



USING MULTI-POSITION INTERFEROMETRY TO DETERMINE THE POSITION OF OBJECTS

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An accurate analytical solution for positioning technologies based on both the difference of distances from the object to reference points (TDOA) and the distances themselves (TOA) is considered. The bijection of the obtained coordinate transformation allows reducing the problem of hyperbolic positioning to the Cartesian coordinate system. It is shown that all localization systems of the same rank with different numbers of sensors reduce to a single canonical form with a fixed number of (virtual) sensors corresponding to the dimension of space plus one. The resulting solution allows us the simultaneous observation of many objects, both close and distant, with determination of the distance to them. The possibilities of using positioning systems with a reduced rank have been analyzed. The synthesis of a sensor system with a higher rank from several separate systems is considered. Algorithms for solving the problem are linear and allow direct reconstruction of the image of objects.

Key words: *multi-position interferometry; localization of objects; antenna; TDOA and TOA technologies.*

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1. Introduction

The problem of object localization is of increased interest due to its importance in a wide variety of applications, such as radio, acoustic, hydro and geolocation, target detection, surveillance and tracking, wireless communication and sensor networks, technical vision, in radio astronomy, in ultrasonic flaw detection and in a number of others [1–12]. Source localization can be performed using TOA (Time of Arrival) or TOF (Time of Flight) measurements, TDOA (Time Difference of Arrival), FDOA (Frequency Difference of Arrival), or AOA angle of arrival (Angle of Arrival), DOA (Direction of Arrival). The application of TDOA is intensively discussed [2, 5, 6, 9, 12] due to its high accuracy and lack of time synchronization with the source. Many different approaches are proposed, mainly focused on the necessary (and sufficient) set of sensors. At the same time no generalizations have been made, from which a number of important conclusions could be drawn.

The work considers a general approach to solving the problem, including the possibility of obtaining an approximate solution under the condition of an inappropriate set of sensors.

In view of the very wide range of possible technical implementations of sensors and the fact that in most cases the difference in arrival times is determined using the interferometry technique, for the sake of presentation we will call the sensor an antenna and the sensor system an interferometer. This in no way reduces the generality of the obtained results.

The basis of interferometry is a two-element interferometer. The antenna serves as a registering element – a sensor. We will consider idealized isotropic antennas, that is, those that receive radiation equally from all directions. If one isotropic receiving antenna were placed in a space filled with elementary incoherent emitters, its response would correspond to the sum of the wave powers from all emitters (taking into account the distances to them). For such a receiving system, there is no specific orientation in space, it perceives space as a zero-dimensional space.

The situation changes radically when there is a system of two antennas, the outputs of which, for simplicity, are multiplied. Two points (antennas placement points) define a straight line in space. The dimension of such a system is one. A selected axis appears in space, and the entire space (no matter how dimensional) is perceived by such a system as a projection on this axis, that is, on a line that passes through the antennas placement points.

Three antennas, placed not on the same line, define the plane on which the entire space is projected. Continuing this reasoning, we can conclude that four antennas placed not on the same plane will add another dimension, that is, to determine the coordinates of radiating elements in three-dimensional space, a four-element interferometer is necessary and sufficient.

Direct reconstruction of the radiation field registered by the antennas can be performed by estimating the radiation power from the element with given coordinates for each pixel (voxel) in the entire area of interest. In the approximation of the incoherence of elementary sources, the desired power corresponds to the value of the correlation function of the antenna signals taken with the appropriate offsets (delays). A similar problem is solved with multilateration technology. Thus, the task of reconstruction is reduced to determining the required delay values for each antenna. At the same time, it should be noted that formally this is an inverse problem. And inverse problems may not have a solution, may have many solutions, the solution may be unstable, etc. The direct reconstruction method has a unique solution, and its stability corresponds to the stability of the correlation function estimate. In this form, the task of reconstruction is seen as correctly posed according to Hadamard [13]. The existence and uniqueness of the solution are ensured by the bijection of transformation of spatial and temporal coordinates (in differences of arrival time), which we will consider below.

2. Geometry of 3D-interferometry

Consider the Fig. 1, which shows the radiation element S and $n + 1$ antennas: the antenna A_0 , on which the origin of coordinates is set to and the antennas A_1, A_2, \dots, A_n , which together with the antenna A_0 form a system of base vectors $\vec{d}_1, \vec{d}_2, \dots, \vec{d}_n$ with the rank of 3.

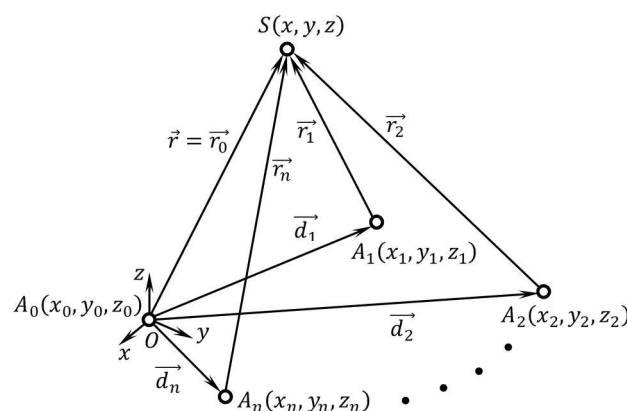


Fig. 1. Scheme of a multi-element interferometer

We denote by $\tau_1, \tau_2, \dots, \tau_n$ the corresponding differences in the propagation times of the waves from point S to the antennas in pairs $(A_0, A_1), (A_0, A_2), \dots, (A_0, A_n)$. The length of the vector will be denoted by the same symbol as the vector, but without the vector icon. If the speed of wave propagation is c , then, using the notation shown in Fig. 1, the following systems of equations can be written:

$$\begin{cases} r - r_1 = c\tau_1 \\ r - r_2 = c\tau_2 \\ \dots \\ r - r_n = c\tau_n \end{cases}, \quad (1)$$

$$\begin{cases} \vec{r} - \vec{d}_1 = \vec{r}_1 \\ \vec{r} - \vec{d}_2 = \vec{r}_2 \\ \dots \\ \vec{r} - \vec{d}_n = \vec{r}_n \end{cases}. \quad (2)$$

After squaring the equations of systems (1) and (2) and extracting r_i^2 , we get

$$\begin{cases} \vec{d}_1 \vec{r} = \frac{1}{2}(d_1^2 - c^2\tau_1^2) + rc\tau_1 \\ \vec{d}_2 \vec{r} = \frac{1}{2}(d_2^2 - c^2\tau_2^2) + rc\tau_2 \\ \dots \\ \vec{d}_n \vec{r} = \frac{1}{2}(d_n^2 - c^2\tau_n^2) + rc\tau_n \end{cases}. \quad (3)$$

For convenience, we introduce the concept of the relative differential coordinate $\eta_i = \frac{c\tau_i}{d_i}$. Note that

η_i is a dimensionless coefficient in the range $-1 \leq \eta_i \leq 1$. In addition, let's divide each equation by d_i^2 , then the system of equations (3) will take the form

$$\begin{cases} \frac{1}{d_1^2} \vec{d}_1 \vec{r} = \frac{1}{d_1} r\eta_1 + \frac{1}{2}(1 - \eta_1^2) \\ \frac{1}{d_2^2} \vec{d}_2 \vec{r} = \frac{1}{d_2} r\eta_2 + \frac{1}{2}(1 - \eta_2^2) \\ \dots \\ \frac{1}{d_n^2} \vec{d}_n \vec{r} = \frac{1}{d_n} r\eta_n + \frac{1}{2}(1 - \eta_n^2) \end{cases}. \quad (4)$$

It is convenient to present the obtained system of equations in matrix form. To do this, let's form a matrix $T(n \times 3)$ by writing in the i -th row the components of the corresponding base vector \vec{d}_i divided by the square of its length:

$$T = \begin{bmatrix} \frac{x_1}{d_1^2} & \frac{y_1}{d_1^2} & \frac{z_1}{d_1^2} \\ \frac{x_2}{d_2^2} & \frac{y_2}{d_2^2} & \frac{z_2}{d_2^2} \\ \dots & \dots & \dots \\ \frac{x_n}{d_n^2} & \frac{y_n}{d_n^2} & \frac{z_n}{d_n^2} \end{bmatrix}. \quad (5)$$

and auxiliary vectors \vec{p}_T and \vec{q}_T , the components of which can be written in the form

$$\vec{p}_T = \begin{bmatrix} \frac{1}{d_1} \eta_1 \\ \frac{1}{d_2} \eta_2 \\ \dots \\ \frac{1}{d_n} \eta_n \end{bmatrix}, \quad (6)$$

$$\vec{q}_T = \frac{1}{2} \begin{bmatrix} 1 - \eta_1^2 \\ 1 - \eta_2^2 \\ \dots \\ 1 - \eta_n^2 \end{bmatrix}.$$

It should be noted that the vector \vec{q}_T can be obtained from the vector \vec{p}_T and we introduce it for convenience. The matrix T will be called the matrix of bases and the index T in the notations \vec{p}_T and \vec{q}_T reflects their correspondence to this matrix.

Denoting the generalized inverse to $T(n \times 3)$ by $T^+(3 \times n)$ and the vectors $\vec{p} = T^+ \vec{p}_T$ and $\vec{q} = T^+ \vec{q}_T$ (whence $\vec{p}_T = T \vec{p}$ and $\vec{q}_T = T \vec{q}$, since $T^+ T = I$ based on the fact that $\text{rang}(T) = 3$ from the condition of the problem), based on the system of equations (4), we write down the equation that gives the way to find the solution of a problem:

$$T \vec{r} = r \vec{p}_T + \vec{q}_T = r T \vec{p} + T \vec{q}. \quad (7)$$

After multiplying (7) on the left by T^+ , we get the main mapping equation

$$\vec{r} = r \vec{p} + \vec{q}. \quad (8)$$

Equation (8) fully describes the problem; it is convenient to interpret it as follows. For a set of values of η in space, there is an auxiliary point Q from which the source S is visible in the direction indicated by the vector \vec{p} . This is a geometric problem about a triangle (Fig. 2), in which one side (q), the angle at it (between vectors \vec{p} and \vec{q}) and the ratio of unknown sides p are known. From this triangle, through the roots of the quadratic equation, the positive value r (with plus sign at the root of the discriminant) is determined, and after substituting it into equation (8) we get the actual vector \vec{r} , that is, the sought coordinates of the source S .

Thus, the system of equations (1) and equation (8) determine the bijection of spatial and relative differential coordinates (delays) in the area of definition of the latter ($-d_i \leq c\tau_i \leq d_i$).

Equation (8) corresponds to the case when the coordinate origin is set on one of the interferometer antennas. In the general case, the following system of equations will be valid for an arbitrary position of the coordinate origin

$$\begin{cases} \vec{r} = \vec{r}_0 + a \\ \vec{r}_0 = r_0 \vec{p} + \vec{q} \end{cases}, \quad (9)$$

where the vector \vec{a} indicates the displacement of the interferometer (that is of the selected antenna A_0) relative to the origin, as shown in Fig. 3. When $\vec{a} = 0$ this system reduces to equation (8), so let's consider it in more detail.

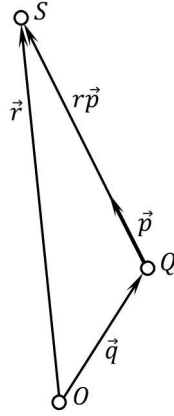


Fig. 2. Triangle for determining the coordinates of the source S from equation (8)

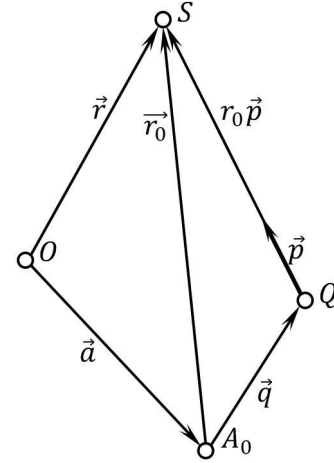


Fig. 3. Determining the coordinates of the source S in the general case of placing the origin of coordinates O

The components of the vector \vec{p} have the content of the direction cosines of the asymptote in equation (8) when the distance r goes to infinity, which is easy to verify by dividing equation (8) by $r \neq 0$

$$\frac{1}{r} \vec{r} = \vec{p} + \frac{1}{r} \vec{q} \quad (10)$$

and putting $r \rightarrow \infty$. The term $\frac{1}{r} \vec{q}$ can then be omitted, taking into account the limited length of the vector \vec{q} , which determines the displacement of the source position from the asymptote.

The essence of the matrix T^+ is that with its help all antennas systems, the rank of the base vector system of which is equal to n , are reduced to a single canonical form of $n+1$ (virtual) antennas and are equivalent to each other. Let us call a canonical interferometer a system with $n+1$ antennas, the first of which placed at the coordinate origin, and the others at unit distances on the coordinate axes. For the canonical interferometer, the lengths of all the bases are equal to unity (the distances are measured in base lengths). If the rank of the base vector system is 3 the matrices T and T^+ are identity 3×3 matrices. Components of the vector $\vec{p} = (\eta_x, \eta_y, \eta_z)$ are numerically equal to the differences in the distances from the source to the corresponding antennas (but their dimension is $length^{-1}$), taken with a plus sign in the positive direction of the corresponding coordinate. That is, for a canonical interferometer

$$\vec{d}_x = (1,0,0), \quad \vec{d}_y = (0,1,0), \quad \vec{d}_z = (0,0,1), \quad T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I. \quad (11)$$

The radius vector of the source \vec{r} in this case is a solution of the mapping equation $\vec{r} = r\vec{p} + \vec{q}$, where the vectors

$$\begin{aligned} \vec{p} &= (\eta_x, \eta_y, \eta_z), \quad -1 \leq \eta_{x,y,z} \leq 1, \\ \vec{q} &= \frac{1}{2} (1 - \eta_x^2, 1 - \eta_y^2, 1 - \eta_z^2). \end{aligned} \quad (12)$$

Equation (8) corresponds to the solution of the problem of multilateration using the TDOA positioning technology, that is, by the differences in the distances from the object to the reference points. It is important to note that, without any changes, it is also valid for TOA technology, that is, in the case when the distance from the object to the reference points is known, since it is not difficult to calculate the necessary differences. At the same time, the solution of equation (8) is only simplified, since the distance r to the object is known a priori. Equation (8) then simply becomes a coordinate transformation rule.

It should be emphasized that no approximations were made when deriving the ratios, the obtained analytical solution of the problem is accurate. As a result, the interferometric system considered by us is not limited to a narrow field of view, does not require the flatness of the object scene, allows simultaneous observation of both distant and nearby objects and, at the same time, determines the distance to them.

3. Interferometer of rank n in n -dimensional space

3D-interferometer in three-dimensional space was considered above. The obtained relations and conclusions remain valid in other dimensions as well. 1-D interferometer has certain features, since in a one-dimensional space (a thin rod, for example, with ultrasonic transducers placed on it), the vector has only one component – x . Therefore, it can be interpreted as a number with a sign, and the length of the vector – as the absolute value of this number – $|x|$.

Determining the coordinates of the source in one-dimensional space has meaning only within the framework of the interferometric system base, that is, when the source lies between the antennas. For all sources that are placed outside the base, the difference in distances to the interferometer antennas is either $+1$ or -1 , depending on where the source lies – to the right or to the left (from both antennas).

Since the canonical 1D-interferometer is formed by two antennas, one of which is placed at the point $x = 0$ and the second at the point $x = 1$, then $x \geq 0$ and $|x| = x$. Matrix $T = [1] = I$, therefore equation (8) will be written in the following form

$$x = x\eta_x + \frac{1}{2}(1 - \eta_x^2). \quad (13)$$

Its solution

$$x = \frac{1}{2}(1 + \eta_x). \quad (14)$$

4. Interferometer of rank $(n-1)$ in n -dimensional space

Decreasing the rank of the base system relative to the dimensionality of space leads to an insufficient number of equations in system (4), so its solution will be ambiguous. Consider a canonical 1D-interferometer in two-dimensional space (this is acoustic emission in a sheet material or ultrasonic defectoscopy on surface waves, for example). In this case, the basis vector is $\vec{d}_1 = (1, 0)$, the basis matrix is $T = [1 \ 0]$, its generalized inverse is $T^+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and their product is $T^+T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq I$, so it cannot be omitted as we did in equation (7). In this case

$$T^+T\vec{r} = rT^+\vec{p}_T + T^+\vec{q}_T \quad (15)$$

or in coordinate form

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \sqrt{x^2 + y^2} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \times [\eta_x] + \frac{1}{2} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \times [1 - \eta_x^2]. \quad (16)$$

The resulting vector equation (16) is equivalent to a system of two equations, where the first of them is:

$$x = \eta_x \sqrt{x^2 + y^2} + \frac{1}{2}(1 - \eta_x^2) \quad (17)$$

and the second is the identity $0=0$. By simple transformations, equation (17) is reduced to the equation

$$(1 - \eta_x^2) \left(x - \frac{1}{2}\right)^2 - \eta_x^2 y^2 = \frac{1}{4} \eta_x^2 (1 - \eta_x^2), \quad (18)$$

which is the equation of a hyperbola shifted along the x -axis by $\frac{1}{2}$. The foci of the hyperbola correspond to the position of the antennas, i.e. $(0,0)$ and $(1,0)$, the parameter η_x sets its eccentricity, and its sign allows you to choose one of the two branches of the hyperbola.

Reasoning similarly, a similar result can be obtained for a 2D-interferometer in three-dimensional space.

In these cases, equation (10) allows us to obtain a one-valued approximate solution under the condition that $r \gg 1$, that is, when large distances are approximated. The error $\frac{1}{r} \vec{q}$ can then be neglected, and equation (10) will take the form

$$\frac{1}{r} \vec{r} \approx \vec{p}. \quad (19)$$

Thus, accepting the approximation of large distances, it becomes possible to obtain the angular coordinates of the source – the components of the vector \vec{p} divided by its length can be approximately taken as the direction cosines of the direction to the source.

5. 1D-interferometer in three-dimensional space

Let's write down the matrix of bases of a canonical one-dimensional interferometer in three-dimensional space, its generalized inverse, and their product:

$$T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad T^+ = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad T^+ T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (20)$$

Reasoning in the same way as it was done above, we will get the first equation of the system of mapping equations (the other 2 equations are the identities $0 = 0$):

$$x = \eta_x \sqrt{x^2 + y^2 + z^2} + \frac{1}{2} (1 - \eta_x^2), \quad (21)$$

which boils down to the following:

$$-(1 - \eta_x^2) \left(x - \frac{1}{2}\right)^2 + \eta_x^2 y^2 + \eta_x^2 z^2 = -\frac{1}{4} \eta_x^2 (1 - \eta_x^2). \quad (22)$$

The obtained equation (22) defines a two-cavity hyperboloid with the center at the point $(\frac{1}{2}, 0, 0)$ and the real axis lying on the coordinate axis, i.e., the solution is ambiguous.

Ensuring the unambiguity of the solution becomes possible by synthesizing a higher-rank interferometer by formally combining several interferometers into one system and reducing the coordinates using the system of equations (9). At the same time, provided the object is stationary, simultaneous observations on the component interferometers are not even required.

Conclusions

In the paper, the problem of location based on positioning technologies is considered and solved in a general form, both by the differences in distances from the object to reference points (TDOA) and by the distances themselves (TOA). An exact analytical solution was obtained, from which the bijection of the transformation of spatial and differential coordinates in spaces of different dimensions follows. This allows you to move the consideration of the problem from hyperbolic positioning to the Cartesian coordinate system. The solution is obtained using the matrix of bases, which is formed by the components of the

vectors of the base system, the vector of asymptotes, and the displacement vector, the components of which depend only on the differences in the distances from the object to the sensors. The base matrix describes the positioning system in an arbitrarily chosen coordinate system, and the corresponding pair of asymptote and displacement vectors determine the position of each of the objects.

It is shown that all localization systems of the same rank with many sensors are reduced to a single canonical form with virtual sensors placed: one at the origin of the coordinates and the others at unit distances on the coordinate axes. Their number corresponds to the dimension of space plus one. Corresponding direct and inverse transformation of coordinates, calculation of new components of the vector of asymptotes and vector of displacement are carried out using the base matrix and the matrix generalized inverse to it.

The resulting solution is not limited to a narrow field of view, does not require the flatness of the object scene, and allows simultaneous observation of many objects, both close and distant, with the determination of the distance to them. The possibilities of using positioning systems, the rank of the base vector system which is lower than the dimension of the space per one, have been analyzed. The synthesis of several separate sensor systems into one with the possibility of rank increase is considered. At the same time, provided the object is stationary, non-simultaneity of observations on component sensor systems is even allowed.

Algorithms for solving the problem are linear and allow direct reconstruction of the image of the object.

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ВИКОРИСТАННЯ БАГАТОПОЗИЦІЙНОЇ ІНТЕРФЕРОМЕТРІЇ ДЛЯ ВИЗНАЧЕННЯ ПОЛОЖЕННЯ ОБ'ЄКТІВ

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Розглянуто точне аналітичне рішення для технологій позиціонування на основі як різниці відстаней від об'єкта до опорних точок (TDOA), так і самих відстаней (TOA). Бієкція отриманого перетворення координат дає змогу звести задачу гіперболічного позиціонування до декартової системи координат. Показано, що всі системи локалізації одного рангу з різними кількостями сенсорів зводяться до єдиної канонічної форми із фіксованою кількістю віртуальних сенсорів, що відповідає розмірності простору плюс один. Отримане рішення дає можливість одночасно спостерігати багато об'єктів, як близьких, так і далеких, з визначенням відстані до них. Проаналізовано можливості використання систем позиціонування з пониженим рангом. Розглянуто синтез сенсорної системи з вищим рангом із кількох окремих систем. Алгоритми розв'язання задачі є лінійними й уможливають пряму реконструкцію зображення об'єктів.

Ключові слова: багатопозиційна інтерферометрія; локалізація об'єктів; антена; TDOA і TOA.