\right) + \phi_T^3(b_i)\log\left(\phi_T^3(b_i)\right)\right)\Bigg] \\
= & \ \Omega_{ff}(S,T).
\end{aligned}
$$

Similarly, $\Omega_{ff}(S\cap T,T)\leqslant\Omega_{\beta}(S,T)$ can be proved. From property (1) we get,

$$
\Omega_{ff}(S\cup T,S\cap T)=\sum_{K_1}\Omega_{ff}(T,S)+\sum_{K_2}\Omega_{ff}(S,T)=\Omega_{ff}(S,T).
$$

Similarly, we obtain

$$
\Omega_{ff}(S\cap T,S\cup T)=\sum_{k_1}\Omega_{ff}(T,S)+\sum_{k_2}\Omega_{ff}(S,T)=\Omega_{ff}(S,T).
$$

2. Proof of the property (3)

$$
\begin{aligned}
\Omega_{ff}(S,S\cup T)+\Omega_{ff}(S,S\cap T) &= \sum_{K_1}\Omega_{ff}(S,T)+\sum_{K_2}\Omega_{ff}(S,S)+\sum_{K_1}\Omega_{ff}(S,S)+\sum_{K_2}\Omega_{ff}(S,T) \\
&= 2\Omega_{ff}(S,T).
\end{aligned}
$$

3. The proof of property (4) comes directly from the proof of property (3).
4. To prove the property (5), consider

$$
\begin{aligned}
\Omega_{ff}(S,R)+\Omega_{ff}(T,R)-\Omega_{ff}(S\cup T,R) &= -\frac{1}{n}\sum_{i=1}^{n}\Bigg[\left(g_S^3(b_i)+g_R^3(b_i)\right)\log\frac{g_S^3(b_i)+g_R^3(b_i)}{2} \\
&+ \left(f_S^3(b_i)+f_R^3(b_i)\right)\log\frac{f_S^3(b_i)+f_R^3(b_i)}{2} + \left(\phi_S^3(b_i)+\phi_R^3(b_i)\right)\log\frac{\phi_S^3(b_i)+\phi_R^3(b_i)}{2} \\
&- \Big(g_S^3(b_i)\log\left(g_S^3(b_i)\right) + f_S^3(b_i)\log\left(f_S^3(b_i)\right) + \phi_S^3(b_i)\log\left(\phi_S^3(b_i)\right)\Big) \\
&- \Big(g_R^3(b_i)\log\left(g_R^3(b_i)\right) + f_R^3(b_i)\log\left(f_R^3(b_i)\right) + \phi_R^3(b_i)\log\left(\phi_R^3(b_i)\right)\Big)\Bigg] \\
&- \frac{1}{n}\sum_{i=1}^{n}\Bigg[\left(g_T^3(b_i)+g_R^3(b_i)\right)\log\frac{g_T^3(b_i)+g_R^3(b_i)}{2} \\
&+ \left(f_T^3(b_i)+f_R^3(b_i)\right)\log\frac{f_T^3(b_i)+f_R^3(b_i)}{2} + \left(\phi_T^3(b_i)+\phi_R^3(b_i)\right)\log\frac{\phi_T^3(b_i)+\phi_R^3(b_i)}{2} \\
&- \Big(g_T(b_i)\log\left(g_T^3(b_i)\right) + f_T^3(b_i)\log\left(f_T^3(b_i)\right) + \phi_T^3(b_i)\log\left(\phi_T^3(b_i)\right)\Big) \\
&- \Big(g_R^3(b_i)\log\left(g_R^3(b_i)\right) + f_R^3(b_i)\log\left(f_R^3(b_i)\right) + \phi_R^3(b_i)\log\left(\phi_R^3(b_i)\right)\Big)\Bigg] \\
&+ \frac{1}{n}\sum_{K_1}\Bigg[\left(g_T^3(b_i)+g_R^3(b_i)\right)\log\frac{g_T^3(b_i)+g_R^3(b_i)}{2} \\
&+ \left(f_T^3(b_i)+f_R^3(b_i)\right)\log\frac{f_T^3(b_i)+f_R^3(b_i)}{2} + \left(\phi_T^3(b_i)+\phi_R^3(b_i)\right)\log\frac{\phi_T^3(b_i)+\phi_R^3(b_i)}{2}
\end{aligned}
$$

$$
\begin{aligned}
&-\Big(g_T^3(b_i)\log\left(g_T^3(b_i)\right)+f_T^3(b_i)\log\left(f_T^3(b_i)\right)+\phi_T^3(b_i)\log\left(\phi_T^3(b_i)\right)\Big)\\
&-\Big(g_R^3(b_i)\log\left(g_R^3(b_i)\right)+f_R^3(b_i)\log\left(f_R^3(b_i)\right)+\phi_R^3(b_i)\log\left(\phi_R^3(b_i)\right)\Big)\Big]\\
&+\frac{1}{n}\sum_{K_2}\Big[\left(g_S^3(b_i)+g_R^3(b_i)\right)\log\frac{g_S^3(b_i)+g_R^3(b_i)}{2}\\
&+\left(f_S^3(b_i)+f_R^3(b_i)\right)\log\frac{f_S^3(b_i)+f_R^3(b_i)}{2}+\left(\phi_S^3(b_i)+\phi_R^3(b_i)\right)\log\frac{\phi_S^3(b_i)+\phi_R^3(b_i)}{2}\\
&-\Big(g_S(l_i)\log\left(g_S^3(b_i)\right)+f_S^3(b_i)\log\left(f_S^3(b_i)\right)+\phi_S^3(b_i)\log\left(\phi_S^3(b_i)\right)\Big)\\
&-\Big(g_R^3(b_i)\log\left(g_R^3(b_i)\right)+f_R^3(b_i)\log\left(f_R^3(b_i)\right)+\phi_R^3(b_i)\log\left(\phi_R^3(b_i)\right)\Big)\Big]\geqslant 0.
\end{aligned}
$$

Therefore $\Omega_{ff}(S,R)+\Omega_{ff}(T,R)\geqslant\Omega_{ff}(S\cup T,R)$.

We can also prove properties (6) and (7) in same way. Properties (8), (9) and (10) are also easily can be prove. ∎

5. MAGDM approach using the TODIM–VIKOR Method based on specified measures

TODIM and VIKOR approaches are integrated in this part to address the MAGDM problem in Fermatean fuzzy settings. Figure 1 shows the proposed integrated TODIM–VIKOR technique's schematic framework. We can see that there are four stages here. The FFNs are utilised in stage 1 to display the original decision-making data, after which the decision matrix is normalised. The objective criteria weights are determined in step 2 by combining the entropy weights model and the divergence measure. The method TODIM is used to prepare the dominance matrix in stage 3. In stage 4, the VIKOR approach is chosen to more efficiently resolve the ranking order of the possibilities.

Stage 1: Obtain decision-making data

A matrix can be used to illustrate multi-attribute decision-making situations. Alternatives and attributes are represented in the decision matrix's rows and columns, respectively. Experts / DMs' first Fermatean fuzzy information is acquired and described as FFNs or performance values. The concept of benefit and cost type attributes is used to create a normalisation matrix.

Consider $Y=\{Y_1,Y_2,\ldots,Y_p\}$ is a set of traits and $\delta=\{\theta_1,\theta_2,\ldots,\theta_n\}$ are alternatives. Let the matrix $W=[w_{ij}]_{p\times n}=(g_{ij},f_{ij})_{p\times n}$; $1\leqslant i\leqslant p, 1\leqslant j\leqslant n$ represented Fermatean fuzzy information. The matrix form for the MAGDM problem with FFNs is as follows:

$$
W=[w_{ij}]_{p\times n}=\begin{array}{c} \\ Y_1\\ Y_2\\ \vdots \\ Y_p\end{array}\begin{array}{c}\begin{array}{cccc}\theta_1 & \theta_2 & \ldots & \theta_n\end{array}\\ \begin{bmatrix}(g_{11},f_{11}) & (g_{12},f_{12}) & \cdots & (g_{1n},f_{1n})\\ (g_{21},f_{21}) & (g_{22},f_{22}) & \cdots & (g_{2n},f_{2n})\\ \vdots & \vdots & \ddots & \vdots\\ (g_{p1},f_{p1}) & (g_{p2},f_{p2}) & \cdots & (g_{pn},f_{pn})\end{bmatrix}\end{array}.
$$

Step 1: We constructed the following normalise algorithm for each attribute to make it dimensionless and easier to use. The normalised FFS decision matrix is displayed by o_{ij} as follows:

$$
o_{ij}=\begin{cases}\operatorname{neg}(w_{ij}), & \text{when cost attribute,}\\ w_{ij}, & \text{when benefit attribute,}\end{cases}
$$

where $\operatorname{neg}(w_{ij})=(f_{ij},g_{ij})$.

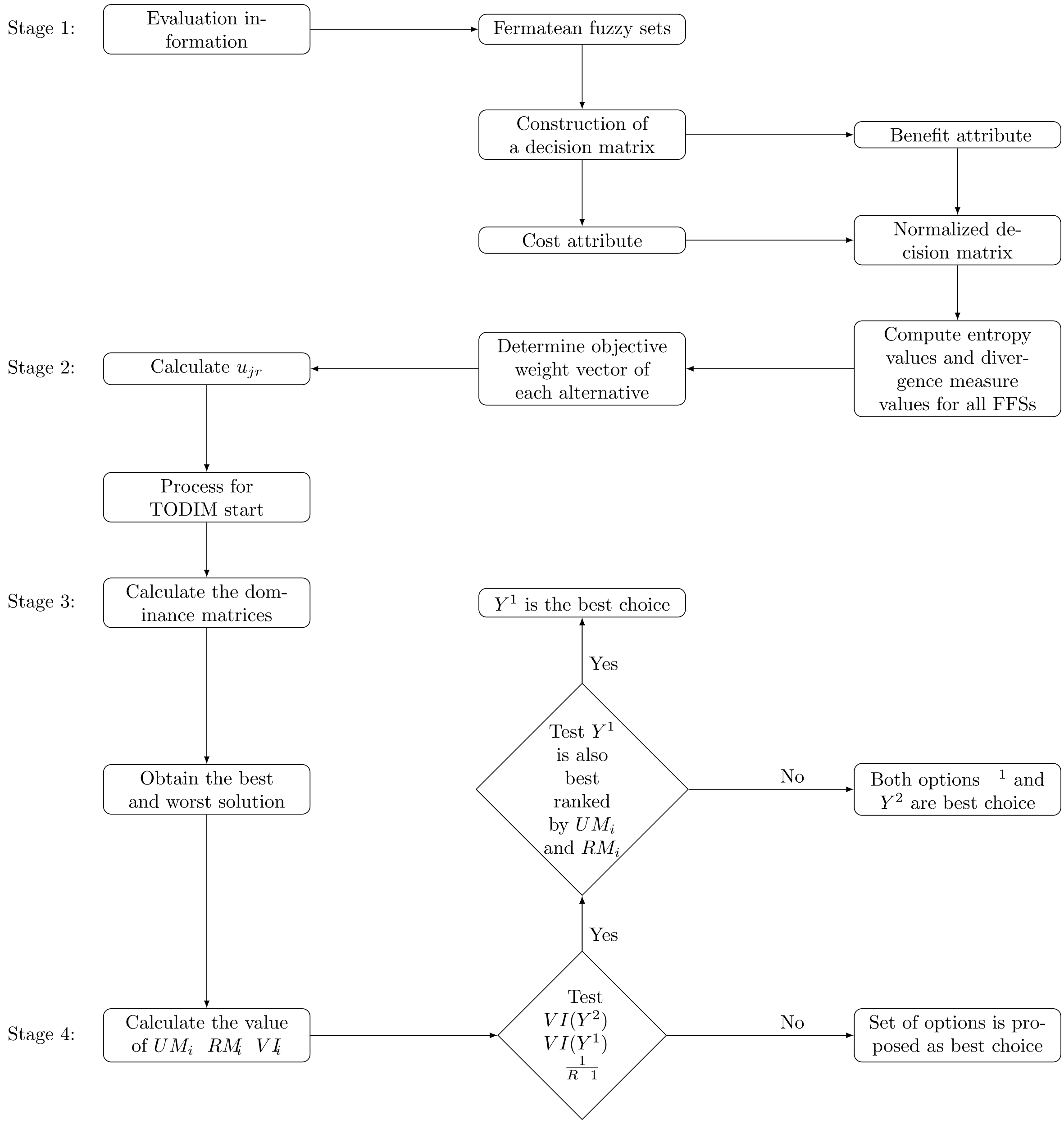

Fig. 1. TODIM–VIKOR technique's schematic framework

Stage 2: Obtain the attribute's goal weights

Step 2: The following method, as a result of proposed divergence measure (5) and entropy measure (1), can be used to compute the attribute's objective weights:

$$u_j^{obj} = \frac{\sum_{i=1}^{p} \left[\frac{1}{p-1} \sum_{t=1}^{p} \Omega_{ff}(o_{ij}, o_{tj}) + M_{fd}(o_{ij}) \right]}{\sum_{j=1}^{n} \sum_{i=1}^{p} \left[\frac{1}{p-1} \sum_{t=1}^{p} \Omega_{ff}(o_{ij}, o_{tj}) + M_{fd}(o_{ij}) \right]}.$$

The relative weights for each attribute are then calculated as follows:

$$u_{jr} = \frac{u_j^{obj}}{u_r}, \quad j, r = 1, 2, \ldots, n, \tag{7}$$

here u_j^{obj} describes the weight of the attribute, u_r is the maximum of $\{u_1, u_2, \ldots, u_n\}$ such that $0 \leqslant u_{jr} \leqslant 1$.

Stage 3: Using TODIM, create a dominance matrix

Step 3: The computation of dominance degree matrix (DDM) for alternatives Y_j concerning each attribute θ_j is determined by the following expression using relative weights acquired from (7):

$$C_j(Y_i, Y_t) = \begin{cases} \sqrt{\frac{u_{jr}\Omega_{ff}(o_{ij}, o_{tj})}{\sum_{j=1}^{n} u_{jr}}}, & \text{if} \quad o_{ij} - o_{tj} > 0; \\ 0, & \text{if} \quad o_{ij} - o_{tj} = 0; \\ -\frac{1}{\gamma}\sqrt{\frac{\left(\sum_{j=1}^{n} u_{jr}\right)\Omega_{ff}(o_{ij}, o_{tj})}{u_{jr}}} & \text{if} \quad o_{ij} - o_{tj} < 0, \end{cases} \tag{8}$$

where $\Omega_{ff}(o_{ij}, o_{tj})$ is the divergence between the two FFNs. o_{ij}, o_{tj} and the parameter $\gamma > 0$ define as the attenuation term of the losses.

Step 4: Calculate the DDM for every alternative Y_i, in relation to each attribute θ_j is shown below:

$$C_j = [C_j(Y_i, Y_t)]_{p \times p} = \begin{array}{c} \\ Y_1 \\ Y_2 \\ \vdots \\ Y_p \end{array} \begin{array}{c} \begin{array}{cccc} Y_1 & Y_2 & \dots & Y_p \end{array} \\ \begin{bmatrix} 0 & C_j(Y_1, Y_2) & \cdots & C_j(Y_1, Y_p) \\ C_j(Y_2, Y_1) & 0 & \cdots & C_j(Y_2, Y_p) \\ \vdots & \vdots & \ddots & \vdots \\ C_j(Y_p, Y_1) & C_j(Y_p, Y_2) & \cdots & 0 \end{bmatrix} \end{array}.$$

Compute the final DDM of alternatives Y_i under attributes θ_j in relation to another alternatives Y_t $(t = 1, 2, \dots, p)$ as given below:

$$\Delta_j(Y_j, Y_t) = \sum_{t=1}^{p} C_j(Y_i, Y_t).$$

Stage 4: For all alternatives acquire the ranking orders with VIKOR

Step 5: To build up the best L^+ and poorest L^- solution for each attribute the continuing to follow:

$$L^+ = (L_1^+, L_2^+, \dots, L_n^+) = \left(\max \sum_{t=1}^{p} C_1(Y_i, Y_t), \max \sum_{t=1}^{p} C_2(Y_i, Y_t), \dots, \max \sum_{t=1}^{p} C_n(Y_i, Y_t)\right)$$

and

$$L^- = (L_1^-, L_2^-, \dots, L_n^-) = \left(\min \sum_{t=1}^{p} C_1(Y_i, Y_t), \min \sum_{t=1}^{p} C_2(Y_i, Y_t), \dots, \min \sum_{t=1}^{p} C_n(Y_i, Y_t)\right).$$

Step 6: Calculate utility measure (UM_i) and regret measure (RM_i) for each alternatives as:

$$UM_i = \sum_{1 \leqslant j \leqslant n} u_j^{obj} \times \frac{d(L_j^+, t_{ij})}{d(L_j^+, L_j^-)},$$

$$RM_i = \max_j \left(u_j^{obj} \times \frac{d(L_j^+, t_{ij})}{d(L_j^+, L_j^-)}\right),$$

where

$$d(L_j^+, t_{ij}) = \max_{1 \leqslant i \leqslant n} \sum_{t=1}^{m} C_j(Y_i, Y_t) - \sum_{t=1}^{p} C_j(Y_i, Y_t),$$

$$d(L_j^+, t_{ij}) = \max_{1 \leqslant i \leqslant n} \sum_{t=1}^{p} C_j(Y_i, Y_t) - \min_{1 \leqslant i \leqslant n} \sum_{t=1}^{p} C_j(Y_i, Y_t).$$

Step 7: Calculate $\bar{UM}$, UM^-, $\bar{RM}$, RM^- values and VIKOR index VI_i based on Equations (9) and (10), respectively,

$$\bar{UM} = \max(UM_i), \quad UM^- = \min(UM_i), \quad \bar{RM} = \max(RM_i), \quad RM^- = \min(RM_i), \tag{9}$$

$$VI_i = e\frac{UM_i - UM^-}{\bar{UM} - UM^-} + (1-e)\frac{RM_i - RM^-}{\bar{RM} - RM^-}. \tag{10}$$

Coefficients e, $1-e$ are proposed as a weights for "maximum group utility" UM_i and "individual regret" (RM_i), respectively.

Step 8: With the minimum of (UM_i), (RM_i) and (VI_i) determine best alternative in decreasing order.

Step 9: for offering the optimal compromise solution, taking the following two conditions into consideration:

C1: If $VI(Y^{(2)}) - VI(Y^{(1)}) \geqslant \frac{1}{R-1}$, where $Y^{(1)}$ and $Y^{(2)}$ will be taken at primary and secondary places in the rank list of VI_i and R stands for the number of alternatives.

C2: The alternative $Y^{(1)}$ must be rated first in the list of UM_i or/and RM_i to be ideal. If from C1 and C2 no condition is satisfied, then solutions set is proposed as below:

(a) **Adequate advantage** $Y^{(1)}$ and $Y^{(2)}$ are compromise Alternatives when only C2 does not hold.

(b) **Adequate stability** When C1 fails to hold, we attempt to discover the greatest value P by the following inequality:

$$VI(Y^{(M)}) - VI(Y^{(1)}) < \frac{1}{R-1}. \tag{11}$$

It also establishes that the compromised solution is the collection of alternatives $Y^{(i)}$ $(i = 1(1)P)$.

6. An illustration

By planning, regulating, and coordinating capital flow and logistical information across manufacturers, distributors, suppliers, and retailers. Supply chain management allows both internal and external resources to be integrated (SCM). Supplier-traditional enterprise relationships are evolving beyond conventional commerce relationships; suppliers are increasingly becoming tactical partner companies. Because Supplier selection is a significant problem in SCM since it creates a win scenario. The basic notion of SCM has been applied to the problem of medical supply supplier selection [7, 14, 34, 51–53]. The difficulty of selecting a supplier to supply the medical products is a classic MAGDM problem. As a result, this part uses FFS data to illustrate the technique presented in this study by providing a numerical example of a supplier problem in medical supply items. A collection of five possible medical consumption product vendors is shown as Y_1, Y_2, Y_3, Y_4, Y_5. For evaluating the five potential suppliers of medicinal consumption items, the experts chose five attributes given by (θ_1) quality of environmental improvement, (θ_2) cost of transportation suppliers, (θ_3) green image, (θ_4) environmental competencies, (θ_5) financial conditions. Only transportation costs are included in cost type attribute, whereas the rest four are benefit attributes.

Step 1: The initial FFS matrix shown in Table 2 displays the normalised matrix obtained from the original choice matrix, as the above described application has one cost attribute (θ_2), Table 3 shows the normalised matrix produced from the original decision matrix.

Table 2. Original decision matrix.

Decision value	θ_1	θ_2	θ_3	θ_4	θ_5
Y_1	(0.8,0.1)	(0.4,0.6)	(0.4,0.7)	(0.3,0.4)	(0.6,0.5)
Y_2	(0.6,0.7)	(0.5,0.3)	(0.5,0.2,)	(0.9,0.4)	(0.7,0.4)
Y_3	(0.7,0.5)	(0.2,0.4)	(0.4,0.8)	(0.7,0.7)	(0.4,0.8)
Y_4	(0.6,0.5)	(0.2,0.7)	(0.8,0.6)	(0.2,0.6)	(0.4,0.1)
Y_5	(0.4,0.3)	(0.6,0.5)	(0.4,0.3)	(0.2,0.9)	(0.3,0.5)

Step 2: The divergence, values of entropy and weights vector are shown for each property in Table 4.

Table 3. Normalized decision matrix.

Decision value	θ_1	θ_2	θ_3	θ_4	θ_5
Y_1	(0.8,0.1)	(0.6,0.4)	(0.4,0.7)	(0.3,0.4)	(0.6,0.5)
Y_2	(0.6,0.7)	(0.3,0.5)	(0.5,0.2,)	(0.9,0.4)	(0.7,0.4)
Y_3	(0.7,0.5)	(0.4,0.2)	(0.4,0.8)	(0.7,0.7)	(0.4,0.8)
Y_4	(0.6,0.5)	(0.7,0.2)	(0.8,0.6)	(0.2,0.6)	(0.4,0.1)
Y_5	(0.4,0.3)	(0.5,0.6)	(0.4,0.3)	(0.2,0.9)	(0.3,0.5)

Table 4. Values are weighted dependent on entropy measure and divergence measure.

Alternatives	values of Divergence	Entropy values	Objective weights
Y_1	0.7542	3.9734	0.2398
Y_2	0.2538	2.9433	0.1733
Y_3	0.6727	3.2832	0.1989
Y_4	1.4902	3.1142	0.2009
Y_5	0.4246	3.1401	0.1871

Step 3: Estimated dominance degree matrices for alternatives Y_i using Eq. (8) are follows from Table 5 to Table 10.

Table 5. Dominance degree under attribute θ_1.

Attribute (δ_1)	Y_2	Y_2	Y_3	Y_4	Y_5
Y_1	0.0000	0.2536	0.1473	0.1848	0.4252
Y_2	-0.4229	0.0000	-0.2086	-0.2127	-0.4316
Y_3	-0.2456	0.1249	0.0000	0.0686	0.2032
Y_4	-0.3082	0.1275	-0.11385	0.0000	0.1536
Y_5	-0.4110	0.2499	-0.3390	-0.2438	0.0000

Table 6. Dominance degree under attribute θ_2.

Attribute (θ_2)	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1	0.0000	0.1249	0.1134	-0.0926	0.0959
Y_2	-0.2863	0.0000	-0.2451	-0.4443	-0.2216
Y_3	-0.2617	0.1062	0.0000	-0.3303	0.1557
Y_4	0.08	0.1925	0.1446	0.0000	0.1655
Y_5	-0.2193	-0.096	-0.3594	-0.3819	0.0000

Table 7. Dominance degree under attribute θ_3.

Attribute (θ_3)	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1	0.0000	-0.4201	0.0747	-0.4478	-0.3762
Y_2	0.242	0.0000	0.2713	-0.5481	0.0538
Y_3	-0.1504	-0.545798	0.0000	-0.4503	-0.511
Y_4	0.2227	0.2726	0.2238	0.0000	0.2897
Y_5	0.1870	-0.1080	0.2542	-0.5824	0.0000

Table 8. Dominance degree under attribute θ_4.

Attribute (θ_4)	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1	0.0000	-0.6842	-0.5491	0.0995	0.3087
Y_2	0.3437	0.0000	-0.3684	0.3541	0.3818
Y_3	0.2759	-0.3684	0.0000	0.2423	0.2256
Y_4	-0.19842	-0.7030	-0.4822	0.0000	0.2261
Y_5	-0.6146	-0.7602	-0.4493	-0.4500	0.0000

Table 9. Dominance degree under attribute θ_5.

Attribute (θ_5)	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1	0.0000	−0.1429	0.1845	0.1594	0.1277
Y_2	0.0671	0.0000	0.2326	0.1760	0.1811
Y_3	−0.3901	−0.4971	0.0000	−0.5898	−0.4004
Y_4	−0.3407	−0.3729	0.2759	0.0000	0.1249
Y_5	−0.2727	−0.3869	0.1873	−0.2676	0.0000

Table 10. Total dominance degree.

	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1	1.0109	0.2416	−1.1694	−0.8251	0.3267
Y_2	−1.2758	1.1993	0.017	0.7112	0.6574
Y_3	0.1511	−0.3301	−1.6651	0.37542	−1.8694
Y_4	−0.1409	0.5826	1.0088	−1.1595	−0.3128
Y_5	−0.7439	−1.0566	−0.2492	−2.2741	−0.7483

Step 4: The best solution L^+ and the poorest L^- are displayed in Table 11 and score function is calculated in Table 12.

Table 11. Best and worst solutions.

Alternatives	Y_1	Y_2	Y_3	Y_4	Y_5
L^+	1.0109	1.1993	1.0088	0.7112	0.6574
L^-	−1.2758	−1.0566	−1.66518	−2.2741	−1.8694

Table 12. Scores of $\mathbf{u}_j^{inte} \times \frac{d(L_j^+,t_{ij})}{d(L_j^+,L_j^-)}$.

Alternatives	θ_1	θ_2	θ_3	θ_4	θ_5
Y_1	0.0000	0.0736	0.1620	0.1034	0.0245
Y_2	0.2398	0.0000	0.0738	0.0000	0.0000
Y_3	0.0901	0.1175	0.1989	0.0226	0.1871
Y_4	0.1012	0.04743	0.0000	0.1259	0.0718
Y_5	0.1840	0.1733	0.0936	0.2009	0.1041

Step 5 and 6: The UM_i and RM_i are identified in a decreasing order. Table 13 and Table 14 explain results.

Table 13. Calculated values of UM_i, RM_i, VI_i.

Alternatives	UM_i	RM_i	VI_i ($e = 0.5$)
Y_1	0.3635	0.1620	0.2149
Y_2	0.3136	0.2398	0.5
Y_3	0.6162	0.1989	0.6626
Y_4	0.3463	0.1259	0.0370
Y_5	0.7559	0.2009	0.8292

Table 14. By UM_i, RM_i, and VI_i, Y_i (alternatives) are ranked.

Alternatives	By UM_i	By RM_i	By VI_i ($e = 0.5$)
Y_1	3	2	2
Y_2	1	5	3
Y_3	4	3	4
Y_4	2	1	1
Y_5	5	4	5
Pref. Sequence	$Y_2 \succ Y_4 \succ Y_1 \succ Y_3 \succ Y_5$	$Y_4 \succ Y_1 \succ Y_3 \succ Y_5 \succ Y_2$	$Y_4 \succ Y_1 \succ Y_2 \succ Y_3 \succ Y_5$

Step 7: We may find the values of VI based on UM_i, RM_i and VM_i in Table 15 and determine the ranking of alternatives and the results are presented numerically in Table 16 and graphically in Figure 3.

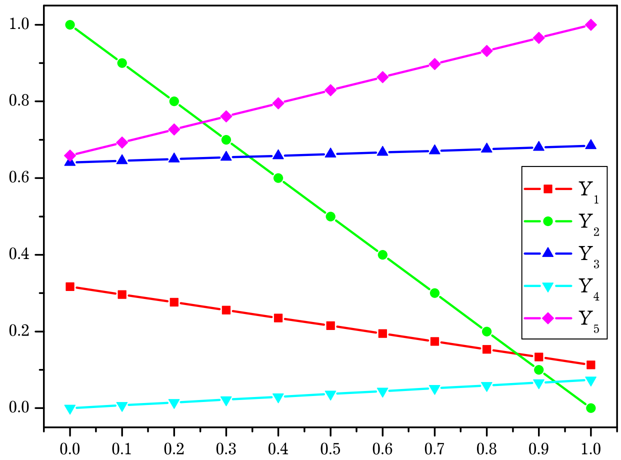

Fig. 2. Different values of e are represented by a line graph.

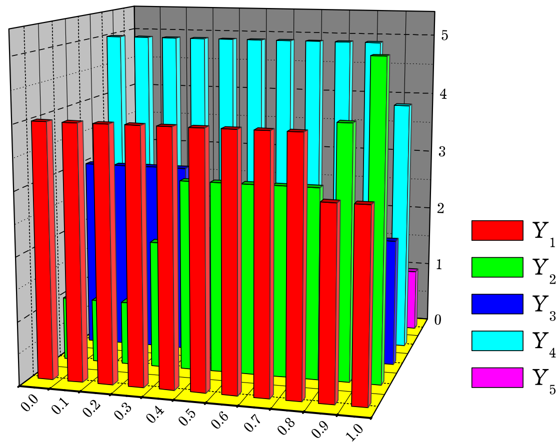

Fig. 3. Ranking of V_i for various weights.

Table 15. The $\mathbf{UM_i}$, $\mathbf{RM_i}$, $\mathbf{VM_i}$ weight e modifications yielded results, as well as a compromise solution.

	Y_1	Y_2	Y_3	Y_4	Y_5
0	0.3169	1.0000	0.6409	0.0000	0.6585
0.1	0.2965	0.9	0.6452	0.0074	0.6926
0.20	0.2761	0.8	0.6496	0.0148	0.7268
0.30	0.2557	0.7	0.65391	0.0222	0.7609
0.40	0.2353	0.6	0.6582	0.0296	0.7951
0.50	0.2149	0.5	0.6625	0.0370	0.8292
0.60	0.1945	0.4	0.6669	0.0444	0.8634
0.70	0.1741	0.3	0.6712	0.0518	0.8975
0.80	0.1536	0.2	0.6755	0.0591	0.9317
0.90	0.1332	0.1	0.6798	0.0665	0.9658
1.00	0.1128	0.0000	0.6842	0.0739	1

Table 16. The ranking order of V_i for weight $0 \leqslant e \leqslant 1$.

	Y_1	Y_2	Y_3	Y_4	Y_5
0	4	1	3	5	2
0.1	4	1	3	5	2
0.20	4	1	3	5	2
0.30	;4	2	3	5	1
0.40	4	3	2	5	1
0.50	4	3	2	5	1
0.60	4	3	2	5	1
0.70	4	3	2	5	1
0.80	4	3	2	5	1
0.90	3	4	2	5	1
1.00	3	5	2	4	1

In Table 16, the best alternative is Y_2 for $0 \leqslant e \leqslant 0.2$ and Y_5 for $0.3 \leqslant e \leqslant 1$.

Step 8 and 9: As well from the Table 16, the alternative Y_4 is readily apparent has been placed first and Y_1 has been placed second in the VI_i ranking. We also have $VI(Y^{(1)}) - VI(Y^{(4)}) = 0.2149 - 0.0370 = 0.1779 < 0.25$ and as a result, the condition $C1$ is not checked. Then there is a list of choices $\{Y_1, Y_2, \ldots, Y_R\}$ is consider a compromise solution and Y_R is depicted by Equation (11). But

$$VI(Y^{(2)}) - VI(Y^{(1)}) = 0.5 - 0.2149 = 0.2851 > \frac{1}{5-1} = 0.25,$$

where $Y^{(2)}$ has been placed third in the order of VI. As just a result, the combination $(Y^{(1)}, Y^{(4)})$ might be thought of as a compromise solution.

Comparative study

We conduct a comparison study by handling the decision problem using the three FFS MADM approaches that are currently available [54–56]. The method [54] is totally reliant on Hamacher Interactive Geometric Operators and [56] approach is entirely dependent on the aggregate operator. To obtain the best ranking order, [55] use the TOPSIS approach in an FFS environment. The proposed FF–TODIM–VIKOR approach is an outranking method that disregards whether the qualities are interconnected or independent. The intensity of liking for the four approaches is, however, determined by the distance between FFNs. When comparing two FFNs, the divergence is preferable to the distance between them. To characterise the following comparative analysis, the quality weights are based on the above-mentioned computation result. The order of ranking of the alternatives can then be obtained by using these four distinct operators for FFS, as shown in Table 17.

Table 17. Results of other methods of ranking are compared in this study.

Researchers	Method approach	Best value	Optimize ranking
[54]	Hamacher Interactive Geometric Operators	Y_4	$Y_4 \succ Y_2 \succ Y_1 = Y_3 \succ Y_5$
[55]	Aggregation operator	$Y4$	$Y_4 \succ Y_1 \succ Y_2 \succ Y_5 \succ Y_3$
[56]	TOPSIS	Y_4	$Y_4 \succ Y_2 \succ Y_3 \succ Y_1 \succ Y_4$
Proposed method	TODIM VIKOR	Y_4	$Y_4 \succ Y_1 \succ Y_2 \succ Y_3 \succ Y_5$

In Table 17, we verify that results of ranking generated using four approaches that existing are differ minutely from those obtained using the suggested method. These five methods will assist you in determining the best alternative, Y_4.

7. Conclusion

This paper has conducted extensive research on the mechanisms of entropy and FFS differentiation. A new degree of entropy is proposed and by extending the idea of Shannon entropy from the theory of chance to the ambiguous set of Fermatean, a few favourable areas are also being investigated. Also defined and confirmed is a novel measure of variation between FFSs. In addition, the proposed divergence measure's requirements are discussed. The unique model to discover the vector weight of characteristics was then detailed utilising the supplied entropy and severity values. Following that, the classic VIKOR based on Fermatean fuzzy entropy and separation measures was combined with the ancient TODIM approach. With significant weight information, we have presented a new technique for dealing with MAGDM difficulties. Finally, we used the supply chain management challenge to evaluate the current decision-making process's feasibility and efficacy. Sensitivity analysis tests were carried out to determine the extent of influence of the parameters on the final outcomes. The advantages of the suggested integrated method are also weighed against a variety of alternatives. Furthermore, the TODIM–VIKOR integrated strategy is straightforward, versatile, and simple to implement. The newly created ambiguous MAGDM approach solves uncertainty decision-making challenges coming from community risk, provider selection, medical diagnosis, cluster analysis, and other factors. As a result, the proposed technique in this study, which is very near to actual decision-making, can represent a wide range of uncertainties and imprecise information. This aids us in dealing with MAGDM issues and correctly drawing realities. The final results indicate the proposed technique is more dependable and resilient than the alternatives.

In the future, we will try to analyse alternative FFS structures utilising information levels and dimension similarities and differences in blurred photo sets and use them in various settings, including the financial market, pattern recognition, emergency management, and decision-making under multiple circumstances.

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Фундаментальною метою цього дослідження є розробка MAGDM (багатоатрибутне групове прийняття рішень) проблеми продуктів медичного споживання. У цій статті пропонується підхід TODIM–VIKOR, який поєднує в собі процедури TODIM (інтерактивне та багатокритеріальне прийняття рішень) і VIKOR (оптимізація за всіма критеріями та компромісне рішення) у межах ферматівської нечіткої інформації. Для роботи зі задачами порівняння подано нову ферматівську функцію нечіткого скорингу. Крім того, введено нову міру ентропії для оцінки ступеня нечіткості, яка пов'язана з ферматівською нечіткою множиною (ФНМ). Також запропоновано міру розбіжності Дженсена–Шеннона для ферматівської нечіткої множини, яку можна використовувати для порівняння інформації про відмінності двох ФНМ. Ця запропонована міра відповідає всім математичним вимогам, щоб вважатися мірою. Введено міри ентропії та розбіжності для визначення об'єктивної ваги в підході TODIM–VIKOR. Тим часом, щоб мати справу з прийняттям рішень щодо кількох груп атрибутів, була запропонована нова процедура прийняття рішень на основі запропонованої ентропії та міри розбіжності Дженсена–Шеннона в ферматівському нечіткому середовищі. У цій статті TODIM має на меті з'ясувати загальний ступінь домінування, а VIKOR — визначити компромісне рішення. Врешті, вирішено задачу вибору постачальника, щоб перевірити ефективність запропонованого ферматівського нечіткого методу TODIM–VIKOR шляхом порівняння рішення ранжування з ранжуванням існуючих методологій.

Ключові слова: *ентропія, міра розбіжності, ферматівська нечітка множина, MAGDM.*