COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 12, No. 2, 2022

MOTION DYNAMICS OF A MULTICHARGING SYSTEM IN AN ELECTRIC FIELD

Vasil Tchaban

Lviv Polytechnic National University, Ukraine

vtchaban@polynet.lviv.ua

https://doi.org/10.23939/jcpee2022.

Abstract. In electrotechnical research there is a problem of analysis of the interaction of moving charged bodies on their trajectories. Its practical solution is possible only on the basis of an adequate mathematical model. To this end, we have adapted the law of force interaction of stationary charges by Charles Coulomb in the case of motion at all possible speeds. This takes into account the finite rate of propagation of the electrical interaction. Differential equations of motion of a closed system of charged moving bodies in their electric field are obtained. On this basis, the transients in a three-charge proton-electron system are simulated, such as the electromechanical equilibrium of an atom of a periodic table of elements. The simulation results are attached.

Keywords: Coulomb's law of moving charges, finite speed of propagation of the electrical interaction, differential equations of motion, multicharged moving system, Euclidean space, physical time.

1. Introduction.

Much attention has long been paid to the problem of taking into account the interaction of the forces of interaction of individual moving physical bodies on their trajectories. This is especially observed in the study of gravitational fields. One of the most famous problems on this topic is the justification for the precession of the perihelion of Mercury's orbit [1,2]. But, frankly, the real cause of the phenomenon has not been established yet. As for the study of electric fields, it is worth mentioning the problem of constructing electromechanical models of atomic structures of elements of the periodic table in Euclidean space and physical time. Unfortunately, the methods of classical electricity proved to be powerless to solve it, and were practically pushed out of the microworld [3,4]. However, the analysis showed [5,6] that this was done in a hurry. Which has only damaged the unity of nature at the micro, macro and mega levels. The methods of electricity adapted to the case of the motion of charged bodies in an electric field in the case of the microworld do not claim to replace the postulates of quantum physics, but can be used to understand many physical processes in this world and quantify them. This study proposes differential equations of electromechanical motion in a closed system of charged bodies under the force of their resulting electric field. To provide strict mathematical support for the real physical process, we had to, despite the meager geometric dimensions of the microworld (when it comes to it) unconditionally take into account the finite rate of propagation of electrical force interaction! From a cognitive point of view, this study is a continuation of a number of works published on the pages of this journal, such as the last [7,8].

The aim of the work is to develop on a strict mathematical basis nonlinear differential equations of motion of a system of multicharged bodies in an electric field taking into account the finite rate of propagation of electrical interaction. According to the results of numerical integration to identify the interaction of individual charges on their trajectories.

2. Equation of motion of charged physical bodies.

Successful mathematical modeling of transients of moving charged bodies in an electric field in Euclidean space and physical time is possible only on the basis of differential equations of motion, which involve the adapted law of Charles Coulomb in the case of moving charges [6]. The inertial equations of moving interconnected electrical force interaction n masses m are obvious

$$\frac{d\mathbf{v}_i}{dt} = \frac{1}{m_i}\sum_{k=1}^{n}\mathbf{F}_{ik}; \quad \frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i, \; i,k = 1,2,...,n, \quad (1)$$

where $\mathbf{r}_i, \mathbf{v}_i$ are the radius-vector of the trajectory and the vector of the velocity of the i-th mass m_i; $\mathbf{F}_{ik}$ - the vector of the force of electrical interaction of the i-th and k-th bodies; t is the time.

The gravitational interaction in (1) is neglected as insignificant compared to the electrical one.

The force vector is written in general form [6]

$$\mathbf{F}_{i,k} = g\frac{q_i q_k}{r_{ik}^2}\left(1 + \frac{v_{ik}^2}{c^2} + 2\frac{v_{ik}}{c}\mathbf{r}_{ik0}\cdot\mathbf{v}_{ik0}\right)\mathbf{r}_{ik0}, \quad (2)$$

where r_{ik} represents the radius of the distance between charges; v_{ik} is the mutual instantaneous speed of movement; g is the fundamental electric constante; $\mathbf{r}_{ik0}, \mathbf{v}_{ik0}$ are the unit vectors of distance and speed of movement.

The modulus of force vector (2) is described in component elements:

$$F_{Nik} = g\frac{q_i q_k}{r_{ik}^2}, \quad (3)$$

$$F_{Lik} = g\frac{q_i q_k}{r_{ik}^2}\frac{v_{ik}^2}{c^2}, \quad (4)$$

$$F_{Tik} = 2g\frac{q_i q_k}{r_{ik}^2}\left(\frac{v_{ik}}{c}\mathbf{r}_0 \cdot \mathbf{v}_0\right), \quad (5)$$

where F_{Nik} is Coulomb's force, F_{Lik}, F_{Tik} are the velocity tangential and radial components of the force of electrical interaction. It is clear that when $v_{ik} \to 0$ the modulus of force interaction (2) degenerates into (3).

The marginal share in the force interaction of components (4) and (5), based on the speed and orientation characteristics, is obvious

$$\mathbf{F}_L = (0 \div 1)\mathbf{F}_N; \quad \mathbf{F}_T = ((-2) \div (+2))\mathbf{F}_N. \quad (6)$$

In [6,7] it is proved that the component of force (4), due to the tangential component of velocity, coincides completely with the Lorentz force, which in classical electrodynamics presents the force of the magnetic field, or the so-called velocity (relativistic) effect in the electric field. Prolonged to mechanical interaction, it presents the corresponding gravitomagnetic force [9,10].

The functional dependence of the force y (5) on the velocity is higher than in (4), since when the condition $v \le c$ is met, the multiplier v/c in (4) is raised to the second power, and in (5) - to the first. It is component (5) that closes the hitherto unknown triune essence of the forces of electrical interaction and makes it possible to solve the problem on a rigorous mathematical basis.

Applied part. For the sake of certainty, consider the transition process of the interaction of three charged bodies: the helium nucleus q_1 and its two orbital electrons q_2 and q_3 placed in the same orbit, which rotate in the same direction with an angular shift of 180^0. This model brings us closer to the real system of the second firstborn of the periodic table of elements. But it does not claim its adequacy to the physics of the process. Because the physics of the helium microworld is far from being studied like hydrogen. We will only find ourselves in the field of convenient mathematical modeling with a certain approximation to a possible real physical process. In order to find interests, several unrealistic transitional processes will be considered! Our goal is to create a tool for analysis, and the subject area of its application can be anything. In general, our examples are all surreal, because in reality we are dealing with established processes. And the problem of finding space-velocity initial conditions that exclude the transient reaction has not solved yet.

The balance of forces (1) in the system of a fixed nucleus is written as

$$\begin{aligned} &\frac{d\mathbf{v}_2}{dt} = \frac{1}{m_2}(\mathbf{F}_{21} + \mathbf{F}_{23}); \quad \frac{d\mathbf{r}_2}{dt} = \mathbf{v}_2; \\ &\frac{d\mathbf{v}_3}{dt} = \frac{1}{m_3}(\mathbf{F}_{31} + \mathbf{F}_{32}); \quad \frac{d\mathbf{r}_3}{dt} = \mathbf{v}_3. \end{aligned} \quad (7)$$

Vectors between electrons and their mutual velocity are found by the results of integration (7)

$$\mathbf{v}_{23} = \mathbf{v}_2 - \mathbf{v}_3; \quad \mathbf{r}_{23} = \mathbf{r}_2 - \mathbf{r}_3. \quad (8)$$

To simplify the analysis, we will solve the problem in 2D space due to the logical orientation of the Cartesian coordinate system with the center coinciding with the center of the virtual atom,

$$\begin{aligned} &\frac{dv_{2x}}{dt} = \frac{1}{m_2}(F_{21x} - F_{23x}); \quad \frac{dr_{2x}}{dt} = v_{2x}; \\ &\frac{dv_{2y}}{dt} = \frac{1}{m_2}(F_{21y} - F_{23y}); \quad \frac{dr_{2y}}{dt} = v_{2y}; \\ &\frac{dv_{3x}}{dt} = \frac{1}{m_3}(F_{31x} + F_{32x}); \quad \frac{dr_{3x}}{dt} = v_{3x}; \\ &\frac{dv_{3y}}{dt} = \frac{1}{m_3}(F_{31y} + F_{32y}); \quad \frac{dr_{3y}}{dt} = v_{3y}. \end{aligned} \quad (9)$$

Projections of forces of electrical interaction will be written down according to (2)

$$\begin{aligned} F_{21k} &= -g\frac{q_1 q_2 r_{21k}}{r_{21}^3} \times \\ &\times\left(1 + \frac{v_{2k}^2}{c^2} + 2\frac{r_{21x}v_{2x} + r_{21y}v_{2y}}{cr_{21}^2}\right); \\ F_{31k} &= -g\frac{q_1 q_3 r_{31k}}{r_{31}^3} \times \\ &\times\left(1 + \frac{v_{3k}^2}{c^2} + 2\frac{r_{31x}v_{3x} + r_{31y}v_{3y}}{cr_{31}^2}\right); \\ F_{23k} &= -g\frac{q_2 q_3 r_{23k}}{r_{23}^3} \times \\ &\times\left(1 + \frac{v_{23k}^2}{c^2} + 2\frac{r_{23x}v_{23x} + r_{23y}v_{23y}}{cr_{23}^2}\right); \\ F_{32k} &= -F_{23k}; \quad k = x, y; \end{aligned} \quad (10)$$

where

$$r_{23k} = r_{21k} - r_{31k}; \quad v_{23k} = v_{21k} - v_{31k}, \quad k = x, y. \quad (11)$$

$$r_k = \sqrt{r_{kx}^2 + r_{ky}^2}, \quad k = 21, 31, 23;$$
$$v_k = \sqrt{v_{kx}^2 + v_{ky}^2}, \quad k = 2, 3, 23. \tag{12}$$

Expressions (9) - (12) form a complete system of algebraic-differential equations for the analysis of transients in a closed system of three moving charges. To obtain the desired unambiguous solution, it is necessary to set constant parameters $g, q_1, q_2, q_3, m_2, m_3$ and space-velocity initial conditions

$$r_{2k}(0),\ r_{3k}(0);\quad v_{2k}(0),\ v_{3k}(0),\quad k = x, y. \tag{13}$$

Under the condition $F_{23} = F_{32} = 0$ of differential equations (9) describe the independent physical processes of interaction of individual charges [8].

Note that expressions (9), 10) are written taking into account the signs of individual charges, so in the practical analysis (10) should use only their modules.

The results of the simulation. The results of the joint implementation (9) - (12) by the numerical method are shown in Fig. 1 - Fig. 6 at constant parameters:

$$g = 8.987742 \cdot 10^{19};\ m_2 = m_3 = 9.1093826 \cdot 10^{-31};$$
$$q_1 = 2q_2;\ q_2 = q_3 = 1.602176629 \cdot 10^{-19},$$

corresponding to the nucleus and electron shell of the helium atom. All dimensions are in SI.

Fig. 1 shows the time dependence of the hodograph of spatial radius $r_{21}(t)$, obtained under the initial conditions:

$$v_{x2}(0) = 4.0420 \cdot 10^6;\ v_{y2}(0) = 0;\ r_{x2}(0) = 0;$$
$$r_{y2}(0) = 31 \cdot 10^{-12};$$
$$v_{x3}(0) = -v_{x2}(0);\ v_{y3}(0) = 0;\ r_{x3}(0) = 0;$$
$$r_{y3}(0) = -r_{y2}(0).$$

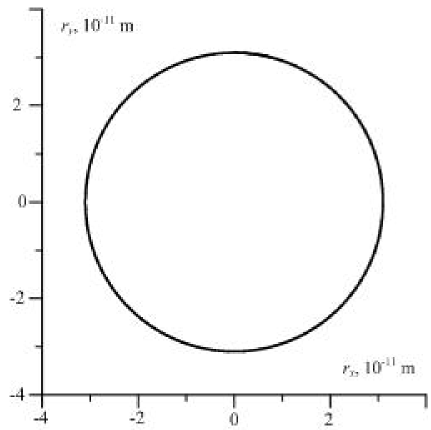

Fig. 1. Hodograph of the distance $r_{21y}(r_{21x})$ of the orbital electron. Duration $T = 0.15 \cdot 10^{-15}$ s.

The duration of the transitional process is $0.15 \cdot 10^{-15}$ s.

In the microworld, only steady-state processes are of practical interest. Such an electromechanical process of helium atom is shown in Fig. 1. We obtained it in accordance with the above spatial-velocity initial conditions, which exclude the transient reaction. They are calculated according to the expression [6, 8]

$$v_{ik}(0) = \sqrt{\frac{g q_i q_k c^2}{m_i c^2 r_{ik}(0) - g q_i q_k}}. \tag{13}$$

Since our task is to calculate the transients, then in the future we will unbalance the original system. First of all, in order to accelerate the course of transients, we will double the charge of the nucleus $q_1 = 2q_{1He}$, where q_{1He} is the charge of the nucleus of the helium atom! Under the same initial conditions, the transient process of the system can be traced as the time dependence of the spatial radius $r_{21}(t)$ in Fig. 2.

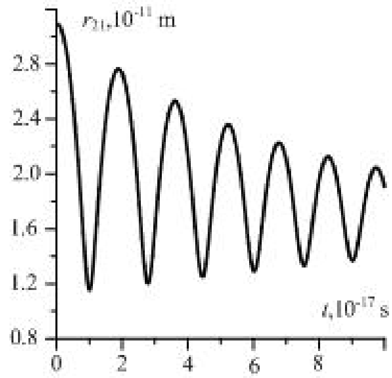

Fig. 2. The time dependence of the radius $r_{21}(t)$ in the transition process shown in Fig. 1 (the charge of the nucleus is doubled).

And now to strengthen the transition process in the system, we will unbalance the initial conditions yet

$$v_{x2}(0) = 0.5469 \cdot 10^5;\ v_{y2}(0) = 0;\ r_{x2}(0) = 0;$$
$$r_{y2}(0) = 8.467 \cdot 10^{-10};$$
$$v_{x3}(0) = -v_{x2}(0);\ v_{y3}(0) = 0;\ r_{x3}(0) = 0;\ r_{y3}(0) = -r_{y2}(0).$$

The course of the corresponding process under such initial conditions is shown in Fig.3.

Previous graphic materials were subject to one goal - a qualitative assessment of the simulation results, because the physical processes behind them were close to the real physical system. Now consider an unreal system similar to the cosmic planetary system. To do this, simply place the orbit of the electron q_3 slightly above the orbit of the electron q_2, as a result of which the initial conditions change

$$v_{x3}(0) = 3.1500 \cdot 10^6;\ v_{y3}(0) = 0;\ r_{x3}(0) = 0;$$
$$r_{y3}(0) = 51 \cdot 10^{-12}.$$

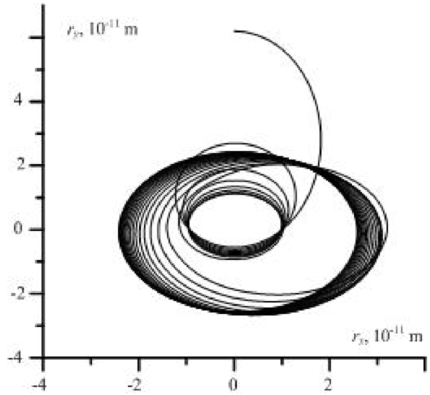

Fig. 3. Hodograph of the distance $r_{21y}(r_{21x})$ corresponding to the transient process of Fig. 1, but calculated with an imbalance of initial conditions.

Fig. 4 shows a hodograph of the trajectory of the closest electron to the nucleus at the planetary location of orbits during the transition process - $1{\cdot}2{\cdot}10^{-15}$ s.

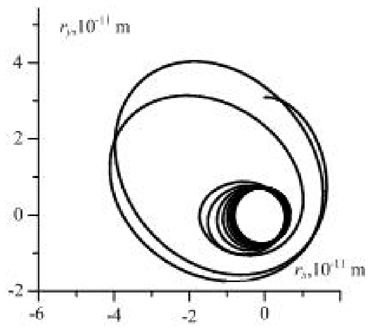

Fig. 4. Hodograph of distance $r_{21y}(r_{21x})$ of the electron at the planetary location of the orbits of electrons:

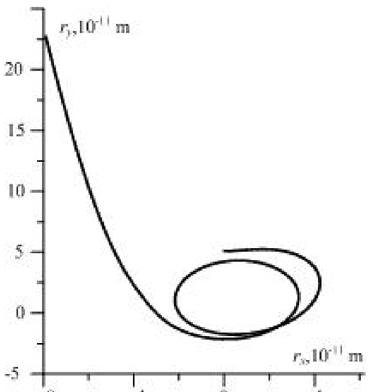

Fig. 5. Hodograph of the distance $r_{31y}(r_{31x})$ of the electron in the transition process, which corresponds to the process of Fig. 4.

In Fig. 5 shows a hodograph of the trajectory of the farthest from the nucleus of the electron in the planetary arrangement of orbits in the transition process, which corresponds to the process of Fig. 4

It turns out that in the harmony of the universe a special place is occupied by the distance between the moving interacting bodies in the force field. This is a hodograph of the distance $r_{23}(t)$ shown in Fig. 6 in the transitional process, which corresponds to the process of Fig. 4 and Fig. 5. Who would have thought of such mathematical beauty. In the electric field, due to the strong interaction, this beauty is somewhat blunted, and in the gravitational field, whose force interaction is much smaller than the electric, this beauty cannot fail to enchant. To do this, just look at the corresponding hodograph of the planets Earth-Venus, shown in Fig. 7.

Doesn't it show a Ukrainian ripe sunflower? Even in Fig. 6, if you look closely, you can see in it the contours of a sea jellyfish over the Russian Black Sea flagship. From this point of view, it is worth rethinking what the great H. Poincare said a hundred years ago: "The objective reality, after all, is what thinking beings have in common. This common side can only be harmony expressed by mathematical laws. this harmony is the only objective reality, the only truth we can achieve, and if I add that the universal harmony of the world is the source of all beauty, it will be clear how we must appreciate those slow and difficult steps forward that are few. little by little they open it to us".

The curve of Fig. 7 is obtained in the same way as a result of solving algebraic-differential equations (9) - (11) under the conditions of electrical and mechanical analogies [6] $q \to m$, $g \to G$, where G is the gravitational constant, and of the corresponding initial conditions.

3. Conclusions.

1. The presented results of calculations of model transients of the force interaction of a system of three moving charged bodies clearly illustrate the possibilities of a new approach in solving a number of fundamental problems of electrodynamics that are impossible by methods of classical physics.
2. The theoretically discovered third component of the electric field force (5), which depends not only on the velocity of the body but also on the spatial orientation of its trajectory, in particular, plays an important and indispensable role in the mathematical description of chaotic motion of charged bodies in an electric field.
3. As for the calculation of precision quantitative characteristics of natural phenomena on the basis of the proposed differential equations of motion, the difficulties are associated only with finding the initial space-velocity conditions laid down by Nature, or the difficulty of finding the appropriate initial conditions that exclude the transiental response.
4. The new results obtained in the work are not only of purely scientific interest, but also epistemological in favor of the unity of the universe at all its levels - mega-, macro- and microworld.

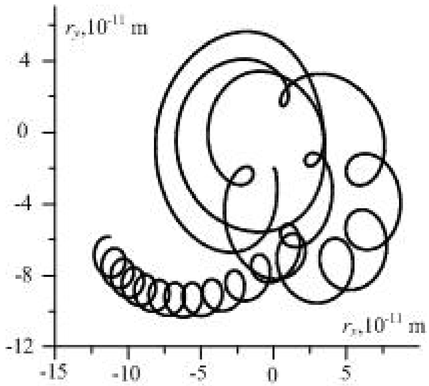

Fig. 6. Hodograph of the distance between the moving electrons at the planetary location of their orbits at a prolonged time of the transitional process $T = 5 \cdot 10^{-15}$ s.

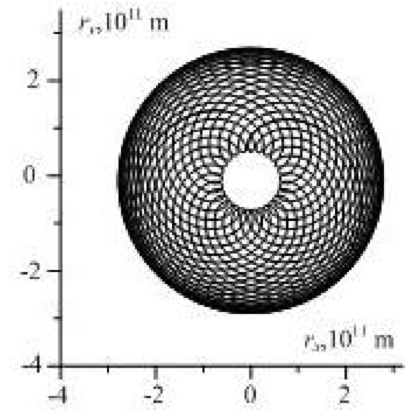

Fig. 7. Hodograph of the distance between the moving planets Earth-Venus. The duration of the transition process $T = 0.75\ 10^{8}$ s, which corresponds to 2.37 terrestrial tropical years.

4. References

[1] N.Roseveare, *Mercury's perihelion. From Le Verrier to Einstein*, Moscow: Mir, p. 244, 1985.

[2] J.Earman and M. Janssen, “Einstein’s Explanation of the Motion of Mercury’s Perihelion”, *The Attraction of Gravitation: New Studies in the History of General Relativity: Einstein Studies*, Boston : Birkhouser, vol. 5, pp. 129–149, 1993.— ISBN 3764336242.

[3] P. A. M. Dirak, *The Principles of Quantum Mechanics*, Moscow: : Nauka, p. 440,1979.

[4] I. O. Vakarchuk, *Quantum mechanics*, Lviv: LNU of Ivan Franko, p.872, 2012. (Ukrainian)

[5] V.Tchaban, “Dynamic of Motion of Electron in Electrical Field”, *Meassuring, Equipment and Metrology*, vol 81, no 2, pp. 39-42, 2020. (DOI https://doi.org/ 10.23939/istcmtm2020. 02.039).

[6] V.Tchaban, *Movement in the gravitational and electric fields*, Lviv: "Space M", p.140, 2021. ISBN 978-617-8055-01-1. (Ukrainian)

[7] V.Tchaban, “Radial Componet of Vortex Ektctric Field Force”, *Computational Problems of Electrical Engineering*, vol. 11, no 1, pp. 32–35, 2021.

[8] V.Tchaban, “Electric intraction of electron-proton tandem”, *Computational Problems of Electrical Engineering*, vol. 11, no 2, pp. 38-42, 2021.

[9] M. L.Ruggiero and A.Tartaglia, *Gravitomagnetic effects. Nuovo Cim*. vol. 117, pp. 743—768, 2002. (gr-qc/0207065).

[10] S.J.Clark and R.W. Tucker, “Gauge symmetry and gravito-electromagnetism”, *Classical and Quantum Gravity : journal*, 2000.

ДИНАМІКА РУХУ ТРИЗАРЯДНОЇ СИСТЕМИ В ЕЛЕКТРИЧНОМУ ПОЛІ

Чабан Василь

В електротехічних дослідженнях постає проблема аналізу взаємовпливу рухомих заряджених тіл на їхні траєкторії. Її практичне розв’язання можливе лише на підставі адекватної математичної моделі. З цією метою нами було адаптовано закон силової взаємодії нерухомих зарядів Ш. Кулона на випадок руху у всеможливому діапазоні швидкостей. При цьому враховано скінчену швидкість поширення електричної взаємодії. Одержано диференціальні рівняння руху замкнутої системи заряджених рухомих тіл у їхньому електричному полі. На цій основі просимульовано перехідні процеси в тризарядній протонно-електронній системі – на зразок електромеханічної рівноваги атома періодичної системи елементів. Результати симуляції додаються.

Vasil Tchaban – D.Sc, full professor at Lviv Polytechnic National University (Ukraine), as well as Lviv Agrarian National University, Editor-in-Chief of Technical News journal. His Doctor-eng. habil degree in Electrical Engineering he obtained at Moscow Energetic University (Russia) in 1987. His research interests are in the areas of mathematical modelling of electromechanical processes in electric and gravity fields theories, surrealistic short story writter. He is the author of 550 scientific publications and 800 surrealistic short stories including 51 books (13 monographs, 17 didactic, 10 humanistic and 11 of the arts), 1500 aphorisms. The total number of publications is over 1500.