Vol. 9, No. 3, 2023

Viacheslav Pasika¹, Andy Augousti², Olena Lanets³, Oleh Parashchyn⁴

Department of Technical Mechanics and Dynamics of Machines, Lviv Polytechnic National University, 12, S. Bandery str., Lviv, Ukraine, e-mail: paswr@meta.ua, ORCID 0000-0002-2226-7414
 Department of Mechanical Engineering, Kingston University, United Kingdom, London, Penrhyn Road, Kingston-upon-Thames Surrey KT1 2EE, e-mail: augousti@kingston.ac.uk, ORCID 0000-0003-3000-9332
 Department of Technical Mechanics and Dynamics of Machines, Lviv Polytechnic National University, 12, S. Bandery str., Lviv, Ukraine, e-mail: olena.v.lanets@lpnu.ua, ORCID 0000-0001-7149-0957
 Department of Technical Mechanics and Dynamics of Machines, Lviv Polytechnic National University, 12, S. Bandery str., Lviv, Ukraine, e-mail: tmdm.dept@lpnu.ua, ORCID 0000-0002-2970-8163

REDUCING THE KINETIC POWER OF THE CRANK PRESS MACHINE

Received: August 22, 2023 / Revised: September 5, 2023 / Accepted: September 26, 2023

© Pasika V., Augousti A., Lanets O., Parashchyn O., 2023

https://doi.org/10.23939/ujmems2023.03.021

Abstract. Crank presses belong to the class of machines in which the payload (stamping force) acts for a short period of time at the end of the working stroke. Since the power of a force is the product of the force times the speed, it is possible to reduce the power of a specific force only by reducing the speed of the point of action of the force. The kinematic characteristics of slider-crank mechanisms (SCMs), which are the main mechanism of crank presses, are qualitatively the same and cannot be changed. The speed of the slider, as a working body, is the most influenced by the rotation frequency and the crank's length. However, reducing the frequency of rotation leads to a decrease in the productivity of the press, and a decrease in the length of the crank is limited by the design possibilities and technological process of stamping.

The purpose of this work is to reduce the kinetic power of the main working mechanism of crank presses by redistributing the start-up and run-out phases of the working body and designing the corresponding structural diagram of the SCM.

Research methodology. A non-standard approach to reducing the kinetic power of crank presses is proposed. It is suggested to change the distribution of the run-up and run-out phases of the slider (punch) in order to reduce its speed in the range of the punching force to achieve this. To change the run-up and run-out time, a slider-crank mechanism with a programmable change in the length of the crank with a fixed cam is proposed.

As a result of this research, an asymmetric cosine law of the movement of the slide was analyzed and it was synthesized into a variable crank length that ensures the movement of the slider according to the determined law. As far as we aware this is the first research that has been carried out on crank presses with the aim of reducing the kinetic power without reducing the value of the stamping force and press productivity. For a given punching force and an asymmetric cosine law for the punch motion, the kinetic power of the press is reduced by 31.4 %. This will reduce the mass and/or radius of the flywheel.

A list of planned further studies is: the influence of different functions for punch motion, functions for changing the punching force, and an investigation of the value and position of the force interval on the kinetic power of the press.

Keywords Crank presses, synthesis of the laws of periodic motion, slider-crank mechanisms.

Introduction

Crank presses are categorized as machines in which the stamping force operates momentarily at the culmination of the working stroke. The link between force and power, defined as the product of force and velocity, underpins the fundamental principle that reducing power necessitates the attenuation of the speed at which force is applied. Today, there is a huge number of crank presses for various technological purposes [1]. The main working mechanism of crank presses is a slider-crank mechanism, and for crankshaft presses there are multi-link lever mechanisms. Presses always contain a flywheel, the kinetic energy of which is used to overcome the forces of resistance to plastic deformation of bodies. Flywheels are the largest movable part of presses in terms of their size and mass [2]. In the work [3] authors research minimizing the weight of the press while maximizing its stiffness. There is a specific example involving a mechanical press with a nominal force of 80 MN. The authors emphasize their aim to create lighter and stiffer press designs using modern computational optimization methods.

Reducing of the kinetic energy of the flywheel for a given stamping force (normal force), which is regulated, simultaneously leads to a decrease in the power of the drive, and adds to the sustainability of the manufacturing process.

Problem Statement

The peculiarity of crank presses is that the pressing process takes place at the end of the working stroke during a short period of time with the angle of the crank continuing to change. The kinetic energy of the moving masses of the main mechanism is not enough to ensure the required coefficient of motion irregularities. In such cases a flywheel is used, the moment of inertia of which is proportional to the power of the press drive. Standard crank presses have a number of disadvantages:

- it is impossible to reduce the kinetic power of the press, and therefore the mass-dimensional characteristics of the flywheel without reducing the power of the press;
- it is impossible to reduce the speed of the slide (punch) due to the design characteristics of the main mechanism without reducing the productivity of the press, which therefore does not allow one to obtain significant forces on the slider.

It is possible to reduce the slider's speed in the SCM during the run-out phase if the time of the run-up and run-out phases is redistributed. If the run-up time is reduced, and the run-out time, accordingly, increases, then during the interval in which the punching force is applied (during the run-out phase) then the speed of the slider will be lower, and, accordingly, the required kinetic power of the press will be lower. A flywheel that provides the necessary kinetic power will have a smaller mass and/or dimensions. Currently it is impossible to ensure the desired movement of the SCM slider by using asymmetric law (eccentricity does not significantly affect asymmetry). This problem is solved by crankshaft presses, but they are multi-linked, have a lower efficiency, are more complex structurally, and have greater weight and dimensions.

Review of Modern Information Sources on the Subject of the Paper

In the work [4] an innovative wireless communication-based crank slider mechanism for multiparameter friction approaches is investigated and dynamic modeling and, the system's variables are analyzed, where the performance evaluation shows an accuracy score of 98.45 %, a sensitivity rating of 97.34 %, a formidable 96.34 % for the crank slider's effectiveness, and an impressive 97.34 % for recall.

In the reference [5] the vibration modes of the punching press were investigated. With the aim to predict the kinematic state during different conditions and the effects of on the motor speed, multidomain model of the punching press was established, which allowed to improve the design of the press due to

creation of multidomain model what allowed to reduce the vibration. The issue of press machines design and new design concepts is most comprehensively considered in references [8, 9]. Research on the basic systems and components of the presses was carried out here in order to improve the technical level of the entire press design. The structural schemes of multi-link presses, in which the movement of the slider is smaller and closer to a given fixed speed, are investigated. Such performance characteristics for the slider can only be provided by multi-link mechanisms. The research focuses on the problem of press design, which concentrates on reducing the volume of metal utilised, on more efficient use of energy and ensuring additional requirements for the movement of the slider according to production requirements. Reference [10] used two varints to describe the spatial slider—crank mechanism to obtain a geometric constraint equation in the spatial coordinates. Reference [11] represents a new identification method for the parameters of a slider-crank mechanism. The dynamic behavior has been studied in [12, 13], including research on the effect of various parameters of the mechanisms including crank length, and the development of a mathematical model that takes into account the dynamics of the slider-connecting rod–crank system.

The analysis of the processed literature showed that the issue of reducing metal consumption while simultaneously providing improved characteristics for the slider movement in crank presses has not been identified in the literature.

Objectives and Problems of Research

The purpose of the research is to reduce the kinetic power of the main working mechanism of crank presses by redistributing the start-up and run-out phases of the working body and designing the corresponding structural diagram of the CSM.

To achieve the purpose of the research, it is necessary to solve the following tasks:

- synthesize an appropriate periodic law of motion, which ensures movement of the slider according to an asymmetric law;
- to suggest and synthesize a structural diagram for the main mechanism of the press, by which it is possible to obtain an asymmetric law of slider motion;
- to carry out a kinematic analysis of the proposed structural scheme for which the kinetic power of the press will be smaller compared to the unmodified mechanism.

Main Material Presentation

This research belongs to the first two stages of machine design, which are decisive in the creation of new, highly efficient machines with low weight, size characteristics, and energy consumption.

In order to achieve the purpose of the research, it is necessary to solve two tasks: to create an asymmetric form for the law of periodical motion (LPM) of a CSM, which will ensure the motion of the slider according to the synthesized form. As is well-known for existing systems, the law of motion of the main CSM slider is symmetric harmonic, in which the start-up time and run-out time are the same. Therefore, we synthesize the simplest harmonic law, which is a cosine asymmetric law, the start-up and run-out of which are described by different cosines. The necessary law of motion of the working body is synthesized in an invariant form for a complete, combined, asymmetric harmonic law.

Run-up phase.

Let us write an expression for the change in acceleration during the run-up phase in the form of an acceleration invariant (Fig. 1) [6]

$$c_{k_{\rm l}} = C_{\rm p} \cos \left[\frac{\pi}{2k_{\rm p}} k_{\rm l} \right],\tag{1}$$

where C_p is the acceleration peak constant, and $0 \le k_1 \le k_p$ is dimensionless time in the run-up phase.

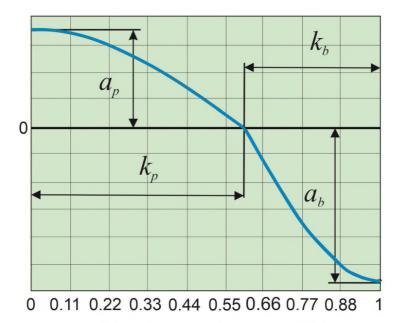


Fig. 1. Cosine asymmetric homogeneous LPM

Lets integrate twice, under the initial conditions $k_1 = 0 \rightarrow a_{k_1} = 0$, $b_{k_1} = 0$, and obtain dependencies for the invariants of speed b_{k_1} and displacement a_{k_1} :

$$a_{k_{1}} = C_{p} \frac{4k_{p}^{2}}{\pi^{2}} \left[1 - \cos\left(\frac{\pi}{2k_{p}}k_{1}\right) \right]; \qquad b_{k_{1}} = C_{p} \frac{2k_{p}}{\pi} \sin\left[\frac{\pi}{2k_{p}}k_{1}\right]$$
(2)

At the end of the period for $k_1 = k_p$

$$a_{\rm p} = c_{\rm p} \frac{4k_{\rm p}^2}{\pi^2},$$
 $b_{\rm p} = a_{\rm p} \frac{2k_{\rm p}}{\pi}.$

Run-out phase.

During the run-out period, we present the change in the acceleration invariant in the form:

$$c_{k_2} = -C_{\rm B} \cos\left(\frac{\pi}{2} - \frac{\pi}{2k_{\rm B}} k_2\right),\tag{3}$$

where $0 \le k_2 \le k_{\rm B}$ represents dimensionless time in the run-out phase.

We obtain next expressions for the invariants of velocities and displacements in the run-out phase after double integration of (3) and taking into account the boundary conditions for $k_2 = 0 \rightarrow c_2 = 0$ and for $k_2 = k_{_{\rm B}} \rightarrow c_2 = -c_{_{\rm B}}$:

$$a_{k_{2}} = C_{p} \frac{4k_{p}^{2}}{\pi^{2}k_{ac}} \left[k_{ac} + \sin\left(\frac{\pi k_{ac}}{2k_{p}}k_{2}\right) \right], \quad b_{k_{2}} = C_{p} \frac{2k_{p}}{\pi} \cos\left(\frac{\pi k_{ac}}{2k_{p}}k_{2}\right), \quad c_{k_{2}} = -C_{B} \sin\left(\frac{\pi}{2k_{B}}k_{2}\right)$$

$$(4)$$

where $C_p = \frac{\pi^2 k_{ac}}{4k_p^2 (1+k_{ac})}$, $C_B = k_{ac} C_p$ - acceleration peak constants during the run-up and run-out phases;

 $k_{\rm ac} = k_{\rm p}/k_{\rm B}$ – coefficient of asymmetry.

The synthesized law of motion unites arrays $a_{k_1}(2)$ and $a_{k_2}(4)$

$$a_k = a_{k_1} \cup a_{k_2}. \tag{5}$$

The main value of crank machines, which is regulated by standards, is the nominal force $P_{\rm H}$ developed by the slider of the CSM at some interval before its extreme lower position. We also present this force and kinetic power of the press in an invariant form. If the invariant of the normal force is p_k , therefore, we calculate the invariant of the kinetic power of the force from the dependence $u_k = p_k b_k$ [6, 7].

To build specific graphic dependencies, we will take the nominal force as having parabolic dependence on time $P_{\rm H} = 10^3 t^2 kN$. Let's move from the variable t to the variable k_f taking into account that $t = k_f[T]$

$$p_{k} = \frac{P_{H}}{10^{3} \lceil T^{2} \rceil} = \frac{k_{f}^{2} \lceil T^{2} \rceil 10^{3}}{10^{3} \lceil T^{2} \rceil} = k_{f}^{2},$$

where $k_f = 0...1$ is relative time, t is current time, and [T] is the time of the stamping force action.

The real value of the nominal force is equal to

$$P_{\mu} = p_{\nu} 10^{3} [T^{2}]$$
.

For an asymmetric law, as an example, we take k_p =0,15; k_p =1- k_p =0,85, and the beginning of force action take as a_F =0,95.

Let's determine the necessary values for further calculations

$$k_{\rm ac} = k_{\rm p}/k_{\rm b} = 0.15/0.85 = 0.176; \quad C_{\rm p} = \frac{\pi^2 \, 0.176}{4 \cdot 0.15^2 \, (1 + 0.176)} = 16.45 \; ,$$

The results of calculations are shown in Fig. 2.

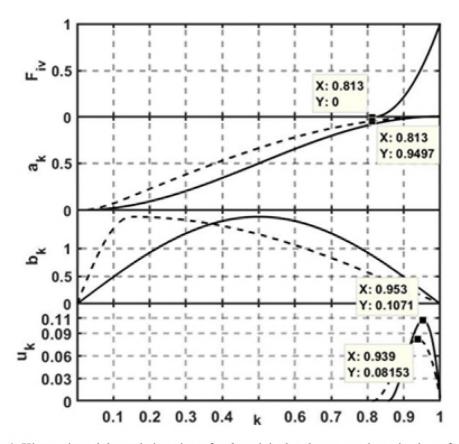


Fig. 2. Kinematic and dynamic invariants for the original and asymmetric cosine law of motion initial, ----- asymmetric

Viacheslav Pasika, Andy Augousti, Olena Lanets, Oleh Parashchyn

In order to plot invariant graph of the force p_k , it is necessary to determine such initial point k_F at which the force will start to act. We will determine the initial position using the example of the asymmetric law. According to the condition of the problem, the stamping force acts during the run-out period in the interval from 95% of the stroke [S] and untill the end. The entire stroke corresponds to the variable k=1, so the force will start to act when the displacement invariant equals $a_k=0.95[S]=0.95$. From the first expression (4) we determine k_2 at the run-up period under the condition that $a_{k_2}=0.95$, and add the run-up period k_p to the obtained value k_2 .

$$\begin{split} k_F &\equiv k_2 + k_{\rm p} = \frac{2k_p}{\pi k_{\rm ac}} \arcsin \left[k_{\rm ac} \left(\frac{0.95\pi^2}{4C_{\rm p}k_{\rm p}^2} - 1 \right) \right] + k_{\rm p} = \\ &= \frac{2.0.15}{\pi \cdot 0.176} \arcsin \left[0.176 \left(\frac{0.95 \cdot \pi^2}{4.16.45 \cdot 0.15^2} - 1 \right) \right] + 0.15 = 0.813. \end{split}$$

We check the correctness of the calculations from the force invariant graph (Fig. 2), where we see, that the force invariant is $p_k \equiv y = 0$ for $k \equiv x = 0.831$. In addition, on the graph of displacements a_k we see that $k \equiv x = 0.831$ of the displacement $a_k \equiv y = 0.9495 \approx 0.95$, which corresponds to the conditions given in problem. Therefore, the initial position of the action of the force is determined correctly.

For a force that depends parabolically on time, the coefficient of reduction of kinetic power (Fig. 2) is equal to $n_{\text{m}} = u_{k_{\text{l}} \text{max}} / u_{k_{\text{l}} \text{max}} = 1,314$, where $u_{k_{\text{l}} \text{max}} = 0,1071$ and $u_{k_{\text{l}} \text{max}} = 0,08153$ are maximum values of the power invariant for the original and asymmetric cosine law of motion. Therefore, changing the law of the slider motion from the initial cosine to an asymmetric form will lead to a decrease in the kinetic power by 31,4% for the same stamping force. Research shows that the coefficient n_{m} increases as the asymmetry coefficient increases k_{ac} . As the interval of force action decreases, the coefficient n_{m} behaves differently, but the general tendency is towards an insignificant decrease. For another law of change of stamping force obtained in Fig. 2 the power invariant will not change qualitatively.

The research carried out shows that the power reduction factor practically does not change for the stamping force, which grows faster, that is, it has a cubic and higher degree of dependence on time.

Synthesis of the structural scheme of KSM.

To ensure that the motion of the slider is indeed according to the synthesized invariant a_k (5), a combined CSM with a variable crank length was used. Fig. 3 shows one of the possible structural schemes. In such mechanisms, connecting rod 2 and crank 1 are connected by an additional slider 4, to which a roller 5 is hinged, which rolls around a fixed cam and thereby changes the length of the crank OA. Such a mechanism has two degrees of freedom: the rotation of the crank with a constant angular velocity ω_1 =const and the motion of the slider 4 relative to the crank. The task of CSM synthesis is to determine such a law of displacement for slider 4 (radius-vector of the fixed cam) for which the motion of slider 2 will occur according to the synthesized law (5).

We will determine the variable length of the crank according to [9], where we accept the eccentricity of the mechanism e=0, and combine the beginning of the right coordinate system with the point of rotation of the crank. The abscissa is directed down. With these remarks, the length of the crank in real sizes will be as follows:

$$r_{B} = x_{B}(\varphi_{1}) \cdot \cos(\varphi_{1}) - \operatorname{sign}(\varphi_{\pi} - \varphi_{1}) \sqrt{\left[x_{\pi}(\varphi_{\pi}) \cdot \sin(\varphi_{\pi})\right]^{2} - \left[x_{B}(\varphi) \cdot \sin(\varphi_{1})\right]^{2}}$$
(6)

where $x_B(\varphi_1) = l_2 - l_1 + a_k[S]$ is the abscissa of the slider and it is expressed through the synthesized law of motion a_k and the stroke of the slider is $S = 2l_1$, l_2 and l_1 are the length of the connecting rod and the

crank at the beginning of the working stroke respectively, and φ_l is the angle of the crank turning, which is counted from the axis of the abscissa, φ_n and $X_n(\varphi_n)$ are the angle of the crank turning and the abscissa of the slider, when the crank and the connecting rod are mutually perpendicular.

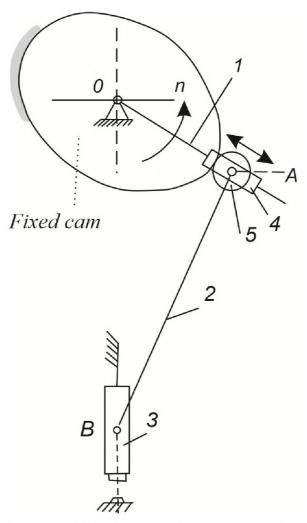


Fig. 3. Structural scheme of the combined mechanism

The research carried out in [6] shows that the angle of the crank inclination $\phi_l = \phi_\pi$ for which the crank and connecting rod are mutually perpendicular, corresponds to the minimum of the nonlinear transcendental equation

$$l_2^2 - \left[x_B(\varphi_1) \sin(\varphi_1) \right]^2 = 0, \tag{7}$$

which we solve numerically. We build the left part of the equation at the stage of the working stroke for changes in the angle $\pi \le \phi_l \le 2\pi$ and determine the angle $\phi_l = \phi_n$ at which the left part of the equation take a minimum value (Fig. 4, curve 2). We can see in the figure 4, that the crank and connecting rod are mutually perpendicular at the angle $\phi_n = 279.3^{\circ}$. We substitute the angle ϕ_n value in (7) and calculate the synthesized length of the connecting rod

$$l_{2c} = x_B(\varphi_{\Pi})\sin(\varphi_{\Pi}). \tag{8}$$

Substitute the expressions for $x_B(\varphi_1)$, φ_{π} and l_{2e} in (6) and obtain the necessary variable of the length of the crank (radius vector of the theoretical profile of the fixed cam) for the movement

of the slider according to the synthesized law (5). When we substitute $l_2 = l_{2c}$ in (7), then we get curve 1 (Fig. 4). Incidentally, for the original CSM the left part of (7) always qualitatively corresponds to curve 1.

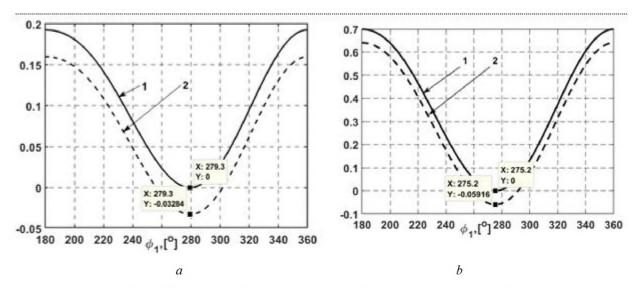


Fig. 4. For determination of the synthesized length of the connecting rod a - l_{2c} =0,4391m, b - l_{2c} =0,8362m

As an example of the application of the obtained analytical dependencies, a synthesis of the combined CSM was carried out for the characteristics the ratios of which are most common for crank presses: $l_1=0.1 \, m$, $l_2=0.4 \, m$ ($l_2=0.8 \, m$). According to the developed computer software, the synthesis can be carried out for arbitrary lengths of the crank and connecting rod. As a result of the synthesis, the synthesized length of the connecting rod was obtained $l_{20}=0.4391 \, m$ and $0.8362 \, m$ (Fig 4).

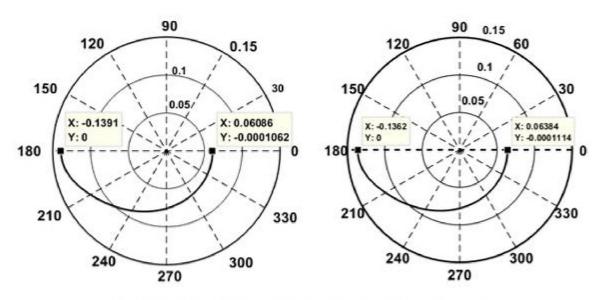


Fig. 5. The change of the crank length at the stage of the working stroke for a - l_{2c} =0,4391m b - l_{2c} =0,8362m

To determine the kinematic characteristics of the combined and output CSMs [6, 7] can be used. Thus, all the parameters included in (6) are defined.

The theoretical profile of the fixed cam at the stage of the working stroke, which ensures the motion of the slider according to the synthesized law is shown in Fig. 5. At the beginning of the working stroke, the length of the crank is equal to $l_{10}=0,1391m$, and at the end of the stroke $l_{1e}=0,06086m$ for $l_{2e}=0,4391m$. Respectively $l_{10}=0,1362m$ and $l_{1e}=0,06384m$ for $l_{2e}=0,8362m$. Purely visually, the pressure angles of the cam do not exceed the permissible ones $a_{\text{max}} \le \approx 45^{\circ}$, which allows us to assert the possibility of press design based on combined CSMs. However, it will be possible to speak more specifically after conducting additional research, namely:

- supplement the profile of the cam on a non-working stroke;
- determine the maximum values of cam pressure angles;
- analyze the influence of the mechanism geometry, the law of change of the normal force on the maximum angle of pressure;
 - determine the kinematic and power characteristics of the designed mechanism.

Conclusions

The following conclusions can be made based on the research undertaked:

- a structural scheme of the crank press mechanism is proposed, the slider of which moves at a speed qualitatively similar to the speed of crankshaft press slider;
- it is shown in the example of the synthesized simplest harmonic asymmetric law of the slider motion, that for a coefficient of asymmetry $k_{\rm ac}$ =0,176 the kinetic power decreases $n_{\rm n}$ =1,314 times at the parabolic change of the stamping force;
- a kinematic synthesis of the combined CSM was carried out and such geometric dimensions, which provide the motion of the slider according to the synthesized asymmetric law, were obtained;
- it has been proven that a crank press, the slider (punch) of which moves according to a synthesized asymmetric law, for the same stamping force, will have a lower drive power, and the flywheel will have a smaller mass and/or radius;
 - tasks for further research are determined.

References

- [1] Recep Halicioglu, L. Canan Dulger, A. Tolga Bozdana, "Mechanisms, classifications, and applications of servo presses: A review with comparisons", Journal of Engineering Manufacture, Vol. 230, Is. 7, pp. 1177–1194, July 2016
- [2] Hlavac J., Dekastello J., Cechura M., Volejnicek M., Kubec V., "Sawing energy in mechanical crank press drives", Advances in Mechanical Engineering, Vol. 14(2), pp. 1–12, 2022.
- [3] Raz K., Cechura M., "Usage of topological optimization in design of mechanical forging presses." MM Science Journal, pp. 2581–2584, ISSN: 18031269, DOI:10.17973/MMSJ.2018_11_201877, 2018.
- [4] Qian Jing, "Dynamic Modelling and Calculation of Crank Slider Mechanism of Multiparameter Friction Model in Wireless Communication", Wireless Communications and Mobile Computing, Vol. 2022, Article ID 7449392, 2022. https://doi.org/10.1155/2022/7449392.
- [5] Teng Xu, Qinxiang Xia, Xiaobin Long, Gianluca Buffa, "Vibration Control of a High-Speed Precision Servo Numerically Controlled Punching Press: Multidomain Simulation and Experiments", Shock and Vibration, Vol. 2017, Article ID 4593546, 2017. https://doi.org/10.1155/2017/4593546.
- [6] Pasika V. R., Heletiy V. M., Solohub B. V., "Kinematychne syntezuvannya zakoniv periodychnoho rukhu." ["Kinematic synthesis of laws of periodic motion"], V Monohrafiya, L'viv: Levada., 123 s., 2021.

Viacheslav Pasika, Andy Augousti, Olena Lanets, Oleh Parashchyn

- [7] Pasika R. V., Roman D. A., Kharzhevskyi V. O., Kinematic analysis and synthesis of cutter movement od slotting machine. Problems of Tribology, Vol. 27, No 2/104-2022, pp. 33–38, 2022.
 - [8] Rider, M.J., "Design and Analysis of Mechanisms: A Planar Approach", p. 315. Wiley, Hoboken (2015).
- [9] Arakelian, V., Briot, S., "Simultaneous inertia force/moment balancing and torque compensation of slider-crank mechanisms.", Mechanics Research Communications 37(2), pp. 265–269, March, 2010, DOI: 10.1016/j.mechrescom.2009.11.007.
- [10] Huang, M.-S.; Chen, K.-Y.; Fung, R.-F., "Comparison between mathematical modeling and experimental identification of a spatial slider–crank mechanism." Applied Mathematical Modelling, Applied Mathematical Modelling, Vol. 34, Is. 8, pp. 2059–2073, 2010.
- [11] Ha, J.-L.; Fung, R.-F.; Chen, K.-Y.; Hsien, S.-C., "Dynamic modeling and identification of a slider-crank mechanism." Journal of Sound and Vibration, Vol. 289(4), pp. 1019–1044, 2006.
- [12] Akbari, S.; Fallahi, F.; Pirbodaghi, T., "Dynamic analysis and controller design for a slider-crank mechanism with piezoelectric actuators." Journal of Computational Design and Engineering, Vol. 3, Is. 4, pp. 312–321, 2016.
- [13] Daniel, G.B.; Cavalca, K.L., "Analysis of the dynamics of a slider-crank mechanism with hydrodynamic lubrication in the connecting rod-slider joint clearance.", Mechanism and Machine Theory, Vol. 46 (10), pp. 1434–1452, 2011.