COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 12, No. 2, 2022

DETERMINATION OF PARAMETERS OF MAGNETICALLY COUPLED COILS BASED ON MATHEMATICAL MODELS OF THEIR MAGNETIC CIRCUITS

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https://doi.org/10.23939/jcpee2022.

Abstract: Calculation of electromagnetic parameters of static and dynamic magnetically coupled inductors based on mathematical models of their magnetic circuits is proposed. The inductance and mutual inductance of such coils are determined using the geometric dimensions of the coils themselves, their relative location, and the physical parameters of the environment.

Keywords: inductor, mathematical model, magnetic circuit, inductance, mutual inductance.

1. Introduction

Electric devices that are components of power transmission and control systems contain windings with magnetic coupling. Magnetically coupled coils have found their application in other fields, including electronics, electromechanics, and medicine. When forming mathematical models of devices with coils implemented on the basis of electric circuits, there is a need to determine their electromagnetic parameters.

In general, modeling various modes of complex electrotechnical devices is a difficult task, primarily due to the complexity of modeling magnetic systems. When developing adequate mathematical models based on the electromagnetic field theory, problems arise with the determination of boundary conditions and the application of numerical methods for solving high-dimensional problems [1].

Mathematical models of devices implemented on the basis of the theory of electromagnetic circuits are much more effective. In such models, the electromagnetic parameters of the windings calculated as accurately as possible can ensure the appropriate adequacy. Therefore, in our opinion, the calculation of electromagnetic parameters of static and dynamic magnetically coupled coils is an actual and promising task.

2. Mathematical formulation of the task

In classical theoretical electrical engineering, the voltage on a real inductor with magnetic couplings is determined by the formula [2]

$$u = Ri + Ldi/dt + \sum_{k=1}^{n} M_k di_k/dt = 0, \qquad (1)$$

where i is the current of this coil, i_k is the current of another k-th coil with which there is a magnetic coupling, R is the active resistance of the coil wire, L is the inductance of the coil (coefficient of self-induction), M_k is the mutual inductance of the given and the k-th coil (coefficient of mutual induction).

Values R, L, M_k are the parameters of the inductor. The resistance of the coil R without taking into account the skin effect can be determined quite simply by the formula known from the course of physics

$$R = l/(\gamma S), \qquad (2)$$

where l is the length of the coil wire, γ is the specific conductivity of the wire material, and S is the cross-sectional area.

Determining the other two electromagnetic parameters L and M_k is a difficult task primarily due to the eddy nature of the magnetic field created by the winding currents. Only in some cases, for example, for a coil wound on a toroidal magnetic conductor made of a ferromagnet, where the direction of the magnetic flux is localized, it is possible to determine the magnetic resistance, and therefore obtain a formula for the inductance of such a coil. But when there are magnetic couplings with other coils, not only through sections of the ferromagnet, then the calculation of mutual inductances already becomes a difficult task. The same problems arise when calculating the inductances of most coils. As mentioned above, this is caused primarily by the complex distribution of magnetic field lines and the difficulties of calculating magnetic fluxes.

To determine the inductances and mutual inductances of coils with magnetic couplings, we have proposed an approach based on modeling the magnetic fields of coils in the form of a planar magnetic circuit with a single magnetic flux [3]. To obtain such a circuit, we divide the entire space into elementary volumes. Based on the geometric dimensions of these volumes and the physical parameters of the medium, we determine their magnetic resistances. The calculation of magnetic circuits with alternate setting of currents allows us to obtain the flux coupling of the coils, and, based on them and the currents, determine their electromagnetic parameters.

The mathematical model of the magnetic circuit is formed in the coordinates of contour magnetic fluxes and currents as a system of finite equations

$$\mathbf{\Gamma_m R_m \Gamma_{mt}} \vec{\Phi}_\kappa = \mathbf{W}_t \vec{i}\ , \qquad (3)$$

where *Гm* is the second incidence matrix of the magnetic circuit; *Rm*– the diagonal matrix of magnetic resistances of magnetic circuit branches; *Гmt* – the transposed second matrix of incidences of the magnetic circuit; $\vec{\Phi}_k$ – the column vector of contour magnetic fluxes; *Wt* is the transposed matrix of turns of the elementary circuits of the magnetic circuit; $\vec{i}$ is the column vector of coil currents.

Having determined the contour magnetic fluxes from equation (3)., we calculate the flux linkage of the coils

$$\vec{\Psi} = \mathbf{W}\vec{\Phi}_k\ , \qquad (4)$$

where $\vec{\Psi}_k$ is a column vector of flux linkages of coils; W is the matrix of turns of the elementary circuits of the magnetic circuit.

Matrix **W** has *m* rows corresponding to the number of coils and q×s columns corresponding to the contours of the magnetic circuit. The matrix element W at the intersection of the lth row and jkth column is determined by the number of turns of the lth coil, which is in the jkth contour of the magnetic circuit

Calculation of flux linkages at a given current of the *l*-th coil allows you to determine the following electromagnetic parameters:

$$L_l = \Psi_l / i_l\,; M_{pl} = \Psi_p / i_l\,, \qquad (5)$$

where L_l is the inductance of the lth coil, Mpl is the mutual inductance between the pth and lth coils, Ψ_l is the flux coupling of the lth coil, Ψp is the flux coupling of the pth coil.

If you alternately set the current values for other coils, you can determine the remaining electromagnetic parameters of all coils.

Let us give examples of parameter calculation for two magnetically coupled coils.

Example 1.

There are two static magnetically coupled coils in the air, the copper turns of which have a cylindrical shape (Fig. 1). The number of turns of each coil and all geometric dimensions (radius of wires, radius of turns, distance between axes of coils) are specified. The task is to determine the parameters of the coils.

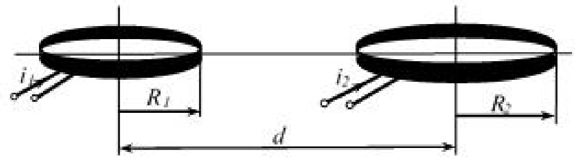

Fig. 1. Two static magnetically coupled coils.

As you can see, the spatial arrangement of the coils in our example resembles the configuration of a two-winding transformer.

The magnetic field created by the currents of each coil has a certain symmetry relative to their axes. Given this, to obtain a calculation diagram of the magnetic circuit in the form of a two-dimensional planar grid (Fig. 3), the entire space is divided into elementary volumes, which are basically hollow half-cylinders (Fig. 2).

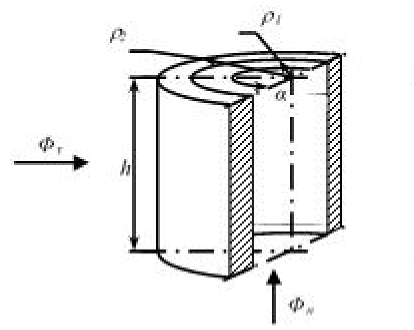

Fig. 2. Elementary space volume of two static cylindrical coils.

Magnetic fluxes pass through these volumes in the vertical and horizontal directions. Their resistances are determined in accordance with the following formulas [4]:

$$R_{mw} = h/(\mu \int_{\alpha_1}^{\alpha_2}\int_{\rho_1}^{\rho_2} \rho d\alpha d\rho) = 2h/(\mu\pi(\rho_2^2 - \rho_1^2)\ ; \qquad (6)$$

$$R_{mh} = \int_{\rho_1}^{\rho_2} d\rho/(\mu h\alpha\rho) = \ln(\rho_2/\rho_1)/(\mu h\pi)\ , \qquad (7)$$

where $h, \alpha, \rho_1, \rho_2$ are the geometric dimensions of the elementary volume, μ represents the absolute magnetic permeability of a section of the elementary volume (in our case $\mu=\mu 0$).

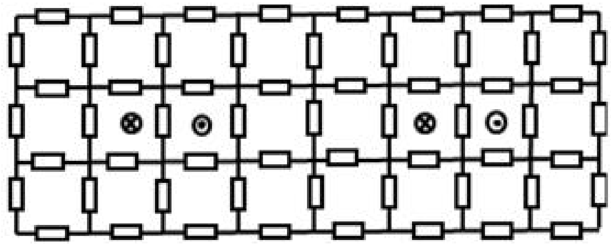

Fig. 3. Magnetic circuit diagram of two static magnetically coupled coils.

Example 2.

In the air, on the same axis there are two magnetically coupled coils, the copper turns of which are in the form of rectangular frames (Fig. 4). The number of turns of each coil and all geometric dimensions are given. The smaller coil rotates inside the larger one at a constant frequency ω. The task is to determine the parameters of the coils.

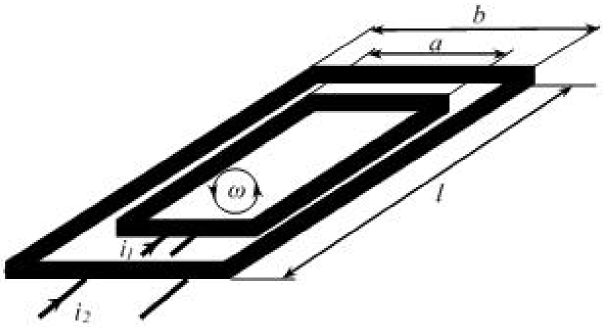

Fig. 4. Two dynamic magnetically coupled coils.

In this case, the arrangement of the coils resembles a simplified configuration of the windings of an electric machine.

If the length of the coil frames l is significantly greater than their width a and b, then the magnetic field can be considered plane-parallel, and then it is expedient to divide the space into elementary volumes by concentric surfaces and diametrical planes along the entire length of the frames. The elementary volumes in this case will be sectors of hollow cylinders (Fig. 5).

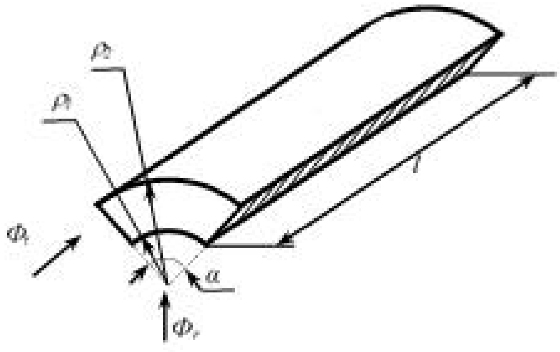

Fig. 5. Elementary space volume of two dynamic rectangular coils.

Magnetic fluxes pass through these volumes in the tangential and radial directions. Their resistances are determined according to the following formulas:

$$R_{m\tau} = \int_{\alpha_1}^{\alpha_2} \rho d\alpha \,/\, (\mu \int_{\rho_1}^{\rho_2} l d\rho) = \frac{\alpha(\rho_1 + \rho_2)}{(2\mu l(\rho_2 - \rho_1))}; \quad (8)$$

$$R_{mr} = \int_{\rho_1}^{\rho_2} d\rho \,/\, (\mu l \alpha \rho) = ln(\rho_2 / \rho_1) / (\mu l \alpha); \quad (9)$$

where l, α, ρ_1, ρ_2 are the geometric dimensions of the elementary volume, μ is the absolute magnetic permeability of a section of the elementary volume (in our case $\mu = \mu 0$).

The planar diagram of the magnetic circuit of two dynamic coils in this case has the form shown in Fig. 6.

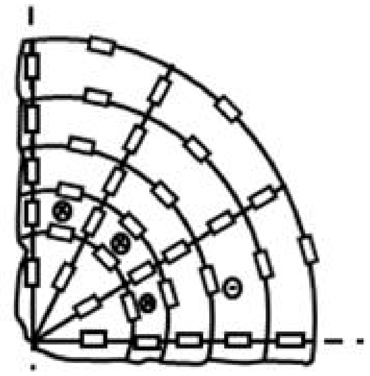

Fig. 6. Fragment of the magnetic circuit diagram of two dynamic magnetically coupled coils.

In mathematical models of dynamic objects, which are moving coils, the elements of the matrices W and Wt are determined by the rotation coordinate γ. In the case of a constant angular frequency of rotation ω

$$\gamma = \omega t. \quad (10)$$

If the inductances of the moving linear coils are constant values, then their mutual inductances are functions of the rotation coordinate γ. This value, as can be seen from (10), depends on the time t. To determine mutual inductances of coils, when one coil rotates inside the other, it is advisable to choose such a time step when γ is equal to the angular sector of the equivalent volume α. (Fig. 5). Then all the turns from the previous circuit pass into the next circuit tha greatly simplifies the determination of the elements of the matrix W.

Based on the proposed method, we have calculated the inductances and mutual inductances of the coils, which are described in examples 1 and 2.

In example 1, the following initial data were specified:

the diameter of the copper wires of the coils is 1 mm, the first coil has 100 turns, the second – 100 turns, the radius of the turns of the first coil – 3 sm, the second – 4 sm. The distance between the axes of the coils – 10 sm.

As a result of the calculation, the following parameters of the coils were obtained:

$R_1 = 0.42$ Ohm, $L_1 = 1.023$ mHn, $M_{12} = 0.018$ mHn;

$R_2 = 0.56$ Ohm, $L_2 = 1.735$ mHn, $M_{21} = 0.018$ mHn.

As we can see, the principle of reciprocity for static coils with magnetic coupling has been confirmed by calculation, i.e. M12 = M21.

Conclusion

The use of magnetic circuits for simulating magnetic fields created by the currents of inductors makes it possible to calculate the electromagnetic parameters of these coils. This allows creation of completely adequate and, first of all, economical circular mathematical models of devices containing inductors. The developed mathematical models with this approach can be used to design devices and study various modes.

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ВИЗНАЧЕННЯ ПАРАМЕТРІВ МАГНІТОЗВ’ЯЗАНИХ КОТУШОК НА ПІДСТАВІ МАТЕМАТИЧНИХ МОДЕЛЕЙ ЇХ МАГНІТНИХ КІЛ

Всеволод Горячко, Орест Гамола, Тарас Рижий

Запропоновано обчислення електромагнітних параметрів статичних і динамічних магнітозв’язаних котушок індуктивності на підставі математичних моделей їх магнітних кіл. Індуктивність і взаємоіндуктивність таких котушок визначаються з використанням геометричних розмірів самих котушок, взаємного розташування та фізичних параметрів середовища.

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Received: 19.09.2022. Accepted: 05.10.2022