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Modeling of the error determination in accepting financial decisions in conditions of uncertainty

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The article presents the modeling mechanism of the error determination when making financial decisions in conditions of uncertainty and risk. Researched modeling is based on using the assessing of the diagnosed level of enterprises' economic security with help of mathematical description and calculations of probability transitions matrix and using the research method, the method of direct and reverse interpolation, averaging, statistical and index method, root mean square error method, simulation modeling, probability theory method, matrix method, etc. It has been studied that the most difficult is the process of modeling financial decision-making in conditions of uncertainty, since due to various types of risk there is a high threat of insufficiency or inaccuracy of information that distorts technical calculations, the indicator of which is real financial costs and timely detection of permissible and unacceptable deviations of parameters.

Keywords: *uncertainty; risk; modeling; error; correlation deviation; financial decisions; economic security.*

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1. Introduction

The construction of models is carried out with the aim of learning socio and economic as well as financial processes that take place during the financial and economic activity of a person or in the life of society. Models are used to describe the adoption of various management decisions and, in particular, financial ones. Modeling helps to find ways to improve phenomena, patterns and manage final financial results using mathematical description and calculations. An important point today is modeling of deviations and errors when making financial decisions in conditions of uncertainty and risks. This is due to the fact that the process of modeling financial decisions itself has certain features and refers to the abstraction of a number of factors influencing financial processes in the country, the introduction of certain restrictions, as a result of which individual models only schematically characterize original events.

Obtaining the necessary information during modeling is achieved due to the fact that the model, on the one hand, abstracts secondary (from the standpoint of the given goal) factors, and on the other hand, summarizes the main aspects of the studied phenomenon or process, making it possible to concentrate efforts on solving the main issues. Therefore, the problem of modeling the amount of deviations and errors when making financial decisions in conditions of uncertainty and risk becomes important.

2. Theoretical aspects

Each model is based on certain elements that are interconnected by a structure. The structural scheme of modeling may include: abstract (conceptual); material (physical); descriptive and optimization models. These models, in turn, are divided into components that form a classification system. Abstract models include: verbal, numerical, mathematical, graphic; physical models include: natural, model, analog; optimization models are normative ones.

Abstract modeling, as a rule, does not preserve the nature of the phenomena and processes that are in the original, so the results of the modeling itself have significant deviations. In these models, only the identity of relationships and the scale of components and phenomena in the studied objects or processes are preserved. Among the abstract models directly used in economic and, in particular, financial research, including research on foreign economic activity, the most common are mathematical and graphical models.

However, experience shows that the most common models used in practice are still mathematical models combined with economic methods, forming economic and mathematical modeling.

Many domestic and foreign scientists were engaged in economic and mathematical modeling, especially this concerns stability preservation and protection of the financial and economic security of the country's system, in particular: Voinarenko M. P., Dovbnya S. B., Zabrodskyi V. A., Kapystin M. M., Lobza G. S., Lyashenko O. M., Malyuta L. Ya., Pasternak–Taranushchenko G. A., Yaremenko O. F. and many others. The scientific view of each of these authors deserves appropriate attention in the aspect of a versatile approach to the assessment and modeling of the relevant process and phenomenon through the proposed assessment of deviations of threshold indexes that act as indicators of various spheres of the enterprise's state. However, none of these scientists approached the quantitative assessment of the enterprise's economic security state through the determination of the absolute value of the fluctuation of the permitted limits of the corresponding protection state (high, medium, low) in one direction or another through the determination of the permissible error (deviation) of the diagnosed level of the main parameter, which in real conditions would make it possible to correctly establish a qualitative transition from one state of security of an economic entity to another with maximum accuracy for further decision-making regarding the need for its stabilization.

3. The purpose and methodology of the research

The purpose of the article is to outline the aspects of modeling values' error, which usually occurs when making financial decisions in the zone of uncertainty and risk, and to be able to manage it for the accurate assessment of the performance indicator that falls under the process of further diagnosis and control.

The decision taken by the entrepreneur is almost always associated with risk, which is due to the presence of a number of uncertainty factors that were not foreseen in advance. Therefore, the following methods are used in the article: probability theory, matrix method, research method, direct and reverse interpolation method, averaging method, statistical method, index method, root mean square error method, permissible deviation method, simulation modeling method, etc.

Applied methodology for uncertainty determination and error element generation. As it is known, uncertainty is a rather broad concept that reflects the objective impossibility of obtaining absolute knowledge about the internal and external conditions and is characterized by the ambiguity of parameters. The most difficult is the process of modeling financial decision-making in conditions of uncertainty, since uncertainty is the incompleteness or inaccuracy of information about the conditions of preparation and implementation of financial decisions, including the risks and costs associated with them when obtaining the final effective values. Uncertainty is always associated with the possibility of adverse situations and consequences occurring during a specific project implementation, and is characterized by the presence of financial and economic risk and needs to be taken into account. The reasons for the uncertainty are diverse, namely: the uncertain nature of scientific and technological progress; random errors in forecasting; chaotic dynamics of internal and external conditions of financial processes development; probabilistic nature of important economic parameters; presence of business conflicts, etc.

The basis of economic and mathematical modeling is a mathematical model, which is a schematic representation of an economic phenomenon or process, in particular a financial one, which is the result of a scientific abstraction of the characteristic features of the surrounding economic life and its management mechanism.

Apparently, quantitative uncertainty can act as the possibility of the financial result deviating from the expected (or average) value both in the smaller and in the greater direction, or the possibility of only negative deviations of the event's final result. Decisions made under conditions of risk include decisions whose outcomes are not determined, but the probability of each possible outcome can be determined. Probability is determined in the interval from 0 to 1 and represents the degree of possibility of the event realization. The sum of the probabilities of all alternatives must be equal to one.

A decision is made under conditions of uncertainty, when it is impossible to estimate the probability of potential outcomes. This happens when the factors are so new and complex that it is impossible to obtain enough relevant information to help objectively determine the probability, or the existing situation does not obey known patterns. Therefore, the probability of a certain consequence cannot be predicted with a sufficient degree of reliability. Uncertainty is characteristic of some decisions made in rapidly changing conditions.

If individual financial values require averaging, they often resort to models of the type:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \overline{x} = \frac{\sum_{i=1}^{n} x_i m_i}{\sum_{i=1}^{n} m_i}, \quad \sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} |x_i - \overline{x}|^2}. \tag{1}$$

The species model is used to model the difference indices:

$$I = \frac{1}{2}\sum_{i=1}^{n} |x_i^* - y_i^*|. \tag{2}$$

It is these models that already contain an element of error during real calculations, which affects the accuracy of the final calculations.

Applied methodology for determining the coefficient of model inconsistency and interpolation method with an element of interpolation error. It is possible to assert, the larger the sample, the higher the probability of obtaining a correct representation of the population under study. However, the relationship between the size of the sample and its representativeness (accuracy) does not have a simple linear character. As mathematical statistics show, as the sample size increases, the representativeness increases sharply at first, and then more slowly. Therefore, depending on the required degree of research results' representativeness, the sample size can be large or small. The nature of the relationship between sample size and representativeness in different types of sample research should be known. This makes it possible to determine in advance exactly what volume of sampling will be required for information processing. For a random sample, the average error or error of the obtained estimate of the entire population is approximately inversely proportional to the square root of the number of units in the sample [1].

In order to determine what regularities are inherent in data distribution, near which values are grouped the majority of indicators deviating from this value in one direction or another, what is the general picture of the distribution, a certain ranking in a certain order of the entire series of researched data is necessary.

Therefore, to assess the suitability of the model, the following is used: the mismatch coefficient determination; verification of the validity of the model based on a small series of experiments and verification of the model based on the selection of significant factors.

To determine the coefficient of inconsistency, some set of retrospective values is formed of m-th indicator for $t = \overline{1,T}$ periods y_{mt}, y_{mt}^* is a set of calculated values of the same indicator for the same period. Then the root mean square error of the calculation of m-th indicator for the entire period has the form:

$$\sigma_m = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_{mt} - y_{mt}^*)^2}. \tag{3}$$

The coefficient of discrepancy is the value:

$$U_m^{(1)} = \frac{\sigma_m}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} y_{mt}^2}}. \tag{4}$$

If the following requirements are present: 1) if $U_m^{(1)} = 0$, then the model (4) completely reproduces the analyzed indicator, the values of which in the observation interval coincide with the observations; 2) if $U_m^{(1)} = 1$, then the model (4) does not reproduce the dynamics of the studied indicator and should be rejected; 3) if $U_m^{(1)} > 1$, then the model (4) gives worse results than a simple extrapolation of the initial value of the studied indicator and should be rejected from the point of view of the analyzed indicator. So, with growth of $U_m^{(1)}$ the adequacy of the model decreases.

The discrepancy indicator can also be calculated using the formula:

$$U_m^{(2)} = \frac{\sigma_m}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} y_{mt}^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T} y_{mt}^{*2}}}. \tag{5}$$

Under the conditions: 1) if $U_m^{(2)} = 0$, then the model (5) is completely adequate; 2) if $U_m^{(2)} = 1$, then the model (5) is inadequate. If the model (5) contains P endogenous variables, then the discrepancy coefficient is calculated based on all indicators, and the general adequacy characteristic is found as the average value of all discrepancy coefficients, i.e.,

$$U = \frac{1}{P}\sum_{m=1}^{P} U_m. \tag{6}$$

The closer U is to zero, the more accurate in retrospect is the model used for calculations.

Next, the model validity is checked based on a small series of experiments. If in the conducted series of tests quite good results are obtained from the point of view of real values, then it can be concluded that in similar situations the model (6) can be used, although there is no guarantee that a situation will not be encountered when this model turns out to be completely unsuitable. This approach is used very limitedly and only in relatively well-established processes, the performance indicator of which has minimal deviations [2].

Additionally, an important role is played by checking the model based on the selection of significant factors, which allows to filter out assumptions that are not essential for the final result formation; highlight the most significant assumptions in the model; check hypotheses about the linearity of dependencies, as this will allow the use of the linear optimization apparatus. However, a linear or reverse interpolation device is used for those processes for which the linear optimization device cannot be used.

A significant spread in practice is the interpolation method, which allows one to take into account the interpolation deviation, that is, the correction. In conditions of uncertainty and risk, it has found wide application in finding the return on an investment portfolio due to its risk value [3]. If the risk value of the investment portfolio is between the values given in the table x_0 and $x_1 = x_0 + h$ (let us assume that there is dynamics of risk growth $(+h)$ for some fixed period of time), they correspond to the yield values of the investment portfolio y_0 and y_1, where $y_0 = f(x_0)$ and $y_1 = f(x_0) + \Delta f$. Then it should be assumed that:

$$f(x) \approx f(x_0) + \frac{x - x_0}{h} \cdot \Delta f, \tag{7}$$

where $\frac{x-x_0}{h} \cdot \Delta f$ is interpolation correction and $\Delta f = f(x_1) - f(x_0)$.

These values are easiest to calculate using tables. If set the nodal points of the overall risk level (x) of investment portfolio on a certain date t and the admissible expected value of income (y) for a given risky situation at these points:

x	0.25	0.30	0.35
y	22 thousand hryvnias	38 thousand hryvnias	49 thousand hryvnias

then it is possible to use linear interpolation (7) to more accurately calculate the return on risk $x = 0.283$, that is

$$y = f(0.283) \approx 22 + \frac{0.283 - 0.25}{0.05} \cdot 16 = 32.56\,(\text{thousand hryvnias}),$$

where 16 thousand hryvnias $= 38 - 22 = f(x_1) - f(x_0) = \Delta f$.

Similarly, if it is necessary to find the approximate value of the argument (risk value) based on the given values of the function, for example, profitability, then it is necessary to perform a reverse interpolation, changing the places of the variables x and y

Then:

$$\varphi(y) = \varphi(y_0) + \frac{y - y_0}{h}\Delta\varphi, \tag{8}$$

where $\varphi(y)$ is the unknown value of the inverse function (risk value).

Otherwise, it is necessary to determine the risk level x. If it yields $f(x) = 42$ thousand hryvnias, we will have: $y_0 = 38$ thousand hryvnias; $\varphi(y_0) = 0.3$;

$$y_1 = 49\,\text{thousand hryvnias}; \varphi(y_1 = 0.35); h = y_1 - y_0 = 49 - 38 = 11\,\text{thousand hryvnias};$$
$$\Delta\phi = \phi(y_1) - \phi(y_0) = 0.35 - 0.3 = 0.05; \quad x = \phi(42) \approx 0.3 + \tfrac{42-38}{11} \cdot 0.05 \approx 0.318.$$

However, sometimes the accuracy of finding unknown values using linear interpolation can be insufficient, then quadratic interpolation is used.

In such cases, dispersion indicators are used to measure risk, therefore: the greater the spread of possible income, the greater the danger that the expected income will not be received. The measure of dispersion is the root mean square deviation of the species:

$$\sigma_i = \sqrt{\sum_{j=1}^{n} P_{ij} \cdot (R_{ij} - m_i)^2}. \tag{9}$$

The risk measure of the investment portfolio is the root mean square deviation of the return on the expected value of the species:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i \cdot X_j \cdot \sigma_{ij}}, \tag{10}$$

where σ_p is a measure of portfolio risk; σ_{ij} is covariance between returns of i-th and j-th securities; X_i, X_j are shares of the total investment, which are accounted for i-th and j-th securities; n is the number of securities in the investment portfolio.

Covariance of returns on securities (σ_{ij}) in formula (10) is equal to the correlation between them multiplied by the product of their standard deviations. It should be also taken into account that for $i = j$, the covariance is equal to the variance of the portfolio stock. If we consider the theoretically limiting case in which an infinite number of securities can be included in the investment portfolio, then the variance (a measure of portfolio risk) will be asymptotically approaching the average value of the covariance.

4. Results and discussion

The method of determining the value of the permissible calculation error. It is possible to propose a method for determining the value of the permissible calculation error ($\pm\Delta$) during the adoption of a financial decision regarding the structural and functional diagnosis of the enterprise's economic security, within which, at any value of its level, the corresponding state of economic security will remain unchanged. In this situation, the modeling under conditions of uncertainty is based on the matrix of probabilistic transitions, which carries an informational memory about the past and present states of the transition system of the business entity in order to overcome various threats that periodically enter its space from the internal and external environments.

Making a decision about the state of the enterprise's economic security requires constant control, the quality of which can be maintained only by continuously improving the diagnostic system and clarifying the quantitative assessment of the permissible range of fluctuations of this integrated value, which combines the quantitative characteristics of various components of economic security. Due to the incompleteness of information regarding various types of threats that constantly enter the space of the enterprise from internal and external environments and measures for their complete avoidance or partial neutralization according to certain diagnostic methods, an error in the accuracy of calculations may occur, which significantly affects the perception of the general situation regarding the change in the state of economic security. Therefore, it is important to determine the exact permissible error of the calculated value of the enterprise's economic security level, within which, for a certain diagnosed value, the overall state of its security will remain stable without sudden changes in the direction of improvement or deterioration.

In our understanding, the integrated economic security of business entities is achieved by determining the absolute value of the fluctuation of the permitted limits of the corresponding state of their security: high, medium, low with a permissible error (deviation) of the diagnosed level of economic security. In real conditions, this makes it possible to establish with maximum accuracy the moment of qualitative transition from one state of security to another.

In connection with the fact that each enterprise must function in a strict system of control over the level of economic security, an effective method of determining the value of its permissible error should be proposed, within which the corresponding general state of security will remain unchanged, which will make it possible to more rationally manage minor transient states of the economic stability even under conditions of uncertainty based on the quantitatively calculated value of this error.

Each value of the diagnosed level of the enterprise's economic security, characterizing the corresponding state of its security, may have a certain range of fluctuation of the total permissible error, which in practice must be divided into at least two components: 1) the error that occurs automatically during mathematical calculations, primarily during rounding values; 2) the error, which is recommended to be additionally introduced, determining it and adding it to the calculated value of economic security level. In this situation, the method of determining the error, which will not depend on the usual mathematical error of values rounding, is of particular importance.

In the conditions of simulation modeling, taking into account the correlation relationships between the parameters, the method of determining the permissible deviation can be applied. This method is based on the maximum correlation deviation $D(n_p, t)$ of the effective indicator, the value of which is equal to the product of the coefficient of transfer of the factor indicator growth rate n_f in the growth rate of the effective indicator n_p, multiplied by the correction coefficient value of the factor indicator $k_{kop}(n_f, t)$ at a point in time t [4].

That is exactly why the permissible error, which in most cases can be determined from a table of random numbers in a predetermined interval, occurs in the case of calculating the correction coefficient of the performance indicator $k_{kop}(n_p, t)$ and is included in the value of the correlation deviation of the species:

$$k_{kop}(n_p, t) = \frac{P(n_p, t, s)\,\{A(n_p, t, s); B(n_p, t, s)\}}{100} \cdot D(n_p, t); \tag{11}$$

$$D(n_p, t) = k_c(n_D, n_p) \cdot k_R(n_D, n_p) \cdot k_{kop}(n_D, t) \cdot \left(1 \pm \frac{\Delta(n_D, n_p)}{100}\right), \tag{12}$$

where $\Delta(n_D, n_p)$ is permissible deviation or error, %; $k_c(n_D, n_p)$ is the analytical coefficient; $k_R(n_D, n_p)$ is the coefficient of calculation; $k_{kop}(n_D, t)$ is the correction coefficient of the factor indicator; $P(n_p, t, s)$ is probability (according to the table of random numbers) of n_p-th indicator in the interval s by t-th time ($\sum_1^s P(n_p, t, s) = 100$); $A(n_p, t, s)$ and $B(n_p, t, s)$ are deviations of the calculated indicator at the beginning and end of the interval s.

Accordingly, after transformations from formula (12), the permissible percentage error can be calculated as follows:

$$\pm\Delta(n_D, n_p) = \left(\frac{D(n_p, t)}{k_c(n_D, n_p) \cdot k_R(n_D, n_p) \cdot k_{kop}(n_D, t)} - 1\right) \cdot 100\,\%. \tag{13}$$

This method of detecting the permissible error by the magnitude of the correlation deviation between the indicators cannot be used in this form to correct the diagnostic assessment of the fluctuation range of the enterprise's economic security level, because the more integrated the final calculated indicator is, the more difficult it is to determine the magnitude of the correlation deviation and manage the correction coefficient of the factor indicator. In addition, there may be several factor indicators, depending on the number of introduced components of economic security.

It should be understood that the permissible error of the diagnosed level of the enterprise's economic security may also involve minor deviations of the threshold values of the main financial and economic indicators. However, in practice, there are situations when it is difficult to calculate the limit values of some indicators, although it has been proven that they can significantly affect the transitional states of the enterprise's economic security. In addition, if these indicators constitute a significant array of input data and are derived only by an expert method, then due to the lack of permanent statistical accounting and there is not always a controlled influence on the accuracy of economic security state, they should be replaced by universal system security factors and factorial research methods. For example, these factors can include: 1) the factor of a sufficient degree of adaptability of the enterprise's economic potential; 2) factor of flexibility and maneuverability; 3) factor of efficiency of use and economic potential profitability; 4) factor of economic potential mobility.

Mostly, all these factors are completely or partially abstracted by the effects of minor changes in the normative values of the main financial and economic indicators and make it possible to simplify the approach to protecting the enterprise from threats, rationally increasing or decreasing the indicators compared to the normative values.

Evidently, the error modeling, as an element of research, is influenced by the systematization of the entire set of main factors that affect the change in the state of the enterprise's economic security, but the obtained results do not allow to determine the permissible error of fluctuation of the quantitative assessment of the enterprise's economic security state due to its diagnosed level, since they do not contain elements of probability calculation overcoming internal and external threats to the enterprise, which affect the level of each component of its economic security.

In our case, the permissible error ($\pm\Delta$) is understood as the difference between the estimated value of the enterprise's economic security level R on the current date according to a certain diagnostic method and any conditionally approximated value in the range of the same state of economic security under the predicted probability of overcoming threats under conditions of uncertainty. In real conditions, only by determining this permissible error, it is possible to control the unchanged state of the enterprise's economic security and its economic structure, and if its value is slightly exceeded, the transition to another state should be recorded.

Therefore, in order to further determine this error, it should be first form a structured quantitative model for assessing the state of the enterprise's economic security in such a form that is most suitable for visual perception and further application, and then it can be allocated a place in it to the permissible error of fluctuation in the value of the enterprise's economic security level, which will not affect the change in the state of its economic security.

Identification of the permissible error of the diagnosed level of the enterprise's economic security. As a rule, the diagnostic model of the enterprise's economic security level allows forming an array of structural elements for diagnosing; to carry out their detailing in the space of diagnostic assessment according to selected main levels: according to the innovation, financial, credit and investment security level; to differentiate blocks of functional diagnostics of these levels through the construction of the corresponding functional dependencies of each of these levels for their further calculation; to single out the general calculation apparatus for structural and functional diagnosis of economic security level using the geometric mean method (although this method gives the largest mathematical error in the rounding of values, however, it is quite common in economic calculations); to identify the place of the permissible error of the diagnosed level of the enterprise's economic security, at which the state of its security will not change, and finally to make a decision to change the state of economic security if the value of this permissible error is exceeded.

In this situation, the integrated level of the enterprise's economic protectobility (L_{EP}), which is affected by the levels of innovative, financial, credit and investment security, as well as the amount of additional destabilizing influence (ε_V), which may arise if the enterprise has proven and solved economic crimes as a result of illegal actions, and the correction factor (ψ) to clarify the quantitative measurement of economic security level within all components of economic security, which will be equal to the annual average probability of business conflicts occurrence with certain negative consequences, in particular with bifurcation nodal points can be represented by the following mathematical relationship [5–7]:

$$L_P = \sqrt[n]{\prod_{i=1}^{n}\left(L_r^V - \sum_{j=1}^{z}\frac{|\Delta_j^i|}{N_j^i} - \varepsilon_V\right)} - \psi \pm \Delta, \qquad (14)$$

where L_r^V is localized r-measurement scale for measuring levels of components and the general level of the enterprise's economic security ($L_r^V = 10$); $|\Delta_j^i|$ are absolute deviations of calculated indicators from normative values; N_j^i are normative values of indicators; n is the number of diagnosed security components ($n = 3$); z is an optimal number of selected indicators; $\pm\Delta$ is a margin of error that does not change the security state within low, medium, or high level fixations.

In turn, taking into account the relative value of the total destabilizing effect of factors on each component of economic security, formula (14) is transformed into a functional dependence of the type:

$$R_{ES} = \sqrt[3]{\left(L_r^V - \sum_{j=1}^{k}\frac{|\Delta_j^i|}{N_j^i} - \varepsilon_1\right)\cdot\left(L_r^V - \sum_{j=1}^{i}\frac{|\Delta_j^{fk}|}{N_j^{fk}} - \varepsilon_2\right)\cdot\left(L_r^V - \sum_{j=1}^{m}\frac{|\Delta_j^{in}|}{N_j^{in}} - \varepsilon_3\right)} \pm \Delta \qquad (15)$$

or in a collapsed form:

$$R_{ES} = \sqrt[n]{\prod_{i=1}^{n}\left(L_r^V - \sum_{j=1}^{z}\frac{|\Delta_j^i|}{N_j^i} - \varepsilon_i\right)} \pm \Delta, \qquad (16)$$

where $|\Delta_j^i|$ are absolute deviations by the module of the calculated values of the recommended indicators from their normative values, which participate in the process of diagnosing the state of economic security by innovative, financial, credit and investment components; N_j^i are normative values of the recommended group of indicators, which participate in the process of diagnosing the state of economic security according to similar components; ε_V is the amount of additional destabilizing influence that occurs if economic crimes objectively exist at the enterprise or are proven and revealed as a result of illegal actions (for each component of the security state the value ε_i is calculated individually); n is the number of diagnosed components of economic security; z is the number of selected indicators for diagnosis the enterprise's economic security state (in the case of each component of economic security, their number may be different); $\pm\Delta$ is an acceptable error when calculating the enterprise's economic security level, which does not change the qualitative state of economic security within the limits of fixing the corresponding low, medium or high level of it.

Applied technique of using the matrix of probability transitions to determine the individual permissible error. It is possible to apply a matrix of probabilistic transitions to determine the individual permissible error during the diagnosis of the unchanged state of the enterprise's economic security. For each business entity, it will be different depending on the potential ability to quickly respond to overcoming the relevant set of threats. The matrix of probabilistic transitions is used to describe the functioning of a certain transitional system. In our situation, we are dealing with a dynamically transitional system — a change in the state of the enterprise's economic security, which depends on the probability of timely neutralization of a separate number of types of threats that enter its space from the internal and external environments at the beginning and end of the studied period. This is reflected in the calculated measurement of individual levels of economic security, where their integrated value with a permissible error is characteristic only for one unchanging state of security.

The matrix of probabilistic transitions (P) is always a matrix of the same dimension ($n \times n$), the elements of which are the corresponding transition probability p_{ij}, which represents the conditional probability that the system will pass from the state i due to the next test to the state j, where, accordingly, the index i will characterize the state of the transitional system at the previous moment of time, and j at the next moment of time with a finite number of states n. The elements of each row of the matrix represent the probabilities of all possible transitions in one step from the selected state, in particular, the possible probability of no transition with equal indices, and the sum of all elements within each separate row of the matrix must be equal to 1, i.e. [8]:

$$\sum_{j=1}^{n} p_{ij} = 1 \quad (i = 1, 2, \ldots, n). \tag{17}$$

This means that the elements of one corresponding row of the matrix, by summing their values, form a complete closed group. In our case, with regard to the desired result of one hundred percent overcoming of threats, which are fixed by the corresponding level of the enterprise's economic security, and the elements of the matrix columns set the probabilities of transitions of the system in only one step on the corresponding date due to the abstractly modeled state of the system in terms of the calculated level of economic security by the method of calculating the geometric mean value, i.e.:

$$R_{ES} = \sqrt[3]{R_i \cdot R_{fk} \cdot R_{in}}. \tag{18}$$

Since as the basis of the study we take one unchanging state of the enterprise's economic security, which is based on the assessment of the three aforementioned levels on the current date due to the probability of neutralizing the corresponding number of threats affecting them under conditions of uncertainty, then the matrix of probabilistic transitions of dimension (3×3) will have the form:

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}, \tag{19}$$

where p_{11}, p_{12}, p_{13} are the transitional probability of neutralizing threats that affect the level of innovative security of the enterprise, in particular, p_{11}, p_{12} are the real transitional probability of neutralizing threats, which is fixed by the level of innovative security, respectively, at the beginning and end of the reporting period, p_{13} is an additional probability value that closes the system at the desired 100% overcoming of the existing threats affecting the state of innovative security;
and p_{21}, p_{22}, p_{23} are transitional probability of neutralizing threats that affect the level of financial and credit security of the enterprise, in particular; p_{21}, p_{22} are the real transitional probability of threat neutralization, which is fixed by the level of financial and credit security at the beginning and end of the reporting period; p_{23} is an additional probability value that closes the system in case of the desired 100% overcoming of existing threats affecting the state of financial and credit security;
and p_{31}, p_{32}, p_{33} are transitional probability of neutralizing threats that affect the level of investment security of the enterprise, in particular, p_{31}, p_{32} are the real transitional probability of neutralizing threats, which is fixed by the level of investment security at the beginning and end of the reporting period; p_{33} are an additional probability value that closes the system for the desired 100% overcoming of existing threats affecting the state of investment security [8].

If it is considered that the matrix of probabilistic transitions carries in its general form an informational memory about past and present events of overcoming threats at the enterprise, then with its help it is possible to determine the individual permissible error of fluctuations in economic security level, during which the state of security will not change. This fact can be applied inversely during the analysis of business activity, namely, if the enterprise's economic security level fluctuates within the predetermined value of the permissible error, then the state of its economic security should be considered unchanged.

To do this, let us transform the matrix P, which generally captures a certain state of economic security due to the probability of overcoming threats during the period $[0, t_1]$, into the square of the matrix of probabilistic transitions $P^2 = P \cdot P$ and further into the cube of the matrix of probabilistic transitions $P^3 = P \cdot P^2$ (if necessary, this process of transforming the initial matrix can be further extended) for a more accurate representation in dynamics unchanged state of economic security, while fixing the basic probability of neutralization of internal and external threats. The square of the matrix of probabilistic transitions makes it possible to ascertain the fact of information preservation of the economic security state for the period $[t_1, t_2]$, the cube of the matrix of probabilistic transitions — for one more period $[t_2, t_3]$. The total time $[0, t_3]$, consisting of at least the three segments discussed above, gives the most optimal idea of the model of the unchanged state of the enterprise's economic security and will be an indicator of its corresponding stable economic potential (sufficient or insufficient) to avoid threatening situations of various degrees of risk in conditions of uncertainty.

Other calculations are possible on the example of an abstractly modeled business entity, which in market conditions, under the constant influence of internal and external threats, will never be fully protected.

Sources of external threats can be destruction during wars, criminal structures, industrial espionage, intelligence companies, direct and indirect competitors, corrupt representatives of authorities, law enforcement agencies, regulatory bodies, mass media, as well as natural disasters, industrial accidents and catastrophes. Internal threats include: low technological level of the enterprise, deformation of the production process structure, high production losses, low quality of the manufactured products and, as a result, low competitiveness of these finished products on the market, insufficient solvency, high level of bankruptcy, low innovation and investment activity, increased criminalization of the enterprise due to tax evasion, income concealment, in particular foreign exchange earnings, etc. [9].

It is important to build a matrix of probabilistic transitions P for a conditional enterprise on the basis of predetermined probabilities of neutralizing threats, using the ratio of the avoidance of a certain number of threats at the corresponding moment in time to their total value, which is reflected in the calculated values of the sets of structural elements of each of the three types of components of economic security on the beginning and end of the reporting period (the first and second columns of the matrix) and additional probabilities that close the system to preserve the condition $\sum_{j=1}^{n} p_{ij} = 1$, $i = 1, 2, \ldots, n$ (under this condition, it is possible to later check the correctness of all intermediate calculations and determine the permissible error of diagnostic assessment of the enterprise's economic security state) [6]:

$$P = \begin{pmatrix} 0.21 & 0.32 & 0.47 \\ 0.12 & 0.26 & 0.62 \\ 0.18 & 0.30 & 0.52 \end{pmatrix}.$$

Then the square of probability transitions matrix by the operation of multiplication of two matrices will take the form:

$$P^2 = \begin{pmatrix} 0.21 & 0.32 & 0.47 \\ 0.12 & 0.26 & 0.62 \\ 0.18 & 0.30 & 0.52 \end{pmatrix} \cdot \begin{pmatrix} 0.21 & 0.32 & 0.47 \\ 0.12 & 0.26 & 0.62 \\ 0.18 & 0.30 & 0.52 \end{pmatrix} = \begin{pmatrix} 0.1671 & 0.2914 & 0.5415 \\ 0.1680 & 0.2920 & 0.5400 \\ 0.1674 & 0.2916 & 0.5410 \end{pmatrix}$$

and, accordingly, the cube of probabilistic transitions matrix of economic security state:

$$P^3 = \begin{pmatrix} 0.21 & 0.32 & 0.47 \\ 0.12 & 0.26 & 0.62 \\ 0.18 & 0.30 & 0.52 \end{pmatrix} \cdot \begin{pmatrix} 0.1671 & 0.2914 & 0.5415 \\ 0.1680 & 0.2920 & 0.5400 \\ 0.1674 & 0.2916 & 0.5410 \end{pmatrix} = \begin{pmatrix} 0.167529 & 0.291686 & 0.540785 \\ 0.167520 & 0.291680 & 0.540800 \\ 0.167526 & 0.291684 & 0.540790 \end{pmatrix}.$$

The calculation correctness is confirmed by the condition mentioned above $\sum_{j=1}^{n} p_{ij} = 1$, $i = 1, 2, \ldots, n$. Now it should be compared all the finite elements of the matrices P and P^3 by the subtraction operation, and later grouped all the calculation results by the modular value into a matrix:

$$P^{\Delta} = \begin{pmatrix} 0.042471 & 0.028314 & 0.070785 \\ 0.047520 & 0.031680 & 0.079200 \\ 0.012474 & 0.008316 & 0.020790 \end{pmatrix}.$$

Accordingly, matrix P^{Δ} contains allowable deviations of elements under an unchanged state of economic security over time $[0, t_3]$, where the correctness of intermediate calculations is confirmed by the fulfillment of equalities: $\Delta p_{11} + \Delta p_{12} = \Delta p_{13}$; $\Delta p_{21} + \Delta p_{22} = \Delta p_{23}$; $\Delta p_{31} + \Delta p_{32} = \Delta p_{33}$. This means that the margin of error can be calculated as the first two columns of the matrix P^{Δ}, as well as by the third column, which closes and integrates the system of deviations of previous transitional states probabilities. In this situation, the recommended method of calculating the permissible error should be the arithmetic mean method, which, in contrast to the geometric mean, makes it possible to expand the range of permissible error for one hundred percent verification of the accuracy of maintaining the constancy of the corresponding state of the enterprise's security, which is diagnosed through the calculated value of its level. This makes it possible not to make mistakes in making effective decisions regarding even a minor transition of a business entity to another state of economic security. We will receive:

$$\tfrac{0.042471+0.047520+0.012474}{3}+\tfrac{0.028314+0.031680+0.008316}{3}=\tfrac{0.070785+0.079200+0.020790}{3};$$

$$0.034155+0.02277=0.056925; \quad 0.056925=0.056925.$$

That is, with the corresponding probabilities of neutralizing threats that affected the economic security of the economic entity, the individual permissible error ($\pm\Delta$), at which the unchanged state of the system will be preserved, will be equal to 0.056925.

If its value is exceeded at a certain point in time, the fact of a mandatory transition to another state of economic security should be established, diagnosing its improvement or deterioration. The size of the permissible error of the enterprise's economic security state can be controlled by specifying changes in the probabilities of neutralizing threats at a certain point in time.

5. Conclusions

The article presents various modeling approaches to determining the error in making financial decisions under conditions of uncertainty. In particular, the applied use of the matrix of probability transitions made it possible to calculate the individual permissible error ($\pm\Delta$), at which the unchanged state of the enterprise's economic security will be preserved when making individual financial decisions, which is equal to the value: 0.056925. Accordingly, if this calculated value is exceeded, for any value of the diagnosed level of economic security, the fact of the mandatory transition of the business entity to another state should be established, diagnosing either its improvement or deterioration.

Further research in this direction requires the construction of a differentiated scale of the permissible range of permissible errors for the diagnostic assessment of various states of economic security for a number of enterprises, especially in the conditions of a full-scale war, since the constant growth of prices for resources, raw materials and components and generally high financial costs keep most enterprises in conditions of uncertainty and high risk.

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Моделювання визначення похибки при прийнятті фінансових рішень в умовах невизначеності

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У статті представлені процеси моделювання визначення похибки при прийнятті фінансових рішень в умовах невизначеності та ризику на прикладі оцінювання діагностованого рівня економічної безпеки підприємств з допомогою математичного опису та обчислень за участі матриці ймовірнісних переходів та використанням науково-дослідного методу, методу прямої та звороної інтерполяції, усереднення, статистичного та індексного методу, методу середньоквадратичної похибки, імітаційного моделювання, методу теорії ймовірностей, матричного методу тощо. Досліджено, що найскладнішим є процес моделювання прийняття фінансових рішень в умовах невизначеності, так як із-за різних видів ризику існує висока загроза недостатності або неточності інформації, яка викривляє технічні розрахунки, індикатором яких є реальні фінансові витратами та вчасне виявлення допустимих і недопустимих відхилень параметрів.

Ключові слова: *невизначеність; ризик; моделювання; похибка; кореляційне відхилення; фінансові рішення; економічна захищеність.*