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# ABOUT METRIC AND ANGULAR DEPENDENCIES OF SPATIAL STRAIGHT-LINE NOTCHES AND THEIR USE IN ENGINEERING AND GEODETIC WORKS

In applied geodesy tasks, it may be necessary to determine spatial angles. When bringing a 3D design of buildings and structures to the field with the help of an electronic total station (ES), it is important to verify the spatial angles between different elements of building structures such as roof overlaps, inclined anchors, and more, using the characteristic points' spatial coordinates. Modern geodetic instruments provide sufficiently high measurement accuracy (up to 1" and 1 mm, respectively). However, measuring the required angles with surveying instruments is not always possible for various reasons. First of all, it is impossible to place the device at the vertex of an angle if the location is not accessible. This paper develops a method for determining a spatial angle whose vertex is not available for measurement. Methods and results. To achieve this goal, we consider one of the options for its determination through the application of the cosine theorem with preliminary measurement or calculation of adjacent sides and vertical angles. This article also presents an algorithm for solving the problem with an estimation of the accuracy of establishing the required parameters. The basic formulas for determining the angles of a spatial triangle with an estimate of their accuracy are proposed. The paper studies the influence of the linear measurement values of the lengths of the sides on the values of the angles of a spatial triangle with the corresponding accuracy assessment. In particular, the root mean square errors of angle calculation were determined based on these calculations and mathematical modeling, namely, the ratio of the sides of the triangle. Through indirect measurements of the tower crane boom and roof spire, the spatial angle values were determined. The inclination of the crane boom to the base resulted in α=910.712±51", while the angle of the roof spire was α=150.109±35". Scientific novelty and practical significance. On the basis of the proposed methodology and numerical experiments, spatial angles were determined and their a priori accuracy was analyzed. This confirms the influence of linear measurements of side lengths on the values of spatial angles. The obtained results make it possible to apply the proposed method in engineering and geodetic works using BIM technologies in 3D space. This method can be used in the application software of electronic total station manufacturers to determine spatial angles in space when solving engineering problems.

*Key words:* angular measurements; spatial angle; error; electronic total station; accuracy.

## Introduction

The main tasks in engineering and geodetic works include determining the geometric parameters of building structures while performing technical measurements. This is a linear or angular value of a construction object to geodetic control. This control includes verification of straightness, alignment, horizontality, elevation, inclination, verticality, parallelism, flatness, perpendicularity, and curvature, which is traditionally carried out in 2D space. The following devices are used: indicators, autocollimators, theodolites, tape measures, levels, optical quadrants, intrometers, cameras, and ET [Baran, 2012; Vivat et al., 2018]. Recently, laser scanners and robotic electronic total stations have been increasingly used for modeling in 3D space [Trevogo & Balandyuk, 2009; Trevogo, 2016; Vivat & Nazarchuk, 2019]. The use of ET is effectively applied in determining the geometric parameters of engineering structures [Baran, 2012; Vivat et al., 2018], defining the straightness of rotation of large-sized continuous-acting units [Moroz et al., 2011], monitoring hydraulic structures [Staroverov, 2020; Baran, 2011], as well as in examining linear objects in the areas affected by underground mining using ET and ground-based laser scanners [Naminat, 2020]. Methods and required accuracy are regulated by DSTU and DBN. Regulatory documents determine the work accuracy, which ranges from tenths of a millimeter to one centimeter. This depends on the control elements of geometric parameters or the establishment of deformations of structures in general [DSTU-N BV.1.3-1:2009, DBN B.1.3-2:2010, ISO 17123-1, ISO 17123-5].

Calculating the angle between two directions at an observation point is crucial for technical measurements and geodetic monitoring tasks. It involves determining the horizontal and vertical displacement values, angular elements of the trajectory, and the directional angle of the working sign's movement [Goryainov, 2018; Gargula, 2009]. In the practice of as-built surveying of constructed building elements, it may also be necessary to determine the spatial coordinates of characteristic points to estimate their geometric dimensions (in particular, spatial angles). Such tasks are best solved with the use of scanners and 3D modeling. When transferring a 3D design of buildings and structures to the field using the spatial coordinates of their characteristic points using (ET), it may also be necessary to check the spatial angles between various elements of building structures, e. g., structures that form roof overlaps, inclined anchors, etc. Almost all tasks of technical measurements and engineering and geodetic monitoring can be performed by ET [Baran, 2011; Staroverov, 2020]. Global manufacturers of geodetic equipment are developing application software for ET that solves problems of coordinate geometry and intermediate measurements [Trevogo, 2016]. It should be noted that measurements of angles and lines using ET can be performed with accuracy (up to 1" and 1 mm, respectively), i. e. these measurements can be used to calculate the dependent quantities with high accuracy. The determination of an arbitrary angle in space is one of the most pressing practical problems that need to be solved in connection with the introduction of BIM technologies. Obviously, it is impossible to measure such an angle directly using (ET). The only exception may be the case when its vertex is located in the horizontal plane and it is necessary to measure the horizontal direction or vertical angle to the point of aiming the optical beam.

The theory of spherical functions describes a method for calculating such an angle. For example, the theorem of the sum of spherical functions uses the concept of a spherical angle, which inherently defines a spatial angle. However, its use in engineering and geodetic works has remained unnoticed by surveyors.

## Purpose

To propose a method for calculating a spatial angle whose vertex is inaccessible and, as a result, its determination is complicated due to the impossibility of any direct measurements.

## Methods

To accomplish this goal, let us consider one of the options for determining it by applying the cosine theorem with preliminary measurement or calculation of the sides. The algorithm for solving this problem with an estimate of the accuracy of determining the required parameters will also be presented in this article. Thus, the distances between two inaccessible points in space can be found only by the median measurements of additional elements with their subsequent use in the corresponding mathematical formulas. Fig. 1 shows a geometric illustration of the solution to the problem.

Let the intersection of the vertical and horizontal axes be at point **M**, and at points **A** and **C** visionary objectives are set. It is necessary to determine the angle $x$ between these two points. The figure also shows the horizontal angle – $\gamma$ as a reflection of the angle projection $\boldsymbol{x}$ to the horizontal plane, as well as two vertical corners – $\mathbf{V_1}$i $\mathbf{V_2}$, and measured distances – $\mathbf{S_1}$, $\mathbf{S_2}$, and two horizontal directions – $\boldsymbol{\beta_1}$, $\boldsymbol{\beta_2}$ (ET).

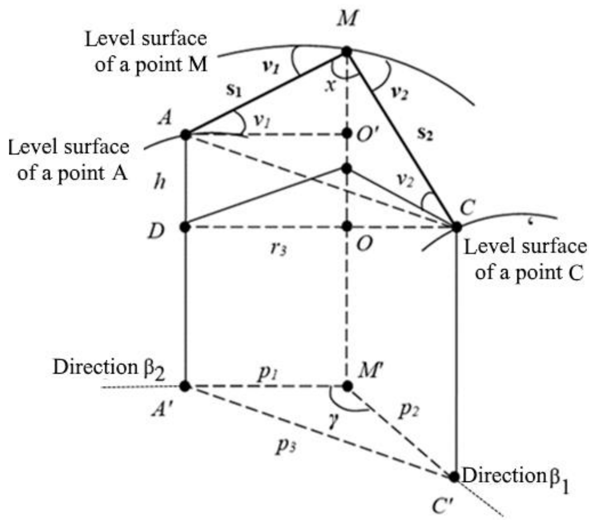


*Fig. 1. Characteristics of the spatial position of a point*

Using the notation in Fig. 1, we sequentially define the following linear elements:

$$p_1 = S_1 \cos v_1, p_2 = S_2 \cos v_2; \quad (1)$$

$$h = MO - MO', MO' = S_1 \sin v_1, \quad (2)$$
$$MO = S_2 \sin v_2, h = S_2 \sin v - S_1 \sin v_1.$$

From the triangle A'M'C' we get:

$$p_3^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\gamma. \quad (3)$$

Substituting the values of the projections from (1), we have:

$$p_3^2 = S_1^2\left(\cos v_1\right)^2 + S_2^2\left(\cos v_2\right)^2 - \quad (4)$$
$$-2S_1S_2\cos v_1\cos v_2\cos\gamma,$$

and from the triangle ACD

$$\left(AC\right)^2 = h^2 + r_3^2. \quad (5)$$

Expression (5), considering (2) and (4), will be as follows:

$$(AC)^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\gamma + \quad (6)$$
$$+(S_2\sin v_2 - S_1\sin v_1)^2,$$

or in another form

$$S_1^2 + S_2^2 - 2S_1S_2(\cos v_1\cos v_2\cos\gamma + \sin v_1\sin v_2). \quad (7)$$

Because

$$(AC)^2 = S_1^2 + S_2^2 - 2S_1S_2\cos x, \quad (8)$$

then

$$S_1^2 + S_2^2 - 2S_1S_2(\cos v_1\cos v_2\cos\gamma + \quad (9)$$
$$+\sin v_1\sin v_2) = S_1^2 + S_2^2 - 2S_1S_2\cos x.$$

From expression (9) we obtain:

$$\cos x = \sin v_1\sin v_2 + \cos v_1\cos v_2\cos\gamma \quad (10)$$

and, accordingly, the formula for calculating the value of the desired angle:

$$x = \arccos(\sin v_1\sin v_2 + \cos v_1\cos v_2\cos\gamma). \quad (11)$$

Note that formula (10) can also be obtained using the methods of vector algebra, but we will not focus on this.

However, the question arises as to how accurately the angle is determined by the formula (11). To do this, we define and a priori estimate the accuracy. Simple differentiation with corner replacement $v_1$ with $\alpha$, and $v_2$ with $\beta$ for traditional perception gives the following:

$$m_x^2 = \frac{\left((\cos\alpha\sin\beta - \sin\alpha\cos\beta\cos\gamma)^2 m_\alpha^2 + (\sin\alpha co\beta - \alpha\cos\alpha\sin\beta\cos\gamma)^2 m_\beta^2 + (\cos\alpha\cos\beta\sin\gamma)^2 m_\gamma^2\right)}{1 - \left((\sin\alpha\sin\beta + \cos\alpha\cos\beta\cos\gamma)\right)^2}. \quad (12)$$

It is also interesting to assess the range of change in this error. It should be noted that at $\cos x \to 0$ the error value increases infinitely, i. e., a singularity occurs. Let us find out in which cases this is true. So, if we consider the expression:

$$\sin\alpha\sin\beta + \cos\alpha\cos\beta\cos\gamma = 1, \quad (13)$$

then by performing the transformation:

$$\sin\alpha\sin\beta + \cos\alpha\cos\beta\cos\gamma = \sin^2\alpha + \cos^2\alpha, \quad (14)$$
$$\left(tg\alpha\sin\beta + \cos\beta\cos\gamma\right)^2 = tg^2\alpha + 1,$$

we finally get a quadratic equation and its solution:

$$tg^2\alpha\left(\sin^2\beta - 1\right) + 2tg\alpha\sin\beta\cos\beta\cos\gamma +$$
$$+\cos^2\beta\cos^2\gamma - 1 = 0,$$
$$tg^2\alpha\left(\cos^2\beta\right) - 2tg\alpha\sin\beta\cos\beta\cos\gamma - \quad (15)$$
$$-\left(\cos^2\beta\cos^2\gamma - 1\right) = 0,$$

$$\left(tg\alpha\right)_{1,2} = \frac{\sin\beta\cos\beta\cos\gamma \pm \sqrt{\left(\sin\beta\cos\beta\cos\gamma\right)^2 + c\mathrm{os}^2\beta\left(\cos^2\beta\cos^2\gamma - 1\right)}}{\cos^2\beta}, \quad (16)$$

$$\left(tg\alpha\right)_{1,2} = \frac{\sin\beta\cos\beta\cos\gamma \pm \cos\beta\sqrt{\left(\sin\beta\cos\gamma\right)^2 + \left(\cos^2\beta\cos^2\gamma - 1\right)}}{\cos^2\beta},$$

$$\left(tg\alpha\right)_{1,2} = \frac{\sin\beta\cos\gamma \pm \sqrt{\cos^2\gamma - 1}}{\cos\beta}.$$

So, a solution can exist when $\cos\gamma = \pm 1$, $,\gamma = 0^0, 180^0$.

Then $tg\alpha = tg\beta, \alpha = \beta + k\pi$. Note that there is inequality if $\cos\gamma = \pm 1$, which is a consequence of the shape of the parabola with a negative determinant. The next step is to obtain the conditions for optimal values. To do this, we first confirm the conclusion about the unlimited growth of the angle calculation error for points in the same vertical plane. The results of the calculations are presented in Table 1 for one quarter of the circle, that is, in the range of angle changes from 0° to 90°.

As can be seen from Table 1, when the points coincide (with the same horizontal directions $\beta_1$ and $\beta_2$), the error in calculating angles increases significantly. Therefore, such cases should be excluded from consideration. But in these situations, there is no need to apply formula (11).

However, the inaccuracy of the calculations can occur for points lying in close vertical planes. Let us also study the variations in angle calculation errors for different values of horizontal angles $\gamma$ and present the results of the calculations in Table 2.

*Table 1*

**MSE for calculating the angle between two directions in space for different point positions**

| $\beta_2$, ° \ $\beta_1$,° | 0 | 20 | 40 | 60 | 80 |
|---|---|---|---|---|---|
| 0 | 0.0 | 1.0 | 1.2 | 1.3 | 1.4 |
| 20 | 1.4 | 61,876.3 | 1.1 | 1.3 | 1.4 |
| 40 | 1.4 | 1.7 | 33,915.9 | 1.6 | 1.7 |
| 60 | 1.4 | 1.5 | 1.2 | 135,695.7 | 2.6 |
| 80 | 1.4 | 1.4 | 1.1 | 1.2 | 418,820.2 |

*Table 2*

**MSE calculating the angle between two directions in space for different positions of points and angle γ in different quarters**

| | $\gamma = 30°$ | | | | | $\gamma = 60°$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_2$, ° \ $\beta_1$,° | 0 | 20 | 40 | 60 | 80 | 0 | 20 | 40 | 60 | 80 |
| 0 | 1.0 | 1.0 | 1.1 | 1.2 | 1.3 | 1.0 | 1.0 | 1.0 | 1.1 | 1.1 |
| 20 | 1.1 | 1.1 | 1.0 | 1.2 | 1.3 | 1.0 | 1.0 | 1.0 | 1.0 | 1.1 |
| 40 | 1.2 | 1.2 | 0.8 | 1.2 | 1.5 | 1.1 | 1.0 | 0.9 | 0.9 | 1.1 |
| 60 | 1.3 | 1.3 | 1.0 | 1.0 | 2.1 | 1.1 | 1.1 | 0.9 | 0.6 | 1.2 |
| 80 | 1.3 | 1.3 | 1.0 | 0.9 | 7.5 | 1.1 | 1.1 | 0.9 | 0.4 | 1.9 |
| | $\gamma = 120°$ | | | | | $\gamma = 150°$ | | | | |
| 0 | 1.0 | 1.1 | 1.0 | 1.1 | 1.1 | 1.0 | 1.0 | 1.1 | 1.2 | 1.3 |
| 20 | 1.0 | 1.2 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.2 | 1.3 | 1.4 |
| 40 | 1.1 | 1.3 | 1.1 | 1.2 | 1.2 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 |
| 60 | 1.1 | 1.4 | 1.2 | 1.4 | 1.5 | 1.3 | 1.3 | 1.4 | 1.6 | 1.9 |
| 80 | 1.1 | 1.2 | 1.3 | 1.6 | 2.4 | 1.3 | 1.4 | 1.6 | 2.0 | 3.2 |

Based on the data presented in Tables 1 and 2, it has been confirmed that the most significant errors in determining the spatial angle occur when the points are located in a vertical direction and approach infinity as they get closer. It is advisable to exclude these scenarios whenever possible during measurements. Additionally, the calculations errors observed in Table 2 are found to be comparable to the measurement errors, and in certain cases, they are even smaller than the latter. Therefore, the formula for finding an angle in space can be recommended for use in applied engineering problems. Now let us look at the basic formulas for determining the angles of a triangle in space. If the vertices of a triangle are located in space, then the direct measurement of its sides is not always possible. Consider the situation when the total station is placed outside the plane of the triangle.

Then you can measure the angular elements between the ends of the sides, that is, the angles opposite to the base sides and the horizontal angle between them and the distances from the points of the ends of the sides to the device.

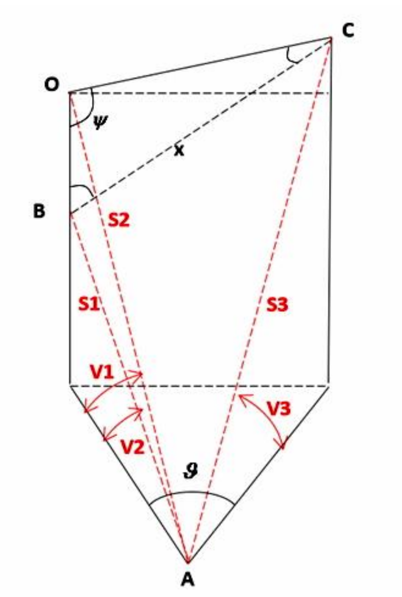


*Fig. 2. Scheme and basic symbols for determining the angles of a spatial triangle*

To calculate the length of the inaccessible segment, we need to determine the angle between two points at the ET's point of origin and measure the corresponding distances to these points.

Let us describe the algorithm for determining the angle. All the necessary elements are shown in Fig. 2. Here $S_1$, $S_2$, $S_3$ – measured inclined distances, $V_1$, $V_2$, $V_3$ – measured vertical angles, $\vartheta$. – measured horizontal angle. So, to implement the proposed method, we accept the following measurement sequence. From point A (device location), measure the distances $S_1$, $S_2$, $S_3$ up to three points (points **B, O, C**) and vertical angles $V_1$,$V_2$,$V_3$ to them. In addition, we will measure the horizontal angle $\vartheta$. For simplicity, we assume that the points **B, O** are in the same vertical plane. According to the measured vertical angles on the points **B, O, C** calculate angles <BAO, <OAC, <CAB and errors of their calculations by formulas (17).

For convenience, along with the calculation formulas, we present an estimate of the accuracy of the required variables.

$$\mu_1 = \arccos(\sin v_1 \sin v_2 + \cos v_1 \cos v_2), \vartheta = 0,$$

$$m_{\mu_1}^2 = \frac{(\cos v_1 \sin v_2 - \sin v_1 \cos v_2)^2 m_{v_1}^2 + (\sin v_1 \cos v_2 - \cos v_1 \sin v_2)^2 m_{v_2}^2}{1 - (\sin v_1 \sin v_2 + \cos v_1 \cos v_2)^2},$$

$$\mu_2 = \arccos(\sin v_2 \sin v_3 + \cos v_2 \cos v_3 \cos\vartheta), \tag{17}$$

$$m_{\mu_2}^2 = \frac{(\cos v_2 \sin v_3 - \sin v_2 \cos v_3 \cos\vartheta)^2 m_{v_2}^2 + (\sin v_2 \cos v_3 - \cos v_2 \sin v_3 \cos\vartheta)^2 m_{v_3}^2 + \cos^2 v_2 \cos^2 v_3 \sin^2\vartheta m_\vartheta^2}{1 - (\sin v_3 \sin v_2 + \cos v_3 \cos v_2 \cos\vartheta)^2},$$

$$\mu_3 = \arccos(\sin v_1 \sin v_3 + \cos v_1 \cos v_3 \cos\vartheta),$$

$$m_{\mu_2}^2 = \frac{(\cos v_1 \sin v_3 - \sin v_1 \cos v_3 \cos\vartheta)^2 m_{v_1}^2 + (\sin v_1 \cos v_3 - \cos v_1 \sin v_3 \cos\vartheta)^2 m_{v_3}^2 + \cos^2 v_1 \cos^2 v_3 \sin^2\vartheta m_\vartheta^2}{1 - (\sin v_1 \sin v_2 + \cos v_1 \cos v_2 \cos\vartheta)^2}.$$

By calculated angles and measured sides from triangles $\Delta BAO, \Delta CAO, \Delta OAC$ calculate the values of the segments using the cosine theory $|BO|,|CO|,|BC|$:

$$a = |BO| = \sqrt{S_1^2 + S_2^2 - 2S_1S_2\cos\mu_1}, m_a = \frac{1}{a}\sqrt{\left(S_1 - S_2\cos\mu_1\right)^2 m_{S_1}^2 + \left(S_2 - S_1\cos\mu_1\right)^2 m_{S_2}^2 + \left(S_1S_2\sin\mu_1\right) m_{\mu_1}^2},$$

$$b = |OC| = \sqrt{S_3^2 + S_2^2 - 2S_3S_2\cos\mu_3}, \mathrm{m}_b = \frac{1}{b}\sqrt{\left(S_3 - S_2\cos\mu_3\right)^2 m_{S_3}^2 + \left(S_2 - S_3\cos\mu_3\right)^2 m_{S_2}^2 + \left(S_3S_2\sin\mu_3\right) m_{\mu_3}^2}, \tag{18}$$

$$c = |BC| = \sqrt{S_1^2 + S_3^2 - 2S_1S_3\cos\mu_3}, m_a = \frac{1}{a}\sqrt{\left(S_1 - S_3\cos\mu_3\right)^2 m_{S_1}^2 + \left(S_3 - S_1\cos\mu_3\right)^2 m_{S_3}^2 + \left(S_1S_3\sin\mu_3\right) m_{\mu_3}^2}.$$

Using the cosine theorem again, we finally determine the desired angle value and the error in its calculation:

$$\psi = \arccos\left(\frac{a^2+b^2-c^2}{2ab}\right), \mathrm{m}_{\psi}^2 = \frac{\left(a^2+c^2-b^2\right)^2 m_a^2 + \left(b^2+c^2-a^2\right)^2 m_a^2 + a^2b^2c^2m_c^2}{\sqrt{c^4-\left(a^2-b^2\right)^2}}. \tag{19}$$

*Table 3*

**Measured values and measurement errors**

| | $S_1$ | $S_2$ | $S_3$ | $v_1$ | $v_2$ | $v_3$ |
|---|---|---|---|---|---|---|
| Value | 34.641 m | 48.990 | 73.655 | $35^0 15' 50.4''$ | $35'' 15' 50.4''$ | $67^0 25' 4.3''$ |
| Error | 1 mm | 1 mm | 1 mm | $1''$ | $1''$ | $1''$ |

*Table 4*

**Calculated values of linear and angular elements of a triangle $\Delta$ BOC and errors in their calculation**

| | a | b | c | $\mu_1$ | $\mu_2$ | $\mu_3$ |
|---|---|---|---|---|---|---|
| Value | 20 m | 48.831 m | 62.488 m | $19^0 28' 16''$ | $41'' 41' 05''$ | $57^0 47' 10''$ |
| Error | 1 mm | 0.8 mm | 0.9 mm | $1.4''$ | $1''$ | $1''$ |

We will test the algorithm on the model of a spatial triangle with the input data shown in Table 3.

The measured horizontal angle was $\vartheta = 90^0$, and its error $1''$.

Using the values of Table 4 and formulae (19), we first determine the value of the slope angle for the model values of the input parameters, and then the error in its calculation. The final result is as follows: $\psi = 125^0 00' 00'' \pm 18''$. Thus, the error in calculating the angle is much larger than the one in measuring the angles. This can be explained by the method of calculating the angle using the cosine theorem. This method is characterized by a small accuracy of calculating corner elements. However, for this task, such accuracy is quite acceptable. Perhaps, for other applications, you should choose the definition of angles with a higher calculation accuracy. For example, you can use the Heron's formula to calculate the area that can connect corner and linear elements. But this is a separate topic for research.

In the technical description of some tower cranes, other possible options are considered in addition to the horizontal placement of the boom. Obviously, the installation and maintenance of tower cranes have their own methods to ensure their reliable operation. Without detracting from the importance of these methods, we will offer another way to determine and, most importantly, control the angle of the boom to the horizon, based on geodetic measurements.


*Fig. 3. Illustration of ET measurements of angles and lines to points on a tower crane*

For example, for a tower crane, it is necessary to determine the angle of inclination of the boom to its supporting base, which is dictated by the rules of its operation.

An illustration of measurements at the site with a tower crane installed is shown in Fig. 3. The measurements made to determine the inclination of the boom to the crane base are listed in Table 5. Tables 6 and 7 present further calculations for determining the angle of inclination of the boom to the crane base.

According to Table 7, we finally calculate the angle of inclination of the tower crane boom and the error value $\alpha = 91.^{0}712 \pm 50^{''}$. The result of the determination is characterized by a relatively low accuracy, which, however, is quite sufficient for this task. To increase the accuracy, as already emphasized, it is necessary to use sides with a longer length or perform additional measurements with a simultaneous change in the method of determining the angle.

Let us look at another example of determining the angle of a spire covered by a church roof. Fig. 4 shows the points at which the ET was viewed. It also demonstrates the corresponding measured distances and vertical angles. Tables 8 and 9 show the input measurements and calculation results, respectively.

*Table 5*

**Measured sides and vertical angles of the reference points and their measurement errors**

| Point number | Horizontal and vertical angles measured | | Measured lengths from ET to point | MSE measurement of angles and distances | | |
|---|---|---|---|---|---|---|
| | $\beta$, ° | $Z$, ° | $S$, m | $m_\beta$, " | $m_v$, " | $m_S$, mm |
| 1 | 221.80045 | 94.159672 | 23.8982 | 2.0 | 2.0 | 2.0 |
| 2 | 221.80340 | 65.140425 | 26.3878 | 2.0 | 2.0 | 2.0 |
| 3 | 328.89003 | 47.172961 | 18.0047 | 2.0 | 2.0 | 2.0 |

*Table 6*

**Preparing data to calculate the sides of a triangle**

| Point number | The name of the horizontal angle | The value of the horizontal angle ° | The value of the tilt angle, ° | Distance from the ET to the point, m | MSE measurements | | |
|---|---|---|---|---|---|---|---|
| | | | | | $m_\beta$, " | $m_v$, " | $m_S$, mm |
| 1 | 1-2 | 0.002953 | -4.15967 | 23.898 | 2.8 | 2.0 | 2.0 |
| 2 | 1-3 | 107.08958 | 24.85958 | 26.388 | 2.8 | 2.0 | 2.0 |
| 3 | 2-3 | 107.08663 | 42.82704 | 18.005 | 2.8 | 2.0 | 2.0 |

*Table 7*

**Calculate the sides and angles of a triangle $\Delta$ 123 and errors in their calculation**

| Point number, vertex of the angle | Calculated angles of the spatial triangle, ° | MSE of angle determination, " | A determined distance, m | MSE distances, mm |
|---|---|---|---|---|
| 1 | 29.019 | 2.8 | 12.827 | 0.9 |
| 2 | 105.323 | 2.1 | 33.507 | 2.2 |
| 3 | 84.822 | 2.2 | 30.574 | 2.0 |

*Table 8*

**Preparing data to calculate the sides of a triangle**

| Point number | The name of the horizontal angle | The value of the horizontal angle, ° | The value of the tilt angle, ° | Distance from the ET to the point, m | MSE measurements | | |
|---|---|---|---|---|---|---|---|
| | | | | | $m_\beta$, " | $m_\beta$, " | $m_\beta$, " |
| 1 | 1–2 | 1.26800 | 16.57277 | 159.168 | 2.9 | 3.0 | 4.4 |
| 2 | 1–3 | 2.51454 | 24.86767 | 159.926 | 6.0 | 0.1 | 7.4 |
| 3 | 2–3 | 1.27355 | 16.65889 | 169.521 | 5.8 | 2.9 | 2.9 |

*Table 9*

**Calculate the sides and angles of a triangle $\Delta$ 123 and the error of their calculation**

| Point number, vertex of the angle | The angles of the spatial triangle, ° | MSE angle determination, " | A certain distance, m | MSE distance, mm |
|---|---|---|---|---|
| 1-2 | 8.379 | 2.986 | 26.138 | 2.6 |
| 1-3 | 2.437 | 5.747 | 6.828 | 4.4 |
| 2-3 | 8.294 | 2.407 | 25.676 | 1.3 |

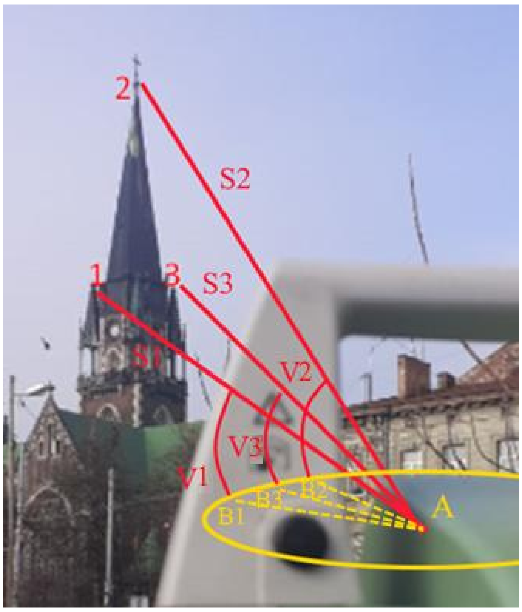


*Fig. 4. Illustration of ET measurements*

According to Table 9, we finally calculate the spatial angle and its error value for the church spire $\alpha = 15.^0 109 \pm 36^{''}$.

## Scientific novelty and practical significance

Based on the numerical modeling experiments, the spatial angles are determined and an analysis of the a priori assessment of their accuracy is conducted. This confirms the influence of the linear measurement values of the lengths of the sides to the points that form a spatial triangle with a determined spatial angle. The obtained ratios of the sides of the spatial triangle make it possible to apply the proposed methods for determining spatial angles using ET measurements of additional elements in engineering and geodetic works. Two examples of determining the spatial angle of inclination of a tower crane and the spatial angle at the top of the spire of a church roof are used to demonstrate the practical application of the proposed method. Such measurements may arise from the need to verify the elements of building structures in their spatial arrangement.

## Conclusions

1. Using the mathematical modeling apparatus, the calculation errors for determining the spatial angle were determined depending on the values of the lengths of the measured sides.
2. The high accuracy of distance measurements provides sufficient accuracy in calculating the angular elements.
3. This method can be used in the application software of manufacturers of electronic total stations to determine spatial angles in solving engineering problems related to the installation/disassembly of technological equipment.

4. This type of algorithm can also be integrated into specialized devices, like modern laser tape measures, in order to tackle architectural measurement challenges and accurately position architectural details in the design.

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# ПРО МЕТРИЧНІ ТА КУТОВІ ЗАЛЕЖНОСТІ ПРОСТОРОВОЇ ПРЯМОЇ ЗАСІЧКИ ТА ЇХ ВИКОРИСТАННЯ В ІНЖЕНЕРНО-ГЕОДЕЗИЧНИХ РОБОТАХ

У прикладних задачах геодезії може виникати потреба у визначенні просторових кутів. Під час виносу 3D проєкту будівель і споруд у натуру за просторовими координатами їх характерних точок з використанням електронного тахеометра (ЕТ) також з'являється необхідність у перевірці просторових кутів між різними елементами будівельних конструкцій (наприклад, конструкцій, які формують перекриття дахів, нахилених анкерів тощо). Сучасні геодезичні прилади забезпечують достатньо високу точність вимірювання (до 1" та 1 мм відповідно). Проте не завжди можна здійснити вимірювання необхідних кутів за допомогою геодезичних приладів з різних причин. Насамперед неможливо розмістити прилад у вершині кута, якщо місце його положення недоступне. Метою цієї роботи є розробка методу визначення просторового кута, вершина якого недоступна для вимірювань. Методика та результати. Для реалізації мети розглянуто один із варіантів його визначення через застосування теореми косинусів із попереднім вимірюванням або обчисленням примикаючих сторін і вертикальних кутів. Алгоритм вирішення поставленої задачі з оцінкою точності визначення необхідних параметрів також наведений в цій статті. Запропоновано основні формули для визначення кутів просторового трикутника з оцінкою їх точності. Виконано дослідження впливу значень лінійних вимірів довжин сторін на величини кутів просторового трикутника з відповідною оцінкою точності. Зокрема, на основі цих обчислень та математичного моделювання, а саме відношення сторін трикутника, було встановлено середньоквадратичні похибки обчислення кутів. На прикладі визначення нахилу стріли баштового крану до основи та визначення кута шпилю даху покриття собору отримано відповідні значення просторового кута: α=910.712±51"та α= 150.109±35" за результатами опосередкованих вимірювань елементів, пов'язаних із цим кутом. Наукова новизна та практична значущість. На основі запропонованої методики та проведених числових експериментів визначено просторові кути та проведено аналіз їх апріорної оцінки точності, що підтверджує вплив значень лінійних вимірів довжин сторін на величини просторових кутів. Отримані результати надають можливість застосувати запропонований метод в інженерно-геодезичних роботах із використанням BIM технологій у 3D просторі. Цей метод може бути використаний у прикладному програмному забезпеченні виробників електронних тахеометрів для визначення просторових кутів у просторі під час вирішення інженерних задач.

*Ключові слова:* кутові вимірювання, просторовий кут, похибка, електронний тахеометр, точність.

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