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Mathematical modeling and statistical analysis of Moroccan mean annual rainfall using EXPAR processes

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In this work, we propose a study of the mean annual rainfall time series in order to evaluate the climate changes pattern over time. If the analysis of this time series is carried out correctly, it can contribute to improve planning and policy development. That is why we consider the problem of mathematical modeling and analysis of the mean annual rainfall of Morocco between 1901 and 2020 using descriptive statistics, structure changes analysis, spectral analysis and a nonlinear Exponential Autoregressive (EXPAR) processes to reproduce the behavior of this time series. The results indicate three main breakpoints and show that the time series has a remarkable cycles about 60, 18 and 6 years with a global decrease tendency about 0.56 mm per year. Furthermore, we have justified the choice of using a non-linear EXPAR processes rather than a linear traditional one and provided a good fitted EXPAR model.

Keywords: *time series; Moroccan rainfall; nonlinearity test; exponential autoregressive models; spectral analysis.*

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1. Introduction

The general evolution of climate during the 20th century, at least, especially in its second half shows a change with tendencies to warming. The extension of dry episodes in Mediterranean perimeter becomes a climatic reality, [1]. Morocco could not avoid these difficult conditions caused by persistence and strictness of dryness since 1970s see [2]. These conditions influenced agrarian activities heavily and considerably reduce means in water subject mobilized for a continuously growing population.

This critical situation, requires to give a good attention to the time series of the Moroccan rainfall in order to evaluate how the climate changes over time in this country. If the forecasts and analysis of these time series are carried out correctly, they can contribute to improve planning and policy development.

The study of rainfall series of northern Morocco, by applying statistical methods of detection of breaks, between 1934 end 2004, showed that the most significant period of rupture appeared during the 1970s, see [3] for more details. The calculation of the standardized rainfall index made it possible to locate a deficit phase before 1956, and a normal phase and/or surplus until the decade of 1970.

In the context of Moroccan rainfall, [4] dealt with the trends and evolution observed in the climate of Morocco through a number of climatic index. Besides, he carried out an assessment of future climate change, this assessment made it possible to state that a change in the distribution of the rainfall coincides with a warming that would manifest on both seasonal and annual scales.

Previous studies attempted to establish regression models and to calculate correlation coefficients, on the one hand, between the spatial variability of the rainfall and the geographical parameters (geographical position according to latitude and longitude), and on the other hand, between the temporal variability of precipitation and large-scale atmospheric circulation index such as the North Atlantic Oscillation (NAO), the West Mediterranean Oscillation (WeMO) and the Southern Oscillation El Niño (ENSO). We cite for example the work of [3, 5–11].

According to [12], this modeling approach introduces a bias into the estimation of the model parameters and makes the generalization of the results across the whole country non credible. That is why we propose in this work a different approach based on time series analysis to study the Moroccan rainfall.

By considering the time series of the mean annual rainfall of Morocco between 1901 and 2020 we seek to determine the main breaks points and the most important periods components contributing in this time series using breakpoints detection methods and spectral analysis tools. By applying a nonlinearity test, see [13, 14], we are looking to justify the use of the EXPAR model rather then the traditional linear Autoregressive one. Then, we will provide the fitted EXPAR model to this time series and analyze its behavior. We stress that our statistical analysis is carried out using R packages or SPSS.

The paper is organized as follows. Section 2 is devoted to a brief mathematical review of EXPAR models, breakpoints detection method and the spectral analysis. Section 3 presents the discussion of the statistical analysis of our time series. Section 4 shows the results of our proposed procedure to model with EXPAR process. Finally, a conclusion is given in Section 4.

2. A brief review of mathematical tools

2.1. Exponential autoregressive models

In this work, we are interested in exponential autoregressive models (in short EXPAR) introduced by [15]. An exponential autoregressive model of order p [EXPAR(p)] is the solution of a stochastic difference equation:

$$X_t = \sum_{i=1}^{p} \left(\pi_i + \beta_i \exp(-\varphi X_{t-1}^2)\right) X_{t-i} + \varepsilon_t,$$

with $\boldsymbol{\vartheta} = (\boldsymbol{\pi}, \boldsymbol{\beta}, \varphi)'$ is the vector of parameters, where $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_p)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)$ are the linear autoregressive parameters, φ is the parameter of exponential component and $\{\varepsilon_t; t \in \mathbb{Z}\}$ is an i.i.d. sequence. Note that the EXPAR(p) may reduce to an AR(p), indeed, for large values $|X_{t-1}|$, the EXPAR(p) becomes an AR(p) with parameters approximately $(\pi_1, \pi_2, \ldots, \pi_p)$ and for small values $|X_{t-1}|$, the EXPAR(p) becomes an AR(p) model with parameters approximately $(\pi_1 + \beta_1, \pi_2 + \beta_2, \ldots, \pi_p + \beta_p)$. Thus, the EXPAR($p$) models may behave as a threshold autoregressive models, though here the coefficients change smoothly between two extreme values, which make the dynamic of the series locally linear and globally nonlinear.

[15] introduced the EXPAR models and discussed necessary conditions to exhibit limit cycle behavior, they described a procedure to estimate EXPAR model parameters and they applied it to fit Canadian lynx data. [16] has shown that EXPAR models are suitable to reproduce nonlinear phenomena like: amplitude-dependent frequency, jump phenomena and limit cycles. [17] discussed EXPAR models on their capability of capturing non-Gaussian characteristics of time series.

[18] proposed an optimal pseudo-Gaussian test for the detection of exponential component in autoregressive of order one which was reconsidered by [13] and generalized for any order, see [14], then a signed rank version test was developed recently by [19].

The first estimation procedure is given by [15]. Then, [20] proposed a method to estimate EXPAR model parameters based on genetic algorithm.

We summary the procedure given by [15] in the following two steps:

1. We fix the value of φ, then we estimate $\pi_1, \beta_1, \pi_2, \beta_2, \ldots, \pi_p, \beta_p$ by least square regression of X_t on
$$\left(X_{t-1}, X_{t-1}\exp(-\varphi X_{t-1}^2) \ldots, X_{t-p}, X_{t-p}\exp(-\varphi X_{t-1}^2)\right).$$

The order p is determined by the minimization of the AIC criterion.

2. The previous procedure is repeated using a value range of φ, and the best φ is selected by the AIC criterion. The φ values considered are such that $\exp(-\varphi X_{t-1}^2)$ varies quite widely over $(0,1)$.

The EXPAR models can exhibit limit cycle whenever the following necessary condition are satisfied (see [15]):

1. The root of $1-\sum_{i=1}^{p}\pi_i\lambda^i=0$ is outside the unit circle.
2. The root of $1-\sum_{i=1}^{p}(\pi_i+\beta_i)\lambda^i=0$ do not all lie outside the unit circle.
3. $\left(1-\sum_{i=1}^{p}\pi_i\right)/\sum_{i=1}^{p}\beta_i>1$ or $(1-\sum_{i=1}^{p}\pi_i)/\sum_{i=1}^{p}\beta_i<0$.

The main motivation of the specific choice of EXPAR model is the fact that we suspect that the rainfall time series has a kind of cycles or amplitude dependent frequency behavior. Since EXPAR models are suitable to reproduce this kind of nonlinearity feature, it will be reasonable to apply this process on the time series of mean annual rainfall. Furthermore, in Section 4, we will justify statistically the use of a such sophisticated model rather than the linear traditional one.

2.2. Breakpoints detection method

A breakpoint is simply a structural change in data in certain point(s). The foundation for estimating breaks in time series regression models was given by [21] and was extended to multiple breaks by [22–24].

R software possesses a whole package named 'strucchange' which contains 'breakpoints function' that we use for estimating breaks or ruptures points in a time series, it implements the algorithm described in [25] for simultaneous estimation of multiple breakpoints. The distribution function used for the confidence intervals for the breakpoints is given in [23].

All procedures in this package are concerned with testing or assessing deviations from stability in the classical linear regression model

$$y_i=x_i'b+\varepsilon_i.$$

In many situations it is straightforward to assume m breakpoints, where the coefficients shift from one stable regression relationship to a different one. Thus, there are $m+1$ segments in which the regression coefficients are constant, and the model can be rewritten as

$$y_i=x_i'b_j+u_i \quad (i=i_{j-1}+1,\dots,i_j,\ j=1,\dots,m+1),$$

where j denotes the segment index. In practice, the breakpoints i_j must be estimated, since they are rarely given. The 'breakpoints function' estimates these breakpoints by minimizing the residual sum of squares (RSS) of the equation above.

Note, that for every potential change point an OLS model is fitted for the observations before and after the potential change point, thus, $2k$ parameters have to be estimated, then, the error sum of squares (ESS) and a statistics test [26] is computed, that allows to test if the change is significant in this point.

2.3. Spectral analysis

Astronomers were the first to use Fourier analysis for time series in order to detect hidden seasonality within their data. Thus, Lagrange used these methods to detect hidden periodicity in 1772 and 1778. In 1847, Buys and Ballot, proposed methods to study the periodicity of astronomical data in 'Periodic changes of temperatures'. However, until 1889 that Sir Arthur Shuster introduced the periodogram, which formed the basis of spectral methods of time series analysis. This theory was then used by several researchers like Whittaker and Robinson in 1924, on the brightness of the star T-Ursa Major, observed over 600 days, and showed that the brightness could be modeled (almost perfectly) using 2 harmonic functions, from respective periods 24 and 29 days.

The idea of this theory is to look for an underlying model of the form:

$$X_t=\sum_j\left(a_j\cos(\omega_j t)+b_j\sin(\omega_j t)\right)+\varepsilon_t=\sum_j\rho_j\cos(\omega_j t-\theta_j)+\varepsilon_t,$$

where ε_t is a sequence of identically distributed independent random variables (a white noise). The factor $\rho_j = \sqrt{a_j^2 + b_j^2}$ corresponds to the amplitude of the j-th periodic component, and indicates the weight of this component within the sum.

In this case, the time series X_t can be seen as the sum of a noise and several sinusoidal functions of different ρ_j amplitudes.

Let consider a sample $X_0, \ldots, X_{T-1}$ of observation of X_t, and let the frequencies $\omega_j = \frac{2\pi j}{T}$, then the periodogram is defined:

$$I(\omega_j) = \frac{2}{T}\left(\sum X_t \cos(\omega_j)^2 + \sum X_t \sin(\omega_j)^2\right).$$

$$I(\omega_j) = \frac{T}{2}(a_j^2 + b_j^2).$$

It is possible to show that $\frac{2}{T} I(\omega_j)$ is a consistent estimator of ρ_j^2, this convergence was wildly studied by Yule in 1927, see [27].

3. Statistical analysis and modeling of the mean annual Moroccan rainfall

3.1. Descriptive statistics

The series of the Moroccan mean annual rainfall includes 120 data taken as the (spatial) average of the cumulative precipitation in each year in the period 1901–2020, measured in mm. The series can be found, among other sources, on the World Data Bank and Trading Economics websites [28, 29].

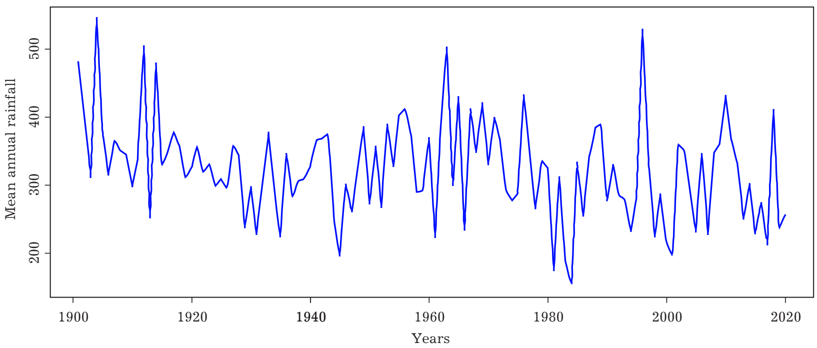

Fig. 1. Mean annual Moroccan rainfall Series 1901–2020.

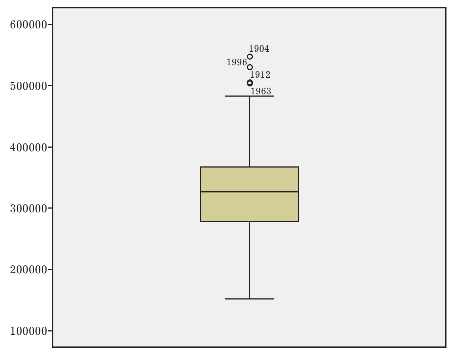

Fig. 2. Boxplot of Moroccan mean annual rainfall for the period 1901–2020.

The descriptive statistics in Table 1 shows that the inter-quartile is 88.8 mm, the minimum value of this time series is 151.29 mm registered in 1984, where the maximum value is 547.5 mm registered in 1904. Thus a considerable range of 396.21.

Table 1. Descriptive statistics of Moroccan mean annual rainfall for the period 1901–2020.

Statistics	Size	Max	Min	Mean	sd	Q_1	Median	Q_3
mean annual rainfall	120	547.5	151.29	323.91	73.9	278.09	327.13	366.91

The box plot 2 shows four extreme observations presenting the best years of rain ever, over the kingdom, three are concentrated before the 1970s which are 1901, 1912, 1963 and only one after 1970 registered in 1996. This may be an indication for a general decrease tendency in the mean annual rainfall which will be studied in the following subsection.

3.2. Breakpoints and tendency analysis

We have performed a breakpoints analysis (3) to investigate the main change in the structure of the mean annual rainfall. The application of the 'breakpoints' procedure described in subsection 2.2 has shown that the main change of structure in this time series are in 1918, 1954 and 1977.

Since the structure changes are estimated, it means they are an approximation of the real change points, we need to analyze the behavior around these ruptures years.

The analysis of Figure 3, including the breakpoints presented by red lines, indicate a small increase registered from 1901 to 1904 followed by a decreasing trend from 1904 to the vicinity of the first breakpoint 1918 then a trend stabilization up to the second breakpoint in 1954. Next, we remark a slight increasing until 1960 followed by a decrease until the third breakpoint of 1977, after that a sort of stabilization is noticed. Consequently, the investigation shows a kind of cycle around 60 years. Indeed, we have noticed a repetition behavior about 18 years of a slight increase followed by a longer decrease, then a stabilized trend of about 42 years, thus a kind of cycle in the vicinity of 60 years. However, this hypothesis based on this analysis needs to be confirmed by spectral analysis we will perform in the next subsection.

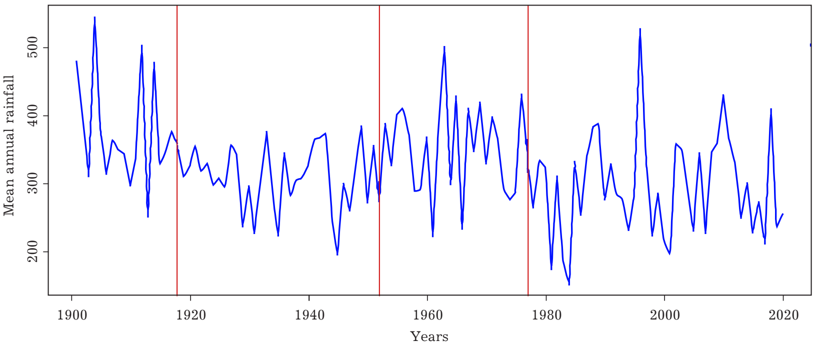

Fig. 3. Main rupture in Moroccan mean annual rainfall for the period 1901–2020.

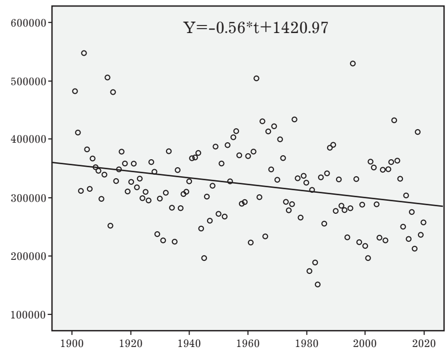

Fig. 4. Trend of Moroccan mean annual time series 1901–2020.

Figure 4 shows the global downward trend in Moroccan annual rainfall which indicates that Morocco is going through a period of drought marked by the lowest value noted in 1984. In addition, we can see by a linear regression of the values over time, that rainfall series decreases, on average, by 0.56 mm annually. Following this pattern we can deduce, theoretically, that the mean annual rainfall will goes under the minimum value ever reached in this time series after the year of 2267, i.e. almost 245 years from now. This theoretical eventual situation should rings the alarm bells for food and water security.

3.3. Stationary test

By applying the Augmented–Dickey–Fuller test on the log transformed and the original data, we obtain a p-value $= 0.01$, which shows the rejection of the null hypothesis (non-stationarity). Consequently, we can consider that the log and original data of Moroccan mean annual rainfall series are stationary. Notice that the log data will be used in the part of constructing a fitted model for this time series.

3.4. Spectral analysis

To illustrate the periodogram 5, we provide Table 2 presenting the first six most important frequencies in the Moroccan mean annual rainfall time series.

Table 2. The six first important periods which contribute the most to the Moroccan mean annual rainfall.

	1	2	3	4	5	6
Periodogram	$5.7735 \cdot 10^4$	$4.3308 \cdot 10^4$	$3.0981 \cdot 10^4$	$2.8624 \cdot 10^4$	$2.7288 \cdot 10^4$	$2.5761 \cdot 10^4$
Frequency	$1.6666 \cdot 10^{-2}$	$3.0833 \cdot 10^{-1}$	$4.9166 \cdot 10^{-1}$	$5.8333 \cdot 10^{-2}$	$3.5833 \cdot 10^{-1}$	0.1500
Periods	60	3.24	2.03	17.14	2.79	6.66

It can be seen from Table 2 that the most important period is 60 years, confirming the analysis breakpoints results which indicate a kind of cycle in vicinity of 60 years.

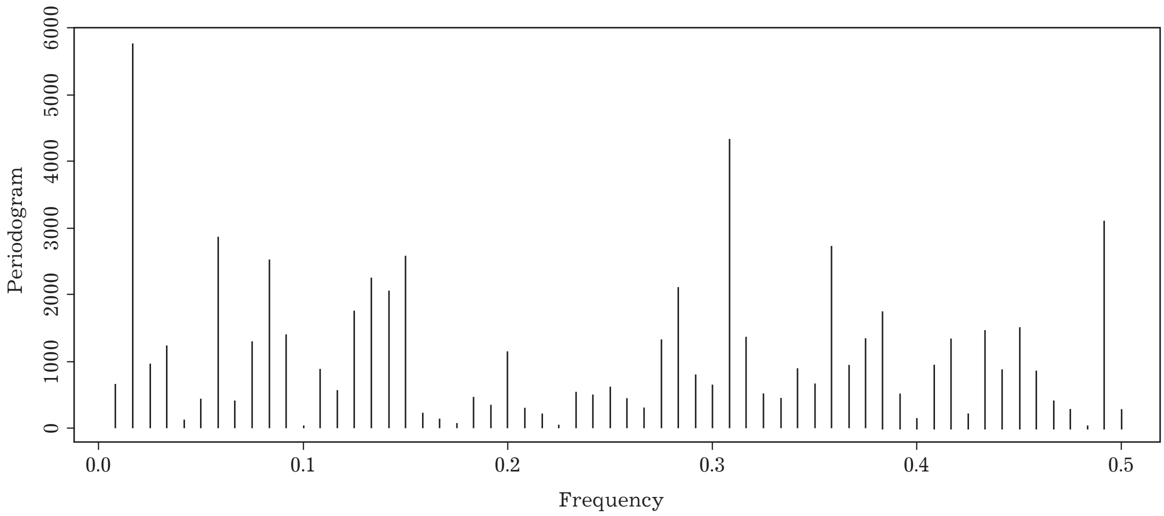

Fig. 5. Periodogram of the time series of Moroccan mean annual rainfall.

Others remarkable periods are noticed, those of 17.14 and 6.66 years. Indeed, they coincide and confirm with the analysis results made in the subsection of breakpoints where we indicate a repetition behavior of 18 years including almost 6 years of increasing and 12 years of decreasing.

Concerning periods 3.24 and 2.03 years, these are hidden cycles which alternate and are of multiple frequencies, possibly dependent on the data, which may call also for an EXPAR model since it is desirable to reproduce the frequencies dependent on the amplitudes.

To summary, the management of matters related to rainfall in Morocco should consider in their policy development the most effective and appeared cycles in the Moroccan mean annual rainfall those around of 60 years 18 and 6 years. Furthermore, since EXPAR processes are suitable for limit cycle behavior it may be considered for modeling this time series, however, we need a mathematical justification to adopt a such nonlinear model rather than a traditional linear one.

3.5. EXPAR non-linearity tests

We propose to model the logarithm of the mean annual rainfall, then, we return to the initial series.

By applying the pseudo-Gaussian test for order 1, developed in [13], we do not reject the linearity hypothesis in favor of that of non-linearity at the level $\alpha = 0.05$. Indeed, the statistic calculated equal to 1.27 is lower than the theoretical one given by 3.84. However, by using the generalization of this last test for order 2, [14], we conclude the rejection of the assumption of linearity in favor of that of the non-linearity at the level $\alpha = 0.05$. Indeed, the calculated statistic equal to 5.71 is higher than the theoretical one given by 2.44. Therefore, we revealed the existence of the exponential component in the Moroccan mean annual rainfall which justifies the use of EXPAR processes to model this time series.

3.6. EXPAR model

EXPAR Model identification and estimation parameters. In this section we are interested to identify the best model which approximate the behavior of the whole mean annual rainfall time series. Based on the breakpoints analysis (3), confirmed by the spectral analysis, there are two phases in a cycle of 60 years. In order to make our model learning from the different behaviors of two phases we decided to investigate a maximum lag of 38 years.

We have used the estimation procedure proposed by [15]. Taking the parsimony and some Mathematical issues (about the singularity problem of the matrix used in the parameters estimation) in consideration we have obtained:

— selected order = **37**;
— MSE = **963.6** (Mean square error);

— selected $\hat{\varphi} = \mathbf{0.1}$ (The exponential parameter);
— $(\pi_1, \beta_1, \pi_2, \beta_2, \ldots, \pi_{37}, \beta_{37}) =$

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 [1]   2.063  -8.825   0.720   0.791   0.878 -32.041  -1.536  44.227  -0.634
[10]   4.067  -0.562  12.745  -0.398  14.868   0.643 -14.845  -1.753  40.486
[19]  -0.714  13.813  -1.720  40.875   2.343 -66.032   1.589 -39.446  -2.101
[28]  65.983   0.589 -14.044  -1.663  44.327   0.603 -18.949  -1.667  29.614
[37]  -1.235  38.169   2.322 -55.871   1.244 -31.783  -3.157  77.926  -1.031
[46]  27.179   2.370 -66.203   2.144 -63.655   0.816 -27.403  -2.129  69.948
[55]  -0.500  12.858   1.473 -50.144   3.090 -88.699   3.866 -92.483  -0.738
[64]  23.703  -1.973  56.095  -2.068  46.289   0.476 -18.898   0.876 -28.600
[73]  -1.847  62.459
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An EXPAR(37) model to fit the Moroccan mean annual rainfall. We present in Figure 6 the fitted model for Moroccan mean annual rainfall by the EXPAR process of order 37. The original series is given in the blue line graph and the estimated one in the red line.

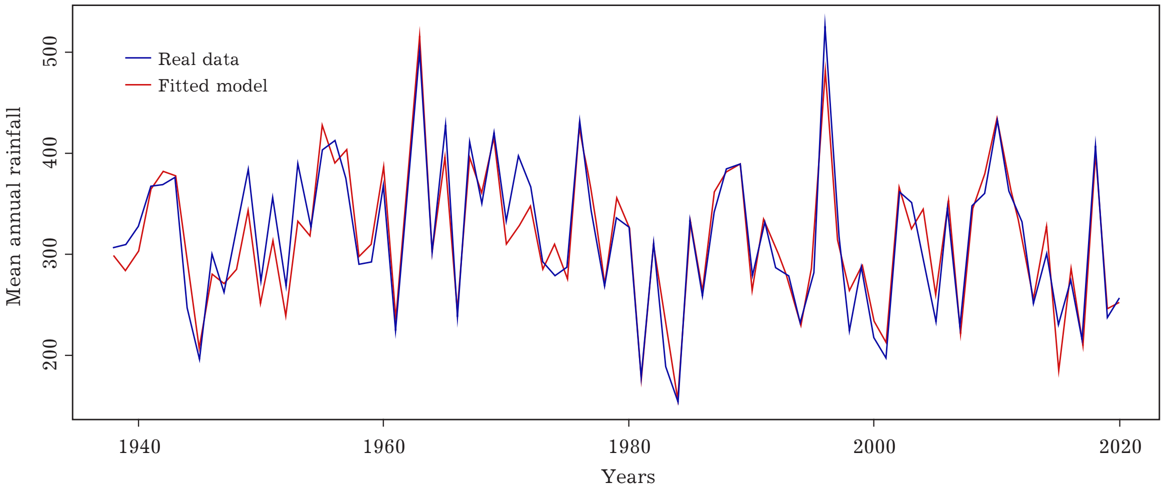

Fig. 6. Fitted model for Moroccan mean annual rainfall by EXPAR(37).

Figure 6 shows the good quality of the fitted model compared to the real data. However, in order to validate this model we need to check the non-autocorrelation of residuals and also the conservation of the stationary behavior.

By applying the Box-Pierce test on the residuals, we obtain a $\mathrm{p-value} = 0.88$. Therefore, we do not reject the null hypothesis of the non-autocorrelation. Thus we can consider the residuals non-autocorrelated. Farther more, by applying the Augmented-Dickey-Fuller test on the EXPAR(37) fitted model we have obtained a $\mathrm{p-value} = 0.01$, thus, the fitted model can be considered as a stationary model. Consequently, we validate the fitted EXPAR(37) model.

EXPAR(37) behaviour.

1. The roots $1 - \sum_{i=1}^{37} \pi_i \lambda^i = 0$ are not all outside the unit circle.
2. The roots $1 - \sum_{i=1}^{37} (\pi_i + \beta_i) \lambda^i = 0$ are not all outside the unit circle.
3. $\left(1 - \sum_{i=1}^{35} \pi_i\right) / \sum_{i=1}^{37} \beta_i = 0.04$.

These results do not allow to conclude the existence of a limit cycle since the first assertion and the third are not satisfied. Consequently, we may conclude that Moroccan mean annual rainfall has rather a chaotic behavior. However, a statistical test should be constructed in this case to check these three necessary conditions rather than a simple deterministic computation.

4. Conclusion

In this work we have analyzed the time series of the mean annual Moroccan rainfall using descriptive methods, breakpoints detection and spectral analysis on a data sample size of 120 from 1901 to 2020. We have justified the use of a nonlinear model called EXPAR rather than a linear one by applying a suitable non-linearity test. Then, we have obtained the corresponding fitted model and analyzed its behavior.

The results of descriptive statistics show the decreasing of 0.56 mm each year in average which, theoretically, leads to deduce that the mean annual rainfall will go under the minimum value ever noticed over 1901 to 2020 in the year of 2267. This catastrophic theoretical projection of our situation should motivate the head of management and policy development to consider a sequence of long time plans to deal with this eventual future crisis.

The investigation of breakpoints have revealed three main ruptures, in 1918, 1954 and 1977. The analysis shows a kind of cycle around 60 years. This cycle is composed by two phases, the first one about 18 years characterized by a slight increase about 6 years followed by a longer decrease. The second phase presents a sort of a stabilized trend of about 42 years. Consequently, a kind of cycle in the vicinity of 60 years was suggested.

The results of spectral analysis confirmed that the most relevant period is 60 years followed by other important periods including 17.14 and 6.66 years which are in vicinity of the results obtained in the ruptures analysis those of 60, 18 and 6 years. Thus, we recommend the vicinity periods of 60, 18 and 6 years to be considered in the future surveillance, analysis and management plan to resist the depletion of water resources.

The resulting EXPAR(37) model has succeed to provide a good adjustment to the mean annual rainfall of Morocco. The analysis of it's behavior using necessary conditions to exhibit a cycle limit [15] leads us to reject the limit cycle behavior, thus, an eventual chaotic behavior of this time series. In fact, the spectral analysis have revealed many important periods components contributing in this time series which automatically do not lead to a cycle limit, or quasi-periodic non a point fix behavior. However, it may leads to another kind of 'multiple cycles' behavior.

This paper has revealed some questions as perspectives in our future works. Indeed, since we consider a sample of the mean annual rainfall, and in the absence of a suitable test, we need to construct a statistical test to validate three necessary conditions developed by [15] rather than a simple comparison of calculated values. Furthermore, a spatio-temporal model should be considered to reach a high level of analysis taking in consideration the spatio-variability of Morocco rainfall. Finally, a prediction model should be studied using several approaches and processes.

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Математичне моделювання та статистичний аналіз середньорічної кількості опадів у Марокко за допомогою процесів EXPAR

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У цій роботі пропонується дослідження часових рядів середньорічної кількості опадів, щоб оцінити характер змін клімату з часом. Якщо аналіз цього часового ряду проведено правильно, то це може сприяти покращенню планування та розробки політики. Ось чому розглядається проблема математичного моделювання та аналізу середньорічної кількості опадів у Марокко між 1901 та 2020 роками з використанням описової статистики, аналізу структурних змін, спектрального аналізу та нелінійних експоненціальних авторегресійних процесів (EXPAR) для відтворення поведінки цього часового ряду. Результати вказують на три основні контрольні точки та показують, що часовий ряд має цикли приблизно 60, 18 та 6 років із загальною тенденцією до зменшення приблизно на 0.56 мм на рік. Крім того, обґрунтовано вибір використання нелінійних процесів EXPAR замість традиційно лінійних та надано добре підігнану модель EXPAR.

Ключові слова: *часові ряди; кількість опадів у Марокко; тест на нелінійність; експоненціальні авторегресійні моделі, спектральний аналіз*