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IDENTIFICATION OF PARAMETERS OF INTERVAL NONLINEAR MODELS OF STATIC SYSTEMS USING MULTIDIMENSIONAL OPTIMIZATION

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Abstract: The article proposes an approach to parametric identification of interval nonlinear models of static systems based on the standard problem of minimizing the root mean square deviation between the values of the modeled characteristics of the static object and the values belonging to the experimental intervals. As a result of expanding the parameter space of nonlinear models by introducing additional coefficients to match the predicted and experimental values into the objective function, a multidimensional optimization problem with a nonlinear multiextremal objective function is obtained. The paper examines the characteristics of the objective function and the convergence of its optimization.

Keywords: interval analysis, interval nonlinear model, static system, parametric identification, multidimensional optimization, objective function, convergence,

1. Introduction

When studying objects, processes, and phenomena in various fields of knowledge, there is a need to establish cause and effect relationships between the observed properties of complex objects and factors reflecting the influence of the external environment on them. At the same time, such objects are considered to be static systems where transient processes are ignored, and models of the "black box" type are developed [1, 2].

To create models of static systems, the interval approach can be used. It considers uncertainty during modeling by operating with interval values of the characteristics of complex objects [4-5]. Interval approach methods have many advantages compared to stochastic methods, namely [6]:

- the interval data analysis does not require the study of the statistical characteristics of factors. In particular, the correlation of factors with the determinant is estimated during the construction of the model: factors that are not correlated with the determinant are removed from the model on the basis of generally accepted hypotheses of interval analysis, for example, when the interval estimate of the factor's weight coefficient includes zero – this indicates the insignificance of this factor;
- interval methods, unlike stochastic ones, make it possible to obtain adequate interval models with given prognostic properties based on small samples of experimental data. It bases on determining the adequacy of the model, which is a consequence of the possibility of solution (compatibility) of the system of interval equations, which ensures the given prognostic properties of the models obtained.

When constructing such models, it is necessary to solve the problems of identifying the structure of the interval model, that is, establishing its general form based on known basis functions and parameters based on the obtained structure. The whole process of building such models can be characterized as a repeated procedure for identifying model parameters in the course of identifying its structure [7]. To date, a number of methods for structural and parametric identification of interval models have been developed for both dynamic and static objects [8-12]. It should be noted that computational problems of both structural and parametric identification of interval models are NP-complex optimization problems. The peculiarities of the problems of structural and parametric identification of interval nonlinear models of static systems are that in the process of optimization it is necessary to search for the global minimum of the objective function and at the same time to bypass or exit from numerous local minima. Therefore, metaheuristic methods of stochastic search are mainly used to solve them [13-17]. In particular, today the most computationally effective methods are those based on swarm intelligence [18-20]. For their implementation, a number of specialized software tools, given, for example, in papers [21-25], have been made. At the same time, researchers of such optimization problems widely use well-known software solutions given in a number of standard packages of application programs, in particular in Global Optimization Toolbox of MATLAB software [26]. The goal of this work is research into the possibility of existing software solutions of Global Optimization

Toolbox of MATLAB in terms of their further application to the problems of structural and parametric identification of interval nonlinear models of static systems. As mentioned above, the problem of structural identification is solved by repeated solution of the problems of parametric identification for theformed candidate structures. Therefore, we will focus our attention only on the problem of parametric identification of interval nonlinear models of static systems, without resorting to the description of the method of forming the structures of candidate models themselves.

2. Features of the formation of the objective function in the optimization problem of parametric identification of interval nonlinear models of static systems

First, let us consider the essence of the optimization problem of parametric identification of interval nonlinear models of static systems.

As a rule, the cause and effect relationships between the observed characteristics of complex objects and factors have a non-linear nature. They are described by nonlinear, both relative to input variables and parameters, algebraic equations in the following form, [8]:

$$Y_0(\vec{X}) = f_{m+1}(\vec{\beta}(\vec{X}))\cdot f_1(\vec{X}) + \ldots + f_{2m}(\vec{\beta}(\vec{X}))\cdot f_m(\vec{X}), \quad (1)$$

where $f_1(X), \ldots, f_m$ is a set of unknown basis functions (of a known class) for input variables; $f_{m+1}(\beta(X)), \ldots, f_{2m}(\beta(X))$ is a set of unknown basis functions (of a known class) for parameters whose values must be estimated; X is the vector of values of input variables, i.e., influencing factors; m is the number of model parameters.

We obtain the results of the experiment under the conditions of uncertainty, which we present in the form of intervals of values [5]:

$$\vec{X}_i \to \left[Y_i^-; Y_i^+\right], i = 1, \ldots, N, \quad (2)$$

where Y_i^-, Y_i^+ are the lower and upper limits of the numerical intervals of the modeled characteristic; N is the total number of observations.

It should be noted that the interval form of presenting the results of experiment (2) is related to the available errors of observation of the output variable Y as a characteristic of the object. In interval data analysis methods, the main assumption is that this error has limited values that are known.

In the interval analysis, the conditions for consistency of the model with experimental interval data are set as follows [1]:

$$Y_0(\vec{X}_i) \in \left[Y_i^-; Y_i^+\right], i = 1, \ldots, N, \quad (3)$$

where $Y_0(X_i)$ means the true unknown value of the initial characteristic of the static object for fixed values of the input variables X_i.

Under the conditions of parametric identification problem, the structure of the interval model is known. In this case, only the parameter values of the model $\beta = (\beta_1, \ldots, \beta_m)(X_i)$ remain unknown. Based on "guaranteedness" condition (3) and expression (1) for the values of the input changes at the i-th observation point, we form a system of interval nonlinear algebraic equations (SILNAR), which are two-sided inequalities at the same time:

$$\begin{cases} Y_1^- \le f_{m+1}(\beta(X_1))\cdot f_1(X_1) + \cdots \\ \quad + f_{2m}(\beta(X_1))\cdot f_m(X_1) \le Y_1^+; \\ \vdots \\ Y_i^- \le f_{m+1}(\beta(X_i))\cdot f_1(X_i) + \cdots \\ \quad + f_{2m}(\beta(X_i))\cdot f_m(X_i) \le Y_i^+; \\ \vdots \\ Y_N^- \le f_{m+1}(\beta(X_N))\cdot f_1(X_N) + \cdots \\ \quad + f_{2m}(\beta(X_N))\cdot f_m(X_N) \le Y_N^+. \end{cases} \quad (4)$$

As you known [27-29], the methods of solving this ISNAR (4) based on the interval data analysis methods are quite complicated, since the domain of solutions itself may be non-convex and even broken.

However, for at least one interval model to be built, it is enough to calculate one solution of SILNAR (4). In this case, interval model (1) on the basis of a point solution is written in the form:

$$\hat{Y}(\vec{X}) = f_{m+1}(\vec{\beta}(\vec{X}))\cdot f_1(\vec{X}) + \ldots + f_{2m}(\vec{\beta}(\vec{X}))\cdot f_m(\vec{X}), \quad (5)$$

Iterative optimization calculation procedures [6] are used to evaluate one solution of SILNAR (4). At the same time, the quality of the obtained parameter estimates is determined using the objective function $\delta(\beta(X))$. The expression for this function is substantiated in works [6, 29]. However, in the case of obtaining a point estimate of model parameters, this approach is unacceptable.

Works [30, 31] propose to modify this criterion function in order to reduce the optimization problem to the standard problem of minimizing the mean square deviation between the values of the modeled characteristics of the static object and the values belonging to the experimental intervals.

To do this, we introduce the notation of the function $I(Y_i, \alpha_i)$ that defines a certain point belonging to the experimental interval in the form of a linear combination of the limits of the experimental values at the measurement points:

$$I(Y_i, \alpha_i) = \alpha_i \cdot Y_i^- + (1 - \alpha_i) \cdot Y_i^+ \quad (6)$$

$$\alpha_i \in [0, 1], i = 1, \ldots, N$$

$$[Y_i^-; Y_i^+], i = 1, \ldots, N$$

Thus, we can write the criterion function in the following form:

$$\Delta(\vec{\beta}(\vec{X}), F, \alpha) = \sum_{i=1}^{N}\left(\hat{Y}(\vec{X}_i, \hat{\vec{\beta}}(\vec{X}_i)) - I(\vec{Y}_i, \alpha_i)\right)^2, \quad (7)$$

where $\beta(X) = (\beta_1(X), \ldots, \beta_m(X))$ is the vector of estimates of model parameters;

$F = \{f_1(X), \ldots, f_m(X), f_{m+1}(\beta(X)), \ldots, f_{2m}(\beta(X))\}$ is a set of structural elements of the model; α_i are the coefficients of the linear combination of the boundaries of the experimental intervals.

Minimizing the value of the function $\Delta(\beta(X), F, \alpha_i)$ ensures obtaining a vector of parameter estimates, which makes it possible to build an adequate interval model, for which condition (3) will be fulfilled. In this case this requirement will have the following form:

$$Y(X_i, \beta(X_i)) \in [Y_i^-; Y_i^+], i = 1, \ldots, N. \quad (8)$$

Thus, [30] shows that if

$$\Delta(\beta(X), F, \alpha_i) = 0, \quad (9)$$

then the obtained solution in the form of a vector of parameters β makes it possible to obtain an adequate interval model.

Thus, the problem of parametric identification of an interval nonlinear model will have the following form of an optimization problem:

$$\Delta(\vec{\beta}(\vec{X}), F, \alpha) \xrightarrow{\hat{\vec{\beta}}(\vec{X}), \hat{\vec{\alpha}}} \min,$$

$$\hat{\vec{\beta}}(\vec{X}) \subset \left[\beta_i^{low}; \beta_i^{up}\right], j = 1, \ldots, m, \quad (10)$$

$$\hat{\vec{\alpha}} \in [0,1], i = 1, \ldots N.$$

Problem (10) is a multidimensional nonlinear optimization problem, even with a linear model structure. Multidimensionality is determined not only by the available parameter vector, but also by the expansion of this parameter space by a set of unknown coefficients $\alpha, i = 1, \ldots, N$. It is also worth noting that if mathematical expression (1) contains a set of nonlinear basis functions for parameters $f_{m+1}(\beta(X)), \ldots, f_{2m}(\beta(X))$, the objective function in the optimization problem of parametric identification of interval nonlinear models of static systems will have a significant number of local minima. Therefore, to solve this problem of parametric identification (10), it is expedient to use the methods implemented in the Global Optimization Toolbox of the MATLAB software [32].

3. Results and their discussion

Let us carry out parametric identification of the models based on known test functions, which will specify the input data, the structure of the model and have a non-linear nature. To consider various cases of the complexity of optimization problem (10) and convergence to the minimum of objective function (7), the test functions [33] were used, in particular, unimodal ones: F1 – Sphere function, F2 – Schwefel's function, F3 – Rosenbroc function; multimodal: F4 – Rastrigin's function, F5 – Ackley's function and multimodal ones with a fixed dimension of input data F_6 and F_7 (see Table 1). The conditions for parameter identification based on optimization problem (10) and test data based on the functions are given in Table 2.

Table 1

Test nonlinear functions

Function	Range of values, X
Unimodal functions	
$F_1(x) = \sum_{i=1}^{n} x^2$	[-100, 100]
$F_2(x) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_j\right)^2$	[-100, 100]
$F_3(x) = \sum_{i=1}^{n-1}\left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	[-30, 30]
Multimodal functions	
$F_4(x) = -20e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x^2}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} + 20 + e$	[-32,768, 32,768]
$F_5(x) = \sum_{i=1}^{n}\left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	[-5, 5]
$F_6(x) = \sum\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	[-5, 5]
$F_7(x) = 4x_1^2 - 2{,}1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	[-5, 5]

For the computational experiment, Global Optimization Toolbox of MATLAB was used [30], in particular, GlobalSearch software tool, which makes it possible to find the global minimum of smooth functions in polynomial time. GlobalSearch uses an interior point algorithm to search for a local solution on the basins of attraction to the optimum (basins). It is used to optimize smooth functions and has a proven quadratic convergence [34]. This uses, a generated set of starting points, which provides the study of all basins of attraction to the optimum of the given search area and finding the global optimum.

The application of GlobalSearch will clearly demonstrate the modality of objective function (7). The results of the experiments are shown in Table 3. The second column of the table displays two-dimensional graphs of the functions that provide the data for modeling. Interval values in the given range (Table 1), obtained on the basis of a relative error of 5%, were used to identify the parameters.

Table 2

Source data for research

Dimensionality, nX	m	Structure of the model
2	2	$Y(X) = x_1^{\beta_1} + x_2^{\beta_2}$
2	2	$Y(X) = x_1^{\beta_1} + (x_1+x_2)^{\beta_2}$
2	3	$Y(X) = \beta_1 (x_{i+1} - x_i^2)^{\beta_2} + (x_i - 1)^{\beta_3}$
2	2	$Y(X) = -\beta_1 e^{-\beta_2 \sqrt{\frac{1}{n}\sum_{i=1}^{n} x^2}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} + \beta_1 + e$
2	2	$Y(X) = \sum_{i=1}^{n} (x_i^2 - \beta_1 cos(\beta_2 x_i) + \beta_1)$
4	2	$Y(X) = \left(\beta_1 - \frac{x_1(\beta_2^2 + \beta_2 x_2)}{\beta_2^2 + \beta_2 x_3 + x_4}\right)^2$
2	11	$Y(X) = \beta_1 x_1^{\beta_2} + \beta_3 x_1^{\beta_4} + \beta_5 x_1^{\beta_6} + \beta_7 x_1 x_2 + +\beta_8 x_2^{\beta_9} + \beta_{10} x_2^{\beta_{11}}$

The third column shows the value of the objective function $\Delta(\beta(X), F, \alpha_i)$ at the optimum points, the last value corresponds to the global minimum of the function. As can be seen for unimodal functions, the number of optima is small and the method demonstrates high convergence, except for the Rosenbroc function, which forms a sprawling area near the optimum, which complicates the convergence of the method. The last column gives the history of the search for the optimal solution in 2D and 3D parameter space.

Fig. 1 and 2 show the graphs of objective function (7) in the 2D space of parameters for the determined coefficients $.\alpha, i = 1, \ldots, N$, which ensure condition (3) of inclusion of predicted values in the experimental corridor. Fig. 1 shows a graph of the function $\beta(X), F, \alpha_i$ for a model based on a two-dimensional unimodal function F1 (Sphere function): $\Delta Y(X) = = x_1^{\beta_1} + x_2^{\beta_2}$, for which a value equal to zero was obtained at the point $\beta(2.0, 2.0)$

In Fig. 2 a graph of the function $\Delta(\beta(X), F, \alpha_i)$ is given, the value of which was obtained during the optimization of the model parameters based on the data and the structure of the Rastrygin multimodal function (F1). The graph demonstrates the presence of several areas with local minima. Therefore, we can conclude about the multimodal nature of objective function (7) in the general case.

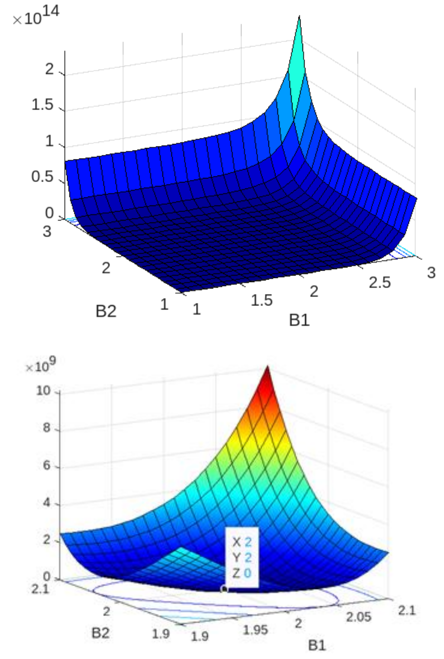


Fig. 1. A function for a model based on the Sphere tion$\Delta(\beta(X), F, \alpha_i)$.

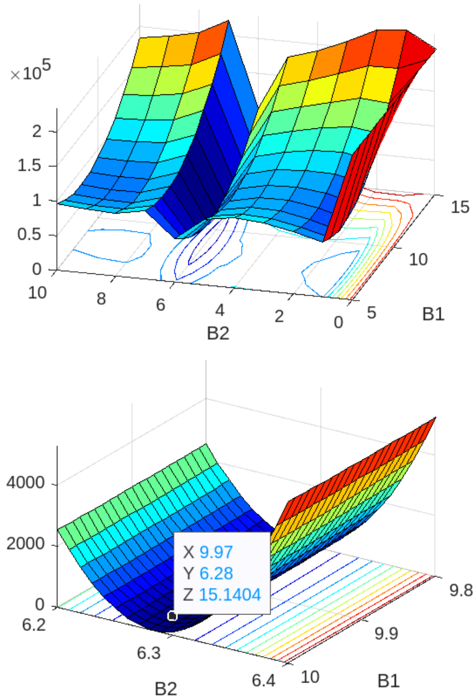


Fig. 2. Function $\Delta(\beta(X), F, \alpha_i)$ for the model based on Rastrigin's function.

Now the issue of choosing computing tools for parametric identification of interval models of static systems based on multidimensional optimization is relevant.

At the same time, the selection criteria are:

– the ability to search for a global minimum,

– low computational complexity,

– independence from input modeling data.

The comparative characteristics of Global Optimization Toolbox of MATLAB tools on the example of finding the global minimum of the Rastrigin function for the two-dimensional case are shown in Table 3.

Table 3

The results of comparative analysis of Global Optimization Toolbox

	Solution		The value of the objective function	The number of objective function calculations
	X1	X2		
patternsearch	19,899	-9.9496	4.9748	174
ga	-0.0042178	-0.0024347	4.7054e-05	9453
particleswarm	-5.2496e-06	2.6546e-05	1.4527e-09	1720
simulannealbnd	0.002453	0.0018028	1.8386e-05	1986
surrogateopt	0.0031	-0.0038	4.7922e-05	300
GlobalSearch	0.1096e-07	0.1096e-07	0	2182

In addition to classical optimization tools based on derivatives, Global Optimization Toolbox contains metaheuristic algorithms, such as genetic, swarm, random search, etc. The genetic algorithms (ga) use a set of starting points (population) and iteratively select the best solutions. The population can migrate on the basins of attraction to the optimum, accordingly, ga can generate a solution based on the best of them.

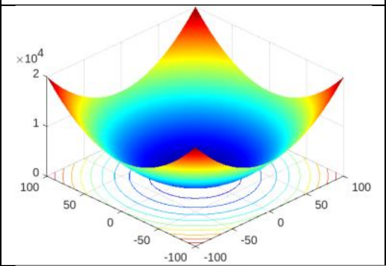
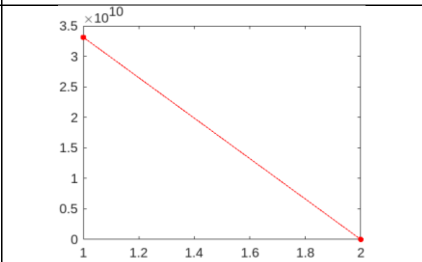
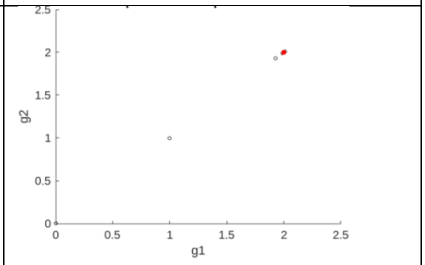
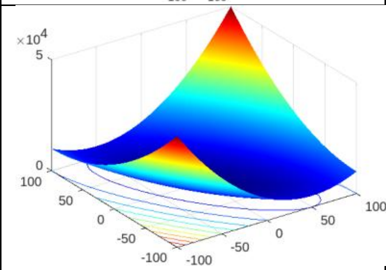
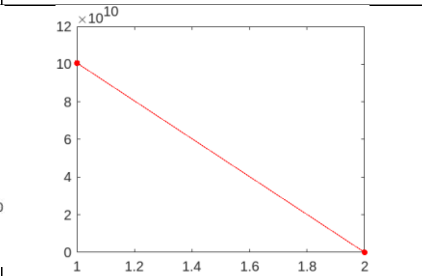
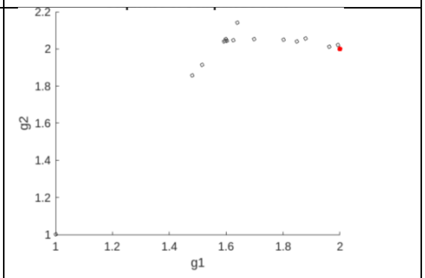
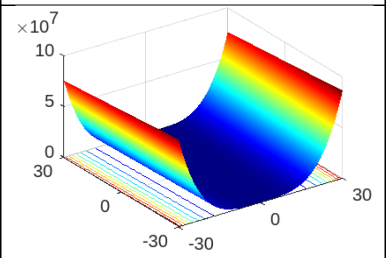
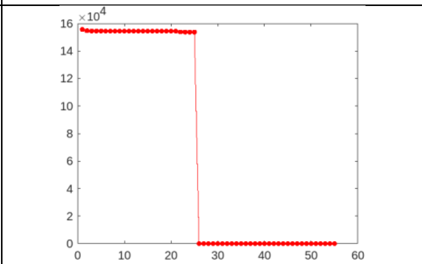
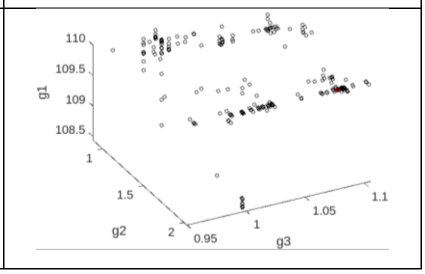
The advantage of swarm intelligence, the particleswarm algorithm (swarm of particles) is that it can simultaneously explore several areas of the optimum based on the presence of researcher particles (scouts).

A random search based on simulannealbnd. As a rule, simulannealbnd chooses a solution if it is better than the previous one. Sometimes a worse point is taken in order to reach another basin of attraction to the optimum.

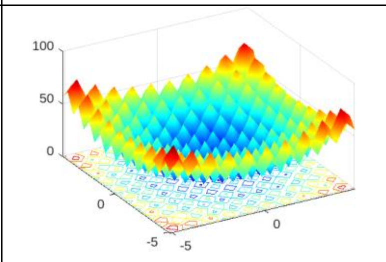
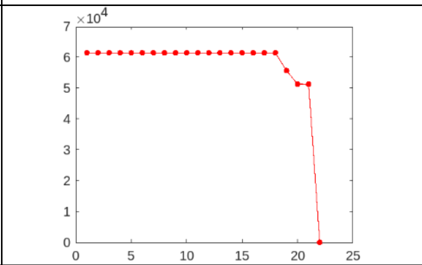
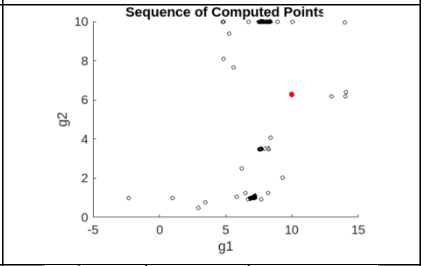
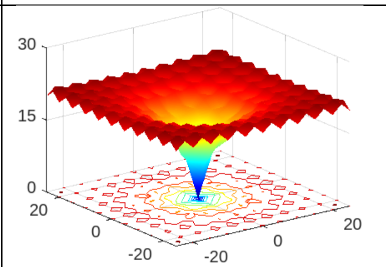
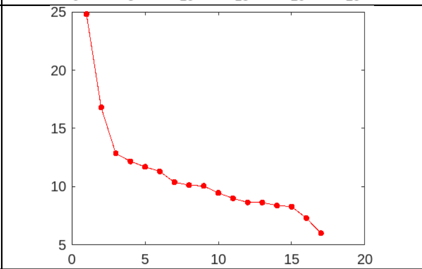
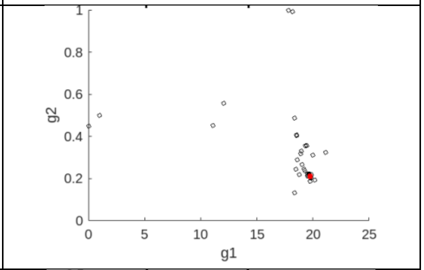
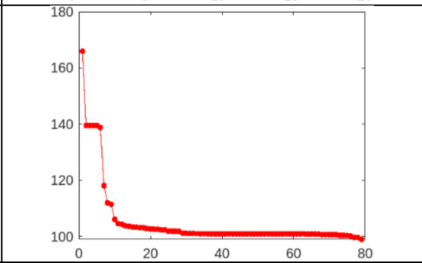
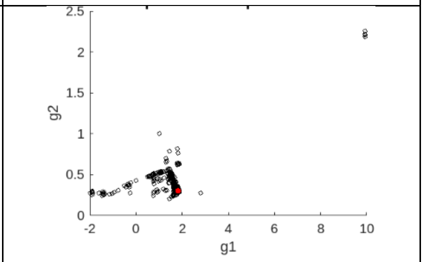
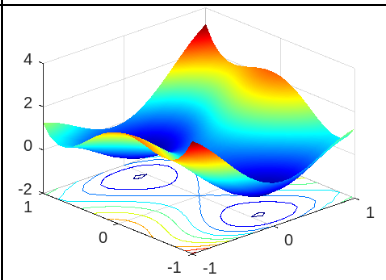
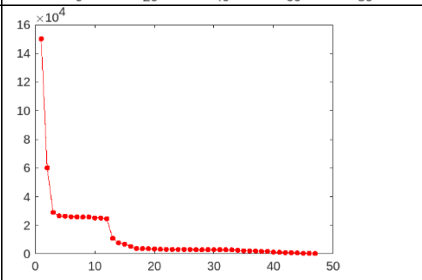
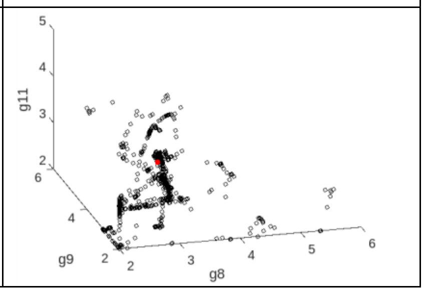
The patternsearch method examines a number of neighboring points before accepting one of them. Under the condition that neighboring points belong to different areas, in fact, the search is carried out in several basins at the same time.

Table 4

Results of numerical experiments of parameter optimization

	A function that specifies data and a non-linear structure	The value of objective function (7) in local and global minima	A sequence of calculated points in the parameter space β
1	2	3	4
F_1			
F_2			
F_3			

Continuation of Table 4

1	2	3	4
F_4			
F_5			
F_6			
F_7			

The surrogate optimization is based on a quasi-random sampling of solutions within given limits based on radial basis functions. Surrogateopt uses a performance function that measures the consistency between the data and the model at the set parameters. Once the current solution cannot be improved, the sample is updated for a wider coverage within the limits. This is another way to find a global surrogateopt solution.

The analysis of the obtained results showed that from the point of view of finding the global minimum, GlobalSearch and the particle swarm method – particleswarm are the best. From the computational point of view, the generally accepted criterion in optimization is the number of calculations of the objective function, and particleswarm – 1720 calculations can also be noted. The optimization based on surrogateopt, which competes with other algorithms, but at the same time is distinguished by computational characteristics, deserves attention and research – 300 calculations of the objective function provided a competitive result: 4.7922e-05 against 1.4527e-09 for particleswarm.

Regarding the third criterion – independence from the input modeling data, it should be noted that GlobalSearch can be legitimately applied to smooth criterion functions for which there is a derivative of search spaces. The multidimensional search space for model parameters of real complex objects is usually unknown and very complex with a large number of local optima, so metaheuristics based on swarm intelligence are a good option for the identification of interval model parameters.

4. Conclusions

The proposed approach to the parametric identification of interval nonlinear models of static systems consists in reducing this identification problem to solving the standard problem of minimizing the root mean square deviation between the values of the modeled characteristics of the static object and the values belonging to the experimental intervals. It is worth noting that this approach leads to the expansion of the parameter space of nonlinear models due to the introduction of additional coefficients α into the objective function, which ensure the consistency of the model-based calculations and experimental data. In this way, we obtain a multidimensional optimization problem with a nonlinear multi-extremal objective function.

Based on numerical experiments, the characteristics of the objective function and the convergence of its optimization were investigated. A comparative analysis of known means of global optimization was carried out in order to choose the optimal method for solving the optimization problem of parameter identification of interval nonlinear models of static systems. It was determined that under the condition of smoothness of the optimization objective function, GlobalSearch based on the interior point method is effective. For conditions of uncertainty of the nature of the objective function, the method of swarm intelligence is optimal.

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ІДЕНТИФІКАЦІЯ ПАРАМЕТРІВ ІНТЕРВАЛЬНИХ НЕЛІНІЙНИХ МОДЕЛЕЙ СТАТИЧНИХ СИСТЕМ ІЗ ЗАСТОСУВАННЯМ БАГАТОВИМІРНОЇ ОПТИМІЗАЦІЇ

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В статті запропоновано підхід до параметричної ідентифікації інтервальних нелінійних моделей статичних систем на основі стандартної задачі мінімізації середньоквадратичного відхилення між значеннями модельованої характеристики статичного об’єкта та значеннями які належать до експериментальних інтервалів. Внаслідок розширення простору параметрів нелінійних моделей за рахунок введення додаткових коефіцієнтів для узгодження прогнозованих та експериментальних значень у функцію мети отримано задачу багатовимірної оптимізації з нелінійною багатоекстремальною функцією мети. В роботі досліджено характеристики функції мети та конвергенцію її оптимізації. Проведено компаративний аналіз відомих засобів глобальної оптимізації з метою вибору оптимального методу розв’язування оптимізаційної задачі ідентифікації параметрів інтервальних нелінійних моделей статичних систем.

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