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A BRIEF OVERVIEW OF STATIONARY TWO-DIMENSIONAL THERMOELASTIC STATE MODELS IN HOMOGENEOUS AND PIECEWISE-HOMOGENEOUS BODIES WITH CRACKS

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Abstract. *Purpose.* A two-dimensional mathematical model of the problem of thermo-elasticity for piecewise-homogeneous component plate containing a crack has been built. The stress intensity coefficients in the vertices of the crack increase affecting strength of the body significantly. This leads to the growth of a crack and, as a result, to further local destruction of a material. Therefore, such a model reflects, to some extent, the destruction mechanism of the elements of engineering structures with cracks.

Methodology. Based on the method of the function of a complex variable we have studied the two-dimensional thermoelastic state for the body with crack as stress concentrators. As result, the problem of thermoelasticity was reduced to a system of two singular integral equations (SIE) of the first and second kind, a numerical solution of which was found by the method of mechanical quadratures.

Findings. The two-dimensional mathematical model of the thermoelastic state has been built in order to determine the stress intensity factors at the top of the crack and inclusion. The systems of singular integral equations of the first and second kinds of the specified problem on closed (contour of inclusion) and open (crack) contours are constructed. The influence of thermophysical and mechanical properties of inclusion on the SIF sat the crack types was investigated. The dependences of the stress intensity factor which characterizes the distribution of the intensity of stresses at the vertices of a crack have been built, as well as its elastic and thermoelastic characteristics of inclusion. This would make it possible to analyze the intensity of stresses in the neighborhoods of crack vertices depending on the geometrical and mechanical factors. As a result, this allows to determine the critical values of temperature in the three-component plate containing a crack in order to prevent the growth of the crack, as well as to prevent the local destruction of the body. It was found that the appropriate selection of mechanical and thermophysical characteristics of the components of a three-component plate containing a crack can be useful to achieve an improvement in body strength in terms of the mechanics of destruction by reducing stress intensity factors at the crack's vertices.

Originality. The solutions of the new two-dimensional problem of thermoelasticity for a specified region due to the action of constant temperature as well as due to local heating by a heat flux were obtained. The studied model is the generalization of the previous models to determine the two-dimensional thermoelastic state in a piecewise homogeneous plate weakened by internal cracks.

Practical value. The practical application of this model is a more complete description of the stress-strain state in piecewise homogeneous structural elements with cracks operating under temperature loads. The results of numerical calculations obtained from the solution of systems of equations and presented in the form of graphs can be used in the design of rational modes of operation of structural elements. This takes into account the possibility of preventing the growth of cracks by the appropriate selection of composite components with appropriate mechanical characteristics.

Keywords: stress intensity factor, singular integral equation, inclusion, heat conduction, thermoselasticity, crack, heat flux, heat source.

Introduction

In the course of operation, elements of numerous structures often work under conditions of non-uniform heating, which leads to the appearance of internal temperature stresses. In the case of piecewise homogeneous bodies, even a uniform temperature distribution may lead to the appearance of these stresses due to the unequal coefficients of linear thermal expansion of components of the body. In some cases, the temperature stresses themselves or in combination with external mechanical stresses may lead to the formation of new cracks and to the propagation of the already existing cracks, i.e., cause the local or final fracture of structures or their elements. Therefore, the investigations of the distributions of temperature stresses near the crack tips in solid bodies are of high theoretical and practical significance. The results of these investigations are used for the strength analysis from the viewpoint of fracture mechanics as well as for the analyses of the serviceability and durability of structural elements with cracks. The realization of these investigations under conditions of plane problem of thermoelasticity was significantly simplified by the development of the method of singular integral equations (SIEs), which proves to be one of the most convenient approaches for practical realization.

Elements of many modern structures are often designed for their operation under conditions of thermal heating, such conditions contribute to the emergence of thermal stresses. This is typical of tools and structures in the mining industry. Their operability, to a great extent, is determined by the level of concentration and intensity of these stresses on some regions, for example, in a neighborhood of non-homogeneities of a technological nature (cracks, inclusions). With this, the fracture of materials is caused by the presence of shar concentrators of stresses such as cracks. Therefore, the study of the thermoelastic state near a crack is necessary for calculations of strength from the point of view of fracture mechanics, which is especially important for structures with highly strong and low plastic materials which are under the action of different kinds of heat loadings.

Plane problems of heat conduction and thermoelasticity for a semi-infinite homogeneous body with a crack

The thermoelastic state of a half-space (plane deformation) with an arbitrarily oriented rectilinear internal or edge crack or a periodic system of such stress concentrators or a curvilinear crack is investigated using the method of singular integral equations and being under the action of local heat flux on its surface.

Two-dimensional stationary problems of heat conduction and thermoelasticity are reduced to systems of singular integral equations of the first kind, solved by a numerical method of mechanical quadratures for a number of new problems in the case of a half-space heated on a part of its free surface by a constant-intensity heat flux. The stress intensity factors (SIFs) at the tips of an arbitrarily oriented internal crack (rectilinear or curvilinear) or an edge rectilinear crack are determined and investigated depending on its slope angle, the distance from the half-space surface or the thermal load. The method of SIEs was used for the analysis of the plane thermoelastic state in a half-space locally heated over a part of its free surface by a heat flux and containing an internal arbitrary oriented rectilinear crack or a periodic system of such cracks [35, 36] or internal curvilinear crack [37] or edge crack [38, 39] or inclusion and a crack [40].

In the articles [35, 37] we consider a two-dimensional problem of stationary heat conduction and thermoelasticity for a half-space with a subsurface rectilinear crack (Fig. 1) and curvilinear crack (Fig. 2) and heated by a heat flux of the constant intensity in a local region of its free surface. We reduced the problem to two SIEs on the crack contour. The numerical solution of the integral equations is obtained by the method of mechanical quadratures. There are obtained graphic dependences of stress intensity factors (SIFs) KI and KII on the angle of inclination of the rectilinear crack (Fig. 1) and curvilinear parabolic crack (Fig. 2) and critical values of the local heat flux for which the body undergoes local fracture. SIFs KI and KII are the real quantities that characterize the distribution of intensity of stresses at the tops of a crack. The absolute value of the SIFs KI and KII at both tips of the curvilinear crack is maximum for the angle of its inclination (the axis of the crack is parallel to the edge of the half-plane) and minimum for the angle (the crack is perpendicular the to edge) as in the case of a rectilinear crack. Note that, for the rectilinear crack SIFs KI and KII are equal to zero, whereas for the curvilinear crack do not equal zero. Note that, according to the criterion by the hypothesis of initial growth of the crack by using the equations of limiting equilibrium we can find the critical values of the local heat flux for which the body undergoes local fracture.

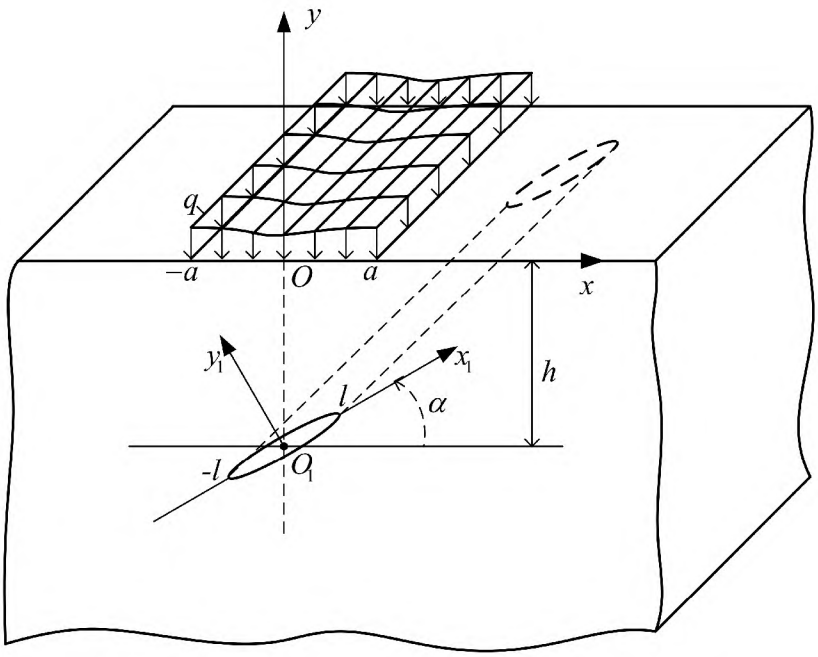

Fig. 1. Schematic diagram of local heating by a heat flow of an elastic half space containing a subsurface rectilinear crack

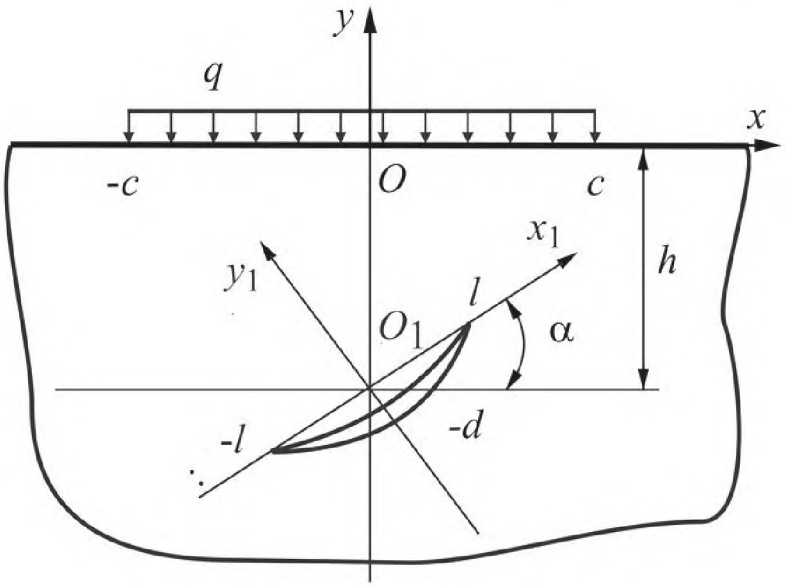

Fig. 2. Geometry of a half plane heated
by a local heat flux and weakened by a curvilinear crack

In the article [36], it has been studied how various shapes of distribution of heat flux over a part of free surface semi-space affect the stress-strain state in the vicinity of the crack tips. There are obtained graphic dependences of SIFs KI and KII on the angle of inclination of the rectilinear crack (Fig. 3) for various shapes (parabolic, elliptical, oscillating) of distribution of heat flux. The paper [38] deals with the two-dimensional problem of local heating for an elastic half-space with an edge crack (Fig. 4).

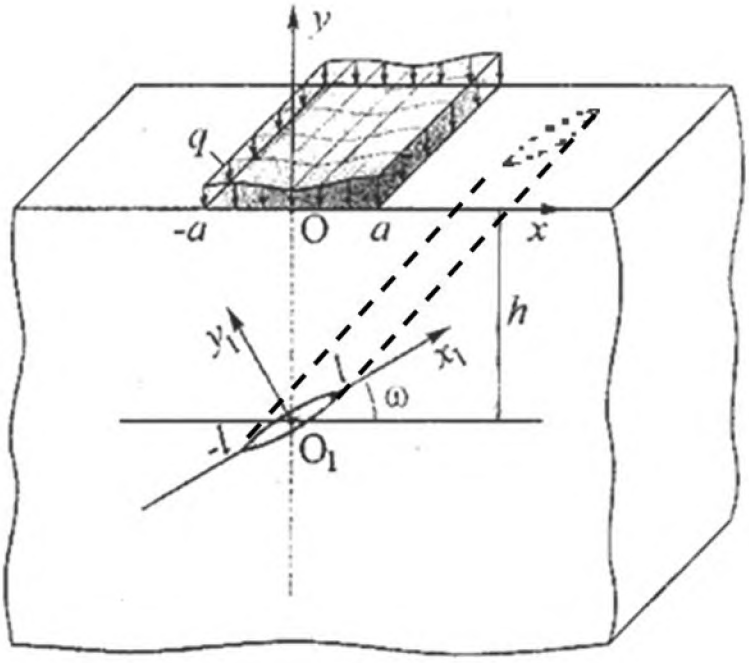

Fig. 3. Schematic diagram of local heating of an elastic half space containing
a crack by heat flux with various shapes of distribution on surface.

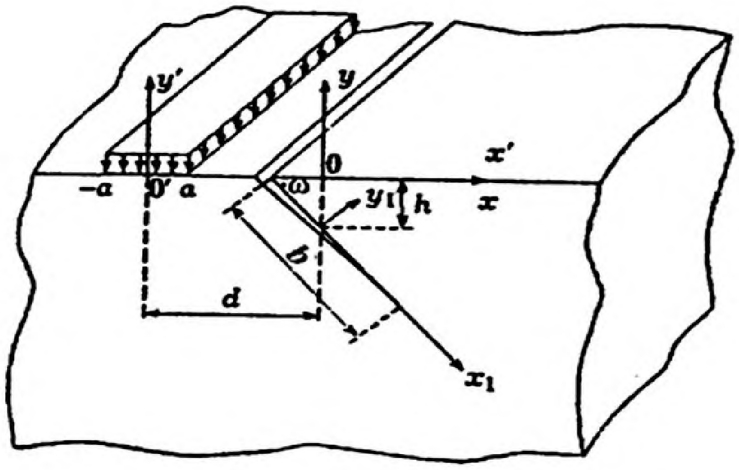

Fig. 4. Schematic diagram of local heating by a heat flux of an elastic half space containing an edge crack

Heat flux on the local area of the body causes changes in temperature and stress in the body, which significantly affects its strength, as it can lead to crack growth and local destruction. The problem is reduced to the solution of SIE on the crack contour. The numerical results present of the dependencies of the SIF KII at the top of a crack on the local heat flux placement and the crack orientation angle. The SIF KII, which characterizes the intensity of shear stresses at the crack tip, increases when the crack tip tends to the boundary plane of the half-space. For every distance of the crack center from the center of heating region there exists the angle inclination for which KII equals zero. Heating of materials by locally distributed head sources is widely used in engineering practice. In the presence of defects of the like of cracks, inclusion, etc., in thermoelastic bodies, one observes considerable singularities of the temperature stresses near the tips of the defects, which often lead to failure of structures.

In the article [39] we consider the problem of stationary thermoelasticity for a semi-infinite plate with an arbitrarily oriented edge crack. The thermal stresses in the body are caused by an internal stationary heat source acting near the tip of the cut (Fig. 5). A study is made of the influence of the internal heat source on the SIFs KI and KII at the tip edge crack.

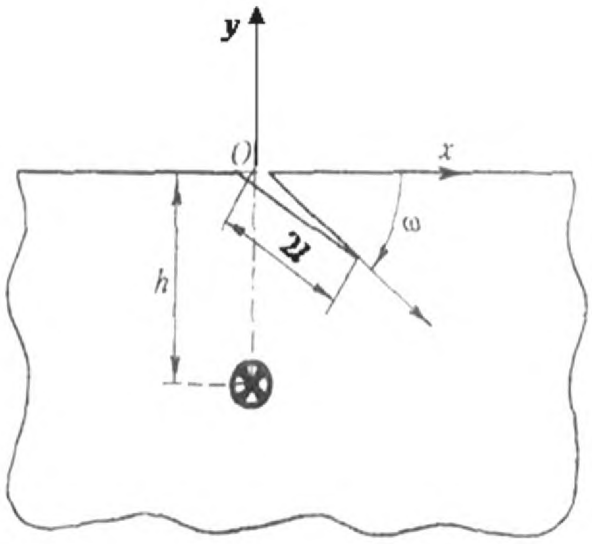

Fig. 5. Geometry of a half plane eated by a stationary heat source and weakened by an edge crack

Plane problems of heat conduction and thermoelasticity for piecewise-homogeneous bodies with cracks

Two-dimensional problems of stationary thermal conductivity and thermoelasticity for an elastic semi-infinite body (half-space), containing a cylindrical inclusion of curvilinear profile and crack of arbitrary configuration (Fig. 6) are formulated and solved within the framework of plane deformation [40].

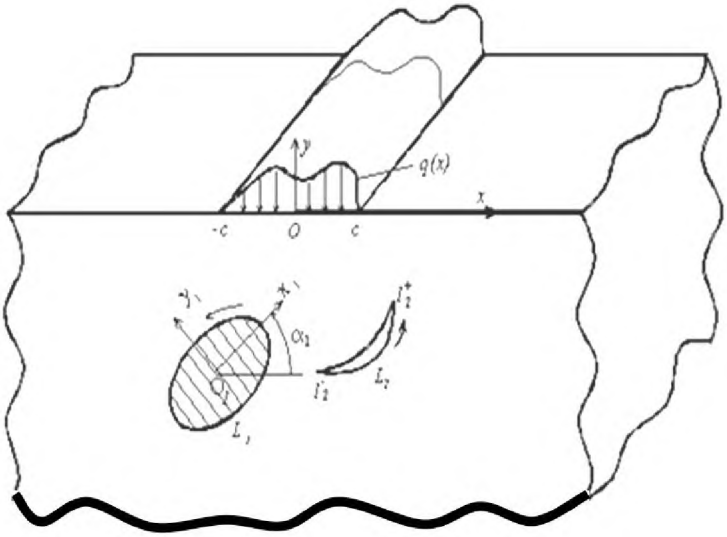

Fig. 6. Schematic diagram of local heating by a heat flow of an elastic half-space containing an inclusion and a crack.

For this purpose, mathematical models of two-dimensional problems of heat conductivity and thermoelasticity within the limits of planar deformation in the form of systems of two singular integral equations of the first and second kind that do not contain an unknown function on the edge of the half-plane (axial section of the half-space) are constructed by the same approach.

A plane thermoelastic state in a half-space that is heated on a part of its surface by a heat flux and contains a cylindrical inclusion of an elliptical or a circle profile and a rectilinear thermally insulated crack (Fig. 7) is studied.

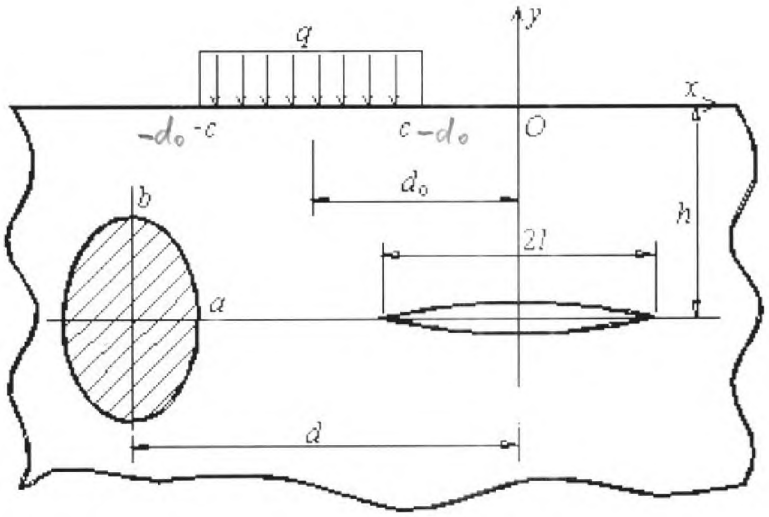

Fig. 7. Geometry of a half plane heated by a local heat flux and containing an elliptical inclusion and a crack.

The problem is reduced to systems of two SIEs on closed (the boundary of the inclusion) and unclosed (crack) contours [40,41]. The numerical solution of these systems is obtained by the method of mechanical quadratures. There are obtained graphical dependencies of SIFs KI and KII on the distance between the centres of the inclusion and crack, on thermal and mechanical properties of inclusion. Based on the analysis of the obtained critical values of the heat flux, it follows: if the more rigid inclusion (steel-inclusion) approaches the crack, then the critical value of the heat flux increases (at which the growth of the crack in the left tip begins); if the less rigid inclusion (aluminum- inclusion) approaches the crack, then, on the contrary, it causes the decreasing of critical value. A similar situation is observed for the right tip of the crack.

The works [42,43,44] are devoted to the formulation and solution of plane problems of thermoelasticity for three-component composite bodies with cracks (Fig. 8, 9,10).

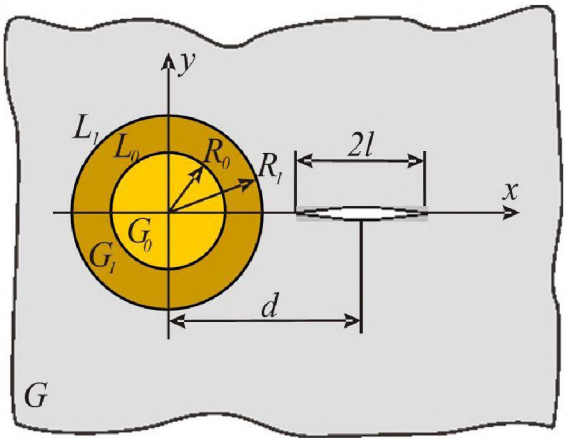

Fig. 8. Geometry of a plane containing
two-component circular inclusion and a crack.

In the presence of a circular contour of the junction of the composite component, a system of modified lower-order integral equations that do not contain an unknown function on the specified contour is obtained. We apply the approach with the help of which it is possible to reduce the order of the initial system of SIEs of the problem of thermoelasticity and, hence, to obtain its solution more efficiently and consider the case where the crack appears on the edge of the hole in the three-layer ring.

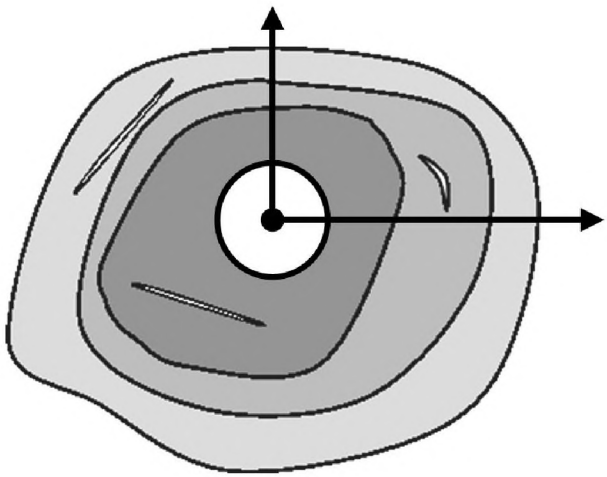

Fig. 9. Geometry of a three-layer annular area with cracks.

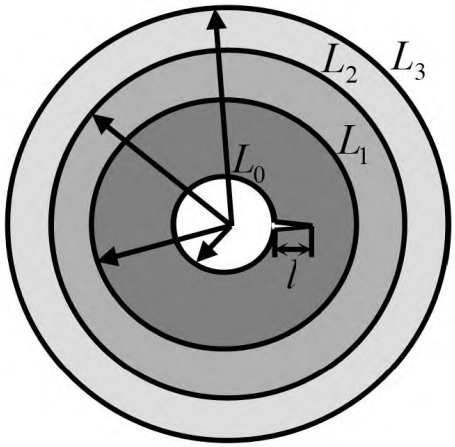

Fig. 10. Geometry of a three-layer annular ring containing edge crack

An effective modification of the method of singular integral equations with respect to plane problems of thermoelasticity for three-component compound bodies with cracks (with a circular line for connecting components) is developed by reducing the number of integral equations in the initial system, which made it possible to obtain numerical solutions of a number of new problems more effectively. The interaction of a two-component circular inclusion and a rectilinear crack in a plate under heating to constant uniform temperature conditions are investigated (Fig. 8). The problem is reduced to a system of two SIEs on closed (the boundary of the inclusion) and unclosed (crack) contours.

We also study the effect of thermal and mechanical properties of the components of the two-component inclusion on the SIFs KI and KII at the crack tip [42]. Under the conditions of constant temperature in the entire three-component plane containing a crack, the influence of the stiffness (shear modulus) of a two–component inclusion on the SIFs KI and KII near the crack tips differs from the influence of ranges of single-component inclusion. In particular, within the analyzed mechanical and thermal parameters, the increase in the stiffness on the disk (an inner component of the inclusion) leads to a decrease in the SIF KI if the CLTE (a coefficient of the linear thermal expansion) of the ring is larger than the CLTE of the matrix and of the disc. And, to an increase in the SIF KI if the CLTE of the disc is larger than the CLTE of the ring and of the matrix. At the same time, for a single-component inclusion, the increase in its stiffness always leads to an increase in the SIF KI if the CLTE of the inclusion is larger than the CLTE of the matrix. Thus, by the proper choice of the shear moduli and the coefficients of linear thermal expansion of the components of the inclusion, one can either increase or decrease the SIF KI at the crack tips, which is important from the viewpoint of control over the crack resistance of materials.

In the article [43] problems of stationary thermal conductivity and thermoelasticity have been formulated and solved for a planar three-layer annular region weakened by curvilinear cracks (Fig. 9). In the presence of a circular contour of the hole in the ring, a system of modified integral equations of the lowest order of the indicated problems not containing an unknown function on the circular contour of the hole is obtained. This made it relatively easy to investigate the stress at the tip of the edge crack, which emerges at the edge of the circular hole of the three-layer concentric ring. The heat conduction and thermoelasticity problem for a three-layer circular concentric hollow cylinder with an edge radial crack (Fig. 10) is solved for the action of a plane axial symmetric temperature field [44]. The cross-section of the cylinder is a circular concentric ring with similar rings of a different material inserted into this ring. The inner ring contains an edge radial crack.

The surface of the cylinder is free of loads and temperature conditions of the first kind are satisfied on these surfaces. On the interface of the media, we impose the conditions of perfect thermal and mechanical contact. The numerical solution is obtained for the case where unequal constant temperatures are satisfied on these surfaces. On the interface of the media, we impose the conditions

of perfect thermal and mechanical contact. The numerical solution is obtained for the case where unequal constant temperatures are given on the inner and outer surfaces of the cylinder. The SIF KI at the tip of an edge crack is found for different values of thermal and mechanical parameters of the components of the cylinder.

Two-dimensional stationary problems of heat conduction and thermoelasticity for infinite elastic bodies containing the periodic system of inclusions and cracks are considered in [45]. The numerical solution of the SIEs is obtained for a plate heated by a heat flux and containing the periodic system of elliptic inclusions and thermally insulated cracks (Fig. 11).

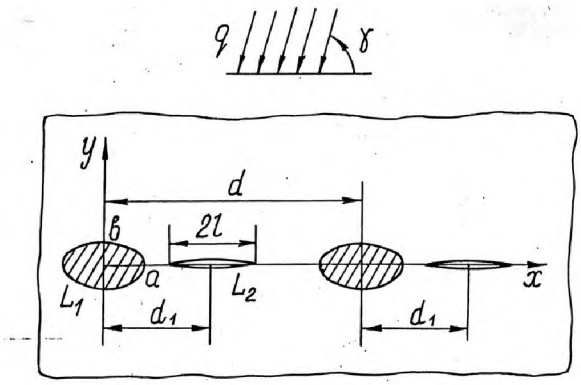

Fig. 11. A periodic system of elliptic inclusions and rectilinear cracks

There are obtained graphic dependences of SIFs KI and KII at the tops of a crack, depending on the length of the crack, elastic and thermoelastic characteristics of inclusion, relative position of crack, and inclusion. which are used to determine the critical values of the heat flux for which the crack begins to grow.

References

[1] Choi, H. J. "Thermoelastic interaction of two offset interfacial cracks in bonded dissimilar half-planes with a functionally grade interlayer", *Acta Mechanica*, Vol. 225, pp. 2111–2131, 2014.

[2] Brock, L. M. "Contours for planar cracks growing in three dimensions: Coupled thermoelastic solid (planar crack growth in 3D)", *Journal of Thermal Stresses*, Vol. 39, pp. 345–359, 2016.

[3] Elfakhakhre, N. R. F., Nik long, N. M. A. and Eshkuvatov, Z. K. "Stress intensity factor for multiple cracks in half plane elasticity", *AIP Conference*, 1795(1), 2017.

[4] Rashidova, E. V. and Sobol, B. V. "An equilibrium internal transverse crack in a composite elastic half-plane", *Journal of Applied Mathematics and Mechanics*, Vol. 81, pp. 236–247, 2017.

[5] Chen, H., Wang, Q., Liu, G.R., Wang, Y. and Sun, J. "Simulation of thermoelastic crack problems using singular edge-based smoothed finite element method", *International Journal of Mechanical Sciences*, Vol. 115–116, pp. 123–134, 2016.

[6] Tagliavia, G., Porfiri, M., Gupta, N. "Elastic interaction of interfacial spherical-cap cracks in hollow particle filled composites", *International Journal of Solids and Structures*, Vol. 48, pp. 1141–1153, 2011.

[7] Chu, S. N. G. "Elastic interaction between a screw dislocation and surface crack", *Journal of Applied Physics*, Vol. 53, pp. 8678–8685, 1982.

[8] Ming-huan, Z., Renji, T. "Interaction between crack and elastic inclusion", *Applied Mathematics and Mechanics*, Vol. 16, pp. 307–318, 1982.

[9] Bower A. F. "The effects of crack face friction and trapped fluid on surface initiated rolling contact fatigue cracks": Techn. Rep. No. CUED \C - Mech. I TR. 40. Univ. of Cambridge, pp. 22–25, 1987.

[10] Bryant H. D., Miller G. R., Keer I. M. "Line contact between a rigid indenter and damaged elastic body", *Quart. J. Mech. Appl. Math.*, Vol. 37, No. 3, pp. 467–478, 1984.

[11] Goshima T., Kamishirna Y. "Mutual interference of multiple surface cracks due to rolling-sliding contact with frictional heating", *JSME Int. J.*, Ser. 1, vol 37, No. 3, pp. 216–223, 1994.

[12] Goshima T., Kamishirna Y. "Mutual interference of two surface cracks in a semi-infinite body due to rolling contact with frictional heating by a rigid roller", *JSME Int. J.*, Ser. 1, Vol. 39, No. 1, pp. 26–33, 1994.

[13] Hills D. A., Barber J. R. "Steady motion an insulating rigid flat-ended punch over a thermally conducting half-plane", *Wear*, Vol. 102, No. 1, pp. 15–22, 1985.

[14] Hills D. A., Nowell D., Sackfield A. "The state stress induced by circular sliding contacts with frictional heating", *Int. J. Mech. Sci.*, Vol. 32, No. 9, pp. 767–778, 1990.

[15] Korovchinski M. V. "Plane contact problem of thermos-elasticity during quasi-stationary heat, generation on the contact surfaces", *Trans. ASME. J.* Basic Eng., Vol. 87, No. 3, pp. 811–817, 1965.

[16] Bryant H. D., Miller G. R., Keer I. M. "Line contact between a rigid indenter and damaged elastic body", *Quart. J. Mech. Appl. Math.*, Vol. 37, No. 3, pp. 467–478, 1984.

[17]. Miller, Gregory R., L. M. Keer, and H. S. Cheng. "On the mechanics of fatigue crack growth due to contact loading", *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences,* pp. 197–209, 1985.

[18] Fan H., Keer I. M., Myra T. *Tribology Transactions*, Vol. 15, no 1, pp.121–127, 1992.

[19] Fujimoto, Koji, Hironobu Ito, and Takashi Yamamoto. "Effect of cracks on the contact pressure distribution", *Tribology transactions*, Vol. 35, no 4, pp. 684–695, 1992.

[20] Evtushenko A.A., Zelenyak V.M. "A thermal problem of friction for a half-space with a crack", *Journal of Engineering Physics and Thermophysics*, Vol. 72, no 1, pp. 170–175, 1999.

[21] Goshima T., Keer I. M. J. Tribology., Vol. 112, No. 2, pp. 382–391, 1990.

[22] Goshima, Takahito, and Toshimichi Soda. "Stress intensity factors of a subsurface crack in a semi-infinite body due to rolling/sliding contact and heat generation", *JSME International Journal Series A Solid Mechanics and Material Engineering*, Vol. 40, no2, pp.263–270. 1997.

[23] Sekine H. "Thermal stress singularities at tips of a crack in a semi-infinite medium under uniform heat flow", *Eng. Fract. Mech.*, Vol. 7, no4, pp.713-729 ,1975.

[24] Sekine H. "Thermal stresses near tips of an insulated line crack in a semi-infinite medium under uniform heat flow", *Eng. Fract. Mech*., Vol. 9, no2, pp.499-507 ,1977.

[25] Tweed I., Lowe S. "The thermoelastic problem for a half-plane with an internal line crack", *Int. J. Eng. Sci.*, Vol. 17, no4, pp.357-363 ,1979.

[26] Kit H. S., Chernyak M. S. "Stressed state of bodies with thermal cylindrical inclusions and cracks (plane deformation)", *Mater Sci.*, Vol. 46, pp. 315–324, 2010.

[27] Cheesman B. A., Santare M.H. "The interaction of a curved crack with a circular elastic inclusion. Int. J. Fract., Vol. 103, pp. 259-278, 2000.

[28] Zelenyak V. M. "Temperature stresses in a circular center-cracked plate induced by head source", *Mater Sci.*, Vol. 30, pp. 272–275, 1995.

[29] Chen H., Wang Q., Liu G., Sun J. "Simulation of thermoelastic crack problems using singular edge-based smoothed finite element method", *Int.J.of Mech. Sci.*, Vol. 115,116, pp.23-134, 2016.

[30] Choi H. J. "Thermoelastic interaction of two offset interfacial cracks in bonded dissimilar half-planes with a functionally graded interlayer", *Acta Mechanica*, Vol. 225, no7, pp.2111–2131, 2014.

[31] Savruk M. P., Zelenyak V. M. "The plane problem of thermal conductivity and thermal elasticity for a finite piecewise uniform body with cracks", *Mater Sci.*, Vol. 23, pp.502 –510, 1987.

[32] Savruk M. P., Zelenyak V. M. "Thermoelastic state of a two-component hollow cylinder with edge radial cracks", *Mater Sci.*, Vol. 30, pp. 470–474, 1995.

[33] Savruk M. P., Zelenyak V. M. "Singular integral equations of plane problems of thermal conductivity and thermoelasticity for a piecewise–uniform plane with cracks", *Mater Sci.*, Vol. 22, pp. 294–307, 1986.

[34] Savruk M. P., Zelenyak V. M. "Plane problem of thermal conductivity and thermal elasticity for two joined dissimilar half-planers with curved inclusions and cracks", *Mater Sci.*, Vol. 24, pp. 124–129, 1988.

[35] Matysiak, S. J., Yevtushenko, A. A., Zelenjak, V. M., "Frictional heating of a half-space with cracks. I. Single or periodic system of subsurface cracks", *Tribology Transactions*, Vol. 32, pp. 237–243, 1999.

[36] Konechnyj S., Evtushenko A., Zelenyak V. "The effect of the shape of distribution of the friction heat flow on the stress-strain of a semispace", *Trenie i Iznos [Friction and Wear]*, Vol. 23, pp. 115–119, 2002.

[37] Zelenyak V. M., Kolyasa L.I. "Thermoelastic state of a half plane with curvilinear crack under the conditions of local heating", *Mater Sci.*, Vol. 52, pp. 315–322, 2016.

[38] Konechnyj S., Evtushenko A., Zelenyak V. "Heating of the semi-space with edge cracks by friction", *Trenie i Iznos [Friction and Wear]*, Vol. 22, pp. 39–45, 2001.

[39] S.Ya.Matysyak, A.A.Evtushenko, V.M.Zelenyak, "Heat□Source□Initiated Thermoelastic State of a Semiinfinite Plate with an Edge Crack", *Journal of Engineering Physics and Thermophysics*, Vol. 76, no 2, pp. 392–396, 2003.

[40] Matysiak S. I., Evtushenko O.O., Zeleniak V.M. "Heating of a half-space containing an inclusion and a crack", *Mater Sci.*, Vol. 40, pp. 466–474, 2004.

[41] Zelenyak V. M. "Mathematical modeling of stationary thermoelastic state in a half plane containing an inclusion and a crack due to local heating by a heat flux", *Mathematical modeling and computing*, Vol. 7, pp. 88–95, 2020.

[42] Zelenyak. V. M. "Thermoelastic interaction of a two-component circular inclusion with a crack in the plate", *Mater Sci.*, Vol. 48, pp. 301–307, 2012.

[43] Zelenyak. V. M. "Integral equations of two-dimensional problems of thermoelasticity for a three-layer annual cracked domain", *Mater Sci.*, Vol. 51, pp. 290–299, 2015.

[44] Zelenyak. B. M. "Thermoelastic equilibrium of a three-layer circular hollow cylinder weakened by a crack", *Mater Sci.*, Vol. 52, pp. 253–260, 2016.

[45] Zelenyak. V. "Mathematical modeling of stationary thermoelastic state for a plate with periodic system inclusion and cracks", *Acta mechanica et automatica*, Vol. 13, No. 1, pp. 11–15, 2019.