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COLLATZ CONJECTURE $3n \pm 1$ AS A NEWTON BINOMIAL PROBLEM

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Abstract. The power transformation of Newton's binomial forms two equal $3n \pm 1$algorithms for transformations of numbers $n \in \mathbb{N}$, each of which have one infinite cycle with a unit lower limit of oscillations. It is shown that in the reverse direction, the Kollatz sequence is formed by the lower limits of the corresponding cycles, and the last element goes to a multiple of three odd numbers. It was found that for infinite transformation cycles $3n-1$ isolated from the main graph with minimum amplitudes of 5, 7, 17 lower limits of oscillations, additional conditions are fulfilled.

Keywords: Collatz conjecture, conjecture $3n \pm 1$, natural numbers, graph

Introduction and problem statement

It is known [1] that various mathematical methods are used in cryptography. One of the promising ones is the discrete transformation of integers by algorithm *a·q±1, a=1,3,5,…* [2], for which the *3q+1* type transformation is known as the Kollatz problem [3].

The classic Kollatz problem is formulated from two arithmetic operations on an arbitrary integer $q \geq 1$: if the number is even, it is divisible by two $q/2$ and if odd, it is converted as $3q+1$:

$$C_q^+ = \quad if \quad q \equiv 0 \quad mod\, 2 \quad then \quad \frac{q}{2} \quad else \quad 3q+1 \ , \tag{1}$$

and ends with an infinite periodic cycle

$$cycle_{1\leftrightarrow 4\leftrightarrow 1}^{3n+1} = \{1 \leftrightarrow 4 \leftrightarrow 2 \leftrightarrow 1\}. \tag{2}$$

However, it is not entirely clear whether a number can come out of the cycle (2), since the number of natural numbers is infinite and it is not possible to test Kollatz's hypothesis for all of them. Therefore, the validity of the hypothesis is established for the finite set of numbers $n \in \mathbb{N}$, the Kollatz problem is not finally solved and continues to be of scientific interest, as in number theory, dynamical systems, algorithm theory, etc. By themselves, these mathematical methods are widely used in the modeling of CAD problems. In this work, the problem $3q \pm 1$ is investigated from the point of view of the power transformation of Newton's binomial $(1+1)^s, .s = 0,1,2,3,4,...$.

The patterns of transformation $3q+1$ are studied even more, but after U. Gosper and R. Schoppel, who, while still in the HAKMEM group, showed that the problem $3q-1$ is equivalent to task $3q+1$ with negative values q [4] interest in the task $3q-1$ increased again [3]. Moreover, recently [5] showed the possibility of using this approach for modeling quantum systems.

Results and discussion

Let's formulate the basic definitions for this work:

Definition 1. A Kollatz sequence (CS) is one whose numbers are calculated according to the rule: let an arbitrary positive integer be a member of the sequence. If it is even, then the next member of the sequence will be the result of dividing it by 2. If the number is odd, then the next member of the sequence will be the number $C_n^{\pm} = 3n \pm 1$.

Definition 2. Count Collatz $\{K_{\theta \geq 1}\}$ with an index θ will be called the sequence $\theta \cdot \{2^s\}, s = 0,1,2,3,4,...$

Definition 3. A graph of type $Graph\ I$ will be called such, whose index is not a multiple of three. The index θ of graph $Graph\ II$ is a multiple of three.

It is known [6] that binomial coefficients C_s^i of power of Newton's binomial

$$(1+1)^s = \sum_{i=0}^{s} C_s^i, \quad s = 0,1,2,3,4,..., \tag{3}$$

form Pascal's triangle (TrP), which is shown in Fig. 1. In a triangle TrP, the sums of the numbers in the rows form a binary sequence

$$\{K_{main}\} = \{2^0,\ 2^1,\ 2^2,.2^3,\ 2^4,\ 2^5,\ 2^6,\ 2^7, 2^8,..,2^s,...\}. \tag{4}$$

The value of each subsequent element is twice the previous one. Therefore, the sequence (4) forms the main graph (stem) of Kollatz, and the numerical sequence (4) represents the possibility of realizing the trajectory of number calculations $n \in \mathbb{N}$ (1a).

Rows of a triangle TrP consist of an odd $\xi = s+1 = 1,3,5,7,...$ and even $\tau = s+1 = 2,4.6.8,...$ the number of elements, which correlates with the type of degree parity s in (4). As shown in Fig. 1, the sums of numbers in rows with odd numbers of elements form a subsequence in (4)

$$K_{m=1}: \quad 1 \leftarrow 4 \leftarrow 16 \leftarrow 64 \leftarrow 256 \leftarrow 1024 \leftarrow\ldots\leftarrow\ldots 2^{\xi-1} \leftarrow\ldots, \xi = 1,\ 3,\ 5,\ 7,\ \ldots. \tag{5}$$

and sums of numbers in rows with even numbers of elements form a subsequence in (4)

$$K_{p=1}: \quad 1 \leftarrow 2 \leftarrow 8 \leftarrow 32 \leftarrow 128 \leftarrow 512 \leftarrow\ldots\leftarrow\ldots 2^{\tau-1} \leftarrow\ldots, \tau = 0,\ 2,\ 4,\ 6,\ 8,\ \ldots. \tag{6}$$

For numbers of subsequences (5) and (6), arithmetic transformations end with odd integers that are multiples of three:

$$\begin{cases} \frac{4-1}{3}=1, \frac{16-1}{3}=5, \frac{64-1}{3}=21, \frac{256-1}{3}=85, \frac{1024-1}{3}=341, \quad \Rightarrow \quad \frac{2^{\xi-1}-1}{3}=m_\xi, \xi=3,5,7,....(a) \\ \frac{2+1}{3}=1, \frac{8+1}{3}=3, \frac{32+1}{3}=11, \frac{128+1}{3}=43, \frac{512+1}{3}=171, \quad \Rightarrow \quad \frac{2^{\xi-1}+1}{3}=p_\xi, \xi=2,4,6,8,....(b) \end{cases}, \tag{7}$$

Thus, on the graphs of subsequences (5) and (6), nodes with numbers (7) are formed, in which the numbers are odd according to the algorithms

$$\begin{cases} 2^{\xi-1} = 3m_\xi + 1, & (a) \\ 2^{\xi-1} = 3p_\xi - 1, & (b) \end{cases}, \tag{8}$$

are converted to numbers that are exactly equal to powers of two. In (8), the transformation (8a) expresses the function $3n+1$, and the transformation (8b) expresses the function $3n-1$. Thus, the transformation of numbers $n \in \mathbb{N}$ by the function (8b) is described by the algorithm

$$\mathrm{C_q^-} = \quad \mathrm{if} \quad \mathrm{q} \equiv 0 \mod 2 \quad \mathrm{then} \quad \frac{\mathrm{q}}{2} \quad \mathrm{else} \quad 3\mathrm{q}-1 \quad , \tag{9}$$

which ends in an endless loop

$$cycle_{1\leftrightarrow 2\leftrightarrow 1}^{3n-1} = \{1 \leftrightarrow 2 \leftrightarrow 1\}. \tag{10}$$

Therefore, the power transformation of Newton's binomial forms two equal Kollatz transformations of numbers $n \in \mathbb{N}$.

In Fig. 1, Kollatz graphs (1) are shown to the right of the triangle TrP, and graphs (9) are shown to the left of the triangle TrP. Nodes with Jacobstal numbers [7,8] (7) are formed on both main graphs $K_{m=1}$ and $K_{p=1}$:

$$\begin{cases} m_\xi: \quad 1,\ 5,\ 21,\ 85, 341, 1365,\ 5461, \ldots \quad (a) \\ p_\xi: \quad 1,\ 3,\ 11,\ 43,\ 171,\ 683,\ 2731, \ldots \quad (b) \end{cases}, \tag{11}$$

The numbers (11a) correspond to even powers in the terms $1 \cdot 2^r$ and determine the nodes of the sequences $\theta \cdot 2^r$ from $m_{\theta,r} = \theta_{1,5}$ which are calculated according to the formulas:

$$\begin{cases} \frac{\theta_5+1}{3} = E, \quad m_{\theta_5,1} = O \\ m_{\theta_5,1} = 2^0 O + 0,\ m_{\theta_5,3} = 2^2 O + 1,\ m_{\theta_5,5} = 2^4 O + 5,\ m_{\theta_5,7} = 2^6 O + 21, \\ \qquad m_{\theta_5,9} = 2^8 O + 85,\ m_{\theta_5,11} = 2^{10} O + 341, .. \\ \\ \frac{\theta_1-1}{3} = O, \quad m_{\theta_1,0} = E \\ m_{\theta_1,0} = 2^0 E + 0,\ m_{\theta_1,2} = 2^2 E + 1,\ m_{\theta_1,4} = 2^4 E + 5,\ m_{\theta_1,6} = 2^6 E + 21, \\ \qquad m_{\theta_1,8} = 2^8 E + 85,\ m_{\theta_1,01} = 2^{10} O + 341, \ldots, \end{cases}, \tag{12}$$

where $\theta = even\,(E) + odd(O)$. Numbers (11b) correspond to even powers in terms

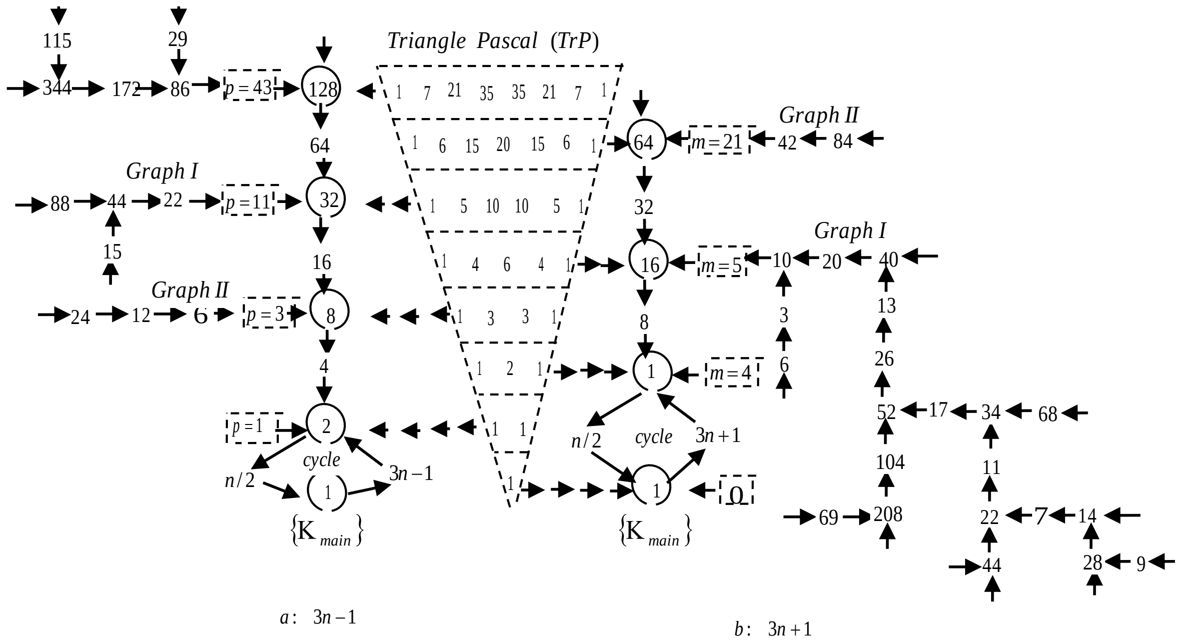

Fig. 1. Illustration of Pascal's triangle, main $\{K_{main}\}$ and lateral $m, p > 1$ graphs of transformations $3n \pm 1$ of numbers $n \in \mathbb{N}$

$1 \cdot 2^r$ determine the sequence nodes $\theta \cdot 2^s$ from $p_{\theta,s} = \theta_{1,5}$ which are calculated according to the formulas:

$$\begin{cases} \frac{\theta_5+1}{3} = E, \quad p_{\theta,0} = E \\ p_{\theta,0} = 2^0 E - 0,\ p_{\theta_5,2} = 2^2 E - 1,\ p_{\theta_5,4} = 2^4 \mathrm{E} - 5, \\ p_{\theta_5,6} = 2^6 E - 21,\ p_{\theta_5,8} = 2^8 E - 85,\ p_{\theta_5,01} = 2^{10} E - 341, \ldots \\ \\ \frac{\theta_1-1}{3} = O, \quad p_{\theta,1} = O \\ p_{\theta_1,1} = 2^0 O - 0,\ p_{\theta_1,3} = 2^2 O - 1,\ p_{\theta_1,5} = 2^4 O - 5, \\ p_{\theta_1,7} = 2^6 O - 21,\ p_{\theta_1,9} = 2^8 O - 85,\ p_{\theta_1,10} = 2^{10} O - 341, \ldots, \end{cases} \tag{13}$$

where the parameters $\theta_{1,5}$ meet the conditions $\theta_1 = \frac{\theta - 1}{3} = \text{integer}$, $\theta_5 = \frac{\theta + 1}{3} = \text{integer}$.

Table. 1.

Numbers $K^{\pm}_{\theta_{1,5},k+1} = 2K^{\pm}_{\theta_{1,5},k} + 1$ ***with indices*** $\theta_{1,5} = 1 \div 25$

r (s)	0	1	2	3	4	5	6	7	8	9	OEIS [11]	$K^{\pm}_{\theta_{1,5},k}$
θ=1	[0]	1	1	[3]	5	11	[21]	43	85	[171]	A001045	$K^{-}_{1,k}$
θ=5	2	[3]	7	13	[27]	53	107	[213]	427	853	A0485573	$K^{+}_{5,k}$
θ=7	2	5	[9]	19	37	[75]	149	299	[597]	1195	A062092	$K^{-}_{7,k}$
θ=11	4	7	[15]	29	59	[117]	235	469	[939]	1877	Unknown	$K^{+}_{11,k}$
θ=13	4	[9]	17	35	[69]	139	277	[555]	1109	2219	Unknown	$K^{-}_{\theta,k}$
θ=17	[6]	11	23	[45]	91	181	[363]	725	1451	[2901]	Unknown	$K^{+}_{\theta,k}$
θ=19	[6]	13	25	[51]	101	203	[405]	811	2389	[3243]	Unknown	$K^{-}_{\theta,k}$
θ=23	8	[15]	31	61	[123]	245	491	[981]	1963	3925	Unknown	$K^{+}_{\theta,k}$
θ=25	8	17	[33]	67	133	[267]	533	1067	[2133]	4267	Unknown	$K^{-}_{\theta,k}$
θ=29	10	19	[39]	77	155	[309]	619	1237	[2475]	4949	Unknown	$K^{+}_{\theta,k}$
θ=31	10	[21]	41	83	[165]	331	661	[1323]	2645	5291	Unknown	$K^{-}_{\theta,k}$
θ=35	[12]	23	47	[93]	187	373	[747]	1493	2987	[5973]	Unknown	$K^{+}_{\theta,k}$
θ=37	[12]	25	49	[99]	197	395	[789]	1579	3157	6315	Unknown	$K^{-}_{\theta,k}$
θ=41	14	[27]	55	109	[219]	437	875	[1749]	3499	6997	Unknown	$K^{+}_{\theta,k}$
θ=43	14	29	[57]	115	229	[459]	917	1835	[3669]	7339	Unknown	$K^{-}_{\theta,k}$
θ=47	16	31	[63]	125	251	[501]	1003	2005	[4011]	8021	Unknown	$K^{+}_{\theta,k}$
θ=49	16	[33]	65	131	[261]	523	1045	[2091]	4181	8363	Unknown	$K^{-}_{\theta,k}$

In Kollatz's problem, numbers are of interest $m(p)_{\theta_{1,5},r(s)}$ for which equalities hold

$$\theta_{1,5} \cdot 2^{r(s)} = 3m_{\theta_{1,5},r(s)} + 1,\ \theta_{1,5} \cdot 2^{r(s)} = 3p_{\theta_{1,5},r(s)} - 1, \tag{14}$$

and differences between adjacent numbers

$$\begin{array}{lccccccc} k & 0 & 1 & 2 & 3 & 4 & 5 & . \\ & m_{\theta_1,0(1)} & 4m_{\theta_1,0(1)}+1 & 4m_{\theta_1,0(1)}+1 & 4m_{\theta_1,0(1)}+1 & 4m_{\theta_1,0(1)}+1 & 4m_{\theta_1,0(1)}+1 & \\ differ & - & ,\ 4^0(3m_{\theta_1,0}+1), & 4^1(3m_{\theta_1,0}+1), & 4^2(3m_{\theta_1,0}+1), & 4^3(3m_{\theta_1,0}+1), & 4^4(3m_{\theta_1,0}+1), & \ldots \\ differ & - & 4^0(3m_{\theta_1,1}+1) & 4^1(3m_{\theta_1,1}+1) & 4^2(3m_{\theta_1,1}+1) & 4^3(3m_{\theta_1,1}+1) & 4^4(3m_{\theta_1,1}+1) & \end{array} \tag{15}$$

$$\Rightarrow differ = m_{\theta_1,k+1} - m_{\theta_1,k} = 4^{k-1}(3m_{\theta_1,0(1)} + 1)$$

Similarly, for $p_{\theta_{1,5},r(s)}$, table (15) is calculated as:

The problem of applying the type rule $4x \pm 1$ in a type problem $3x \pm 1$ is discussed in [7].

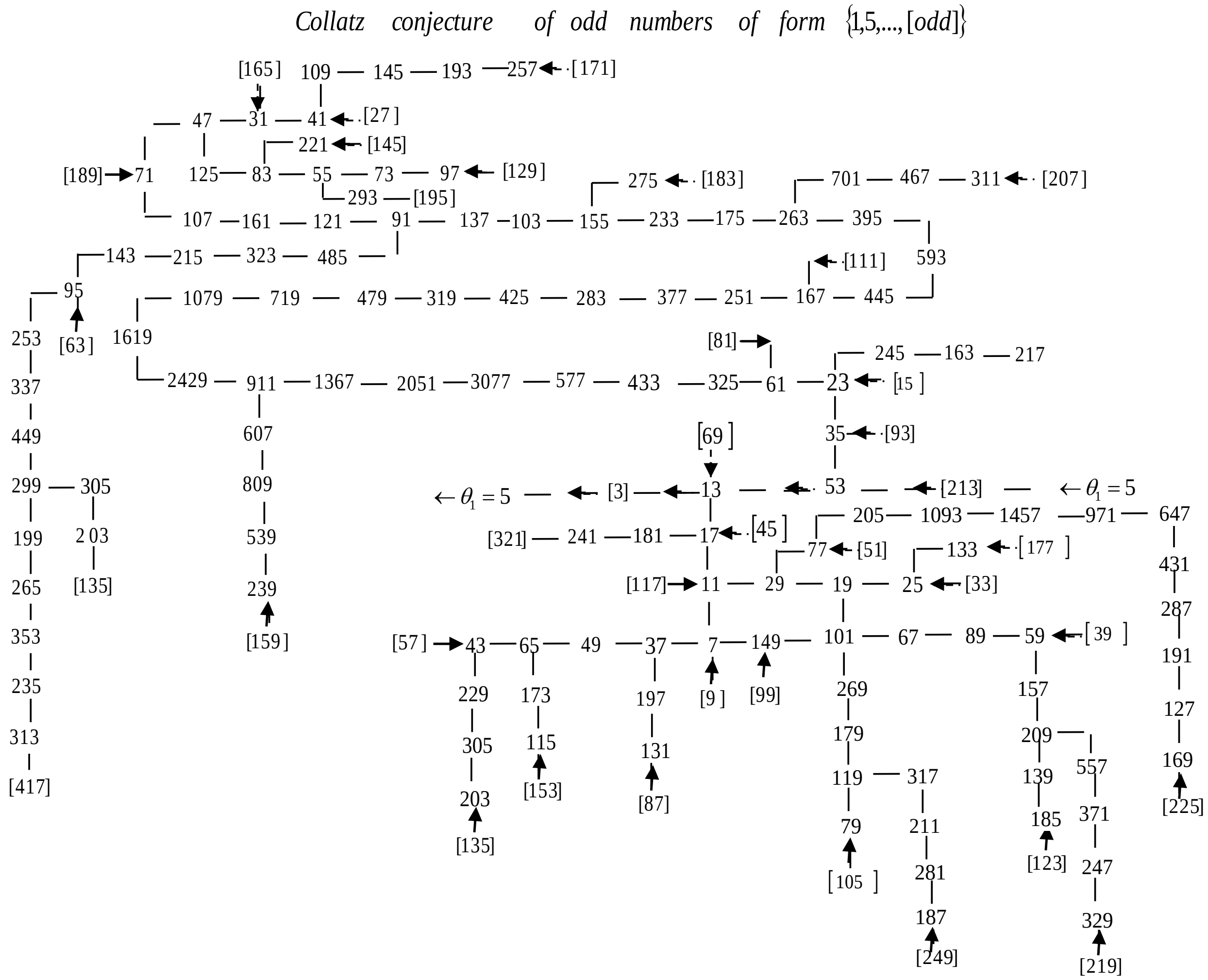

Рис.2. Графи розгалужень параметризованих кратною трьом правою межею послідовностей Коллатца для $\theta = 13$ і $\theta = 53$.

As can be seen from Table 1, for the sequences $\{K_{\theta 6k}\} = \theta \cdot 2^k, \theta, k \in \mathbb{N}$, the graph $Graph_{full}$ of the transformation (1) can be considered as a superposition of two types of graphs:

$$Graph_{full} = Graph\ \mathrm{I} + Graph\ \mathrm{II}. \tag{17}$$

The graph is formed by a sequence of odd numbers of intermediate calculations:

$$Graph\, I = \left\{1, ..., \frac{a_N - 1}{3} = Od_3\right\} \quad , \tag{18}$$

the left limit of which is the infinite loop (2) on the trunk (4). The right limit of (18) is formed by an odd number that is a multiple of three $X_n = Od_3$. Nodes with numbers (11) are formed on the graphs $Graph\, I$, from which side graphs are subsequently generated, so they can be considered active.

The graph $Graph\, II$ is generated by a node with a number $\frac{a_N - 1}{3} = Od_3$, and is further formed from doubled values in the reverse direction $X_n = Od_3$:

$$Graph\ \mathrm{II} = \{2^0 \cdot Od_3,\ 2^1 \cdot Od_3,\ 2^2 \cdot Od_3,\ 2^3 \cdot Od_3\ , \dots\} = Od_3 \cdot \mathrm{K}_{m,p=1}. \tag{19}$$

So, the starting numbers are localized on this graph

$$n = 2^k \cdot Od_3,\ k = 0,1,2,3,\ldots. \quad (20)$$

Unlike a graph $Graph\,I$, a graph $Graph\,II$ has no nodes, so it is passive. The method of directed graphs is widely used to study Kollatz transformations [9]. The dynamics of the formation of graphs with a multiple of three right limits of the sequence is shown in Fig. 2 for two graphs with indices $\theta = 13$ and $\theta = 53$.

Let it show that the numbers m of nodes on the graphs can be represented in the form of a two-dimensional matrix $m_{i,j}$:

$m_{i,1} = 4m_{i-1,1} + 1$ ↑

$m_{i,j} =$ (21), $\rightarrow \begin{cases} m_{i,j} = 4m_{i,j-1} + 1, \\ j \rangle 1. \end{cases}$

i							
8	21845	14563	58253	233013= Od_3	932053	3728213	14912853= Od_3
7	5461	7281= Od_3	29125	116501	466005= Od_3	1864021	7456085
6	1365= Od_3	–	–	–	–	–	–
5	341	227	909= Od_3	3637	14549	58197= Od_3	232789
4	85	113	453= Od_3	1813	7253	29013= Od_3	116053
3	21= Od_3	–	–	–	–	–	–
2	5	3	13	53	213= Od_3	853	3413
1	1						
i / *j*	1	2	3	4	5	6	7

Here, the first column $j = 1$ is formed from numbers (11a), and their values are calculated according to the recurrent formula

$$m_{i,1} = 4m_{i-1,1} + 1, \quad i \rangle 1. \quad (22)$$

The numbers in the second $j = 2$ column are the beginnings of rows with node numbers of lateral active graphs $m > 1$, generated by nodes with numbers in the first column. The values of the numbers in the rows are calculated using a recursive formula

$$m_{i,j} = 4m_{i,j-1} + 1, \quad j \rangle 1. \quad (23)$$

Lines not filled with numbers correspond to passive graphs. Representation (23) is true when multiplying the lateral graph from an arbitrary node for both transformation (1) and transformation (9):

$p_{i,1} = 4p_{i-1,1} - 1$ ↑

$p_{i,j} =$ (24), $\rightarrow \begin{cases} p_{i,j} = 4p_{i,j-1} - 1, \\ j \rangle 1. \end{cases}$

i							
8	43691	58255	233019= Od_3	932075	3728299	14913195= Od_3	59652779
7	10923= Od_3	–	–	–	–	–	–
6	2731	1821= Od_3	7283	29131	116523= Od_3	466091	1864363
5	683	911	3643	14571= Od_3	58283	233131	932523= Od_3
4	171= Od_3	–	–	–	–	–	–
3	43	29	115	459= Od_3	1835	7339	29355
2	11	15= Od_3	59	235	939= Od_3	3755	15019
1	3= Od_3	–	–	–	–	–	–
i / *j*	1	2	3	4	5	6	7

In (24), there is an inversion of the sign in the rules for calculating the numbers of columns and rows in accordance with the transformation ($3n - 1$).

Rules (22)-(23) are universal and are valid for calculating the numbers of an arbitrary pair of adjacent nodes on the graphs of both transformations $3n \pm 1$

$$\begin{cases} 4\dfrac{2^k-1}{3}+1=\dfrac{2^{k+2}-1}{3} & \Rightarrow 4\cdot(2^k-1)+3=2^{k+2}-1 \quad\Rightarrow\quad 4=4,\quad k=2,4,6,... \\ 4\dfrac{2^k+1}{3}-1=\dfrac{2^{k+2}+1}{3} & \Rightarrow 4\cdot(2^{k+1}+1)-3=2^{k+2}+1 \quad\Rightarrow\quad 4=4,\quad k=1,3,5,... \end{cases} \tag{25}$$

In addition, according to (4)-(6), for an arbitrary graph, its sequence $\{K_{m(p)}\}$ and numbers $m(p)$ are related to the main sequence $\{K_{main}\}$ by the relation:

$$\{K_{m(p)}\}=m(p)\cdot\{K_{main}\}. \tag{26}$$

Therefore, power transformations of Newton's binomial (3) give rise to the sequence on the binary basis (4), as superpositions of two subsequences (5) and (6). On the graphs of subsequences (5) and (6), nodes with numbers (11) are formed, thanks to which the transformation of odd numbers of the type $3n+1$ and $3n-1$ ends with the corresponding cycle (2) and (10), the lower bounds of which are equal to:

$$\begin{cases} cycle^{3n+1}_{1\leftrightarrow4\leftrightarrow1}(2): \quad n=[(3n+1):2]:2 \quad \Rightarrow 4n=3n+1 \quad\Rightarrow\quad n_{\lim}=1, \\ cycle^{3n-1}_{1\leftrightarrow2\leftrightarrow1}(10): \quad n=[(3n-1):2] \quad \Rightarrow 2n=3n-1 \quad\Rightarrow\quad n_{\lim}=1. \end{cases} \tag{27}$$

However, if the lowest lateral graph $m=5$ of the transformation $3n+1$ is active, then the lowest lateral graph of the transformation $3n-1$ is passive, and the lowest active lateral graph of the transformation $3n-1$ is generated from node $p=11$.

However, according to rule (7b), in addition to cycle (10), transformations $3n-1$ are accompanied by the formation of other infinite cycles

$$\begin{aligned} &cycle^{3n-1}_{5\leftrightarrow5}=\{5\leftrightarrow7\leftrightarrow5\} && (a)\\ &cycle^{3n-1}_{7\leftrightarrow7}=\{7\leftrightarrow5\leftrightarrow7\} && (b)\\ &cycle^{3n-1}_{17\leftrightarrow17}=\{17\leftrightarrow25\leftrightarrow37\leftrightarrow55\leftrightarrow41\leftrightarrow61\leftrightarrow91\leftrightarrow17\} && (c) \end{aligned} \tag{28}$$

with lower bounds $n_{\lim}$

$$\begin{aligned} &cycle^{3n-1}_{5\leftrightarrow5}: \quad n=([(3n-1):2]\cdot3):2:2 \quad\Rightarrow 8n=(3n-1)\cdot3 \quad\Rightarrow\quad n_{\lim}=5 && (a)\\ &cycle^{3n-1}_{7\leftrightarrow7}: \quad n=[((3n-1):2:2)\cdot3-1]\cdot:2-1]:2 \quad\Rightarrow 8n+4=9n-3 \quad\Rightarrow\quad n_{\lim}=7 && \\ &cycle^{3n-1}_{17\leftrightarrow17}: \quad n=([(([(([(([(3n-1):2]3-1):2]3-1]:2]3-1):2:2]3-1):2]3-1):2]3-1):16 \quad\Rightarrow && (b)\\ &\Rightarrow 1024n+574=243(((3n-1):2)3-1 \quad\Rightarrow\quad 2048n+2363=2187n \quad\Rightarrow\quad n_{\lim}=2363:139=17 && (c) \end{aligned} \tag{29}$$,

Cycles (28) are isolated from the main Kollatz stem $\{K_{main}\}$, and their existence was also noted by T. Tao [10]. In conclusion, we note the following.

Both transformations $3q\pm1$ are based on the same regularities, so they are equivalent to each other. However, the results of the transformations themselves $3q\pm1$ $CT_{3q\pm1}$ are radically different. Thus, the fact that CT_{3q-1} of the numbers $q_5=5, q_7=7, q_{17}=17$ does not reach a single value $1\cdot2^0$, but has the form of final periodic oscillations with minimal amplitudes $q_{\min}=5, 7, 17$, can be due to the property of the triple of recurrent numbers q_5, q_7, q_{17}, which

$$q_5\cdot q_7=2q_{17}+1, \tag{30}$$

where

$$(2^k+1)(2^{k+1}-1)=2^{k+2}+1 \;\Rightarrow\; 2^k=4 \;\Rightarrow\; k=2. \tag{31}$$

The product grows exponentially in the direction $k \to \infty$, so there are no other triples of numbers with properties (30). The triplet of numbers q_5, q_7, q_{17} with properties (30) does not exist in the system $3q+1$, and there are no isolated cycles $cycle^{3n-1}_{5(7)\leftrightarrow 5(7)}, cycle^{3n-1}_{17\leftrightarrow 17}$. of the type for it.

Conclusions

The power transformation of Newton's binomial forms two equal $3n\pm1$ algorithms for transformations of numbers $n \in \mathbb{N}$, which each have one infinite cycle with a unit lower limit of oscillations. It is shown that in the reverse direction, the Kollatz sequence is formed by the lower limits of the corresponding cycles, and the last element goes to a multiple of three odd numbers. It was found that for infinite transformation cycles $3n-1$ isolated from the main graph with minimum amplitudes of 5, 7, 17 lower limits of oscillations, additional conditions are fulfilled.

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ГІПОТЕЗА КОЛЛАТЦА 3N±1 ЯК БІНОМІАЛЬНА ПРОБЛЕМА НЬЮТОНА

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Collatz Conjecture $3n \pm 1$ as a Newton Binomial Problem

Анотація. Степеневе перетворення біному Ньютона формує два рівноправні алгоритми перетворень чисел $\mathbb{N}$, які мають по одному нескінченному циклу із одиничною нижньою межею осциляцій. Показано, що в реверсному напрямку послідовність Коллатца формується нижніми межами відповідних циклів, а останній елемент прямує до кратного трьом непарного числа. Виявлено, що для ізольованих від основного графу безмежні цикли перетворення із мінімальними амплітудами 5,7,17 нижніх межам осциляцій, виконуються додаткові умови.

Ключові слова: Гіпотеза Коллатца, гіпотеза $3n \pm 1$, натуральні числа, графік.